An Introduction to Feedback

- Concept developed by Harold Black (1920's). Initially ridiculed for the idea of lowering gain for other benefits.
- Two varieties of feedback: positive and negative
- Positive feedback used in multivibrators, oscillators and Schmitt triggers.
- Negative feedback mostly used in amplifiers.

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- Advantages of negative feedback in amplifiers:
  - gain stabilization
  - lowers distortion
  - minimizes noise
  - better control of Z_{in}, Z_{out}
  - extends bandwidth
- Basic idea: Trade off some gain to improve other parameters.
- We've seen BJT amplifier quiscent operating point stabilized with the inclusion of R_e in BJT amplifiers.

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- General Structure of an Amplifier with Negative Feedback

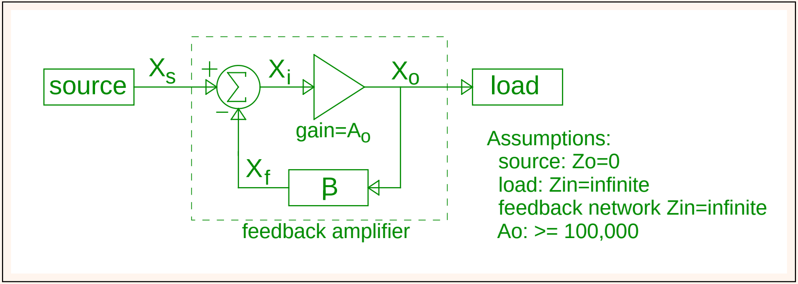

Figure: Signal Flow Diagram, Amplifier with Negative Feedback

- Either voltage or current feedback can be used, hence the use of X_n instead of V_n or I_n.
- A_o must be large, typically 10^5 or greater.

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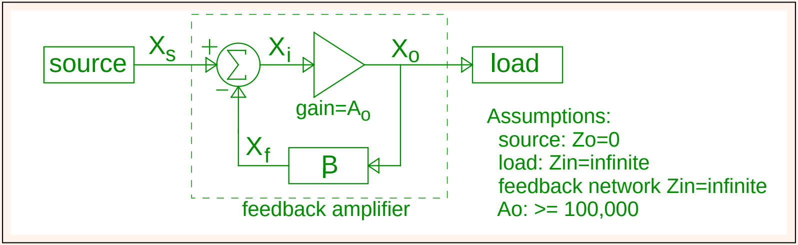

Figure: Signal Flow Diagram of a Amplifier with Negative Feedback

From this diagram, 3 equations emerge:

$$X_o = A_o X_i \quad \text{or} \quad A_o = \frac{X_o}{X_i} \tag{1}$$

$$X_f = \beta X_o \quad \text{or} \quad \frac{X_f}{X_o} = \beta \tag{2}$$

$$X_i = X_s - X_f \tag{3}$$

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We want to find A_f, the closed-loop gain of the amplifier. Or,

$$A_f = \frac{X_o}{X_s} \quad \text{or} \quad \frac{output}{input}$$

So, substitute equation 3 into equation 1

$$A_o = \frac{X_o}{X_i} \quad (1) \qquad\qquad X_i = X_s - X_f \quad (3)$$

$$A_o = \frac{X_o}{(X_s - X_f)} \quad \text{thus,}$$

$$X_o = A_o(X_s - X_f) \quad \text{or}$$

$$X_o = A_o X_s - A_o X_f$$

We can substitute for X_f using equation two, $X_f = \beta X_o$ yielding an equation in X_o and X_s.

$$X_o = A_o X_s - A_o X_f$$

$$X_o = A_o X_s - A_o \beta X_o$$

Now, solve for $A_f = \frac{X_o}{X_s}$ by gathering X_o terms on the left.

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We want to find A_f, the closed-loop gain of the amplifier.

$$
\begin{aligned}
X_o + A_o\beta X_o &= A_o X_s \\
X_o(1 + A_o\beta) &= A_o X_s \\
\frac{X_o}{X_s}(1 + A_o\beta) &= \frac{A_o \cancel{X_s}}{\cancel{X_s}}
\end{aligned}
$$

Note that:

$$A_f = \frac{X_o}{X_s} \quad \text{Thus,}$$

$$A_f = \frac{A_o}{1 + A_o\beta} \quad \text{(closed loop gain)}$$

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Note: If $A_o\beta \gg 1$:

a) A_f will be less than A_o
b) and if $A_o\beta \gg 1$, then:

$$A_f = \frac{\not{A}_o}{\not{1} + \not{A}_o\beta} \approx \frac{1}{\beta} \quad (!!)$$

This means that the gain A_f is dependent only on β, the feedback network and is relatively immune to changes in A_o which could vary considerably.

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From the initial three equations, we can write:

$$X_f = \frac{A_o\beta}{1 + A_o\beta} X_s$$

If $A_o\beta \gg 1$, then $X_f \approx X_s$, which implies the input to the amplifier is reduced to almost zero.

Thus, if large amounts of negative feedback are used, X_f is nearly identical to X_s. An outcome is that both inputs to an OPAMP are at the same voltage.

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Negative feedback greatly reduces the dependency upon OPAMP gain on the non-inverting amplifier gain. Using the feedback amplifier gain equation, let's first determine R1 and R2 with the assumptions: $R_{in} = \infty$, $R_{out} = 0$, $A_f = 10$, $A_o = 10^4$, and that R1 and R2 do not load the amplifier output.

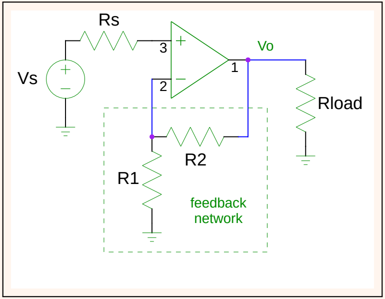

Figure: Example: Non-inverting OPAMP circuit

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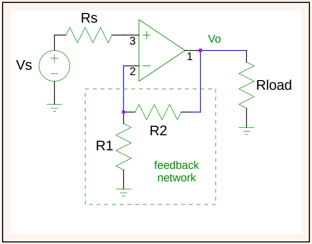

Figure: Example: Non-inverting OPAMP circuit

From the circuit we see that:

$$\beta = \frac{R1}{R1 + R2}$$, and the gain equation is:

$$A_f = \frac{A_o}{1 + A_o\beta}$$

Now, solve for R1 and R2.

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From our feedback amplifier gain equation:

$$
\begin{aligned}
10 &= \frac{10^4}{1 + 10^4\beta} \\
10(1 + 10^4\beta) &= 10^4 \\
10 + 10^5\beta &= 10^4 \\
10^5\beta &= 10^4 - 10 \\
\beta &= \frac{10^4 - 10}{10^5} \\
\beta &= 0.0999
\end{aligned}
$$

So,

$$\frac{R1}{R1 + R2} = 0.0999$$

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Now determine R1 and R2 as a ratio:

$$\begin{aligned}
0.999(R1 + R2) &= R1 \\
0.999R1 + 0.999R2 &= R1 \\
-0.9001R1 &= -0.999R2 \\
\frac{R2}{R1} &= \frac{0.901}{0.099} \\
&= 9.01
\end{aligned}$$

To prevent loading the amplifier output, let R1 be $1K\Omega$. Therefore, R2 would be 9.01KΩ.

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From the prior discussion of OPAMP non-inverting amplifiers,

$$\begin{aligned} A_f &= 1 + \frac{R_f}{R1} \\ &= 1 + \frac{9.01}{1.0} \\ &= 10.01 \end{aligned}$$

This is very close to what was expected. Note that an OPAMP with a gain of 10^4 would be a fairly poor OPAMP. So, what if A_o, the OPAMP gain, was 20 percent lower than expected? What would the feedback amplifier gain be then?

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From the prior discussion of OPAMP non-inverting amplifiers,

$$A_f = \frac{A_o}{1 + A_o\beta}$$
$$A_f = \frac{0.8(10^4)}{1 + (0.8(10^4)) * 0.0999}$$
$$A_f = \frac{8000}{800.2}$$
$$A_f = 9.998$$

Darn close, 0.119 percent error. What if the amplifier gain was 10^6?

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With a A_o of 10^6:

$$A_f = \frac{A_o}{1 + A_o\beta}$$

$$A_f = \frac{10^6}{1 + (10^6 * 0.0999)}$$

$$A_f = \frac{10^6}{99901}$$

$$A_f = 10.01$$

Pretty much a zero percent error. Thus, having an OPAMP with a large gain really helps! Most OPAMPs have gains around 10^6.