Transmission Line Junctions

Sometimes we have a joining of transmission lines of two different $Z_0$'s.

\[
\begin{align*}
\text{incident wave} & \rightarrow V_{1+}^+ & \rightarrow V_{1,2} & \rightarrow \text{transmitted wave} \\
\text{reflected wave} & \leftarrow V_{1-}^- & \end{align*}
\]

- the junction between the lines is so short to be of no consequence.
- It's drawn this way to illustrate the presence of a point discontinuity.

Unlike the previous cases of an endpoint discontinuity, here we have a discontinuity at a midpoint location.

At the moment the incident wave arrives at the junction, if there is a discontinuity, three waves will exist simultaneously; namely, the incident, reflected and transmitted waves.

At the junction: \[\text{transmitted wave} = \text{incident wave} + \text{reflected wave}\]
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Let's expand upon our reflection coefficient notation to accommodate not only reflected waves but transmitted ones.

The notation $P_{ij}$ refers to a wave going into region $i$ from region $j$, or

$P_{i \to j}$ origin

$P_{j \to i}$ destination

$P_{21} \Rightarrow$ wave is going into line 2 from line 1

$P_{12} \Rightarrow$ wave is going into line 1 from line 2

$P_{ii} + P_{zz}$ are similar to $P_s + P_L$ we saw before ($\frac{\text{Reflected}}{\text{Incident}}$)

$P_{12} + P_{21}$ are the portion of the incident wave that is transmitted ($\frac{\text{Transmitted}}{\text{Incident}}$)

Think of $P_{21}, P_{12}$ as a forward reflection coefficient, or more intuitively, a transmission coefficient.
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Let's develop $P_{21} + P_{12}$:

\[ V_{+1}^+ + V_{-1}^- = V_{1,2}^- \]
\[ V_{+1} + P_{11} V_{+1} = V_{1,2}^- \]
\[ V_{+2}^+ (1 + P_{11}) = V_{1,2}^- \]

\[ P_{21} = 1 + P_{11} \text{, or} \]
\[ = 1 + \left( \frac{Z_{2,2} - Z_{2,1}}{Z_{2,2} + Z_{2,1}} \right) \]

These results for $P_{21} + P_{12}$ are true for simple, line-to-line junctions.
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Simple Example:

\[ P_s = 0 \]

\[ P_L = 0 \]

\[ P_{11} = \frac{75 - 50}{75 + 50} = 0.2 \]

\[ P_{21} = 1 + \frac{75 - 50}{75 + 50} = 1.2 \]

\[ P_{22} : \text{who cares, } P_L = 0! \]

Load at the end of the line received a signal of greater amplitude than was sent.
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Now, let's consider this:

We may insert a resistive network between the two lines to eliminate some reflections or reduce them in a digital environment. What about an RF environment?
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Another example:

\[ P_L = \frac{40-75}{40+75} = -0.304 \]

\[ P_{11} : \text{what is the equivalent circuit to a wave entering the junction?} \]

\[ P_{11} = \frac{(25+75)-50}{25+75+50} = 0.333 \]

Here, \( P_{21} \neq (1+P_{11}) \) as this is not a simple junction; it has a resistor across the junction. We have to redefine \( P_{21} \) for this case.

To junction \( L \) as before, \( P_{21} = (1+P_{11}) \), but this is followed by a resistive voltage divider.

\[ V_1^+ (1+P_{11}) \quad \rightarrow \quad \text{Junction L} \quad \rightarrow \quad \text{Junction R} \quad \rightarrow \quad V_1^+ P_{21} \]

so, the transmitted waveform is scaled by the voltage divider:

\[ V_1^+ P_{21} = V_1^+ (1+P_{11}) \left( \frac{75}{75+25} \right) \]

\[ P_{21} = (1+P_{11}) \cdot 0.75 \]

\[ P_{21} = 0.998 \quad \text{scaling caused by voltage division} \]
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Likewise, $P_{22}:

\[ z_{0,2} = 75 \Omega \]

\[ P_{22} = \frac{75 - 75}{75 + 75} = 0 \]

\[ V_{1,2} P_{12} = (1 + P_{22}) V_{1,2} \left( \frac{50}{50 + 25} \right) \]

\[ P_{12} = (1 + P_{22}) \left( \frac{50}{50 + 25} \right) = 1.667 \]