## RC Differentiator

Here is a circuit for a RC differenetiator.


Figure 1: RC Differentiator Circuit
Analysis:

- At time zero, the capacitor is assumed to be discharged and be at zero volts.
- We know that the capacitor will not allow an instantaneous change in voltage across its terminals as this requires infinite current as: $I_{c}=C \frac{d v}{d t}$
- Thus, when the instantaneous change in $V_{i n}$ occurs, since the capacitor cannot allow a change in voltage across its terminals, all of $V_{\text {in }}$ 's voltage will appear across $V_{R}$. Thus, $V_{R}=V_{\text {in }}$ at the moment that $V_{\text {in }}$ transitions to 5 V . This can be seen by writing a KVL loop of the circuit. Here, we see that,

$$
\begin{aligned}
-5+0+V_{R} & =0 \\
V_{R} & =5
\end{aligned}
$$



Figure 2: KVL Loop During Charging

- After a time determined by the time constant of $C 1$ and $R 1$, the voltage across $C 1$ rises exponentially to 5 volts. As it does, $V_{R}$ will decrease exponentially as writing a KVL loop for the circuit shows.

$$
\begin{aligned}
-\operatorname{Vin}+V_{C}+V_{R} & =0 \\
V_{R} & =V_{i n}-V_{c} \quad ; \quad \text { as Vc grows, Vr goes down }
\end{aligned}
$$

- When $V_{i n}$ transitions back to zero volts again as shown in figure $3, V_{i n}$ 's positive terminal is at zero volts, effectively connecting $C 1$ 's left side to ground.


Figure 3: KVL Loop Once $V_{i n}$ Transitions back to Zero Volts

- Thus, the output is now at -5 volts as a KVL loop analysis shows:

$$
\begin{aligned}
+5+V_{r} & =0 \\
V_{r} & =-5
\end{aligned}
$$

- Finally, $C 1$ discharges through the $R 1$ again and $V_{r}$ reaches zero volts again after about $10 \tau$. An ngspice netlist and waveform for the behavior of the circuit is shown below.


Figure 4: RC Differentiator Simulation Output

```
RC differentiator
*pulse must be long relative to RC time constant
*5V input source with 1ns delay, 1nS edges, 10ms pulse width, 20ms cycle time
Vin vin gnd 5.0 PULSE(0 5.0 1ns 1ns 1ns 10ms 20ms)
c1 vin output 1uf
rload output gnd 2k ;1k load resistor
.control
    set hcopydevtype=postscript
    set hcopypscolor=true
    set color0 = rgb:f/f/f
    set color1 = rgb:0/0/0
    tran 0.1ms 30ms
    plot V(vin) V(output) V(vin,output) xl 0.1ms 30ms
* gnuplot rc_diff V(vin) V(output) V(vin,output) xl 0.1ms 30ms ;make plot for latex
    .endc
    .end
```

