## KCL Analysis Again

Below is another circuit to analyze. The goal is to determine the voltage at nodes X and Y .


All the nodes have been named and all currents labeled. The common node is the is the ground node which usually appears as a common lead across the bottom of a circuit diagram.

The next step is to write the KCL equation for each node except the common node, or for nodes that have a voltage source between the node and common. Notice Z is such a node. For node Z , we see by inspection that its value is 5 V .

For node Y , the KCL equation is: $\quad \mathrm{I} 2-\mathrm{I} 3+\mathrm{I} 4=0$
At node X , the KCL equation is: $\quad-.001-\mathrm{I} 1-\mathrm{I} 2=0$

Now do the Ohm's law replacement $(\mathrm{I}=\mathrm{V} / \mathrm{R})$ For example:
$\mathrm{I} 1=\mathrm{Vx} / 3000 \quad($ where Vx is the voltage at node X$)$
$\mathrm{I} 2=(\mathrm{Vx}-\mathrm{Vy}) / 1000$
$\mathrm{I} 3=\mathrm{Vy} / 2000$
$\mathrm{I} 4=(5-\mathrm{Vy}) / 1000$
Now substitute these expressions for the currents in the original equations.
$((\mathrm{Vx}-\mathrm{Vy}) / 1000)-(\mathrm{Vy} / 2000)+((5-\mathrm{Vy}) / 1000)=0 \quad$ @ node Y
$-.001-(V x / 3000)-(V x-V y / 1000)=0$ @ node X
now we have two independent equations with two unknowns. Let's solve.

Simplify both equations at X and Y first
$-.001-(V x / 3000)-(V x-V y / 1000)=0$
$-3-(V x)-(3 V x-3 V y)=0$
$-3-V x-3 V x+3 V y=0$
$-4 V x+3 V y=3$
@ node X equation $-3-(V x)-(3 V x-3 V y)=0$
(multiply through by 3000)
$-4 V x+3 V y=3$
(remove parenthesis)
(gather terms)
$((\mathrm{Vx}-\mathrm{Vy}) / 1000)-(\mathrm{Vy} / 2000)+((5-\mathrm{Vy}) / 1000)=0 \quad(\quad$ @ node Y equation
$(2 V x-2 V y)-(V y)+(10-2 V y)=0$
(multiply through by 2000)
$2 \mathrm{Vx}-2 \mathrm{Vy}-\mathrm{Vy}+10-2 \mathrm{Vy}=0$
(remove parenthesis)
$2 \mathrm{Vx}-5 \mathrm{Vy}=-10$
(gather terms)

Now, with the simplified equation at node $Y$, solve for $V x$ in terms of $V y$ :
$2 \mathrm{Vx}-5 \mathrm{Vy}=-10$
$2 \mathrm{Vx}=5 \mathrm{Vy}-10$
$\mathrm{Vx}=5 / 2 \mathrm{Vy}-5$

Take this form for Vx and substitute into the simplified equation for X :

$$
\begin{aligned}
& -4 V x+3 V y=3 \\
& -4(5 / 2 V y-5)+3 V y=3 \\
& -10 V y+20+3 V y=3 \\
& -7 V y=-17 \\
& V y=17 / 7=2.43 V \\
& \hline \hline
\end{aligned}
$$

simplified $X$ equation
substitute in equation for Vx multiply out

Substitute Vy in the simplified equation for Y to solve for Vx :
$2 \mathrm{Vx}-5 \mathrm{Vy}=-10$
$2 \mathrm{Vx}-5(17 / 7)=-10$
$2 \mathrm{Vx}-85 / 7=-10$
$\underline{\underline{\mathrm{Vx}}=(-10+85 / 7) / 2=1.07 \mathrm{~V}}$
simplified Y equation
substitute in Vy
multiply out

