Transmission Line Parameters & Characteristics

- A T-line will be considered to be a two conductor structure of uniform cross sectional dimensions; AKA, "controlled impedance" line. A uniform cross section gives a uniform characteristic impedance.

- The T-lines cross sectional dimensions must be << the wavelength of the signals it carries. (contrast w/waveguides)

- The T-line may be of any reasonable length. Its length does not affect its characteristic impedance, \( Z_0 \).

Cross-sectional Dimensions

\[ D \ll \lambda \]

A fixed cross-sectional area yields a controlled impedance

\[ L \ll \lambda \]

Lengths vary from fractions of a wavelength to many wavelengths
T-line Parameters and Characteristics

Our T-lines are built from conductor pairs:

\[ V_{sc} \]

We know that any wire carrying current exhibits an inductance. Also, two conductors separated by an insulator and charged by an electric field exhibit a capacitance.

Therefore, it is reasonable to model our T-lines as follows:

\[ V_s \]

The basic T-line model is an infinite series of simple networks.

But for a finite length line we can build the same model by specifying each network to represent an infinitesimally short section of the T-line. Actually, as long as each section is short enough to act as a lumped model at the frequency of interest, model accuracy will be maintained.
A 4n$, 80$Ω$ 20$, lumped element transmission line

\[ \begin{align*}
\text{input} & \quad 12.5 \, \text{pf} \quad 12.5 \, \text{pf} \quad 12.5 \, \text{pf} \quad 12.5 \, \text{pf} \\
\text{output} & \quad \\
\text{delay} & = 4n$
T-line Parameters and Characteristics

\[ R : \text{represents the resistance of the unit length conductor} \]
\[ G : \text{represents the leakage conductance of the dielectric (insulator) per unit length} \]
\[ L, C \text{ are the per unit length inductance and capacitance} \]
\[ \Delta z \text{ is the incremental length of our model} \]

Note: here R, L, G, C represent total values per section

6.4.1 Single Lossy Transmission Line (TXL)

General form:

YYYYYYYY N1 0 N2 0 mname <LEN=LENGTH>

Example:

Y1 1 2 0 ymod LEN=2
.MODEL ymod txl R=12.45 L=8.972e-9 G=0 C=0.468e-12 length=16

n1 and n2 are the nodes of the two ports. The optional instance parameter 1en is the length of the line and may be expressed in multiples of [unit]. Typically unit is given in meters. 1en will override the model parameter length for the specific instance only.

The TXL model takes a number of parameters:

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Units/Type</th>
<th>Default</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>resistance/length</td>
<td>( \Omega/\text{unit} )</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>L</td>
<td>inductance/length</td>
<td>( H/\text{unit} )</td>
<td>0.0</td>
<td>9.13e-9</td>
</tr>
<tr>
<td>G</td>
<td>conductance/length</td>
<td>( \text{mho/\text{unit}} )</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C</td>
<td>capacitance/length</td>
<td>( F/\text{unit} )</td>
<td>0.0</td>
<td>3.65e-12</td>
</tr>
<tr>
<td>LENGTH</td>
<td>length of line</td>
<td>( \text{unit} )</td>
<td>no default</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Model parameter length must be specified as a multiple of unit. Typically unit is given in [m]. For transient simulation only.
T-line Parameters and Characteristics

For many applications we can assume a lossless T-line model:

\[ \Delta z \]

A lossless model

6.1 Lossless Transmission Lines

General form:

\[ TXXXXXX N1 N2 N3 N4 Z0=VALUE <TD=VALUE> <F=FREQ <NL=NRMLLEN> >\]

+ <IC=V1, I1, V2, I2>

Examples:

T1 1 0 2 0 Z0=50 TD=10NS

\[ \text{netlist:} \]

\[ Vp \text{ vin } \phi \text{ PULSE(0 1.0 2e-9 1n 1n 20n 40n)} \]

\[ R_s \text{ vin } a \ 50 \]

\[ T1 \ a \phi \ b \phi \ Z0=50 \ TD=2ns \]

\[ R_L \ b \phi \ 100 \]
**T-Line Parameters & Characteristics**

Applying KVL & KCL to our lossless model:

**KVL:** 
\[-V(z,t) + L \frac{\partial i(z,t)}{\partial t} + V(z+\Delta z,t) = 0\]

\[V(z+\Delta z,t) - V(z,t) = -L \frac{\partial i(z,t)}{\partial t}\]

\[-\frac{\partial V(z,t)}{\partial z} = L \frac{\partial i(z,t)}{\partial t}\]

**KCL at center node:**
\[i(z,t) - C \Delta z \frac{\partial V(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0\]

\[-i(z+\Delta z,t) + i(z,t) = C \Delta z \frac{\partial V(z+\Delta z,t)}{\partial t}\]

\[-\frac{\partial i(z,t)}{\partial z} = C \frac{\partial V(z,t)}{\partial t}\]

\[\text{Remember: } \frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} ; \text{ Let } \Delta z \to 0 \Rightarrow \frac{\partial V(z,t)}{\partial z}\]

The current drawn by the line is related to the time rate of change of voltage across the capacitor.

The "Transmission Line Equations" or "Telegrapher's Equations" (lossless case)
T-Line Parameters and Characteristics

The telegrapher's equations:

\[ \frac{\partial V(z,t)}{\partial z} = L \frac{\partial i(z,t)}{\partial t} + \frac{\partial i(z,t)}{\partial z} = C \frac{\partial V(z,t)}{\partial t} \]

Can be combined to yield an equation entirely in terms of only \(i\) or \(V\).

To eliminate \(i(z,t)\) from the equations, differentiate (1) with respect to \(z\), and (2) with respect to \(t\).

\[ \frac{\partial^2 V(z,t)}{\partial z^2} = L \frac{\partial^2 i(z,t)}{\partial z \partial t} \quad \text{and} \quad \frac{\partial^2 i(z,t)}{\partial z \partial t} = C \frac{\partial V(z,t)}{\partial t^2} \]

Now, substituting for \(\frac{\partial^2 i(z,t)}{\partial z \partial t}\):

\[ \frac{\partial^2 V(z,t)}{\partial z^2} = L \left[ \frac{\partial^2 V(z,t)}{\partial t^2} \right] \]; this is a one-dimensional wave equation

A general solution for this equation is:

\[ V(z,t) = V^+(z-v pt) + V^-(z+v pt) \]

\[ = V^+ (t-\frac{z}{v p}) + V^- (t+\frac{z}{v p}) \]

\[ \text{wave going} \quad \text{wave going} \]

\[ +z \rightarrow \quad -z \leftarrow \]

the notation \(V^+\) refers to a voltage wavefront propagating towards the right.

\(V^-\) refers to a voltage wavefront propagating from right to left.
T-Line Parameters and Characteristics

\[ v(z,t) = v^+(z-v_p t) + v^-(z+v_p t) = v^+(t-z/v_p) + v^-(t+z/v_p) \]

Imagine riding a wave on a surfboard and observing its shape.

Looking at the wave at two different times, we see it is moving in the \( z^+ \) direction, and is maintaining its shape as time increases.

There is no change in the waveform shape over time or distance with an ideal or lossless T-line. Constant shape indicates that all frequency components are delayed identically. This T-line has \( \infty \) bandwidth.

Imagine two observers, one 200yd from shore, one 50yd from shore observing the wave's shape.
T-Line Parameters & Characteristics

There is an "apparent contradiction" between our T-line model and the idea of infinite bandwidth.

Question: What type of filter is this?

![Diagram]

Question: Isn't this our model for an infinite bandwidth T-line?

How can both be true?