Linearity Property

(2) A combination of homogeneity and additivity implies

if (Input is multiplied by a constant) the response to a sum of
then (output is multiplied by the same constant) inputs is ...

ex: $v = ir$

if $i_1$ is scaled by $k$, then

$v$ scales by $k$

$k_i = k_i r$

the sum of the responses to each input applied separately.

ex:

\[ \begin{align*}
  v_1 &= i_1 R \\
  v_2 &= i_2 R \\
\end{align*} \]

then

\[ (i_1 + i_2)R = (v_1 + v_2) \]

A circuit is linear if it is both additive and homogeneous.

Linear circuits consist of linear elements and linear
dependent and independent sources.
Superposition

- Voltage across or current through an element is the sum of each independent source acting alone.
- Only works for linear circuits

Method:

1. Turn off all sources except one
   - Leave dependent sources alone + find I or V
2. Repeat #1 for all sources
3. Sum all the contributions due to each source.

* Cannot use this to directly compute power
Solve for $i_x$ using superposition:

1. Kill 10V source

   $i_x + 3 - (V_a - 2i_x) = 0$
   $i_x + 3 - V_a + 2i_x = 0$
   $3i_x - V_a = -3$
   $3i_x + V_a = -3$
   $5i_x = -3$
   $i_x' = -0.6$

2. Open 3A source

   $-10 + 2i_x + 1.1i_x + 2i_x = 0$
   $5i_x = 10$
   $i_x'' = 2$

Sum the individual current components

   $i_x = i_x' + i_x''$
   $= -0.6 + 2$
   $= 1.4A$
Maximum Power Transfer

- Many times we want to maximize power delivered to a load (radio receivers, 50Ω amplifier match).

- Consider a fixed Thévenin equivalent circuit drawing a variable load.

\[
\begin{align*}
V_{TH} & \quad \text{We desire to maximize the power delivered} \\
R_{TH} & \quad \text{to the load } R_L.
\end{align*}
\]

Maximum power is delivered to \( R_L \) when \( R_L = R_{TH} \)
- We say the load is matched to the source.
- In this case \( P_{\text{max}} = \frac{V_{TH}^2}{4R_{TH}} \)

\[
\begin{align*}
10V \pm & \\
\text{1} & I=0.91, P=0.83 \\
\text{2} & I=0.77, P=1.78 \\
\text{3} & I=0.5, P=2.5W \\
\text{4} & I=0.45, P=2.48W \\
\text{5} & I=0.4, P=2.4W \\
\end{align*}
\]

\[
P_{\text{max}} = \frac{10^2}{4(10)} = 2.5W
\]
Practical Problem 4.13

Find $R_L$ that will draw the most power from the circuit. We will need both $R_{TH}$ and $V_{oc}$

To get pure we need $V_{oc}$

$\text{KCL}\begin{align*}
\frac{V_a}{2} - (V_a - 3V_x) + 1 &= 0 \\
-V_a - 2V_a + 6V_x &= -2 \\
-3V_a + 6V_x &= -2 \\
V_x &= -V_a \\
-3V_a + 6(-V_a) &= -2 \\
V_a &= \frac{3}{2} \\
Vin &= \frac{3}{2} + 4 = 4.5 \text{V}
\end{align*}$

Thus, $Vin = V_a + V_{oc}$ or $V_{oc} = \frac{3}{2} + 4 = 4.22 \text{V}$

If $I_m = 1\text{A}$, $R_{TH} = 4.22 \Omega$

$V_{oc}$ will be same as voltage at $C$

$\text{KVL Loop:} -9 + V_x + I_1 + 3V_x = 0$

$\begin{align*}
4V_x + I_1 &= 9 \\
9V_x + V_x &= 9 \\
8V_x &= 18 \\
V_x &= 2 \text{V} \\
V_{oc} &= 7 \text{V}
\end{align*}$

$P_{max} = \frac{V_{oc}^2}{4R_{m}}$

$= \frac{49}{4(4.22)}$

$= 2.9 \text{W}$