Circuit Analysis Using KCL (node voltage) Method

Below is a circuit to analyze. We shall now determine the voltage at each node.

The first step in the analysis is to label all the nodes except for the common node. Often, the common node is the one connected to the negative terminal of the voltage source. More often than not, it appears as a common lead across the bottom of a circuit diagram.

Secondly, label the currents entering or leaving each node. The direction of the current arrows is totally arbitrary. If you choose the wrong direction, the current magnitude will simply be negative.

The next step is to write the KCL equation for each node except the common node, or for nodes that have a voltage source between the node and common. For node Vz, we see by inspection that its value is 10V.

For node Vy, the KCL equation is: \(-I_1 + I_2 - I_3 + 2 = 0\)

At node Vx, the KCL equation is: \(I_3 + I_4 - 2 = 0\)

The last step in setting up the equations to solve is to utilize Ohm’s law to replace the currents with \(V/R\). (remember \(I = V/R\)) For example:

\[ I_1 = \frac{(V_y - V_z)}{R_1} \Rightarrow I_1 = \frac{(V_y - 10)}{5} \]

Why did we use \((V_y - 10)\)? Since we chose \(I_1\) to be flowing from right to left, we imply by the passive sign convention that \(V_y\) is greater than \(V_x\). (Current flows from higher potential to lower potential.) If our current arrow is in the right direction, this will yield a positive current as we expect.

Continuing with the other currents:

\[ I_2 = \frac{(0 - V_y)}{R_2} \Rightarrow I_2 = -\frac{V_y}{10} \]
(the common node is 0V)

\[ I_3 = \frac{(V_y - V_x)}{R_3} \quad \Rightarrow \quad I_3 = \frac{(V_y - V_x)}{10} \]

\[ I_4 = \frac{(0 - V_x)}{R_4} \quad \Rightarrow \quad I_4 = -\frac{V_x}{10} \]

Now substitute these expressions for the currents in the original equations.

\[ -(\frac{(V_y - 10)}{5}) + (-\frac{V_y}{10}) - (\frac{(V_y - V_x)}{10}) + 2 = 0 \quad \text{@ node } V_y \]

\[ (\frac{V_y - V_x}{10}) + (-\frac{V_x}{10}) - 2 = 0 \quad \text{@ node } V_x \]

now we have two independent equations with two unknowns. Let’s solve.

from the equation for \( V_x \), solve for \( V_y \):

\[ \frac{(V_y - V_x)}{10} + (-\frac{V_x}{10}) - 2 = 0 \quad \text{@ node } V_x \]

\[ V_y - V_x - V_x - 20 = 0 \quad \text{multiply through by 10} \]

\[ V_y = 2V_x + 20 \quad \text{gather terms, solve for } V_y \]

Taking this result for \( V_y \) and substituting into the equation at \( V_y \):

\[ -(\frac{(V_y - 10)}{5}) + (-\frac{V_y}{10}) - (\frac{(V_y - V_x)}{10}) + 2 = 0 \quad \text{@ node } V_y \]

\[ -(\frac{2V_x + 20 - 10}{5}) + (-\frac{2V_x - 20}{10}) \]

\[ -\frac{2V_x + 20 - V_x}{10} + 2 = 0 \quad \text{reduce fractions} \]

\[ -\frac{4V_x + 1}{10V_x} = 4 \]

\[ V_x = -40/7 \]

\[ V_x = -5.71; \quad \text{Thus the voltage at node } V_x \text{ is } -5.71 \text{ volts.} \]

Taking the voltage for \( V_x \) and putting it into the equation for \( V_x \) to solve for \( V_y \) yields:

\[ \frac{(V_y - V_x)}{10} + (-\frac{V_x}{10}) - 2 = 0 \quad \text{@ node } V_x \]

\[ (V_y - 5.71)/10 + (-5.71/10) - 2 = 0 \quad \text{substitute } V_x = -5.71 \]

\[ V_y = 2(-5.71) + 20 \]

\[ V_y = 8.58; \quad \text{Thus the voltage at node } V_y \text{ is } 8.58 \text{ volts.} \]