how to use the Smith Chart

A discussion of the Smith chart with examples of its use in transmission-line problems

Although articles on the Smith chart have appeared in the amateur magazines from time to time, amateurs have made little use of this handy transmission-line calculator — probably because it has been difficult to measure complex impedances with simple homebuilt equipment. However, this problem has been solved with the simple impedance bridge described by W2CTK — at least for the high-frequency range.1 With careful attention to lead dress and component layout this instrument should be usable on six and two meters as well.

A quick glance at the Smith chart suggests a formidable array of curved lines and circles that would cause the most hardened technician to go into fits of despair. On the other hand, if you spend a little time with the chart and look at each of its component parts, it’s not really very complicated. Perhaps the one thing that scares many prospective users is its unfamiliar circular shape; it’s not at all like the straight-line graphs you’re accustomed to. However, when you understand the chart and have mastered its use you’ll be able to solve complex impedance and transmission-line problems much easier and faster than ever before.

layout of the chart

The Smith chart is basically a circle which contains various circular scales. The horizontal line through the center marked resistance component is the only straight line on the chart and is called the “axis of reals” (see fig. 1). Constant resistance circles are centered on the axis of reals, tangent to the rim of the chart at the infinite resistance point. All the points along a constant-resistance circle have the same resistive value as the point where it crosses the axis of reals.

Superimposed over the resistance-circle pattern are portions of other circles tangent to the axis of reals at the infinite resistance point, but centered off the edge of the chart (fig. 2). The large outer rim of the chart is calibrated in relative reactance and is called the “reactance axis.” Any point along the same constant-reactance circle has the same reactive value as the point where it intersects the reactance axis on the rim of the chart. All points on the Smith chart above the axis of reals contain an inductive-reactive component and those below the axis of reals contain a capacitive-reactive component. Since the calibration points go from zero to infinity, any complex impedance can be plotted on the chart.

The impedance coordinates on the Smith chart would be of little use without the accompanying peripheral scales (fig. 3). These scales relate to quantities which change with position along a transmission line. Two scales are calibrated in terms of wavelength along the transmission line: one, in a clockwise direction, is “wavelengths toward generator,” and the other, counter-clockwise, is “wavelengths toward load.” The entire length of the circumference of the chart represents one-half wavelength.

By James R. Fisk, W1HR, (reprinted from the November, 1970, issue of ham radio)
normalized numbers

Normalized values must be used when plotting impedances on the Smith chart. Normalized impedance is defined as the actual impedance divided by the characteristic impedance of the transmission line. Normalizing is done to make the chart applicable to transmission lines of any and all possible values of characteristic impedance. For example, a 50-ohm coaxial transmission has a normalized value of 50/50 or 1. On this basis an impedance of 120 ohms would have a normalized value of 120/50 = 2.4 ohms. Similarly, \( z = 0.8 \) ohms (the lower case indicates a normalized value) would correspond to a value of 0.8 times the characteristic impedance of the line or \( 0.8 \times 50 = 40 \) ohms.

What has been said about coaxial cable with regard to normalized impedance applies equally to waveguide, where a characteristic impedance of 400 ohms at a specific frequency would be considered unity in normalized form. All other values would be related to this value, so that a 560-ohm component would have the value \( 560/400 = 1.4 \) ohms in normalized terminology, while \( z = 0.9 \) in normalized form would actually be \( 0.9 \times 400 = 360 \) ohms.

plotting values on the chart

Any complex impedance, regardless of value, may be plotted on the Smith chart. For example, assume the load on a 50-ohm transmission line is 42.5 - j31.5 ohms. This is equal to 0.85 - j0.63 when normalized. To plot this point on the chart, locate 0.85 on the axis of reals and note the corresponding constant-resistance circle (fig. 4). Next locate 0.63 on the periphery of the chart. The quantity \(-j\) indicates a capacitive-reactive component so the value 0.63 is on the lower half of the chart. Note the constant-reactance circle representing \(-j0.63\). The complex impedance 0.85 - j0.63 is at the intersection of the constant-resistance and constant-reactance circles.

Draw a line from the center of the chart through this point to the outer rim. With the point 1.0 on the axis of reals as the center, scribe a circle that intersects the impedance point. This circle is known as the “constant-gamma circle,” and its radius is equal to the coefficient of reflection. The constant-gamma circle crosses the axis of reals at two points; the point of intersection to the right of center is the standing wave ratio (2.0 in this case).

If the voltage were measured at this point on the transmission line, it would be found to be a maximum. Conversely, the point of intersection one-quarter wavelength away on the left-hand axis of reals is a point of voltage minimum (this point is also equal mathematically to the reciprocal of the swr).

The point at the intersection of the radial line and the angle of reflection coefficient scale represents the phase of the coefficient of reflection. This is the angle by which the reflected wave leads or lags the incident wave. When these two waves add in phase to give maximum voltage, the impedance is resistive and greater than the characteristic impedance of the line and the angle of the coefficient of reflection is zero.

As you move away from the zero-phase-angle point in a clockwise direction toward the generator, the reflected voltage lags the incident voltage, and the phase angle is negative for the first quarter wavelength. The reactive component of the impedance in this region is negative or capacitive.

At the quarter-wavelength (90°) point the incident and reflected waves are out of phase and the angle of the coefficient of reflection is ±180°. As you continue in a clockwise direction the two waves become

*Since 50-ohm systems are standard for military and industrial use, 50-ohm Smith charts are available. On a 50-ohm Smith chart the center point has a value of 50 ohms.
circle on the radial scale labeled “standing wave voltage ratio.” The value of swr in dB may also be determined from this scale.

\[ \text{swr}_{\text{dB}} = 16.1 \, \text{dB} \]

**Example 2.** Finding the reflection coefficient (ρ) and angle of the reflection coefficient (θ) for voltage and current. A 50-ohm transmission line is terminated with a load impedance 65 – j75 ohms. What is the reflection coefficient and angle of reflection coefficient? (See fig. 6).

1. Normalize the load impedance

\[ \frac{65 - j75}{50} = 1.3 - j1.5 \]

2. Locate this point on the chart and draw a line from the center of the chart through it to the outer scale.

3. Construct a constant-gamma circle.

4. The reflection coefficient may be calculated by measuring the radii of the constant-gamma circle and the Smith chart to its first periphery and by computing their ratio. Smith-chart radius = 57/16 inch; constant-gamma radius = 32/16 inch.

\[ \rho = \frac{32}{16} + \frac{57}{16} = 0.56 \]

5. The coefficient of reflection may also be found on the radial nomograph. Simply mark the radius of the constant-gamma circle on the scale labeled “reflection coefficient of voltage.” The constant-gamma radius intersects the radial scale at 0.56. The “reflection coefficient of power” may also be determined from this same scale at 0.314.
6. The angle of the reflection coefficient is defined by the intersection of the radial line plotted in step 2 and the 'angle of reflection coefficient in degrees' scale on the rim of the chart.

\[ \rho = -46^\circ \]

**Example 3.** Finding input impedance. A 50-ohm transmission line 20 feet long is terminated with \( Z_L = 50 - j50 \) ohms. What is the input impedance at the sending end of the line at 14.1 MHz? (See fig. 7.)

1. Normalize the load impedance

\[ \frac{50 - j50}{50} = 1 - j1 \]

2. Find the length of the transmission line in meters by multiplying by 0.3048.*

\[ 20 \text{ feet} \times 0.3048 = 6.096 \text{ meters} \]

3. Find the electrical length of the transmission line at 14.1 MHz. First, determine the wavelength at 14.1 MHz. Free-space wavelength is found by dividing the speed of light by frequency

\[ \lambda = \frac{3 \times 10^8 \text{ meters per second}}{14.1 \times 10^6 \text{ cycles per second}} = 21.276 \text{ m} \]

Calculate the electrical length of the transmission line

\[ \theta = 360^\circ \left( \frac{6.096 \text{ m}}{21.276 \text{ m}} \right) = 102^\circ = 0.28 \text{ wavelength} \]

4. Plot the impedance coordinates from step 1 on the chart and draw a line from the center of the chart through this point to the outer scale.

5. Draw another line from the chart center to the outer scale at a point 0.28 wavelength clockwise (toward the generator) from the line drawn in step 3. Swing an arc from the center of the chart through \( z_L \) to this line. The intersection is at \( z_L = 0.62 + j0.7 \), the normalized input impedance. To find the actual impedance this value must be multiplied by the line's characteristic impedance

\[ Z_L = 50(0.62 + j0.7) = 31 + j35 \]

**Example 4.** Calculating load admittance. The impedance of a load terminating a 50-ohm transmission line is \( 75 + j82 \) ohms. What is the admittance of the load? (See fig. 8.)

1. Normalize the load impedance

\[ z_L = (75 + j82)/50 = 1.5 + j1.64 \]

2. Plot this point and draw a line through the center to the outer scale on the opposite side of the chart.

3. Swing an arc through \( z_L \) to the line on the opposite side of the chart. The point of intersection denotes the normalized admittance

\[ y_L = 0.305 - j0.33 \]

4. Calculate the actual admittance by multiplying the characteristic admittance of the system times the normalized admittance. The characteristic admittance \( (Y_o) \) is equal to the reciprocal of the character-

*Although all the computations may be made in feet (or inches) the metric equivalents are easier to work with. To convert from inches to centimeters, multiply by 2.54.
istic impedance
\[ Y_o = \frac{I}{Z_o} = \frac{1}{50} = 0.02 \text{ mho} \]

Therefore, the admittance is
\[ Y_L = 0.02 \times (0.305 - j0.33) \]
\[ = 0.0061 - j0.0066 \text{ mho} \]

**Example 5.** Determining the effect of a characteristic impedance change. A 50-ohm transmission line, 0.15 wavelength long, is terminated with 100 - j0 ohms. The 50-ohm line is fed from a 72-ohm line. What is the vswr in the 72-ohm line? (See fig. 9.)

![fig. 8. Calculating load admittance (example 4).](image)

is \( 50(0.68 - j0.48) = 34 - j24 \) ohms. Normalize this value to the 72-ohm line
\[ (34 - j24)/72 = 0.47 - j0.33 \]

5. Plot this point on the chart (fig. 9B) and draw a circle through \( z_A \) to the "axis of reals." The vswr in the 72-ohm line is 2.5:1. The vswr can also be found with the radial nomograph as outlined in example 1.

In the upper vhf region ordinary capacitors and inductors cannot be relied upon to act as pure reactances, and sections of transmission line are often used in their place since any input reactance may be obtained with the proper length of open- or short-circuited line.

![fig. 10. Using a transmission line as a circuit element (example 6).](image)
1. Normalize the desired reactance

\[ z = (+j100)/50 = +j2 \]

2. Since the line is short-circuited,

\[ Z_L = 0 + jo, \text{ and } z_L = 0 \text{ ohms}. \]

3. Plot these two points on the chart and draw lines from the center of the chart through each of them. On the ‘wavelengths toward generator’ scale there is a distance of 0.176 wavelength between the two lines. Therefore, a transmission line 0.176 wavelength long is required for a reactance of +j100. (At 144 MHz, +j100 represents an inductance of 0.11 \( \mu \)H.)

Example 7. Finding matching stub length and location. A 50-ohm transmission line is terminated with a load impedance of 32 + j20 ohms. A matching stub is to be used to provide a match to the line. Both the length of the stub \( l_d \) and its distance from the load \( l_{d0} \) are variable; find \( l_d \) and \( l_{d0} \). (See fig. 11.)

1. Normalize the load impedance

\[ z_L = (32 + j20)/50 = 0.64 + j0.4 \]

2. Locate this point on the chart and draw a line through it and the chart center, extending the line through the peripheral scales in the negative, or bottom, portion at 0.336\( \lambda \) (or \( \theta = -62^\circ \))

3. Construct a constant-gamma circle through \( z_L \), on through the admittance point \( y_L \), and intersecting the unity conductance circle (G = 1) at point A.

4. Draw a line from the chart center through point A to the outer scale at 0.348\( \lambda \) (or \( \theta = -71^\circ \)). \( l_d \), the distance from the load to the stub, is the distance from 0.336 to 0.348.

\[ l_d = (0.348 - 0.336) = 0.012\lambda \]

\[ \theta = 71^\circ - 62^\circ = 9^\circ (4.5 \text{ electrical degrees}) \]

5. To find the length of the stub, determine the amount of susceptance necessary to match out the load. The required susceptance is the difference between the susceptance at point A and the susceptance at the center of the chart. The susceptance at point A is \( -j0.67 \). The required stub susceptance is

\[ B = +j0.67 \]

6. Determine the equivalent stub reactance by taking the reciprocal of the susceptance (as described in example 4).

\[ X = -j1.49 \]

7. Locate the reactance \( -j1.49 \) on the rim of the chart (point B). Determine the distance between the short-circuit point and the required reactance (point

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Example 6. Transmission lines as circuit elements. It is desired to obtain +j100 ohms reactance with a 50-ohm short-circuited transmission line as the circuit element. What length is required? (See fig. 10.)
B) along the "wavelengths toward generator" scale. 
\( l_j = 0.344 \lambda \) \( (\theta = 248^\circ, 124 \) electrical degrees).  

For practical reasons it may not be possible to place a shunt stub only 4.5\(^\circ\) from the load. It may be necessary to increase the distance \( l_D \) to the next point where \( G = J \) (not \( R = 1 \)), represented by point C, fig. 11A. In this case \( l_D \) would be measured, clockwise from 0.336 through 0.50 to 0.151. Using the reflection coefficient scale, from \( \theta = -62^\circ \) to \( 180^\circ \) plus \( 180^\circ \) to \( +71^\circ \), which totals \( 227^\circ \)R, or 113.5 electrical degrees. This represents 0.316\( \lambda \). This will require a \( +jX \) stub, length shown as \( L_j \) (C), of the same numerical reactance value as before.

**Lossy lines**

All the examples shown so far have assumed no attenuation in the transmission line. Since all lines have some loss, this must be considered to find the actual case. However, at many amateur frequencies loss is low enough to be neglected. Nevertheless, at 144 MHz and above, line loss should be considered when using the Smith chart.

Attenuation along a uniform transmission line causes the impedance point to spiral inward toward the center of the chart when moving toward the generator; when moving toward the load the impedance point spirals outward toward the rim of the chart. The rate at which the spiral approaches the center (or the rim) depends upon the attenuation as well as the starting point. Impedance points near the rim are affected more per dB of attenuation than points near the center.

The attenuation effect is easily determined with the scale at the bottom of the Smith chart labeled "transmission loss, 1-dB steps." Since the initial point on this scale must apply to any point on the chart, it is laid out without numerical calibration. The opposite attenuation effects of moving toward the load as opposed to moving toward the generator are indicated by arrows on the scale which show the proper direction to move the corrected impedance point. Thus, to determine the effect of 2-dB attenuation, simply mark off two 1-dB intervals in the proper direction along the scale from the initial starting point before reading the actual impedance coordinates.

**Example 8.** Impedance transformation through a lossy line. A 50-ohm transmission line 24 centimeters long is terminated with 10 – j10 ohms. What is the input impedance to the line at 250 MHz if the attenuation of the line is 2 dB? (See fig. 12.)

1. Normalize the load impedance

\[ Z_L = (10 - j10)/50 = 0.2 - j0.2 \]
2. Find the electrical length of the line at 250 MHz.

\[ \lambda = \frac{300 \times 10^8}{250 \times 10^6} = 120 \text{ cm} \]

The electrical length of the line is

\[ \theta = 360^\circ \left( \frac{24 \text{ cm}}{120 \text{ cm}} \right) = 72^\circ = 0.2 \text{ wavelength} \]

3. Plot the impedance from step 1 on the chart and draw a line from the center of the chart through this point to the outer scale.

4. Draw another line from the chart center to the outer scale at a point 0.2 wavelength clockwise (toward the generator) from the line passing through \( z_L \). Swing an arc through \( z_L \) to this line. The intersection point denotes \( z_i = 0.71 + j1.52 \text{ ohms} \). This is the normalized solution for the lossless case. The rf energy from the generator is attenuated 2.0 dB on reaching \( z_L \) and the voltage reflection coefficient is lower than the lossless case. Since the voltage reflection coefficient varies directly with the power ratio of one-way line attenuation, the reflection coefficient is reduced to

\[ \text{antilog} \frac{2.0 \text{ (dB)}}{10} = 0.631 \]

5. The reflection coefficient \( \rho \) for the lossless case is 0.68 (found on the scale at the bottom of the chart). The actual coefficient of reflection may be calculated by multiplying the lossless coefficient of reflection by the power ratio from step 4.

\[ 0.631 \rho = 0.631 \times 0.68 = 0.429 \]

6. Swing an arc equal to the ratio \( \rho = 0.429 \) so it intersects the line drawn through \( z_L \), the radius of this arc can be found on the "voltage reflection coefficient" scale on the bottom of the chart. The normalized impedance for the lossy case is 1.08 + j1.05. The actual input impedance is

\[ Z_i = 50(1.08 + j1.05) = 54 + 52.5 \text{ ohms} \]

**slotted lines**

At frequencies above 300 MHz conventional impedance-measuring instruments give way to the slotted line. A slotted line is essentially a section of transmission line with a small opening so you can use a probe to measure the voltage along the line. Vswr is easy to determine with the slotted line since it's the ratio of the maximum voltage along the line to the minimum. With the known vswr and position of the first voltage minimum, the impedance of the load can be quickly found with the Smith chart.

**Example 9.** Calculate the load impedance from the vswr and position of the first voltage minimum. A 50-ohm transmission line has a vswr of 2.5; the first voltage minimum is 0.1 wavelength from the load. What is the impedance of the load? (See fig. 13.)

1. Draw a radial line from the center of the chart
Through the 0.1 wavelength mark on the "wavelengths toward load" scale.

2. Find the 2.5 point on the axis of reals and draw a constant-gamma circuit through this point to intersect with the 0.1-wavelength line.

3. Read the coordinates of this intersection to obtain the normalized impedance of the load

$$Z_L = 0.56 - j0.57$$

$$Z_L = 50(0.56 - j0.57) = 28 - j28.5 \text{ ohms}$$

If you use twin-lead or open-wire feedline this technique could be used to determine the impedance of your antenna. However, the voltage probe must be held a uniform distance away from the line for all measurements, and must not be so close that it disturbs the electric field around the conductors.

**expanded smith charts**

The more closely an antenna is matched to a transmission line, the closer the impedance points are to the center of the Smith chart. In a well-designed system the impedance points may be so close to the center of the chart that it’s difficult to work with them. When this happens it’s best to use an expanded Smith chart. Two versions are commonly available: one with a maximum swr of 1.59, the other with a maximum swr of 1.12.

The use of the expanded Smith chart is shown in fig. 14. In fig. 14A the impedance plot of a well-matched 10-meter beam over the low end of the phone band falls very close to the center of the chart. When these same impedance points are plotted on the expanded Smith chart in fig. 14B they are much easier to read and work with.

**where to buy them**

Smith charts can usually be purchased at college bookstores in small quantities, or in larger quantities from Analog Instruments Company or General

*Smith charts from Analog Instruments come in packages of 100 sheets, $4.75 the package. For standard charts order 82-BSPR; expanded charts (maximum swr = 1.59), order 82-SPR; highly expanded (maximum swr = 1.12), order 82-ASPR. Analog Instruments Company, Post Office Box 808, New Providence, New Jersey 07974.

Smith charts from General Radio are available in pads of 50 sheets, $2.00 per pad. For standard charts, normalized coordinates, order 5301-7560; 50-ohm coordinates, order 5301-7569; normalized, expanded coordinates, order 5301-7561. General Radio, West Concord, Massachusetts 01781.
Radio.* If you buy directly from the manufacturer, there's a minimum order quantity, so it might be a good idea to get your radio club to sponsor the purchase.

Another solution is the Smith-chart rubber stamp shown in the photo. This stamp is 10 cm (about 4 inches) in diameter and presents an adequately detailed grid structure for most engineering problems. The rubber surface of these stamps is cast from metal dies, and is dimensionally compensated for rocker-mount ellipticity and shrinkage. The capacity is well over a million impressions so you should never be able to wear it out. The stamps are available in standard (vswr = \infty) or expanded form (vswr = 1.59 or 1.121) from the Analog Instruments Company. Cost is $14.75 each.

If you don't need a permanent record of your Smith chart calculations, the calculator shown in fig. 15 provides rapid answers to complex impedance problems. This calculator is constructed from two laminated plastic discs and a radial arm pivoted at the center with a sliding cursor. This calculator, which is 9-1/2 inches (24 cm) in diameter, is priced at $9.95 and is available from the Ham Radio's Communications Bookstore, Greenville, New Hampshire 03048.

references

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