Circuit Analysis Using KCL (node voltage) Method

Below is a circuit to analyze. We shall now determine the voltage at each node.

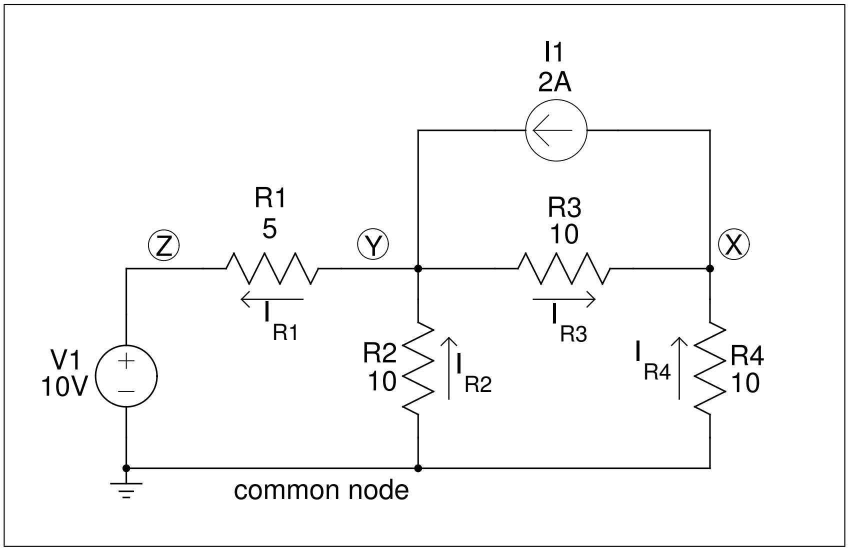

Figure 1: KCL problem.

The first step in the analysis is to label all the nodes except for the common node (often referred to as *ground*). Often, the common node is the one connected to the negative terminal of the voltage source. More often than not, it appears as a common wire across the bottom of a circuit diagram.

Secondly, label the currents entering or leaving each node. The direction of the current arrows is totally arbitrary. If you choose the wrong direction, the current magnitude will simply be negative. The next step is to write the KCL equation for each node except the common node, or for nodes that have a voltage source between the node and common. For node Z, we see by inspection that its value is 10V.

For node Y, the KCL equation is:

$$-I_{R1} + I_{R2} - I_{R3} + 2 = 0$$

At node X, the KCL equation is:

$$I_{R3} + I_{R4} - 2 = 0$$

The last step in setting up the equations for solving is to remember that $I = {}^{V}\!/\!_{R}$. Thus we can replace the currents with ${}^{V}\!/\!_{R}$. For example:

$$\begin{aligned} I_{R1} &= \frac{(V_y - V_z)}{R1} \\ &= \frac{(V_y - 10)}{5} \end{aligned}$$

Why did we use $(V_y - 10)$? Since we chose I_{R1} to be flowing from right to left, we imply by the passive sign convention that $V_y > V_z$. (Current flows from higher potential to lower potential). If our current arrow is in the correct direction, this will yield a positive current as we expect.

Writing out the other currents we get:

$$\begin{aligned} I_{R2} &= \frac{(0 - V_y)}{R2} & I_{R3} &= \frac{(V_y - V_x)}{R3} & I_{R4} &= \frac{(0 - V_x)}{R4} \\ &= \frac{-V_y}{10} & &= \frac{(V_y - V_x)}{10} & &= \frac{-V_x}{10} \end{aligned}$$

Now substitute these expressions for currents I_{R1} through I_{R4} in the original equations.
At node V:

$$\begin{aligned} -\frac{(V_y - 10)}{5} + \frac{-V_y}{10} - \frac{(V_y - V_x)}{10} + 2 &= 0 \\ \frac{(10 - V_y)}{5} + \frac{-V_y}{10} - \frac{(V_y - V_x)}{10} &= -2 \\ 20 - 2V_y - V_y - V_y + V_x &= -20 \\ -4V_y + V_x &= -40 \end{aligned}$$

At node X:

$$\begin{aligned} \frac{(V_y - V_x)}{10} + \frac{-V_x}{10} - 2 &= 0 \\ V_y - V_x - V_x &= 20 \\ V_y - 2V_x &= 20 \end{aligned}$$

Now we have two independent equations with two unknowns. Now we solve them.

From the equation for V_x, solve for V_y.

$$\begin{aligned} V_y - 2V_x &= 20 \\ V_y &= 20 + 2V_x \end{aligned}$$

Taking this result for V_y and substituting into the equation at node Y:

$$\begin{aligned} -4(20 + 2V_x) + V_x &= -40 \\ -80 - 8V_x + V_x &= -40 \\ -7V_x &= 40 \\ V_x &= -\frac{40}{7} \\ &= \underline{\underline{-5.71V}} \end{aligned}$$

Now, substituting this voltage for V_x, (-5.71) into the equation for V_y, gives us V_y.

$$\begin{aligned} V_y &= 20 + 2(-5.71) \\ &= \underline{\underline{8.58V}} \end{aligned}$$

Thus, the voltage at node Y is 8.58 volts.

We can check to see if KCL holds true at Y. Computing the currents around the node we get:

$$\begin{aligned} I_{R1} &= \frac{(8.58 - 10)}{5} & I_{R2} &= \frac{-8.58}{10} & I_{R3} &= \frac{(8.58 - (-5.71))}{10} \\ &= -0.28A & &= -0.858A & &= 1.429A \end{aligned}$$

Now, accounting for the our guesses of current directions, our currents look like:

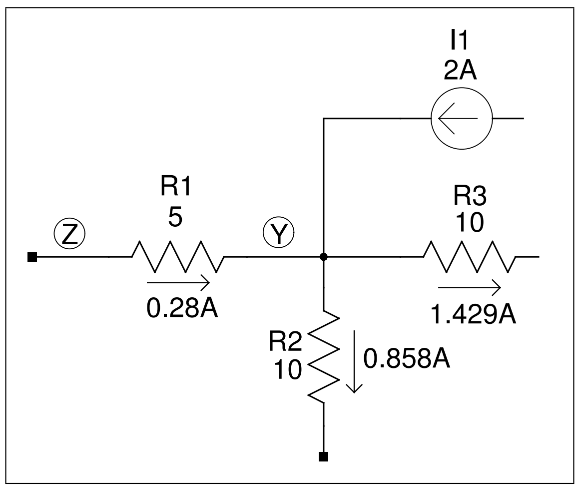

Figure 2: Summing currents at Node Y.

If we sum the currents according to our convention, we get:

$$0.28 + 2 - 0.858 - 1.429 \approx 0$$