Standing Waves

Sometimes when we are attempting to determine how well we are delivering power to the load, we are able to determine the distance from the load to the first minimum of the voltage standing wave pattern. How could we find the phase angle of the reflection coefficient from this?

Earlier we saw that:

\[ V(z') = V_0^+ e^{i\beta z'} + V_0^- |\Gamma| e^{-i\beta z'} \]

And since \( |\Gamma| = |\Gamma| e^{i\Theta} \):

\[ = V_0^+ e^{i\beta z'} + V_0^- |\Gamma| e^{i\Theta} e^{-i\beta z'} \]

\[ = V_0^+ e^{i\beta z'} \left[ 1 + |\Gamma| e^{i\Theta} e^{-2i\beta z'} \right] \]

\[ = V_0^+ e^{i\beta z'} \left[ 1 + |\Gamma| e^{i(\Theta - 2\beta z')} \right] \]

This could also be considered as \( \Gamma(z) \), a generalized voltage reflection coefficient that can be found at any point \( z \) on the line.

Important points:

1) Magnitude of the reflection coefficient is unchanged across a lossless line.
2) The phase of the reflection coefficient changes at a rate of \( 2\pi \) the electrical distance from the termination.

It also follows that:

\[ I(z') = \frac{V_0^+}{Z_0} e^{i\beta z'} \left[ 1 - |\Gamma| e^{i(\Theta - 2\beta z')} \right] \]

We are interested in how \( |V(z')| \) varies with distance from the load, especially looking for minimums or maximums.
$z_{\text{min}} + z_{\text{max}}$ of Standing Waves

From before:

$V(z') = V_0 e^{j\beta z'} \left[ 1 + |\beta| e^{j(\theta - 2\beta z')} \right]$

- We would like to have the standing wave plot $|V(z')|$ and find its maximum.
- The magnitude of $V(z')$, $|V(z')|$, is $|V(z')| = \left[ V(z') V^*(z') \right]^{1/2}$
  where $V^*(z')$ is the complex conjugate of $V(z')$. We find that:

$|V(z')| = |V_0| \sqrt{\left[ (1 + |\beta|)^2 \cos^2(\beta z' - \frac{\theta}{2}) + (1 - |\beta|)^2 \sin^2(\beta z' - \frac{\theta}{2}) \right]}$

Maximize this term to find the maximum of $|V(z')|$

- The maximum will be found where the argument of the $\cos^2$ function is equal to $n\pi$.

- So, we can say the maximum standing wave voltages are found from the condition:

$\beta z'_{\text{max}} - \frac{\theta}{2} = n\pi$ ; $n = 0, 1, 2, 3, ...$

Also $2\beta z_{\text{max}} - \theta = 2n\pi$

- The voltage minimums will be found away or when:

$\beta z'_{\text{min}} - \frac{\theta}{2} = \frac{\pi}{2} + n\pi$ $n = 0, 1, 2$
Z_{\text{min}} + Z_{\text{max}} \text{ of Standing Waves}

Being able to find a voltage max or min, including the first one measured from the load,

1. The wavelength of the signal (\lambda) on the line since points of \( V_{\text{max}} / V_{\text{min}} \) occur at \( \lambda / 2 \) intervals. This yields \( \beta \) then as \( \beta = \frac{2 \pi}{\lambda} \)

2. IF we know \( \lambda \), and thus \( \beta \), and have a relationship such as

\[
Z'_{\text{max}} = \frac{\theta + 2n\pi}{2\beta}
\]

we can find \( \theta \) the phase angle of \( \Gamma \).

Then, if we know the SWR, which is related to \( |\Gamma| \), we can fully characterize a transmission line terminated in an arbitrary load impedance.

\[
\text{SWR} = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \Rightarrow \text{gives } |\Gamma|
\]

\[
Z'_{\text{max}} = \frac{\theta + 2n\pi}{2\beta} \Rightarrow \text{gives } e^{i\theta}
\]

\[
Z'_{\text{min}} = \frac{\theta + (2n+1)\pi}{2\beta} \Rightarrow |\Gamma| e^{i\theta} = \Gamma
\]
Calculate $\Gamma_L$, $|\Gamma_L|$ 

$V_{\text{max}}$

$V_{\text{min}}$

$\text{SWR}$

Sketch $|V(z')|$ as function of $\frac{z'}{\lambda}$

$$\Gamma_L = \frac{Z_L - 50}{Z_L + 50} = \frac{37.5}{62.5} = -0.6 \quad \text{on} \quad 0.6 e^{j180^\circ}$$

So $|\Gamma_L| = 0.6$, $\theta_L = 180^\circ$ or $\pi$.

$$V_{\text{max}} = |V^+| (1 + |\Gamma_L|) = 1 (1 + 0.6) = 1.6V$$

$$V_{\text{min}} = |V^+| (1 - |\Gamma_L|) = 1 (1 - 0.6) = 0.4V$$

So $|V(z')|$ is maximized when $\beta z'_{\text{max}} = \frac{\theta_L}{2} = n\pi$.

Since $\beta = \frac{2\pi}{\lambda}$, and $\theta_L = \pi$, and let $n = 0$.

$$Z_{\text{max}} = \frac{\theta_L}{2} \frac{2\pi}{2\pi} = \frac{\pi}{2} \cdot \frac{\lambda}{2\pi} = \frac{\lambda}{4}$$
An s.w.r. measurement on a line reveals an s.w.r. \( s = \frac{V_{\text{max}}}{V_{\text{min}}} = 5 \).

Distance between voltage minima is 20 cm.

Distance from termination to nearest voltage minima is 4 cm.

What is the load impedance \( Z_L \)?

\[
|Z_L| = \frac{\text{s.w.r.-1}}{\text{s.w.r.}+1} = \frac{5-1}{5+1} = \frac{4}{6} = 0.66
\]

20 cm between voltage minima is \( \frac{\lambda}{2} \); thus \( \lambda = 40 \) cm.

Distance from termination to minima (as function of \( \lambda \)) is \( \frac{4\lambda}{40} = 0.1 \lambda \).

First voltage maxima will be at \( \lambda/4 \) from the minimum at \( 0.35 \lambda \).

At the first maxima \( \beta \times z_{\text{max}} = \frac{\Theta_L}{2} \) (n=0)

\[
\frac{2\pi}{\lambda} 0.35\lambda = \frac{\Theta_L}{2}
\]

\( \Theta_L = 1.47 \pi \) or \( 252^{\circ} \); so \( Z_L = 0.66 e^{j252^{\circ}} \)

\( = -0.206 - j0.634 \)

\[
Z_L = Z_0 \left( \frac{1 + Z_0}{1 - Z_0} \right) = Z_0 \left( \frac{0.794 - j0.634}{1.206 + j0.634} \right) = Z_0 \left( \frac{1.016 e^{j(38.6^{\circ})}}{1.362 e^{j(27.9^{\circ})}} \right) = Z_0 \left( 0.795 e^{j(66.33^{\circ})} \right)
\]

\( = Z_0 (1.299 + j0.683) \)
Standing Wave Patterns for Open, Short and Resistive Terminations

\[ \frac{|V(z)|}{|V_0|} = \begin{cases} V_{\text{max}} = 2V_0^+ & \text{for } V_{\text{min}} = 0 \\ V_{\text{min}} = 0 & \text{for } V_{\text{max}} = 2V_0^+ \end{cases} \]

Currents:
\[ |I(z)| = \begin{cases} V(z) = 2V_0^+ \cos(\beta z') \\ I(z) = \frac{2V_0^+}{Z_0} j \sin(\beta z') \end{cases} \]

\[ Z_0 \]

\[ \Gamma_L = \begin{cases} 1 & \text{Open Circuit} \\ -1 & \text{Short Circuit} \\ \Gamma > 0 & \text{Resistive Load} \\ \Gamma < 0 & \text{Resistive Load} \end{cases} \]
Standing wave patterns for inductive \((+jx)\) and capacitive \((-jx)\) loads.