transforming a capacitor to an inductor with a ¼ line

Smith Chart

Example at 150 MHz + 50 Ohm line:

$\sqrt{1.44} = 1.2 \Rightarrow X_L = 2\pi f L; L = \frac{22}{2\pi (150 \times 10^6)} = 76.4 \text{H}$

$\sqrt{0.7} = 0.8 \Rightarrow X_C = \frac{1}{2\pi f C}; C = \frac{1}{2\pi (150 \times 10^6) (35)} = 30.3 \text{pF}$
To obtain more gain, antennas are often used in "phased arrays". As such, one 50Ω coax must be able to drive multiple antennas.

The quarter wave matching section can do this basically,

2 parallel sections of 75Ω cable is equivalent to one 37.5Ω coax cable

& A simpler, easier to construct implementation

Quarter-wave matching sections are formed by rotating \( \frac{\pi}{4} \) from the pure resistance line back to itself.
System impedance = 75 ohms
Xform 50 → 112 with
75 Ω ¼ wave.
Problem: Match non-reactive load to 50Ω with 7/4 line @ 100MHz.

Load = 40 - 60j (Zc = 0.8 - 1.2j)

Smith Chart

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Smith Chart

\( z_0 = \frac{1}{4} \times 60 \)

50 \( z_0 = \frac{20}{10} \)

26.45

14.2

\( z_0 = \frac{1}{4} \times 180 \)

0

50 \( z_0 = \frac{20}{10} \)

26.45

14.2

\( z_0 = \frac{1}{4} \times 180 \)

0

50 \( z_0 = \frac{20}{10} \)

26.45

14.2

Smith chart solution not shown

\( z = 1.89 \)

2 = 5.29
Matching reactive load w/ \( \lambda/4 \) line (100 MHz)

\[
Z_0 \to Z_m \to Z_0 \\
Z_m = 50 \Omega \quad Z_m = 26.45 \Omega \quad \lambda = 0.158 \\
Z_m = 94.86 \Omega \quad \lambda = 0.408
\]

At this point, for \( \lambda = 0.158 \), real part is 14 \( \Omega \)

\( \lambda = 0.408 \), real part is 180 \( \Omega \)

\[\theta = \frac{1}{2\pi \cdot 100 \times 10^6} \]

\[C = \frac{1}{2\pi \cdot 60 \cdot 100 \times 10^6} \]

\[C = 26.53 \, \text{pF} \, @ \, 100 \, \text{MHz} \]

For \( Z_m = 26.45 \Omega \), use 2 50 \( \Omega \) coax cables in parallel (RG58)

For \( Z_m = 94.86 \Omega \), use 1 92 \( \Omega \) coax cable (RG62)
Quarter-wave matching can be extended to multiple sections.

Matching 75Ω to 300Ω:

\[ Z_{in} = \frac{Z_m}{Z_L} \]

\[ Z_m = \sqrt{Z_L \cdot Z_{in}} \]

- **300Ω**
  - 150Ω, \( R_L = 75 \)
  - **\( \frac{1}{4} \)**
  - 150Ω
  - **\( \frac{1}{4} \)**
  - 105Ω
  - **\( \frac{1}{4} \)**
  - 105Ω
  - **\( \frac{1}{4} \)**
  - 95Ω

\[ Z_{m1} = \sqrt{(300)(105)} = 237.5Ω \]

\[ Z_{m2} = \sqrt{(105)(95)} = 105Ω \]

\[ Z_{m3} = \sqrt{(95)(75)} = 75Ω \]

(This method quickly becomes impractical for coplanar lines. (150Ω is pretty much impractical for PCBs too!)

Advantage is a wider bandwidth, or which matching can be accomplished.

See: Klopfenstein taper, Exponential Taper T-lines

https://microwaves101.com/encyclopedias/klopfenstein
matching resistive 75 ohm load with 300 ohms via one taper (quarter wave matching)

\[ \frac{50 \cdot \text{vm(t1_in)}}{\text{vm(vin,t1_in)}} \]

\[ \text{Hz} \]

- 3 dB from 94-107 MHz
- (13 MHz BW)

matching resistive 75 ohm load with 300 ohms with 3 taper sections (quarter wave matching)

\[ \frac{50 \cdot \text{vm(t1_in)}}{\text{vm(vin,t1_in)}} \]

\[ \text{Hz} \]

- 3.5 dB from 85-110 MHz
- (25 MHz BW)

\[ \text{NOTE Z SCALE IS DIFFERENT} \]
A Klopfenstein tapered line. Y scale is exaggerated so the taper is clearly visible.

Thanks to microwaves101.com