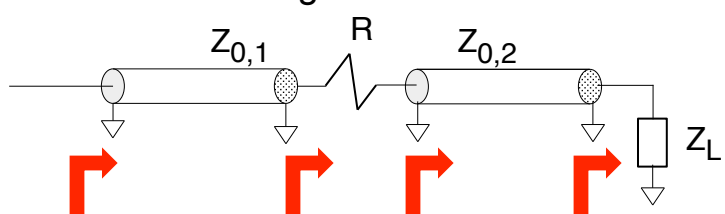
ECE 391

supplemental notes - #11

Adding a Lumped Series Element

Consider the following T-line circuit:



$$z_{in,1} = r_{in,1} + jx_{in,1}$$
$$Z_{in,1} = z_{in,1} Z_{0,1}$$

$$z_{in,2} = r_{in,2} + jx_{in,2}$$

$$z_L = \frac{Z_L}{Z_{0,2}}$$

$$z_{L,1} = r_{in,2}\frac{Z_{0,2}}{Z_{0,1}} + \frac{R}{Z_{0,1}} + jx_{in,2}\frac{Z_{0,2}}{Z_{0,1}}$$

renormalization

➔ **All calculations can be done on Smith chart**

Smith Chart as Admittance Chart

For circuits involving parallel (shunt) connections of circuit elements, it is more convenient to work with admittances.

How can the Smith chart be used with admittances?

➔How is Y=1/Z = G + jB related to reflection coefficient Γ ?

Consider normalized admittance y

$$\mathrm{y} = \frac{Y}{Y_0} = \frac{G + jB}{Y_0} = g + jb = Y\,Z_0 = \frac{Z_0}{Z} = \frac{1}{\mathrm{z}} = \frac{1}{r + jx}$$

Smith Chart as Admittance Chart

Reflection coefficient Γ

$$\Gamma = \frac{\mathrm{z}-1}{\mathrm{z}+1} = \frac{1/\mathrm{y}-1}{1/\mathrm{y}+1} = -\frac{y-1}{y+1}$$

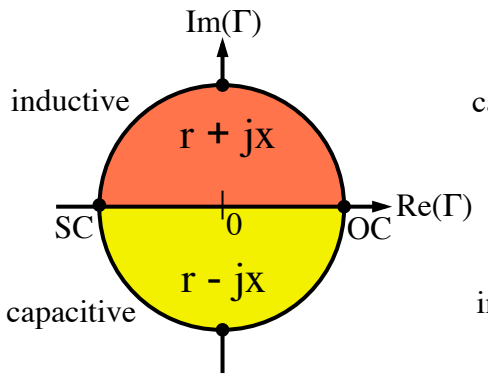
Smith chart can directly be used with y=g+jb after rotating reflection coefficient by 180°.

$$\frac{y-1}{y+1} = -\Gamma = \Gamma\, e^{j\pi}$$

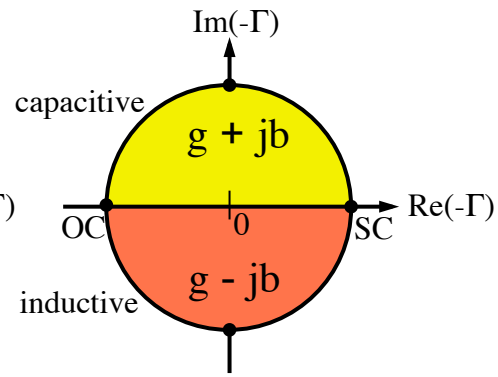
g = const
b = const
circles

Impedance vs. Admittance Coordinates

Impedance Chart



Admittance Chart



Example 4

Given the normalized load admittance

$$y_L = 0.5 + j2.0$$

Determine the normalized admittance at distance

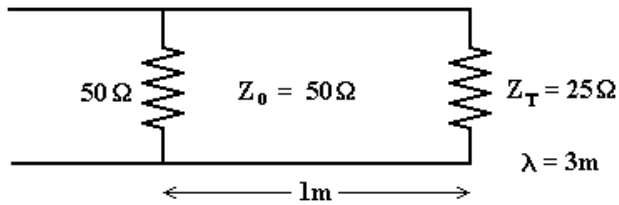
$$d = \lambda/16 = 0.0625\lambda$$

<u>Solution:</u> use Smith chart with admittance coordinates ➔ see web applet (example 4)

Example 5

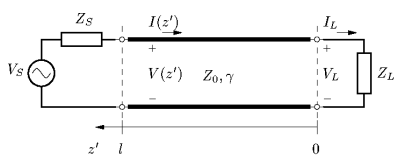
Given a 50Ω T-line that is terminated in $Z_L = Z_T = 25\Omega$ and has a shunt resistance $R_{sh} = 50\Omega$ connected at distance d = 1m from the termination. The wavelength on the line is λ = 3m.

Determine the input impedance at the location of the shunt resistance.

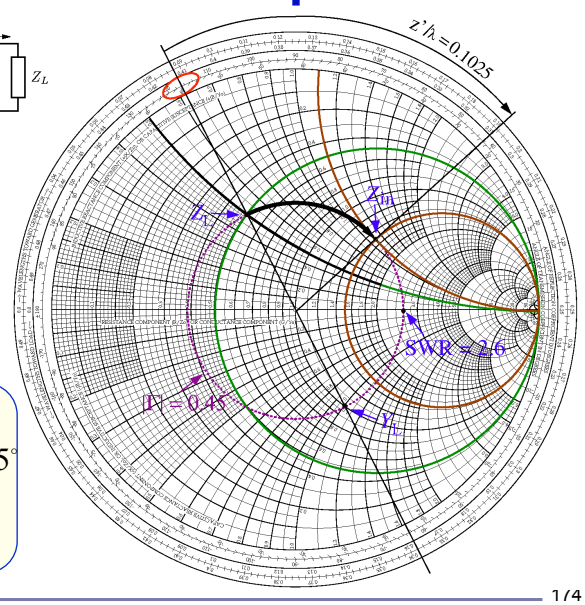


<u>Solution</u>: see web applet, example 5

Additional Example



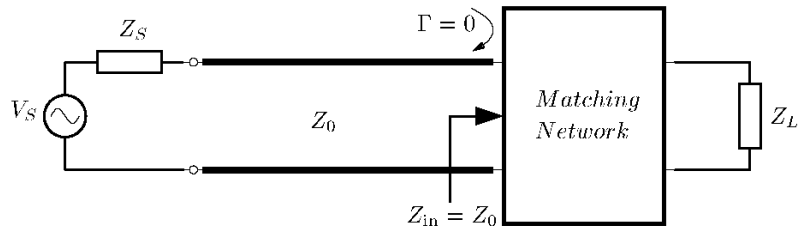
$$Z_L = (25 + j25)\Omega$$
$$Z_0 = 50\Omega$$
$$z_L = 0.5 + j0.5$$
$$l = 0.1025\lambda$$



Results

$$\text{SWR} \approx 2.6$$
$$|\Gamma_L| \approx 0.45 \quad \theta_L \approx 116.5^\circ$$
$$y_L = 1 - j$$
$$z_{in} \approx 1.5 + j1.1$$

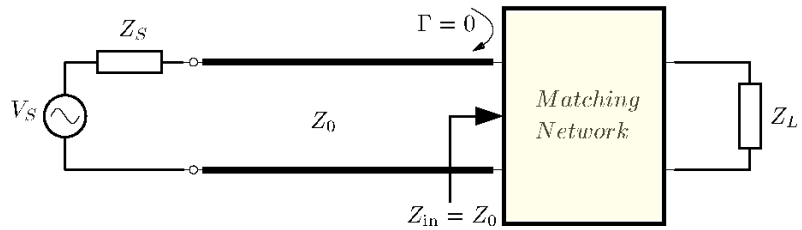
Impedance Matching



Implementation Issues

- Types of Networks
  - lumped elements
  - distributed elements
  - mixed
  - reactive vs. resistive
- Design Criteria
  - bandwidth
  - lossless/lossy
  - complexity
  - parasitics

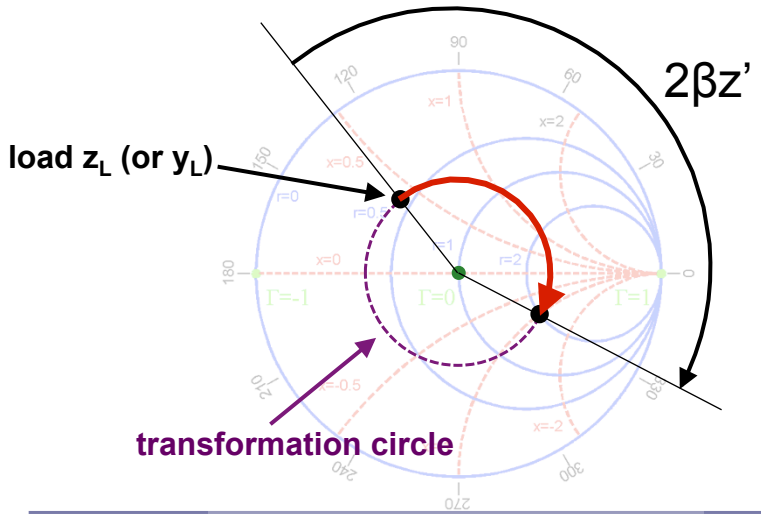
Impedance Matching



➔ Matching network needs to have at least two degrees of freedom (two knobs) to match a complex load impedance

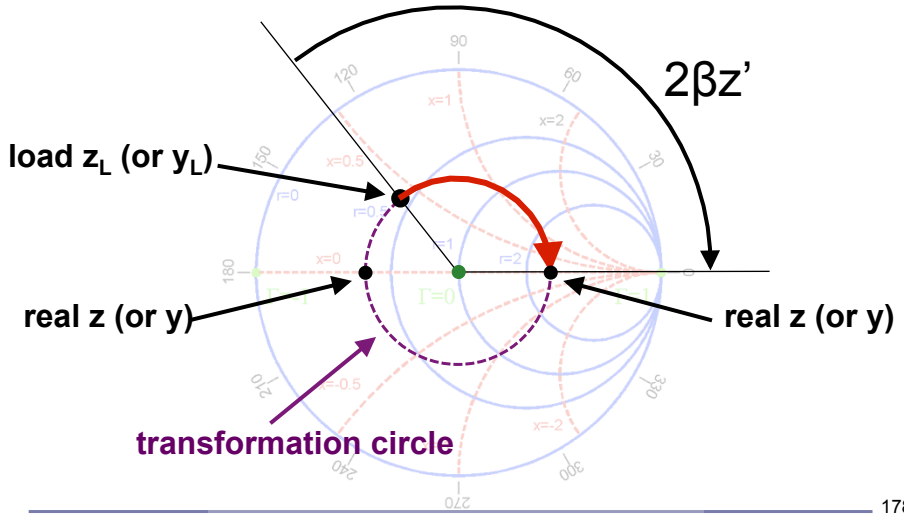
Transmission-Line Matching

Idea: Use impedance transformation property of transmission line



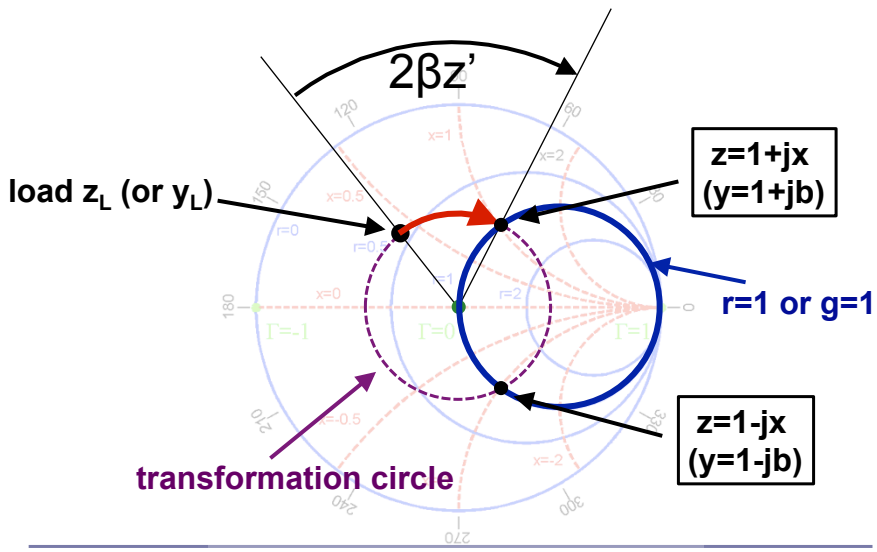
Transmission-Line Matching

Transformation to real z or y



Transmission-Line Matching

Transformation to r=1 (or g=1) circle



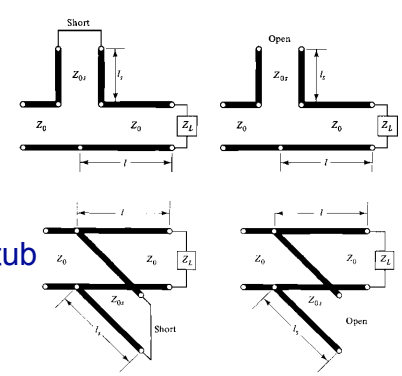
Single Stub Matching

Stub tuners are useful for matching complex load impedances. The two design parameters in single stub matching networks are

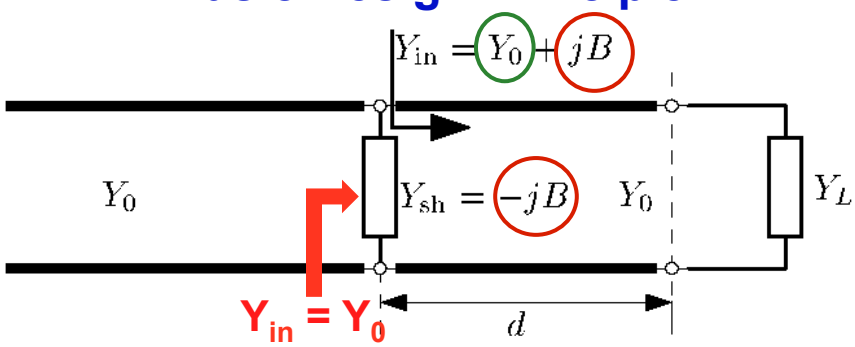
- distance from load at which stub is connected
- length of stub

Other design considerations are

- open or shorted stub
- series or shunt connected stub



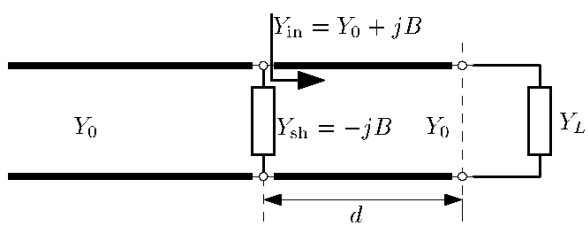
Basic Design Principle



Objective: Match load admittance by

1. transforming along T-line to $Y_{in} = Y_0 + jB$
2. tuning out remaining jB with a parallel shunt element of admittance –jB
3. shunt element can be realized with a T-line stub

Design Steps

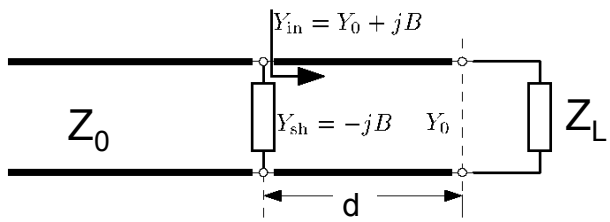


1. Determine **distance d/λ** from the load Y_L (or Z_L) at which the
   - input admittance is Y_0 +jB (for shunt stub matching)
   - input impedance is Z_0+jX (for series stub matching)

2. Realize the stub with a **length l/λ** of open- or short-circuited transmission line (stub).

Example 6

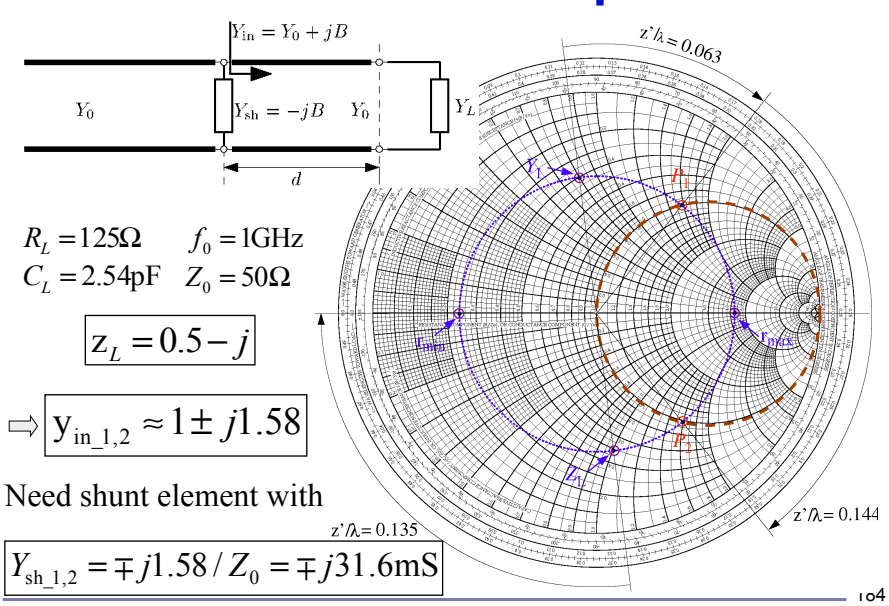
Using a short-circuited stub, match the load impedance Z_L=(80 + j70)Ω to a transmission line with Z_0 = 50Ω.



<u>Solution:</u> use Smith chart with admittance coordinates

➔ see web applet (example 6)

Additional Example



$R_L = 125\Omega \quad f_0 = 1\text{GHz}$

$C_L = 2.54\text{pF} \quad Z_0 = 50\Omega$

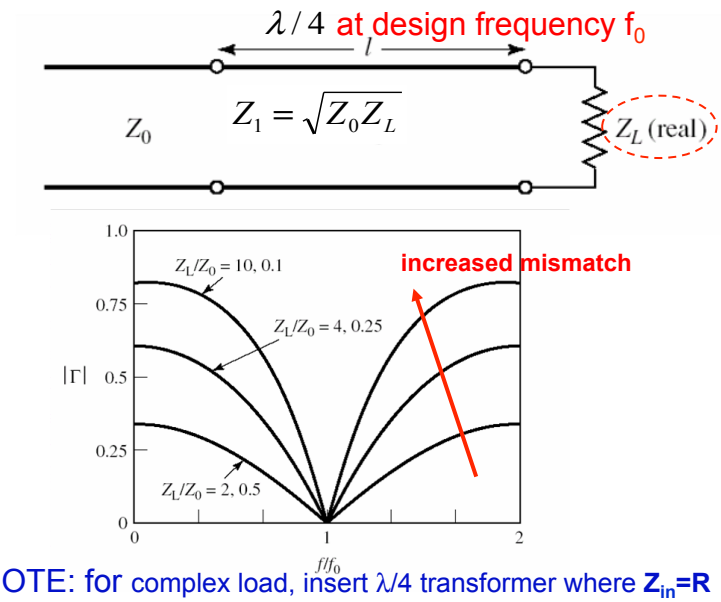
$$\boxed{z_L = 0.5 - j}$$

$$\Rightarrow \boxed{y_{in\_1,2} \approx 1 \pm j1.58}$$

Need shunt element with

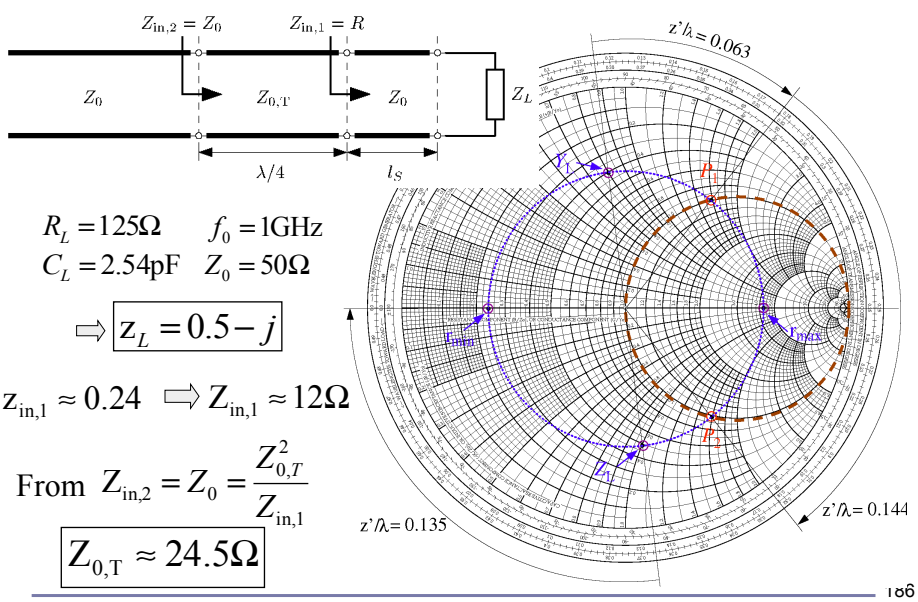
$$\boxed{Y_{sh\_1,2} = \mp j1.58 / Z_0 = \mp j31.6\text{mS}}$$

Quarter-Wave Transformer



NOTE: for complex load, insert λ/4 transformer where $Z_{in}=R$

Example: Quarter-Wave Transformer



$R_L = 125\Omega \quad f_0 = 1\text{GHz}$

$C_L = 2.54\text{pF} \quad Z_0 = 50\Omega$

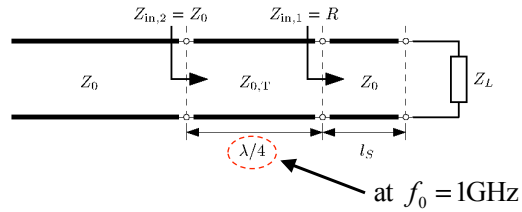
$\Rightarrow \boxed{z_L = 0.5 - j}$

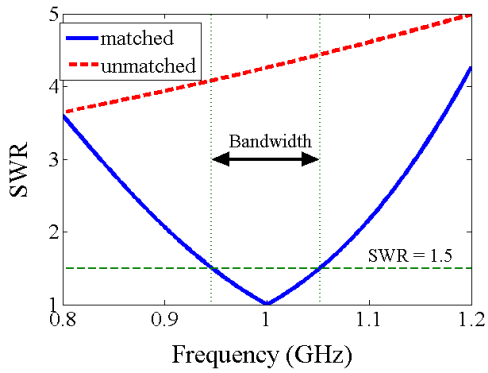
$z_{in,1} \approx 0.24 \Rightarrow Z_{in,1} \approx 12\Omega$

From $Z_{in,2} = Z_0 = \dfrac{Z_{0,T}^2}{Z_{in,1}}$

$\boxed{Z_{0,T} \approx 24.5\Omega}$

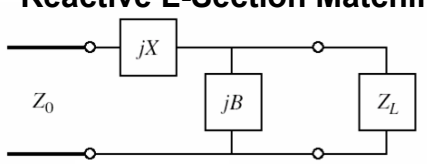
Bandwidth Performance



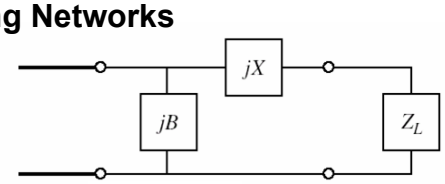


Lumped Element Matching

Reactive L-Section Matching Networks



$R_L > Z_0$



$R_L < Z_0$

$$B = \frac{X_L \pm \sqrt{R_L/Z_0}\sqrt{R_L^2 + X_L^2 - R_L Z_0}}{R_L^2 + X_L^2}$$

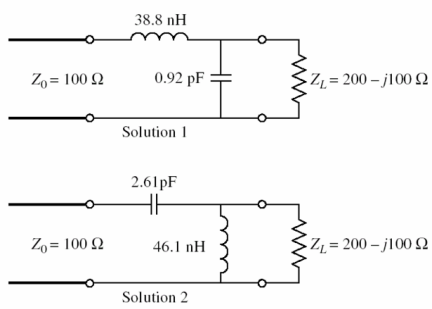
$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L}$$

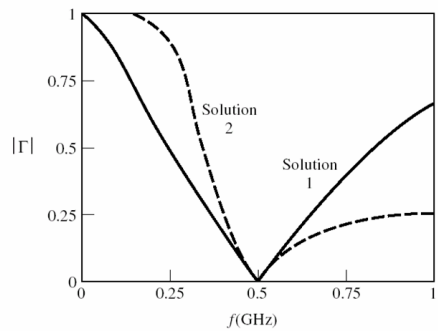
$$B = \pm\frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}$$

$$X = \pm\sqrt{R_L(Z_0 - R_L)} - X_L$$

different solutions ⇨ may result in different bandwidths

Example



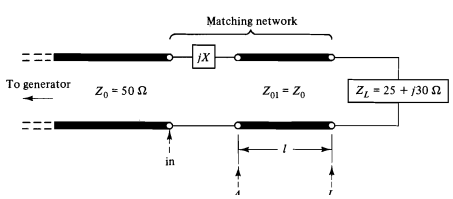


Design criteria

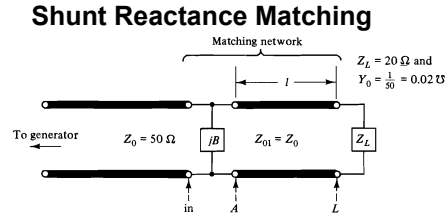
- bandwidth
- realizability of components
- parameter sensitivity
- … cost … ?

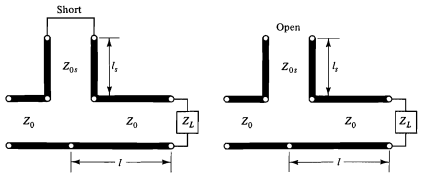
Lumped-Distributed and Distributed Matching Networks

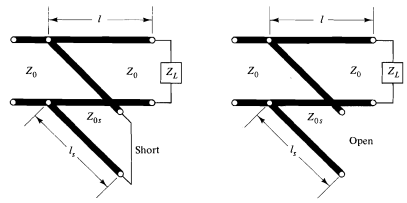
Series Reactance Matching



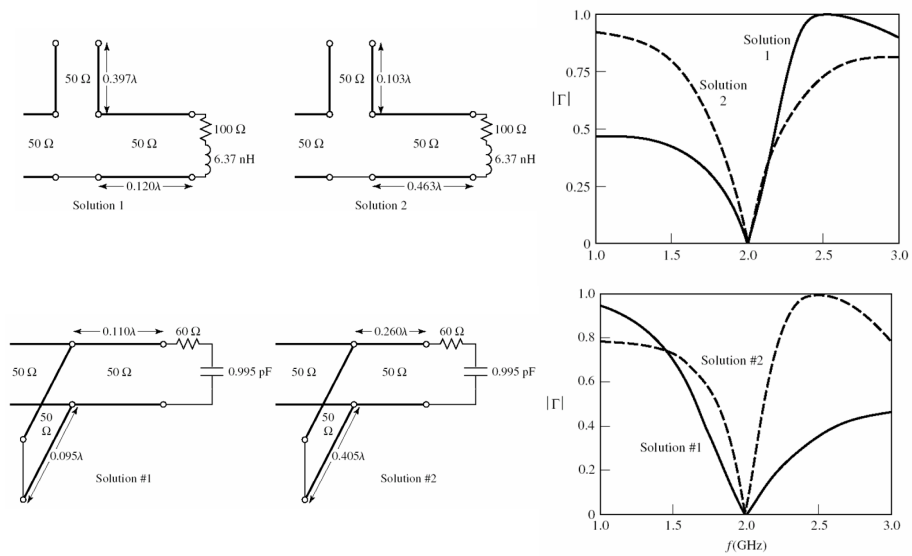
Shunt Reactance Matching





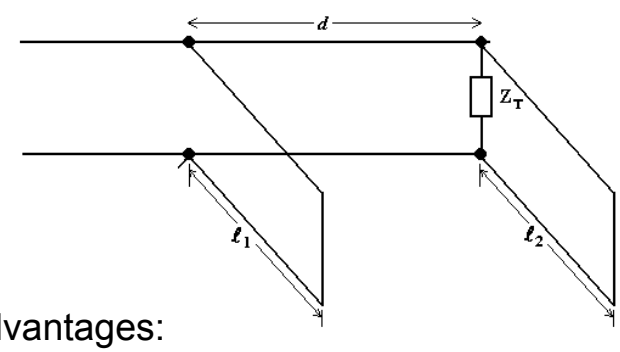


Examples



Double-Stub Tuner

- Double-stub (and multi-stub) tuners use fixed stub locations (typical spacing: λ/8)



- Advantages:
  - easier to accommodate different loads
  - better bandwidth performance

Smith Chart with Lossy T-Lines

Recall for a lossless T-line

$$z_{in}(z') = \frac{Z_{in}(z')}{Z_0} = \frac{1+|\Gamma_L|e^{j\phi(z')}}{1-|\Gamma_L|e^{j\phi(z')}}$$

$$\phi(z') = \theta_L - 2\beta z'$$

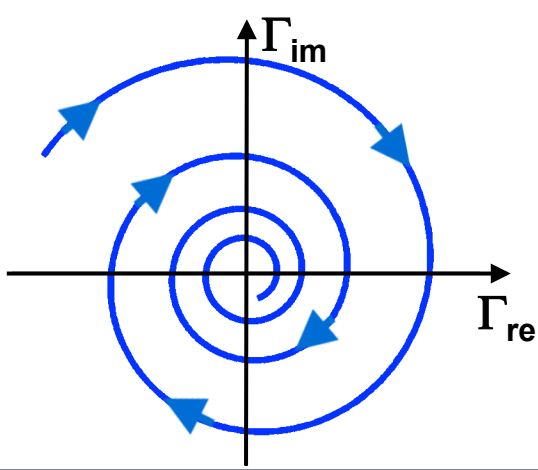
For low-loss T-lines, $Z_0 \approx$ real and $\gamma = \alpha + j\beta$

Then

$$z_{in}(z') = \frac{Z_{in}(z')}{Z_0} = \frac{1+\Gamma_L e^{-2\gamma z'}}{1-\Gamma_L e^{-2\gamma z'}} = \frac{1+|\Gamma_L|e^{-2\alpha z'}e^{j\phi(z')}}{1-|\Gamma_L|e^{-2\alpha z'}e^{j\phi(z')}}$$

Smith Chart with Lossy T-Lines

$$|\Gamma(z')| = |\Gamma_L|e^{-2\alpha z'}$$

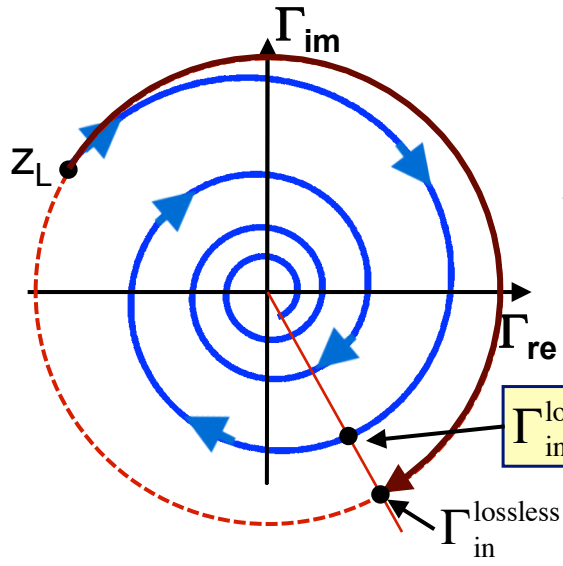


As $z' \to \infty$

$z_{in} \to 1$

$(Z_{in} \to Z_0)$

Approach



1. Assume lossless line and rotate by -2βz'
2. Change |Γ| by factor

$$e^{-2\alpha z'}$$

Example 7

Given a lossy T-line with characteristic impedance Z_0 = 100Ω, length l = 1.5m (l < λ/2), and $Z_{SC}(z'=l)$ = 40 – j280 Ω.

(a) Determine α and β.

(b) Determine Z_{in} for Z_L = 50 + j50 Ω an l = 1.5m.

<u>Solution using Smith chart</u>

➔ see web applet (example 7)