Adding a Lumped Series Element

Consider the following T-line circuit:

\[ z_{\text{in}, 1} = r_{\text{in}, 1} + jx_{\text{in}, 1} \]
\[ z_{\text{in}, 1} = z_{\text{in}, 1} Z_0, 1 \]

\[ z_{\text{in}, 2} = r_{\text{in}, 2} + jx_{\text{in}, 2} \]

\[ z_{L, 1} = r_{\text{in}, 2} \frac{Z_{0, 2}}{z_{\text{in}, 1} Z_0, 1} + \frac{R}{z_{\text{in}, 2} Z_0, 1} + jx_{\text{in}, 2} \frac{Z_{0, 2}}{Z_0, 1} \]

\[ Z_L = \frac{Z_L}{Z_0, 2} \]

\[ \Rightarrow \text{All calculations can be done on Smith chart} \]
Smith Chart as Admittance Chart

For circuits involving parallel (shunt) connections of circuit elements, it is more convenient to work with admittances.

How can the Smith chart be used with admittances?

How is $Y = \frac{1}{Z} = G + jB$ related to reflection coefficient $\Gamma$?

Consider normalized admittance $y$

$$y = \frac{Y}{Y_0} = \frac{G + jB}{Y_0} = g + jb = Y Z_0 = \frac{Z_0}{Z} = \frac{1}{r + jx}$$

Smith Chart as Admittance Chart

Reflection coefficient $\Gamma$

$$\Gamma = \frac{z - 1}{z + 1} = \frac{1}{y + 1} - \frac{y - 1}{y + 1}$$

Smith chart can directly be used with $y = g + jb$ after rotating reflection coefficient by 180°.

$$\frac{y - 1}{y + 1} = -\Gamma = \Gamma e^{j\pi}$$

$g = \text{const}$

$b = \text{const}$

circles
Impedance vs. Admittance Coordinates

Example 4

Given the normalized load admittance

\[ y_L = 0.5 + j2.0 \]

Determine the normalized admittance at distance

\[ d = \lambda/16 = 0.0625\lambda \]

Solution: use Smith chart with admittance coordinates ➔ see web applet (example 4)
Example 5

Given a 50Ω T-line that is terminated in $Z_L = Z_T = 25Ω$ and has a shunt resistance $R_{sh} = 50Ω$ connected at distance $d = 1m$ from the termination. The wavelength on the line is $\lambda = 3m$.

Determine the input impedance at the location of the shunt resistance.

\[ Z_L = (25 + j25)Ω \]
\[ Z_0 = 50Ω \]
\[ z_L = 0.5 + j0.5 \]
\[ l = 0.1025\lambda \]

Solution: see web applet, example 5

Additional Example

\[ Z_L = (25 + j25)Ω \]
\[ Z_0 = 50Ω \]
\[ z_L = 0.5 + j0.5 \]
\[ l = 0.1025\lambda \]

Results

\[ \text{SWR} \approx 2.6 \]
\[ |\Gamma_L| = 0.45 \quad \theta_L \approx 116.5° \]
\[ y_L = 1 - j \]
\[ z_{in} = 1.5 + j1.1 \]
Impedance Matching

Implementation Issues

- Types of Networks
  - lumped elements
  - distributed elements
  - mixed
  - reactive vs. resistive

- Design Criteria
  - bandwidth
  - lossless/lossy
  - complexity
  - parasitics

Matching network needs to have at least two degrees of freedom (two knobs) to match a complex load impedance
Transmission-Line Matching

Idea: Use impedance transformation property of transmission line

transformation circle

load $z_L$ (or $y_L$)

$2\beta z'$

transformation circle

Transformation to real $z$ or $y$

load $z_L$ (or $y_L$)

real $z$ (or $y$)

real $z$ (or $y$)
Transmission-Line Matching

Transformation to \( r=1 \) (or \( g=1 \)) circle

\[ 2\beta z' \]

Load \( z_L \) (or \( y_L \))

\[ z=1+jx \quad (y=1+jb) \]

\[ z=1-jx \quad (y=1-jb) \]

Single Stub Matching

Stub tuners are useful for matching complex load impedances. The two design parameters in single stub matching networks are

- distance from load at which stub is connected
- length of stub

Other design considerations are

- open or shorted stub
- series or shunt connected stub
**Basic Design Principle**

Objective: Match load admittance by
1. transforming along T-line to $Y_{in} = Y_0 + jB$
2. tuning out remaining $jB$ with a parallel shunt element of admittance $-jB$
3. shunt element can be realized with a T-line stub

**Design Steps**

1. Determine **distance** $d/\lambda$ from the load $Y_L$ (or $Z_L$) at which the
   - input admittance is $Y_0 + jB$ (for shunt stub matching)
   - input impedance is $Z_0 + jX$ (for series stub matching)

2. Realize the stub with a **length** $l/\lambda$ of open- or short-circuited transmission line (stub).
Example 6

Using a short-circuited stub, match the load impedance $Z_L = (80 + j70) \Omega$ to a transmission line with $Z_0 = 50 \Omega$.

Solution: use Smith chart with admittance coordinates
⇒ see web applet (example 6)

Additional Example

$R_L = 125 \Omega$  \hspace{1cm} $f_0 = 1 \text{GHz}$
$C_L = 2.54 \text{pF}$  \hspace{1cm} $Z_0 = 50 \Omega$

$z_L = 0.5 - j$

⇒ $y_{in\_1,2} \approx 1 \pm j1.58$

Need shunt element with

$Y_{sh\_1,2} = \mp j1.58 / Z_0 = \mp j31.6 \text{mS}$
Quarter-Wave Transformer

\[ Z_1 = \sqrt{Z_0 Z_L} \]

**NOTE:** for complex load, insert \( \lambda/4 \) transformer where \( Z_{in} = R \)

Example: Quarter-Wave Transformer

- \( R_L = 125 \Omega \)
- \( f_0 = 1 \text{GHz} \)
- \( C_L = 2.54 \text{pF} \)
- \( Z_0 = 50 \Omega \)

\[ Z_L = 0.5 - j \]

\[ z_{in,1} \approx 0.24 \Rightarrow Z_{in,1} = 12 \Omega \]

From \( Z_{in,2} = Z_0 = \frac{Z_{0,T}^2}{Z_{in,1}} \)

\[ Z_{0,T} \approx 24.5 \Omega \]
### Bandwidth Performance

- At $f_0 = 1$ GHz, the SWR (Standing Wave Ratio) is plotted against frequency.
- The SWR dips to 1.5, indicating a bandwidth performance.
- Lumped Element Matching: Reactive L-Section Matching Networks.

#### Lumped Element Matching

**Reactive L-Section Matching Networks**

- For $R_L > Z_0$: 
  
  \[
  B = X_L \pm \sqrt{R_L/Z_0} \frac{R_L^2 + X_L^2 - R_L Z_0}{R_L^2 + X_L^2} 
  \]

  \[
  X = \frac{1}{B} \left( X_L Z_0 - \frac{Z_0}{B} R_L \right) 
  \]

- For $R_L < Z_0$: 

  \[
  B = \pm \sqrt{(Z_0 - R_L)/R_L/Z_0} 
  \]

  \[
  X = \pm \sqrt{R_L (Z_0 - R_L)} - X_L 
  \]

- Different solutions may result in different bandwidths.
Example

Design criteria
- bandwidth
- realizability of components
- parameter sensitivity
- ... cost ... ?

Lumped-Distributed and Distributed Matching Networks

Series Reactance Matching

Shunt Reactance Matching

To generator  

Matching network

$Z_0 = 10 \Omega$  

$Z_0 = Z_0$  

$Z_0 = 25 + 30 \Omega$

$Z_0 = 20 \Omega$ and  

$Y_R = Y_L = 0.02 \Omega$

To generator  

Matching network

$Z_0 = 10 \Omega$  

$Z_0 = Z_0$  

$Z_0$

$Z_0$  

$Z_0$  

Short  

Open  

$Z_0$  

$Z_0$  

$Z_0$  

$Z_0$  

$Z_0$  

$Z_0$  

$Z_0$
Double-Stub Tuner

- Double-stub (and multi-stub) tuners use fixed stub locations (typical spacing: $\lambda/8$)

- Advantages:
  - easier to accommodate different loads
  - better bandwidth performance
Smith Chart with Lossy T-Lines

Recall for a lossless T-line

\[
\frac{z_{in}(z')}{Z_0} = 1 + \frac{\Gamma_L e^{j\phi(z')}}{1 - \Gamma_L e^{j\phi(z')}}
\]

\[
\phi(z') = \theta_L - 2\beta z'
\]

For low-loss T-lines, \(Z_0 \approx \text{real and } \gamma = \alpha + j\beta\)

Then

\[
\frac{z_{in}(z')}{Z_0} = \frac{Z_{in}(z')}{1 + \Gamma_L e^{-2\alpha z'} - \Gamma_L e^{-2\alpha z'} e^{j\phi(z')}}
\]

\[
\phi(z') = \theta_L - 2\beta z'
\]

Smith Chart with Lossy T-Lines

\[
|\Gamma(z')| = |\Gamma_L| e^{-2\alpha z'}
\]

As \(z' \to \infty\)

\[
Z_{in} \to 1
\]

\[
(Z_{in} \to Z_0)
\]
### Approach

1. Assume lossless line and rotate by $-2\beta z'$
2. Change $|\Gamma|$ by factor $e^{-2\alpha z'}$

\[
\Gamma_{\text{lossy}}^{\text{in}} = \Gamma_{\text{lossless}}^{\text{in}} e^{-2\alpha z'}
\]

### Example 7

Given a lossy T-line with characteristic impedance $Z_0 = 100\,\Omega$, length $l = 1.5\,m$ ($l < \lambda/2$), and $Z_{SC}(z' = l) = 40 - j280\,\Omega$.

(a) Determine $\alpha$ and $\beta$.

(b) Determine $Z_{\text{in}}$ for $Z_L = 50 + j50\,\Omega$ an $l = 1.5\,m$.

**Solution using Smith chart**

⇒ see web applet (example 7)