Analysis in the Steady State (A quick review of some basics and a justification for use of phasors)

Sinusoidal sources produce both a transient response and a steady state response.

Transient responses die out with time and leave only the steady state response.

When the transient response becomes negligibly small, we say that a system is operating at sinusoidal steady state.

Sinusoids may be conveniently expressed as phasors. A phasor is a complex number that represents the amplitude and phase of a sinusoid. A phasor represents a time-varying sinusoidal waveform by a fixed complex number.

In RF circuits, we are most interested in delivering power to a load, which is often reactive by nature.
Analysis in the Steady State

Phasor representation is based on Euler's identity

\[ e^{jx} = \cos x + j\sin x \quad e^{-jx} = \cos x - j\sin x \]  
(also, \( e^{j\pi} = 1 \) or \( e^{-j\pi} = -1 \) \{the most beautiful equation\}

We can consider \( \cos x \) as the real part of \( e^{jx} \): \( \cos x = \text{Re} \{ e^{jx} \} \)

and, \( \sin x \) as the imaginary part: \( \sin x = \text{Im} \{ e^{jx} \} \)

time domain  \hspace{1cm} \text{magnitude}  \hspace{1cm} \text{arbitrary reference phase}

Given we have a signal: \( v(t) = V_m \cos(\omega t + \phi) \) then, since \( \cos x = \text{Re} \{ e^{jx} \} \):

\[ v(t) = \text{Re} \{ V_m e^{j(\omega t + \phi)} \} \]

\[ = \text{Re} \{ V_m e^{j\phi} e^{j\omega t} \} \]

\[ = \text{Re} \{ V_m e^{j\phi} V e^{j\omega t} \} \text{ where } V = V_m e^{j\phi} \]

\( V \) is the phasor representation of \( v(t) \)

The phasor "\( V \)" captures the amplitude and phase of a sinusoid but does not provide information about its frequency.

We are assuming a linear circuit where \( \omega \) does not change and no new frequencies are created. This is a definition of a linear system.

The angular frequency \( \omega \) is assumed to be known and is the only frequency present.

We are just interested in phase and magnitude, not how fast the vector is rotating (\( \omega = \text{Angular velocity} \)).

So now, we focus on \( V = Ve^{j\phi} \)
Analysis in the Steady State

For a sinusoidal wave on a transmission line we must keep track of both time and space. From before, (we)

\[ V(z,t) = \cos(2\pi ft - 2\pi \frac{z}{V} + \phi) \]

Let \( R = \frac{Z}{X} \)

Now, allow scaling by a magnitude \( V_m \)

\[ V(z,t) = V_m \cos (wt - \beta z + \phi) \]

\[ \Rightarrow V(z,t) = \text{Re} \{ V_m e^{-j(\beta z - \phi)} e^{jwt} \} \]

And the voltage phasor is: \( V(z) = V_m e^{-j(\beta z - \phi)} \)

- We only need concern ourselves with magnitude and phase for our solutions.
- There is time domain behavior present on a T-Line, but it's of relatively no interest.
- DC circuits had dimensions of only magnitude.
- Lumped AC circuits have dimensions of magnitude and phase (linear, steady state).
- Transmission lines have dimensions of magnitude, phase, and distance.
- We can use phasors alone because of linearity.