Reduction of Logic Equations using Karnaugh Maps

The design of the voting machine resulted in a final logic equation that was:

z = (a*c) + (a*c) + (a*b) + (a*b*c)

However, a simple examination of this equation shows that the last term (a*b*c) is already covered by the three previous product terms. Thus in the realization of the logic for the design the last term can be omitted without any change in functionality.

When manufacturing logic designs, minimizing the number of logic gates is advantageous. This is for server reasons. Fewer gates mean:

*less power consumed
*less heat produced
*greater reliability
*faster design time
*easier to debug
*smaller silicon die area

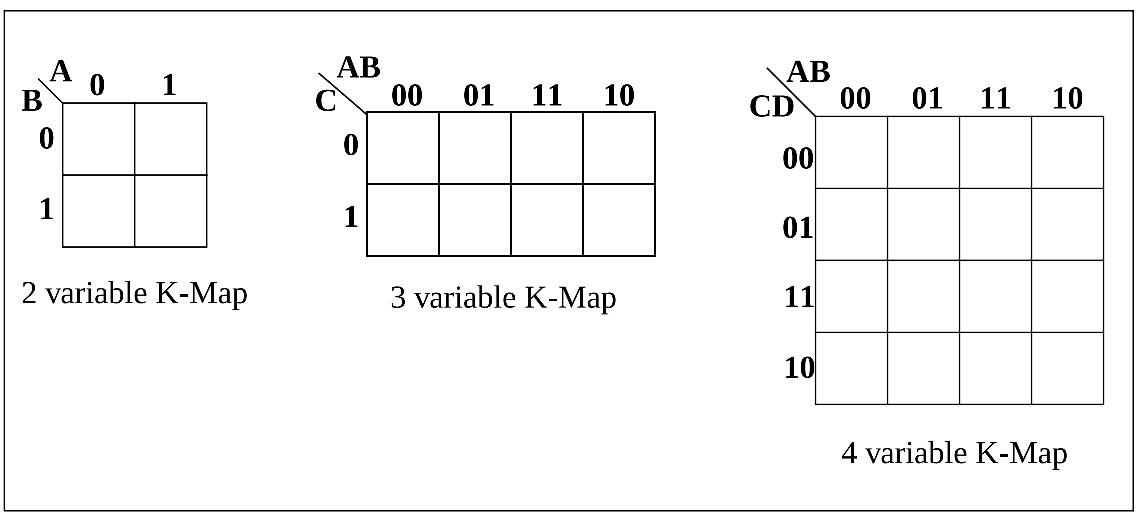
Because of the many motivations to minimize the number of logic gates, several methods for minimizing logic equations (an thus the number of gates) have been devised. One very handy method useful for hand reduction of logic equations is the Karnaugh Map (K-Map). K-Maps order and display a geometrical pattern such that application of the logic adjacency theorem becomes obvious. The logic adjacency theorem contained in the Boolean algebra states:

$$AB + A\overline{B} = A$$

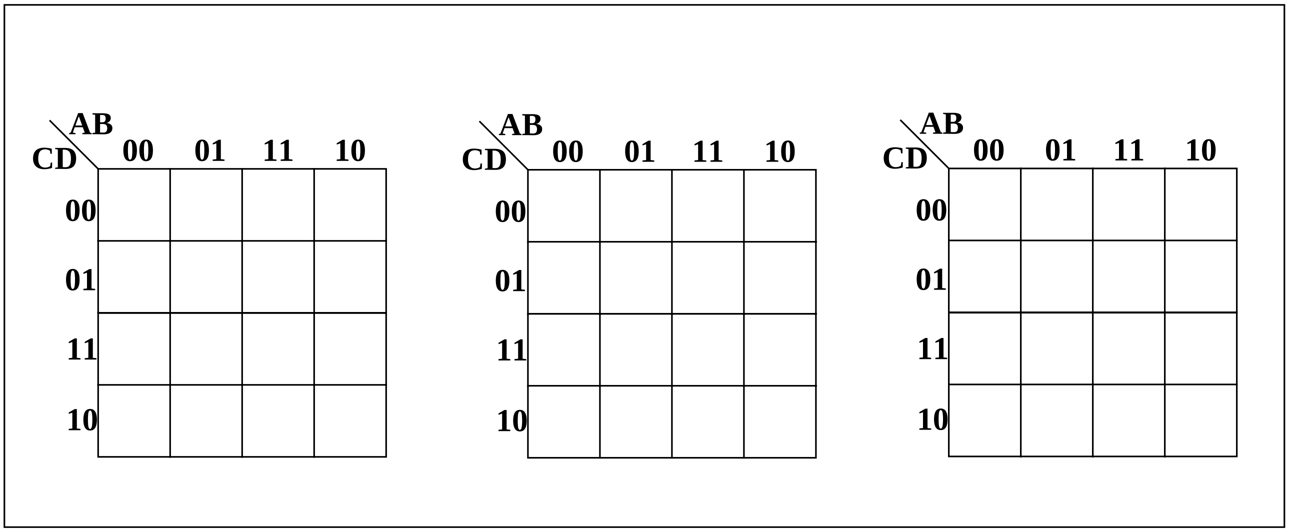
We could state this as: “If any two terms in a SOP expression vary in only one variable, and that variable in one term is the complement of the variable in the other term, then that variable is superfluous to both terms.”

K-maps are organized such that each term in a SOP expression is physically adjacent to its adjacent terms. In other words, each block is placed such that any two adjacent blocks are different by only one variable. First, lets take a look at what cells in the map are adjacent.

Below we see K-Maps for 2, 3, and 4 variables.



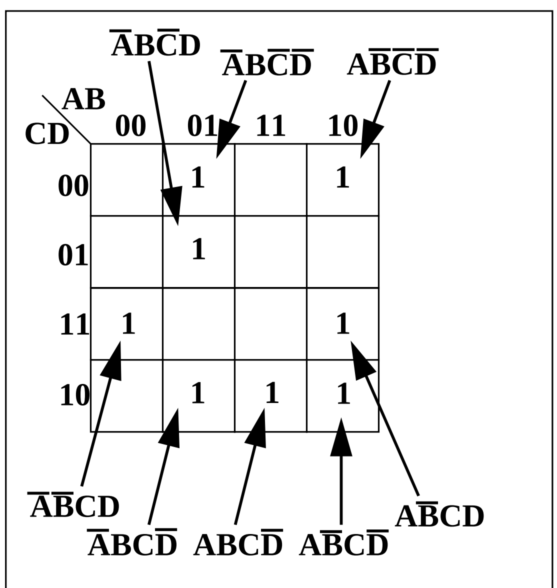
Now lets see where the adjacencies lie. We will look only at the 4 variable map as it contains all the adjacencies the other two do plus some extras.



To see how the adjacencies involving one common term can appear, suppose we have the logic equation below:

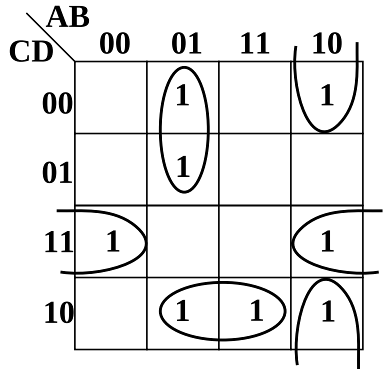
$$F = \overline{A}B\overline{C}\overline{D} + \overline{A}BC\,\overline{D} + A\overline{B}\overline{C}\overline{D} + A\overline{B}CD + ABC\overline{D} + \overline{A}B\overline{C}D + A\overline{B}C\overline{D} + \overline{A}\overline{B}CD$$

Since there are four variables in this equation, we will use a four variable K-Map. Fill in the K-map cells with the pattern for each product term of the equation. This amounts to entering a "1" in each cell required to represent that term in the logic equation. This step is shown below.



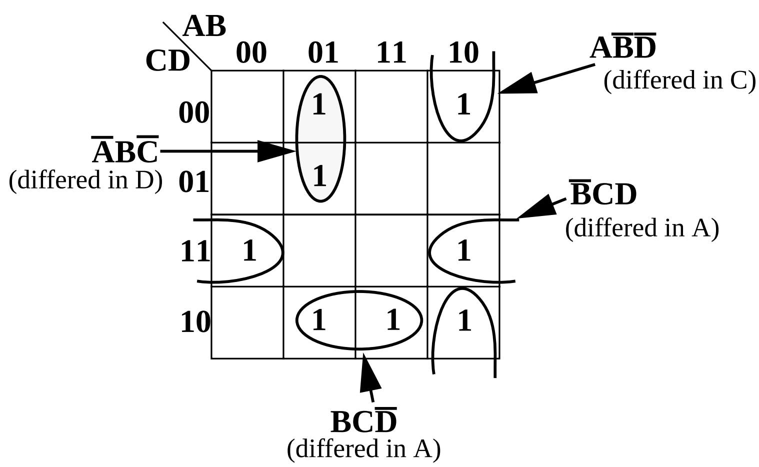
Enter a "1" for each product term

The next step is to encircle the largest number of “ones” that are a power of two until all ones are “covered”. This means 2, 4 or 8 terms. This step is shown below.



Encircle largest number of 1’s that is a power of 2

The reduced product terms are now determined by observing the coverings. If the variables under a covering do not change they will be in the final minimized equation, else they are excluded. For example, under the shaded covering, the variables corresponding to $\overline{A}$, B and $\overline{C}$ do not change. Therefore that term becomes $\overline{A}B\overline{C}$. The horizontal covering at the bottom has variables B, C and $\overline{D}$ do not change. Thus, that minimized term becomes: $BC\overline{D}$



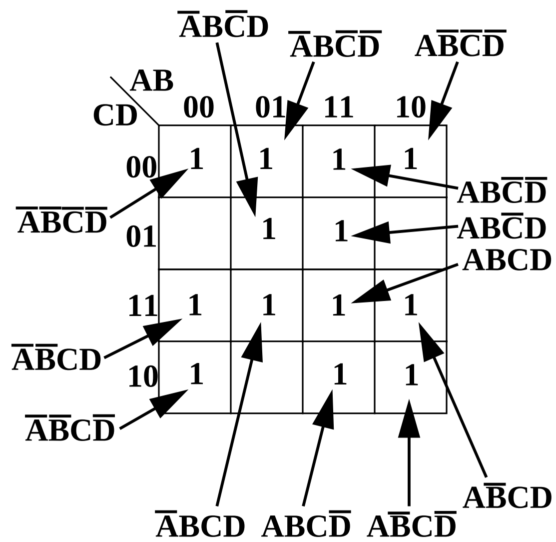
Gather the reduced terms eliminating the adjacent variable

The minimized equation is now written out by summing the reduced product terms. Thus the minimized equation becomes: $F = A\overline{B}\overline{D} + \overline{B}CD + BC\overline{D} + \overline{A}B\overline{C}$ Note that we were only able to encircle two terms. This limits us to a reduction of only one variable. Now we will look at the possibilities for reducing product terms by two variables or four cells.

To see how the adjacencies involving two common term appear, suppose we have the logic equation below:

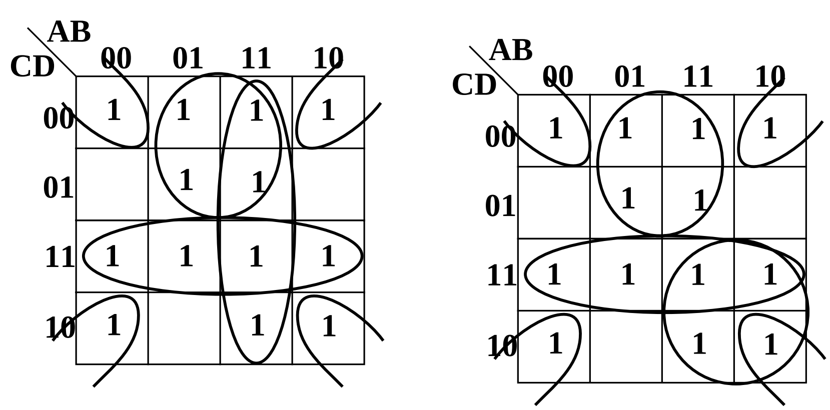
$$F = \overline{A}B\overline{C}\overline{D} + A\overline{B}\overline{C}\overline{D} + A\overline{B}CD + ABC\overline{D} + \overline{A}B\overline{C}D + A\overline{B}C\overline{D} + \overline{A}\overline{B}CD$$
$$\overline{A}\overline{B}\overline{C}\overline{D} + AB\overline{C}\overline{D} + AB\overline{C}D + ABCD + \overline{A}BCD + \overline{A}\overline{B}C\overline{D}$$

Fill in the K-map cells with the pattern for each product term of the equation..



Enter a “1” for each product term

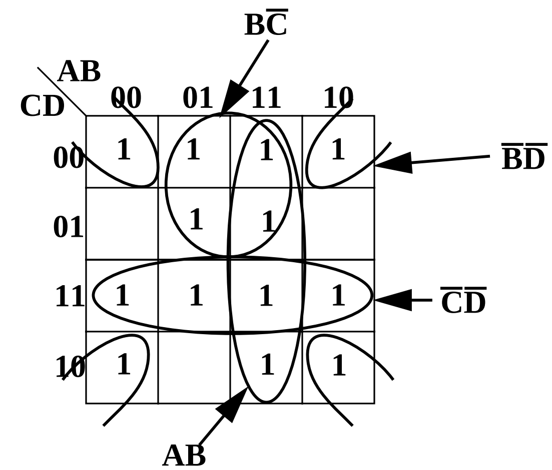
Now encircle the largest number of 1’s that is a power of two to “cover” all the ones with a minimum number of coverings.



Here is one minimal cover **Here is another**

Cover the ones

These coverings can eliminate two variables from each covering. For the first covering shown, we get the following product term reduction.



Gather the reduced terms eliminating the adjacent variable

To obtain the reduced equation, we sum all the reduced product terms yielding:
$F = B\overline{C} + \overline{BD} + \overline{CD} + AB$