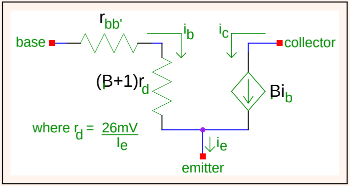
Small Signal BJT Analysis

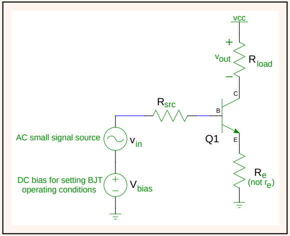
- A common model for the the BJT operating in the small signal region is the ”Hybrid Pi” model. Here we show the low frequency version that is valid through the audio spectrum.



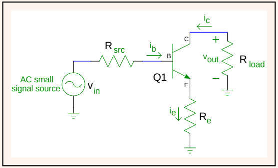
- This model views the BJT as a current controlled current source (CCCS), where β is the current gain, r_d is the dynamic emitter resistance and $r_{bb'}$ is the base spreading resistance.
- $r_{bb'}$ arises from the depletion layer that extends deeply into already thin, lightly doped the base region. This decreases its cross-sectional area, increasing the resistance through the base.
- $r_{bb'}$ can usually be ignored except at high-frequencies or with low noise applications. Its value is typically in the ten's of ohms.

Small Signal BJT Analysis

- The transistor is usually driven by a combination of inputs: a biasing DC voltage and a AC input signal.



(a) AC and DC circuit

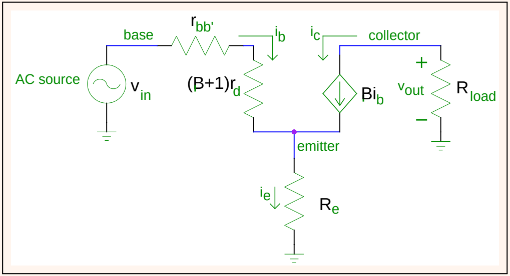


(b) Small-signal AC circuit

- In the AC small-signal model, we short out all the DC supplies. AC signals usually are in lower case.

Small Signal BJT Analysis

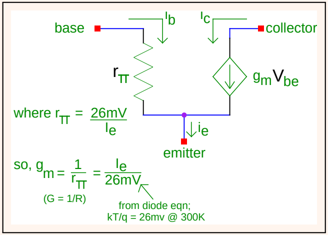
- In a circuit (a common emitter amplifier) putting the small-signal model within the circuit, we get:



- The coupling capacitors disappear as they are assumed to be zero impedance at the signal frequency.
- Now, let's change our model slightly to make it easier to use.
- We will move from a current controlled current source (CCCS) to a voltage controlled current source (VCCS).

Small Signal BJT Analysis

- Here we have the simpler model with the VCCS.
- First, $r_{bb'}$ was removed since its typically small (tens of ohms).



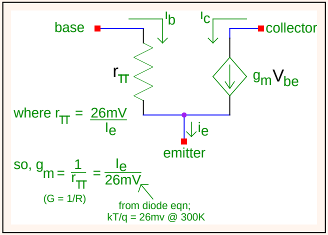
- r_π is the dynamic diode resistance we looked at earlier as the B-E junction *is* a diode.
- β was removed from the dependent current source and replaced with a transconductance multiplied by V_{be}.
- Now, I_e and I_c are accurately set by V_{be} rather than base current. This exponential relationship is accurate over an enormous range of currents unlike β which can vary widely.

Small Signal BJT Analysis

- β varies considerably even for one transistor type. h_{fe} (small-signal β) for the 2N4401 varies from 40-500. This might lead to the misconception that the small-signal gain of an amplifier will accordingly which is not true.
- I_c is related to V_{be} by: $I_c = I_s(T)(e^{\frac{V_{be}}{V_t}} - 1)$; where:
  $I_s = 5 * 10^{-9}$ saturation current
  $q = 1.602 * 10^{-19}$ charge on an electron
  $K = 1.38 * 10^{-23}$ Boltzmann's constant
  $T = 300$ absolute temperature
- If $\beta \geq 100$, $I_c \approx I_e$, thus V_{be} programs both I_c and I_e.
- g_m is well defined by the emitter current I_e in any BJT transistor.

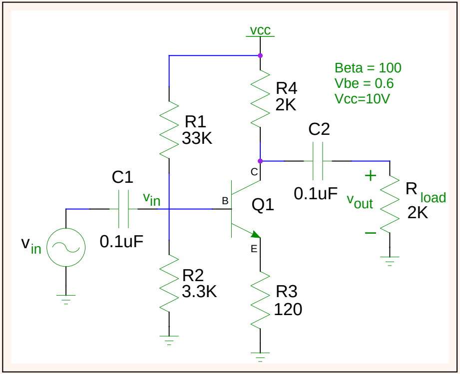
Small Signal BJT Analysis

- Let's step back for a moment and look at the idea of *gain*.
- Gain is a ratio between output and input where input and output can have differing units.
- In our new model, the gain has units of: $\frac{output}{input} = \frac{current}{voltage} = \frac{1}{R}$ or conductance.
- An amplifier with gain expressed with units of conductance is called a *transconductance amplifier.*
- The transconductance of such an amplifier is the ratio $\frac{\Delta I_{out}}{\Delta V_{in}}$ or $\frac{i_{out}}{v_{in}} = g_m$



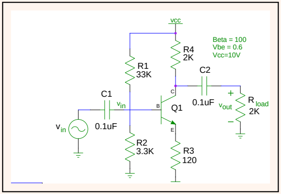
Small Signal BJT Analysis

- Consider the following common emitter amplifier with input v_{in} and load R_{load} as indicated.

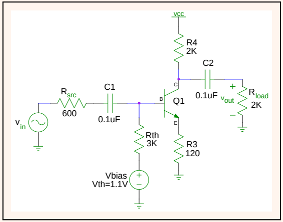


Small Signal BJT Analysis

- Before we can work out the gain and other parameters, we must first determine what the quiescent point (Q-point) is for the transistor.
- First, reduce the base biasing resistor network, $R1$ and $R2$, into a Thevenin equivalent circuit.



(a) Original Circuit

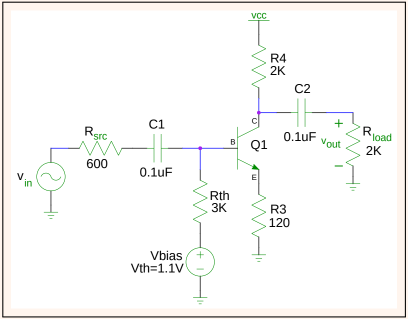


(b) Thevenin Equivalent in place

$$
\begin{aligned}
R_{th} &= \frac{R1 * R2}{R1 + R2} \\
&= 3K\Omega \\
V_{th} &= Vcc * \frac{R2}{R1 + R2} \\
&= 1.1V
\end{aligned}
$$

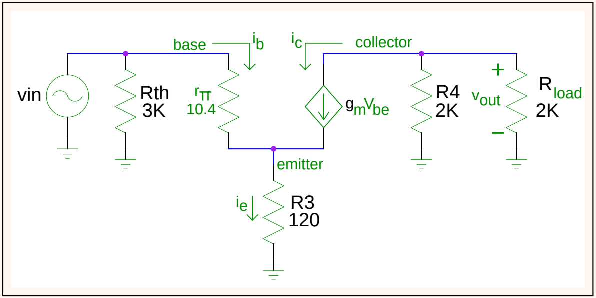
Small Signal BJT Analysis

- Knowing the voltage at the base is 1.1V ($I_b \approx 0$), we know that the voltage at the emitter is one V_{be} less: $V_e = (1.1 - 0.6) = 0.5V$
- If V_e is 0.5V, then I_e is $\frac{0.5}{120} = 4.2mA$
- With the approximation that ($I_b \approx 0$), $I_e \approx I_c$, so $I_c = 4.2mA$
- If $I_c = 4.2mA$ then $V_c = 10 - (4.2mA * 2000) = 3.7V$
- Now we know all the pertinent DC voltages and currents that will help us determine the small-signal behavior.



Small Signal BJT Analysis

- Knowing I_e we can find r_π: $r_\pi = \frac{26mV}{4.2mA} = 6.1\Omega$
- The transconductance, g_m is $\frac{1}{r_\pi} = 0.161S$
- Knowing these, and noting that the 3K R_{th} resistor comes from the paralleled biasing resistors, we can find the input impedance and A_v (voltage gain).
- Putting our model into the circuit at hand, we now have:



Small Signal BJT Analysis

- First determine the voltage gain where: $A_v = \frac{v_{out}}{v_{in}}$.

  $$\begin{aligned} v_{out} &= -g_m * V_{be} * 1000 \text{ \textit{(Rc and Rload are in parallel)}} \\ &= -(0.16)(0.6)(1000) \\ &= -96.9 \end{aligned}$$

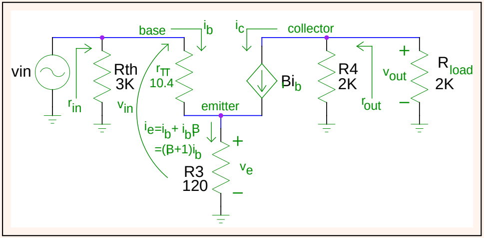
- Now determine v_{in}:

  $$\begin{aligned} v_{in} &= (i_b * r_\pi) + 120 i_c \text{ \textit{(but ib=0, so)}} \\ &\approx 120 g_m V_{be} \\ &\approx 11.52 \end{aligned}$$

- So, the gain is $\frac{-96.9}{11.5} \approx -8.4$ *(what does the minus mean?)*
- Note that $\frac{R4}{R3} = 8.4$
- In this circuit configuration, gain may be approximated by $A_v = \frac{R_c}{R_e}$

Small Signal BJT Analysis

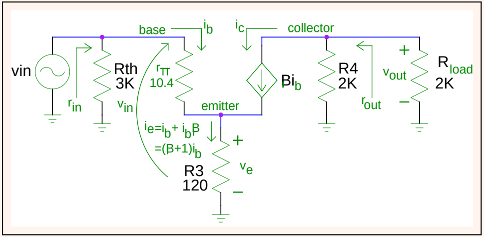
- Now the input resistance can now be found.
- Previously, we used an approximation where $I_b = 0$. We temporarily drop that approximation to determine input impedance. This is because if the *transistor* input impedance was $^{v_{in}}/_{i_b}$ and $i_b = 0$, the transistor input resistance would be infinite, which we know is not true.
- If we replace the VCCS of the hybrid-pi model with the CCCS we previously had, and take into account an input current, we can compute the input resistance.
- Our slightly modified model is shown below.



Small Signal BJT Analysis

- Input resistance:

$$
\begin{aligned}
r_{in} &= \left(\frac{v_{in}}{i_{in}}\right) \,||\, 3000 \\
&= \left(\frac{(\cancel{i_b} r_\pi) + (\beta + 1)\cancel{i_b} R_e}{\cancel{i_b}}\right) \,||\, 3000 \\
&= (r_\pi + (\beta + 1)R_e) \,||\, 3000 \\
&= (10.4 + (101)120) \,||\, 3000 \\
&= 12130 \,||\, 3000 \\
&= 2.4K\Omega \text{ \textit{(note: the bias resistors dominating input impedance)}}
\end{aligned}
$$



Small Signal BJT Analysis

- Output resistance r_{out} is made up of two components, the infinite output resistance of the current source and R4.
- Clearly, $r_{out} = R4 || \infty = 2K\Omega$.