ECE 391: Suggested Homework Problems

Cascaded Lines

1. Two lossless transmission lines are connected in tandem through a series resistance as shown in figure 1.

![Figure 1: Problem 1](image)

(a) Determine the delay time of each line;
(b) determine the reflection and transmission coefficients ($\rho_{11}, \rho_{22}, \rho_{12}, \rho_{21}$) at the junction between the two lines;
(c) draw a lattice diagram and specify the voltage and current of each wave component up to $t = 80\mu\text{sec}$;
(d) plot the voltage across the 10Ω series resistor as a function of time for $0 \leq t \leq 80\mu\text{sec}$;
(e) determine the steady-state voltage across the 36Ω load resistor.
2. Two lossless transmission lines are connected in tandem as shown in figure 2. Initially, assume that the end of line 1 is directly connected to the input of line 2.
(a) Determine the voltage at the junction of the two lines for $0 \leq t \leq 310\text{ns}$;
(b) Design a matching network such that no reflected waves appear at the 150\text{\Omega} terminating resistor. Determine the steady-state voltage across the terminating resistor as well.
3. Two lossless transmission lines with different characteristic impedances are connected through a network consisting of three resistors. The delay times of the lines are unknown. Line two is terminated in an unknown resistance $R_t$.

![Diagram](image)

**Figure 3: Problem 3**

At time $t = 0$, a 12V battery is connected to the input of line 1 through an unknown resistor $R_g$. The voltages at the input of line 1 (A-A'), and line 2, (B-B') are observed on an oscilloscope as seen in figure 4 below.

![Graph](image)

**Figure 4: Problem 3**
(a) Determine the flight times $t_{f1}$, and $t_{f2}$.
(b) Determine the generator resistance $R_g$.
(c) Determine the reflection coefficient $\rho_{11}$ at the junction between the two lines.
(d) Determine the transmission coefficient $\rho_{21}$ at the junctions between the two lines.
(e) Determine the reflection coefficient $\rho_{22}$ and the transmission coefficient $\rho_{12}$ at the junction between the two lines.
(f) Determine the unknown terminating resistance $R_t$.
(g) Determine the steady-state voltages (i.e. for $t \to \infty$) at the input of the first line (A-A’) and at the input to the second line (B-B’).

4. Two transmission lines with different characteristic impedances are to be connected as shown.
   a) You have one variable resistor ($0 \leq R \leq 200$) with two terminals available. Determine the value of R and how you would connect R to the lines order to have a reflectionless system.
   b) Assume now that the value of $R_t$ at the end of the second line is changed such that reflections are generated at the end of the line. An additional resistor will be required at the boundary between the lines to maintain a reflectionless system. Determine the connection and values of a two resistor connection that will maintain a reflectionless system.

![Diagram of transmission lines with different characteristic impedances connected.](image)
5. For the cascaded line given below, find:
   (a) $\rho_S$
   (b) $\rho_L$
   (c) $\rho_{11}$
   (d) $\rho_{21}$
   (e) $\rho_{12}$
   (f) $\rho_{22}$
   (g) Plot the voltage at "A" and "B" for $t = 0$ to $t = 12\text{ns}$ on the graph below.

![Diagram of the cascaded line with labels and voltages](image)

Figure 6: Problem 5
6. Two lossless transmission lines are connected through a matching network at their junction. Draw the voltages at the input terminal (a1) of T1 and the output terminal of T2 (b2).

(a) Incident wave at input to T1:
(b) \(\rho_S\)
(c) \(\rho_L\)
(d) \(\rho_{11}\)
(e) \(\rho_{21}\)
(f) \(\rho_{22}\)
(g) \(\rho_{12}\)

(h) On the graph below, draw the voltages at the input terminal (a1) of T1 and the voltages at the output terminal (b2) of T2. Fill in the X and Y axis values.

(i) [4] Determine the steady-state voltages \((t = \infty)\) at the input to the first line \((a1 \text{ and } a1')\) and the output of the second line \((b2 \text{ and } b2')\).