Analysis of a single-loop circuit using the KVL method

Figure 1 is our circuit to analyze. We shall attempt to determine the current through each element, the voltage across each element, and the power delivered to or absorbed by each element. You will note that the KVL method determines the unknown current in the loop by using a sum of voltages in the loop.

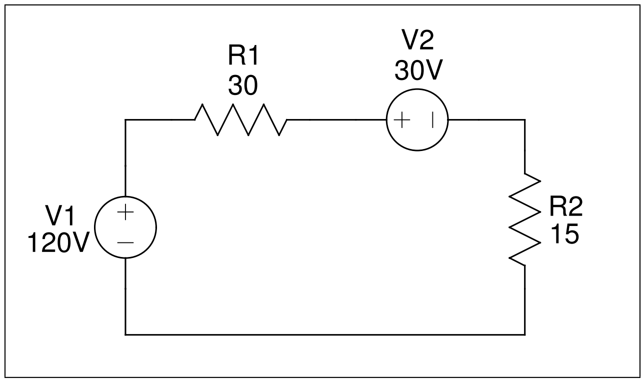

Figure 1: Single loop circuit.

The first step in the analysis is to ***assume a reference direction for the unknown current***. We do not know apriori what the direction is, nor does it matter. If our assumption is wrong, the current will simply have a minus sign associated with it. The circuit below is shown with an arbitrary direction for the current I.

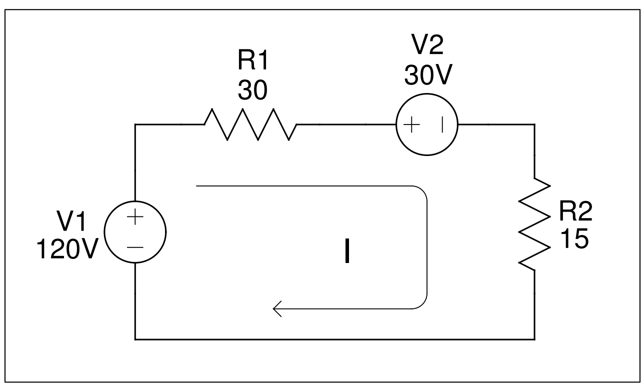

Figure 2: Single loop circuit with I direction.

The second step in the analysis is to ***assign the voltage references*** (where needed) for each element in the circuit. We already know that the passive sign convention for resistors demands that the sense of the current and voltage be selected such that the current enters the more positive terminal. *The choice of direction in which we draw the current arrow is* ***arbitrary****, but once the direction is chosen, the polarity orientation across the resistors is then* ***fixed****.* The voltages across the voltage sources and their polarities are taken as given and cannot be altered as they are independent sources. Therefore, voltages V_r1 and V_r2 in this loop must be assigned as shown in figure 4.

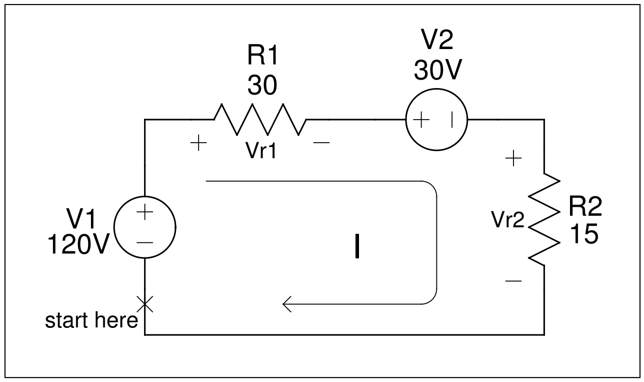

Figure 3: Single loop circuit, with V signs.

Next, we apply ***Kirchhoffs voltage law to the single closed path***. We may sum the voltages by traversing the circuit in either direction, but let's do so in the clockwise direction, beginning at the lower left corner, (where it says `start here`) and write down each voltage first encountered at its positive reference and write down the negative of every voltage encountered at its negative terminal. Thus we get,

$$-120 + V_{r1} + 30 + V_{r2} = 0 \; .$$

This is a single equation with two unknowns. To solve this equation, we must find some way to express V_{r1} and V_{r2} in one variable. Since this is a series circuit and the same current flows through all elements, we may express V_{r1} and V_{r2} in terms of the current I and the individual resistances using Ohms law. So we ***apply the Ohm's law substitution step***.
We know that for R_1 and R_2, Ohms law states that:

$$V_{r1} = I \times 30, \text{and } V_{r2} = I \times 15 \; .$$

If we substitute these expressions for V_r1 and V_r2 into the first equation, we get:

$$-120 + 30I + 30 + 15I = 0 \; .$$

Solving for I, we obtain:

$$45I = 90, \text{ thus } \underline{\underline{I = 2\text{A.}}}$$

Since the solution for current is a positive value, we know that the assumed direction for current is correct. If the answer had been -2A, we would know that the current would be actually flowing the opposite direction.

Before we proceed to compute the power for the components in this circuit, let's repeat the problem but reverse the assumed direction of current flow. Therefore we have the situation shown in figure 4.

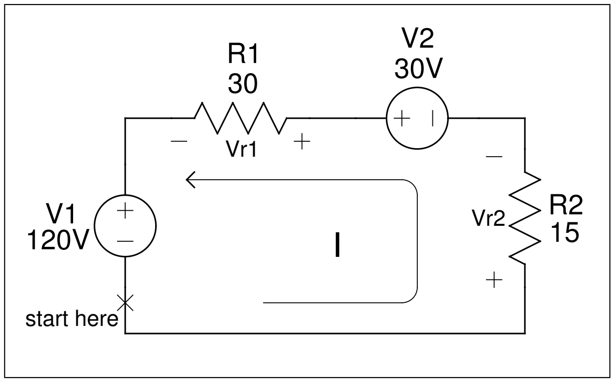

Figure 4: Single loop circuit, with V signs.

We will now write the KVL equation by traversing the circuit in the clockwise direction as we did before. Again we write down each voltage first encountered at its positive reference and write down the negative of every voltage encountered at its negative terminal. Thus we get,

$$-120 - V_{r1} + 30 - V_{r2} = 0 \, .$$

Applying the Ohms law substitution step we get:

$$-120 - 30I + 30 - 15I = 0 \, .$$

Solving for I, we obtain:

$-45I = 90$, thus $\underline{\underline{I = -2\text{A}.}}$

Since the resulting current is negative, we know that the assumed direction for current is incorrect. However, our answer is correct considering the direction of the arrow.

To compute the power dissipation, we will assume that a positive current was obtained as in the first example. Knowing the equation for power dissipation; $P = I^2R$, we can calculate the power dissipated by the resistors.

For the 30Ω resistor: $P = 2 \times 2 \times 30 = 120$ Watts.
For the 15Ω resistor: $P = 2 \times 2 \times 15 = 60$ Watts.

Resistors always dissipate positive power. They can never generate power.

How much power is generated or consumed by the voltage sources? Remember to compute power dissipated, we must take each element and redefine the voltage reference sign and the current reference arrow so that the arrow points into the positive terminal of the component such that it looks like the model for computing power dissipated shown below.

In figure 5 we see the progression of taking the 120V source and transforming its current arrow so that the arrow points into the positive terminal just like the model for computing power.

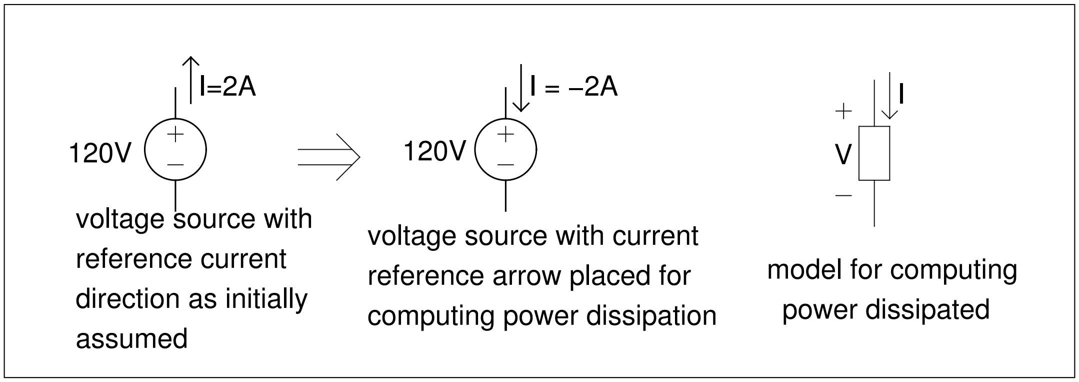

Figure 5: Orient the voltage source to match the PSC model

Note that the current arrow for the 120V source is now drawn as the passive sign convention demands; it points into the positive terminal. The voltage references are identical to the model so they were not changed. Now we can compute the power dissipation directly as:

$$P_{120V} = -2 \times 120 = -240 \text{ Watts.}$$

The negative power dissipation indicates that the source is actually delivering power, not dissipating it. For the 30V source, again the current arrow is drawn as the passive sign convention demands. However in this case, the current into the source is in the same direction as the model dictates, thus:

$$P_{30V} = 2 \times 30 = 60 \text{ Watts.}$$

In this case, the 30 volt voltage source is dissipating power.

A useful check of your calculations may be accomplished by using the power check. This method relies on the fact of the conservation of energy. In other words, the power dissipated by all the elements must equal the power supplied by all the elements. In our example:

$$\begin{aligned} P_{120V} + P_{R_1} + P_{30V} + P_{R_2} &= 0 \\ -240 + 120 + 60 + 60 &= 0 \\ 0 &= 0. \end{aligned}$$

Thus, our calculations are correct.

To summarize the KVL method:

1. Assign a reference direction for the unknown current
2. Assign voltage references to the elements
3. Apply KVL to the closed loop path
4. Substitute in Ohms law where needed to get an equation in I
5. Solve for the current I

Remember that:

1. The assumed direction for the current flowing in the loop does not matter. If the assumed direction is backwards to the actual flow, the magnitude will simply be negative.
2. The polarity across the resistors must conform to the passive sign convention.
3. The direction in which you sum voltages does not matter. Neither does it matter where in the loop you start the summation process.