

## Inductors

Inductors are two terminal, passive energy storage devices. They store electrical potential energy in the form of an magnetic field around the current carrying conductor forming the inductor. Actually, any conductor has the properties of an inductor. Most inductors are formed by fashioning the conductor into a cylindrical coil. As such, inductors are often referred to as "coils". The schematic symbol for a inductor is shown below in figure 1 . The symbol resembles the typical cylindrical construction.

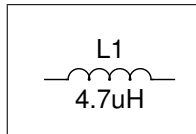


Figure 1: Inductor schematic symbol

In figure 2 we see several different types of inductors. There are three main ways of distinguishing inductors. One of the type of core its wound around. Inductor cores may be simply air or some type of magnetic material that enhances the inductors ability to store energy. The other distinguishing characteristic is the shape that the coil is wound in. Many are in a cylindrical shape, some are wound in circles. No matter the exact shape, the inductor still retains in some fashion the shape of a coil of wire.

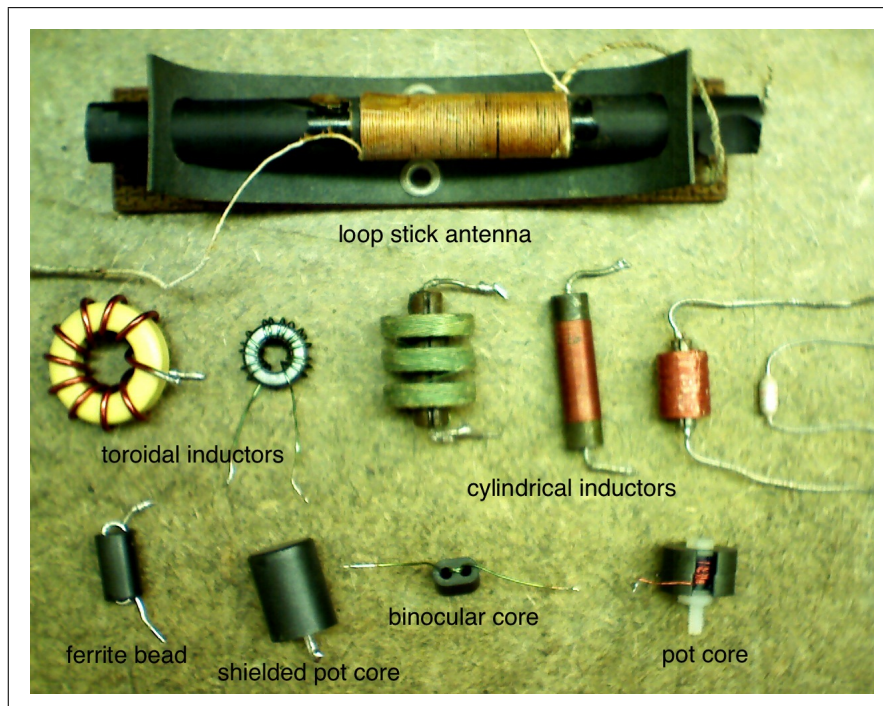


Figure 2: Various inductors

At the top of figure 2 is a loop stick antenna which is an antenna used for the AM broadcast band. Its an inductor with a metallic inner core that increases the inductance and allows the inductor to act as an efficient antenna.

Below it at left are two toroidal inductors. These are efficient inductors used primarily at higher radio frequencies. They excel at keeping their magnetic field within the "doughnut" shaped core. To the right of the toroids are three cylindrical inductors. These are of the shape and configuration of most inductors.

Below the cylindrical inductors are some more specialized inductors. At left is a "ferrite bead" inductor made by winding a wire through several holes in the bead shaped ferrite material. The ferrite iron alloy greatly increases the inductance without requiring more turns of wire on the form. Next to the bead is a shielded audio pot core inductor. It is surrounded by the ferrite material so that nearby 60hz magnetic fields do not couple into the windings and induce an audio frequency *hum*. The binocular core is somewhat like a bead core in that it allows for high inductance with few turns of wire. Lastly is shown a *pot core* as its construction is that of a bobbin of wire enclosed by a pot of ferrite. It is self-shielding and has high inductance in a small size.

Regardless of physical shape, inductors are formed by multiple turns of wire around some form of air or some type of magnetic material. The last distinguishing characteristic is if the inductor is adjustable or variable. Adjustable inductors change their inductance by a movable inner core. As the core moves to the center of the windings, the inductance increases if the core is a magnetic material. If the core is brass, the inductance decreases as the core moves to the center of the windings. A number of variable inductors are see in figure 3.



Figure 3: Variable inductors

The amount of inductance available from an inductor depends on its physical dimensions. For a cylindrical inductor with some core as shown in figure 4, the inductance is given by:

$$L = \frac{N^2 \mu A}{l}$$

where,

- L = the inductance in Henrys (H)
- N = the number of turns around the core
- $\mu$  = the permeability in Henrys/meter (H/m)
- A = the cross-sectional area in meters

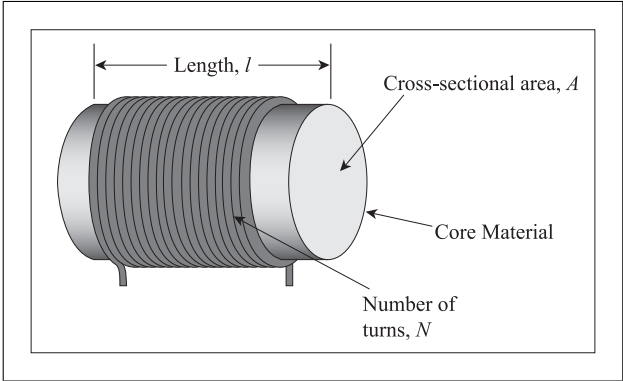


Figure 4: A Cylindrical Coil

**Inductors in Series and Parallel**

When we have networks of inductors in series and parallel, they add in the same way as resistors do. Inductors in series simply add and those in parallel add as the reciprocal of the sum of the reciprocals as shown in figure 5

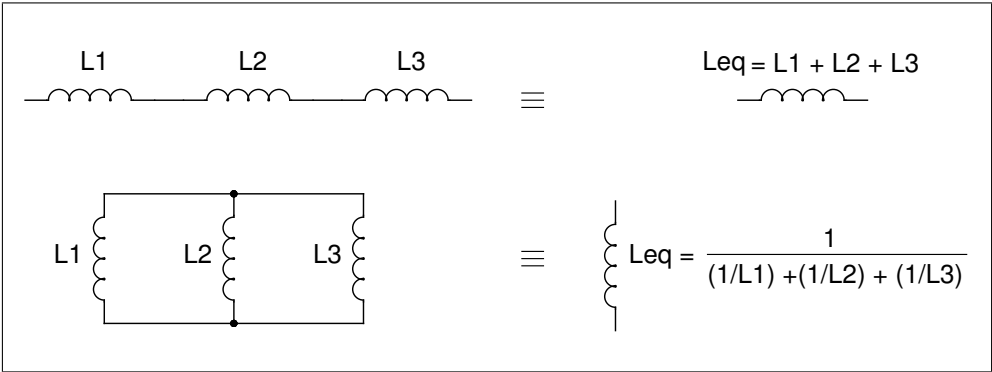


Figure 5: Inductors in Series and Parallel

## The Inductor Current Voltage Relationship

An inductor is formed whenever a current flows through a conductor. The current-voltage relationship for the inductor is:

$$v_L = L \frac{di}{dt}$$

where,

$v_L$  is the voltage across the inductor

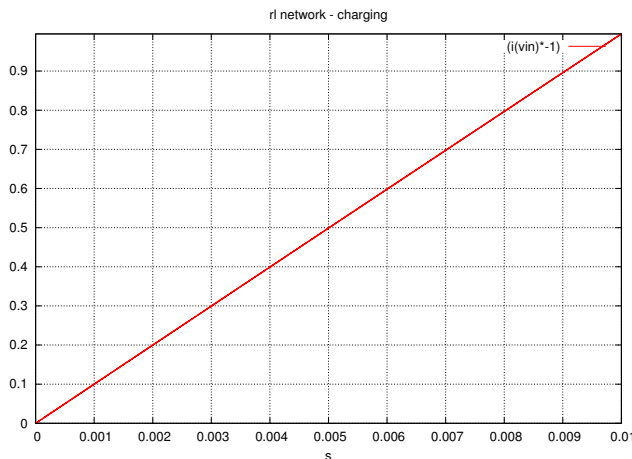
$L$  is the inductance of the inductor in Henrys, and

$\frac{di}{dt}$  is the change in current through the inductor per unit time

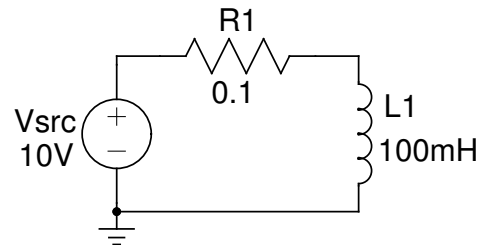
This relationship tells us several things. First, is that with a constant applied voltage, the rate of change in current with respect to time is constant; i.e. a straight line. This is clearly seen from the spice simulation of figure 6. For this simulation to work, ngspice requires a small resistor in series with the inductor to prevent infinite current from being sourced. This resistor is so small it can be ignored.

```
A inductor charging from a constant voltage source
*10V input source with 1ms delay, 1nS edges, 50ms pulse width, 100ms cycle time
Vin  vin      gnd      10.0 PULSE(0 10.0 0ms 1ns 1ns 50ms 100ms)
r1   vin      l_in     0.10
l1   l_in     gnd      100mH
.control
tran 100ns 10ms
plot (I(Vin) * -1) ;Vin current is assumed to follow PSC, must reverse
*gnuplot lr_sim (I(Vin)*-1) xl 1u 10ms ;make lr_sim.eps for latex
set noaskquit
.endc
```

Running the simulation, we see that we get a constant  $di/dt$  with a constant voltage source connected to the inductor.



(a) Constant  $\frac{di}{dt}$



(b) Schematic of ngspice Simulation

Figure 6: Voltage Source Driving Inductor Gives Constant  $\frac{di}{dt}$

Rearranging the equation yields:

$$\begin{aligned}\frac{di}{dt} &= \frac{V_L}{L} \\ \frac{di}{dt} &= \frac{10}{1e^{-3}H} \\ \frac{di}{dt} &= \frac{100A}{S}\end{aligned}$$

This make sense as the graphs shows a voltage ramp of 1A in 10 milliseconds.

Another fact we notice from the defining equation,

$$v_L = L \frac{di}{dt}$$

is that an inductor cannot allow an instantaneous change in current through it. To do so would require an infinite voltage across it.

Finally, if we rearrange the equation above giving;

$$\frac{di}{dt} = \frac{1}{L} v_L$$

we see that if the current through the inductor is not changing, the voltage across the inductor is zero. Therefore, the inductor must be a short circuit to direct current.

## RL Circuit Behavior

When a voltage source in series with a resistor is placed across the terminals of an inductor, we can see a different behavior from the inductor with respect to the current flowing through it. See the schematic below.

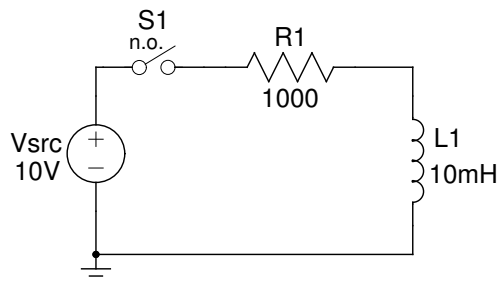


Figure 7: Circuit to determine inductor transient response

When the switch is closed at ( $t = 0$ ), the current flowing through the inductor begins to increase but at a rate that is limited by the inductance. It does not immediately "jump" to some value but begins at zero current and increases exponentially until it reaches the current determined by the source voltage and R1 as the inductor has zero DC resistance.

It has been determined that the current through the inductor in such a circuit is expressed by the equation:

$$i_L(t) = (V_{src}/R)(1 - e^{-tR/L})$$

This relationship can also be seen from a ngspice simulation. Note that although there is no switch in the circuit as the schematic shows, the voltage source is not turned on until *1ns* into the simulation. To sense current in the circuit, a zero volt voltage source is used. An exponential curve of the current increasing through the inductor is clearly seen. Also seen is that as time increases, the voltage across the inductor approaches zero. This is in agreement with our assertion that the voltage across an inductor is zero for DC currents.

```

RL network - charging
*10V input source with 1ns delay, 1nS edges, 100ms pulse width, 200ms cycle time
Vin   vin      gnd      10.0 PULSE(0 10.0 1ns 1ns 1ns 100ms 200ms)
r1    vin      tie      10
vtest tie      l_in     0
l1    l_in     gnd      100mH
.control
  tran 1us 100ms
  plot I(vtest)  xl 1us 50ms
  plot v(l_in)  xl 1us 50ms
  gnuplot rl_sim I(vtest) v(l_in)  xl 1u 100ms ;make rl_sim.eps for latex
  set noaskquit
.endc
*Print the inductor current at one Tau (100mh/10)=10ms
.meas tran attau find i(Vtest) at=10ms
.end

```

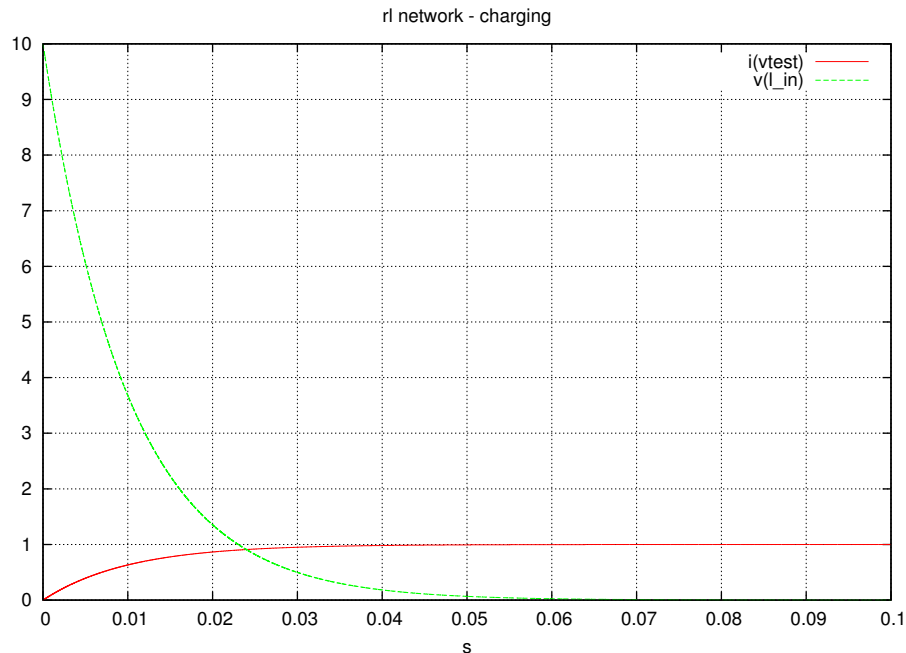


Figure 8: Current through Inductor Building While Voltage Drops

If we choose  $t$  equal to the product of  $R$  and  $L$ , the inductor current expression becomes:

$$i_L(t) = \frac{V_{src}}{R}(1 - e^{-1})$$
$$i_L(t) = V_{src}(1 - 0.368)$$
$$i_L(t) = 0.63 \frac{V_{src}}{R}$$

The fraction  $\frac{L}{R}$  is called the RL *time constant* and has units of seconds. When time  $t$  is equal to  $\frac{L}{R}$ , the current through the inductor will have reached to 63% of the final value of inductor current given by  $\frac{V_{src}}{R}$ .

In the spice simulation above, the statement

```
*Print the inductor current at one Tau (100mh/10)=10ms  
.meas tran attime find i(Vtest) at=10ms
```

gives us the value of the inductor current at 10ms which is, for this circuit,  $\tau$ . The output from the simulation is:

```
attau          = 6.321205e-01
```

So the inductor current had risen to 0.63A which is 63% of the final value of 1A.

## Inductor Applications

Inductors are at a disadvantage relative to other passive components such as capacitors and resistors that is related to their physical size. Inductors are typically avoided if possible in many electronic products since their inclusion results in a bigger product than otherwise possible. However, there are still many applications in which inductors are the only viable solution.

One principle use for inductors is in *filtering*. Since inductors are a short circuit to DC currents and represent an increasing opposition to AC currents of increasing frequency (called reactance), they make excellent frequency filters. For example in a sub-woofer speaker system, the deep base speaker must only see the lowest frequencies. A filter, a *cross-over network* using inductors is typically used to reject all frequencies above a certain cutoff frequency. Only a fairly large inductor would have the power handling capability for high-powered audio systems.

Another indispensable application for inductors is in the use of *switch-mode* power supplies. These power supplies can step an input voltage up or down to a new voltage at its output with high efficiency. The inductor in these power supplies is used as a energy storage device. For example, when used with a transistor to very quickly stop the current through the inductor, a very high voltage can be easily generated.