Capacitors and Inductors

- Both elements are circuit elements that store energy.
- Ideally do not dissipate any energy.
- They are storage elements.

Capacitors

- Stores energy in an electric field.
- Consists of 2 conducting plates separated by an insulator.
- Schematic symbol: \( \cap \) (also called a "cap").

- When connected to a voltage source, the capacitor stores charge.

\[ q = CV \]

- When disconnected, it holds the charge.
- The amount of charge stored is given by \( q = CV \). 
  - \( C \) is the constant of proportionality, called "capacitance".
  - Units are "farads".

1 Farad = 1 coulomb/volt
Capacitance depends on physical dimensions

Parallel plate cap: \[ Q = \frac{\varepsilon A}{d} \]

- \( A \): Surface area of plates
- \( \varepsilon \): Permittivity of insulator
- \( d \): Distance between plates

- Typical caps are 1-10F ( sensors) - 1-10pF
- May be fixed or variable:  \[ \square \]
- Have polarity or not (electrolytic or tantalum)
  \[ +\square -\]

Capacitor current/voltage relationships:

Find the current through a cap:
we know now that \[ q = C V \]

definition of current \[ i = \frac{dq}{dt} \] thus \[ dq = i \, dt \]

differentiate \[ q = C V \]

\[ dq = dC \, dV \]

\[ i \, dt = dC \, dV \]

\[ i = C \frac{dv}{dt} \text{ (current thru a capacitor)} \]

- Tells us that
1. Current is zero when \( V(t) \) is constant.
   \[ \therefore \text{Cap is open circuit & 0C} \]
2. Voltage on a cap cannot change instantaneously as current would be required to change voltage in two time.
To find the voltage across a cap:

\[ i = C \frac{dv}{dt} \]

\[ dv = \frac{1}{C} i \, dt \]

\[ v(t) = \frac{1}{C} \int_{-\infty}^{t} i \, dt \quad \text{or} \]

\[ v(t) = \frac{1}{C} \int_{t_0}^{t} i \, dt + v(t_0) \quad \text{initial voltage on the cap} \]

- \( v(t) \) term expresses memory, voltage in a time past.
- DRAM, SRAM, DDRAM, all use capacitors for memory.

Instantaneous power delivered to the cap is

\[ P = vi = v \cdot C \frac{dv}{dt} \]

\[ \text{energy} (w) = \int_{-\infty}^{t} P \, dt \]

\[ = C \int_{-\infty}^{t} \left( \frac{v}{C} \right) \, dv \]

\[ = C \left[ \int_{v(-\infty)}^{v(t)} \frac{v}{C} \, dv \right] \]

\[ = \frac{1}{2} C v \left|_{v(-\infty)}^{v(t)} \right. \]

\[ = -\infty, \quad v(-\infty) = 0 \quad \text{because cap was discharged at } t = -\infty, \text{ so...} \]

\[ w = \frac{1}{2} C v^2 \quad \text{or} \quad \frac{q^2}{2d} \quad \text{(Joules)} \]

energy stored in electric field between the plates.
Capacitors in Series + Parallel

- in parallel they add directly

\[ C_{eq} = \frac{1}{C_1 + C_2 + C_3 \ldots C_n} \]

- in series they add as a sum of reciprocals

\[ C_{eq} = \frac{1}{C_1 \cdot C_2 \cdot C_3 \cdot C_n} \]

\[ C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_n}} \]
Inductors
- Stores energy in a magnetic field
- Any conductor has properties of an inductor
- More effective inductors are created by fashioning a conductor into a cylindrical coil
- Often called "coils", choke to

- Schematic symbol

- When connected to a current source the inductor creates a magnetic field
- When disconnected, the magnetic field collapses! How do we take advantage of this?
- When current flows through an inductor, the voltage across it is given by

$$V = L \frac{di}{dt}$$

$L$ is the proportionality constant called the inductance of the inductor
- Units are Henrys 1H = \frac{V}{A} sec

- Inductance is a measure of how much an inductor opposes a change in current through it.

- What happens?

$$t = 0$$
$$dt \rightarrow 0$$
Inductance of an inductor depends on its physical configuration and construction.

- **Solenoid** \( L = \frac{N^2 A \mu_0}{l} \)
  
  \( N = \) number of turns
  \( \mu_0 = \) permeability of the core
  \( A = \) cross-sectional area
  \( l = \) length

- **Typical values**: 1-5H \( \rightarrow \) 1-5nH

- **Voltage current relationship**:
  \[ v = L \frac{di}{dt} \]

  \( v \) vs. \( \frac{di}{dt} \)

  - From \( v = L \frac{di}{dt} \) find \( i \) induction

  \[ \frac{di}{dt} = \frac{1}{L} v \]
  \[ di = \frac{1}{L} v dt \]
  \[ i = \frac{1}{L} \int v dt + i(t_0) \]

  - **Power**:
    \[ P = \frac{1}{2} L \left( \frac{di}{dt} \right)^2 \]

  - **Energy**:
    \[ W = \frac{1}{2} Li^2 \] (this is the energy stored in the magnetic field)

  - Tells us that
    1. when current is constant \( v_i \) is zero, (its a short to AC)
    2. current thru inductor cannot change instantaneously
Inductors in parallel and series

In series, inductors simply add:

\[ L_{eq} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4} \]

In parallel, inductors add as sum of reciprocals:

\[ L_{eq} = L_1 + L_2 + L_3 + L_4 \]
OP AMP Configurations w/R+C

Integrator:

\[
\begin{align*}
  \frac{V_i}{R} - C \frac{d[V_i - V_o]}{dt} &= 0 \\
  \frac{V_i}{R} + C \frac{dV_o}{dt} &= 0
\end{align*}
\]

\[
\frac{dV_o}{dt} = -\frac{V_i}{RC} \cdot \frac{1}{C}
\]

\[
v_o = -\frac{1}{RC} \int_{-\infty}^{+\infty} V_i(t) \, dt
\]

Assuming integrator op is reset \( e(t) = 0 \) we get

\[
v_o = -\frac{1}{RC} \int_{0}^{+\infty} V_i(t) \, dt
\]
Differentiator:

\[ i_i \rightarrow \quad i_f \rightarrow \quad \begin{array}{c}
\text{vi} \\
\text{C} \\
\end{array} \quad \begin{array}{c}
\text{R} \\
\text{Vo} \\
\end{array} \]

KCL at (A)

\[ i_i = i_f \]

\[ i_i = C \frac{dV_i}{dt}, \quad i_f = 0 - \frac{V_0}{R} = -\frac{V_0}{R} \]

Substitute in

\[ C \frac{dV_i}{dt} = -\frac{V_0}{R} \]

\[ V_0 = -RC \frac{dV_i}{dt} \]