ECE 391 - Midterm 1a - Spring 2016

Name and last 4 of ID: ____________________________________________

1. [10] A plumber and electrician decide to start a side business making 96Ω coaxial transmission lines. Their first product uses an outer conductor made from copper water pipe with an inside diameter of 0.5 inch. The inner conductor is made from solid copper wire with an outside diameter of 0.1 inch. They will use air as a dielectric.

2 a) Initially, they do not have enough 0.5 inch diameter water pipe. Having not taken ECE391, the plumber thought it would be fine to substitute copper water pipe with an inside diameter 0.625 inch. How will the current and voltage of an incident wave traveling down this transmission line change? Why? (give two reasons) [2]

The inner outside diameter of the outer conductor will decrease the parallel plate capacitance and slightly raise the per unit length inductance, thus Zo will go up. Thus the incident wave voltage will be higher & the incident wave current will be less.

2 b) The electrician thought they could save money and weight by using an inner conductor of hollow tubing with the same outside diameter of the solid copper wire. If they continue to use an outer conductor of 0.625 inch diameter copper pipe, how will the characteristic impedance of this new cable be different from the cable with the solid center conductor? Why?

The Zo of the new cable with hollow center conductor will be the same. This is because the capacitance between inner & outer conductors will not have changed. [2]

6 c) To help stabilize the inner conductor mechanically and to make use of all the Styrofoam packing peanuts they had accumulated, the plumber and electrician decide to replace the air dielectric with the peanuts. This changes the per unit length capacitance to 50pf/meter, and the Zo to 92Ω. For the packing peanut enhanced line, determine the lines:

i) inductance per meter [2]
ii) its velocity of propagation [2]
iii) its εr [2]

\[ L = \frac{1}{\sqrt{2}\varepsilon} \]
\[ L = \frac{1}{2.17\times10^8(50\times10^{12})} \]
\[ L = 429.7\text{mH} \]

\[ V = \frac{c}{\sqrt{\varepsilon_r}} \]
\[ V = \frac{2.17\times10^8}{\sqrt{1.9}} \]
\[ V = 2.17 \times 10^8 \text{m/s} \]

\[ \varepsilon_r = 1.9 \]
2. [12]

For the T-line below, different waveforms are shown as seen at the input to the line. For each waveform, circle the correct circuit parameters.

All waveforms are viewed from here.

- a) \( V_s > Z_0 \) AND \( R_L > Z_0 \)
- b) \( V_s < Z_0 \) AND \( R_L = \infty \)
- c) \( V_s = Z_0 \) AND \( R_L = 0 \)
- d) \( V_s < Z_0 \) AND \( R_L < Z_0 \)
- e) \( V_s > Z_0 \) AND \( R_L = 0 \)
- f) none of the above

(a)

\[ V_A \]

1.0
-0.8
-0.6
-0.4
-0.2

0 2 4 6 8 ns

well over 0.5 V

(a) \( R_s > Z_0 \) AND \( R_L > Z_0 \)
- b) \( Z_0 < R_s < R_L \)
- c) \( Z_0 < R_L < R_s \)
- d) \( R_s < Z_0 < R_L \)
- e) None of the Above
- f) None of the Above

(b)

\[ V_A \]

1.0
-0.8
-0.6
-0.4
-0.2

0 2 4 6 8 ns

less than 0.5 V

- a) \( R_s > Z_0 \)
- b) \( Z_0 < R_s < R_L \)
- c) \( Z_0 < R_L < R_s \)
- d) \( R_s < Z_0 < R_L \)
- e) None of the Above

(c)

\[ V_A \]

-0.05
-0.1
-0.15
-0.2
-0.25
-0.3

0 2 4 6 ns

less than 0.5

- a) \( R_s < \frac{Z_0}{R_L} \)
- b) \( R_s > \frac{Z_0}{R_L} \)
- c) \( R_s = 0 \) AND \( R_L = Z_0 \)
- d) \( R_s = Z_0 \) AND \( R_L = 0 \)
- e) None of the Above
An engineer started the lattice diagram shown above but being interrupted by an important Facebook posting, never finishing his work. Given the information above, find:

a) \( R_L = \frac{150}{25} \)

b) \( R_L = \frac{75}{25} \)

c) \( t_D = 4 \text{ns} \)

d) \( R_S = 25 \)

e) \( V_S = 3.0 \)

\[
Z_0 = \frac{2.25}{10.3} = 75 \Omega \\
R_L = \frac{1.7425}{2.25} = 0.73 = \frac{R_L - Z_0}{R_L + Z_0} \\
3.3 R_L + 3.3 Z_0 = R_L - Z_0 \\
-1.67 R_L = -1.33 (75) \\
R_L = 150 \\
R_S = \frac{-0.3713}{17425} = -0.5 \\
-VS = \frac{R_S - Z_0}{150 + 25} \Rightarrow R_S = 25 \Omega \\
V_{ac} = V_S \left( \frac{R_S}{R_S + Z_0} \right) \\
2.25 = V_S \left( \frac{25}{25 + 75} \right) \Rightarrow V_S = 3.0 \text{V}
\]
1. Consider the circuit below:

\[ \begin{align*}
\text{\( R = 113 \, \Omega \)} & \quad \text{\( L = 400 \, \text{H} \)} \\
\text{\( V_I = 1 \, \mu \text{A} \)} & \\
\text{\( V_0 \)} & \\
\text{\( t = 7.5 \)} & \\
\text{\( t = 2 \pi \)}
\end{align*} \]

a) On the graph below, plot \( V_O \) for \( t = 0 \) to \( t = 10 \mu \text{s} \). Clearly label voltages.

b) Indicate on your plot the voltage and time at:

i) \( t = T \)

ii) \( t = 3T \)

Hint: Note that \( V_O \) is not the voltage across the inductor but across both inductor and resistor. So, \( V_O = V_L + V_{RL} \).

**Equivalent circuit:**

\[ \frac{3 \pi}{2} \begin{array}{c}
\frac{3}{\pi} \quad 400 \, \text{H} \\
\frac{3}{113 \, \Omega}
\end{array} \rightarrow \begin{array}{c}
\gamma = \frac{1}{R} = \frac{400 \times 10^3}{113 \, \Omega} = 2.17 \, \mu \text{s} \\
\beta = 6.4 \, \mu \text{s}
\end{array} \]

**Final voltage at \( V_O \) is a voltage divider,** \( \left( \frac{113}{113+5} \right) V = 0.601 \, V \)

**Waveform will open circuit at the inductor & have full-sized positive reflection followed by decay as current builds in the inductor.**

\[ V_0 (t) = \left[ V_0 (\infty) + \left( V_0 (t) - V_0 (\infty) \right) e^{-t/T} \right] e^{-(t-t_d)/\beta} \]

\[ V_0 (t) = 0.601 + \left[ 1 - 0.601 \right] e^{-t/t_d} \]

\[ V_0 (t) = 0.601 + 0.399 e^{-t/2} \]

\[ e^{-T} = \frac{V_0 (T)}{0.601} = 0.747 \]

\[ e^{-3T} = \frac{V_0 (3T)}{0.601} = 0.621 \]
2. Consider the cascaded line below:

\[ V_0 = 50 \Omega, \ t_0 = 2 \text{ ns} \]

\[ Z_0 = 75 \Omega, \ t_d = 2 \text{ ns} \]

Find:

(a) \( p_2 = 0 \)
(b) \( p_L = 0.33 \)
(c) \( p_{11} = 0.037 \)
(d) \( p_{21} = 0.667 \)
(e) \( p_{12} = 0.444 \)
(f) \( p_{22} = 0 \)

(g) Plot voltage at (a) and (b) for \( t = 0 \) to \( t = 12 \text{ ns} \)

\[ V_{A1}, V_{B2} \]

\[ V_{A1} = \begin{cases} 0.57 \text{ V} & \text{if } t = 0 \\ 0.52 \text{ V} & \text{if } t = 3 \text{ ns} \\ 0.44 \text{ V} & \text{if } t = 6 \text{ ns} \end{cases} \]

\[ V_{B2} = \begin{cases} 0 & \text{if } t = 0 \\ 0.57 \text{ V} & \text{if } t = 3 \text{ ns} \\ 0.44 \text{ V} & \text{if } t = 6 \text{ ns} \end{cases} \]

\[ V_{11} = \left[ 0.5 \times (0.667) \right] + \left[ 0.667 \times 0.5 \times 0.33 \right] = 0.444 \text{ V at } t = 3 \text{ ns} \]

Voltage at (b) remains at this voltage since \( p_{22} = 0 \) and \( p_2 = 0 \)

\[ V_{11} = 0.444 \text{ V at } t = 6 \text{ ns} \]

\[ V_{21} = \left[ 0.5 \times (0.667) \right] = 0.333 \text{ V at } t = 9 \text{ ns} \]

\[ V_{22} = 0 \text{ V at } t = 12 \text{ ns} \]

\[ P_{21} = \left( 1 + p_{11} \right) \left( V_{\text{Division}} \right) = \left( 1 + 0.037 \right) \left( \frac{75^2 + 41.6}{75} \right) = \left( 1.037 \right) \left( 1.693 \right) = 1.667 \text{ V at } t = 3 \text{ ns} \]

\[ P_{12} = \left( 1 + p_{22} \right) \left( V_{\text{Division}} \right) = \left( 1 + 0.444 \right) \left( \frac{100 \times 50}{100 + 50} \right) = 0.499 \text{ V at } t = 6 \text{ ns} \]

\[ V_{\text{Division}} = \left( 1.667 \times 0.037 \right) \frac{41.6}{75} + 0.444 \]

\[ V_{\text{Division}} = 0.333 + 0.444 = 0.777 \text{ V at } t = 9 \text{ ns} \]

\[ V_{\text{Division}} = 0 \text{ V at } t = 12 \text{ ns} \]

At 4 ns, (a) sees the small positive reflection from the network: 0.5 \times 0.33 = 0.165

So the total voltage goes up to 0.15 + 0.165 = 0.315 (52)

At 8 ns (a) will see the positive reflection from the load \( V_{21} \) which will be reduced by the network:

\[ (0.667 \times 0.33) \frac{41.6}{75} = 0.049 \text{ V} \]

The total voltage is then

\[ 0.52 + 0.049 = 0.57 \]
1. [20] A receiver operating at 451Mhz is being interfered upon by a transmitter operating at 461MHz. While no changes can be made directly to the receiver or antenna, the coaxial transmission line between antenna and receiver is accessible. In particular, a coaxial "tee" fitting is accessible directly where the coax is attached to the receiver.

   a) Design the filter using coax cable, 50Ω, \( \varepsilon_r = 2.3 \), that will drastically reduce the 461MHz signal without affecting 451Mhz signal. Show by drawing/schematic diagram how your filter is connected and constructed. Be sure to include coax length and open or short circuits.

   b) Design another filter using lumped elements that will drastically reduce the 461MHz signal without affecting the 451MHz signal. Show by drawing/schematic diagram how your filter is connected and constructed. Give the specific element values.

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Diagram:

- Antenna
- "Tee" connector
- Receiver input
- \( T_1 \)

Calculation:

- \( \lambda = \frac{c}{\varepsilon_r f} = \frac{3 	imes 10^8}{2.3 \times 461 \times 10^6} = 429 \text{ metres} \)
- \( \lambda/4 \) at 461Mhz = 107 metres

Series resonant network with short center pin to ground:

- \( f_0 = \frac{1}{2\pi \sqrt{LC}} \)
- \( 461 \times 10^6 = \frac{1}{2\pi \sqrt{10 \times 10^{-12} \times L}} \)
- \( 2.897 \times 10^{-2} = \frac{1}{10 \times 10^{-12} \times L} \)
- \( \sqrt{10 \times 10^{-12} \times L} = 3.452 \times 10^{-10} \)
- \( L = 11.9 \text{nH} \)
2. [6] A 50Ω transmission line which is \(\frac{1}{4}\) long at 50MHz is driven by a 100MHz generator. What input impedance does the 100MHz generator experience if the line is terminated with 150Ω?

\[
\frac{3\text{ min}}{\frac{1}{4}\text{ long} \cdot 50\text{ MHz}} \quad \text{at} \quad 100\text{ MHz}
\]

Line is \(\frac{1}{2}\) c 100MHz \(\Rightarrow Z_{in} = 2.4 \quad Z_{in} = 150\Omega\)

3. [28] Consider the circuit below.

A source terminated 2\(V_{\text{rms}}\) AC source drives a 75 Ω transmission line with a loss \(\alpha = 0.1 \text{ dB/m}\).

The end of the transmission line is terminated in a 6dB (voltage gain) amplifier that has a input impedance, \(Z_{in} = 75\Omega\). The output of this amplifier drives a 75Ω resistive load. Determine:

a) The RMS voltage at point B
b) The RMS voltage at point C
c) The power delivered to the 75Ω load in dBm.
d) The overall voltage gain (or loss) from point A to point C

\(\text{G} = 20\log_{10}(1.123) = 1.00\text{dB gain}\)
4. [46] For the circuit below, find:
   a) \(|\Gamma_L| = 1.4472\)
   b) \(\Gamma_L = 1.4472 e^{-116.5^\circ}\)
   c) \(\Theta_L = -116.5^\circ\)
   d) \(V_{\text{max}} = 1.9472\)
   e) \(V_{\text{min}} = 0.5528\)
   f) \(V_{\text{SWR}} = 2.62\)
   g) The location in meters to the first (as measured from the load) voltage minimum is 2.64 m.

\[
\begin{align*}
\Gamma_L &= \frac{2e^{-2\alpha}}{2e^{-2\alpha} + 1} = \frac{25 - j25 - 50}{25 - j25 + 50} = 0.4472 e^{-2.034i} \\
\Theta_L &= -116.5^\circ \\
V_{\text{max}} &= V_0 \left(1 + |\Gamma_L|\right) = 0.7386 \text{ V} \\
V_{\text{min}} &= V_0 \left(1 - |\Gamma_L|\right) = 0.2764 \text{ V}
\end{align*}
\]

\[
V_{\text{max}} = \beta z' - \frac{\Theta_0}{2} = n\pi \alpha \quad (\text{let } n=0)
\]

\[
\beta z' - \frac{\Theta_0}{2} = 0
\]

\[
\frac{2\pi}{\lambda} z' = \frac{\Theta_0}{2} ; \quad \lambda = 10 \text{ MHz}, \quad \epsilon_r = 1 \quad \lambda = \frac{\lambda}{\epsilon_r} = 30 \text{ m}
\]

\[
\frac{2\pi}{30} z' = -58.25, \pi
\]

\[
\frac{z'}{30} = 0.1618
\]

\[
2' = -4.854 \text{ m} \quad (\text{from 10 MHz}) \quad (V_{\text{max}})
\]

\[
7.5 \text{ m} + 4.854 \text{ m} = 12.354 \text{ m}
\]