Momentum 1 Review

Fundamental Relationships

\[ 1 \, \text{C} = 6.24 \times 10^{18} \text{ electrons} \]

\[ i = \frac{dq}{dt} \quad \text{[Coulombs/sec]} \]

\[ 1 \, \text{A} = 1 \, \text{C}/\text{sec} \]

Unit Conversions - for example, show that mAh = electrons

Current: A point
   Reference Arrow
   Magnitude

Voltage: two points
   A reference terminal
   Magnitude
   \[ V_{AB} = \frac{dq}{dt} \quad \text{[Coulomb]} \]
   \[ \text{moving charge (Joulles)} \quad \text{work (N-m)} \]

Power:

\[ P = VI = \frac{v^2}{R} = I^2R \quad \text{(instantaneous power)} \]

\[ P = \frac{dw}{dt} \quad \text{the time rate of work} \]

Ohms Law:

\[ V = IR \quad I = \frac{V}{R} \quad R = \frac{V}{I} \]

Passive Sign Convention:
- Current Arrow points into the positively marked terminal of the element in question

\[ \text{Power Dissipated} = V \times I \]

- If \( P > 0 \), element is dissipated power
- If \( P < 0 \), element is generating power
KCL: Sum all currents entering and leaving a node or a closed surface containing multiple nodes. The sum of the currents will be zero.
\[ \sum_{n=1}^{N} i_n = 0 \]

KVL: Sum of all the voltages around any loop sum to zero.
\[ \sum_{n=1}^{M} v_n = 0 \]

Elements in series: 
- One end of each is connected to one end of the other. At that junction no other wires exist.
- In this case, both resistors will pass the same current.

Elements in parallel: 
- Suppose we have two elements with two terminals each, non-oo 1 + 2. If A1 is connected to B1 and A2 is connected to B2 then A + B are in parallel.
- Likewise if A1 is connected to B2 and A2 is connected to B1, A + B are in parallel.

\[ \text{Parallel} \quad \text{Parallel} \]

If elements are in parallel, they will have the same voltage across them.
Resistor Networks

Identify pairs of parallel or series connected resistors.

For each pair, reduce them to a single resistor connected to the same points as the pair.

Continue doing this till you have only 1 resistor.

Resistors in Series: Add directly

Parallel: \( \frac{R_1 \cdot R_2}{R_1 + R_2} \)

Multiple resistors in parallel: \( \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}} \)
Nodal Analysis

**General case:**
- Select the reference node (ground)
- Assign currents through resistors
- Express currents through resistors in terms of $\frac{V}{R}$
- Write KCL equations at non-reference nodes
- Perform Ohm’s law substitution
- Solve equations

**Special case:**
- If a voltage source exists between two non-reference nodes, form a supernode by encircling the voltage source & anything in parallel with it.
- Write a KCL equation summing the currents at the supernode.
- Apply KVL or “stairstep method” to create the constraining equation that relates the voltages between the nodes within the supernode.
Mesh Analysis

General Case:
- Determine planarity
- Assign mesh currents, all CW
- Perform KVL on each mesh, expressing each voltage in terms of i * R
- Solve set of equations

Special Case:
- If a current source exists between two meshes...
  Write a KVL equation around a loop that encompasses the current source but that avoids it. This is the supermesh.
- Write a KVL equation around any other meshes that does not share a current source between meshes.
- Using KCL, determine the relationship between the current source and the two mesh currents that go through it. This is the constraining equation.
Now let's tackle a more complex network. The solution is shown schematically step by step.

Also find $P_{out}$ of 2A src.
\[ I_1 = \frac{V_A - 12}{2} \]
\[ I_2 = \frac{V_A}{4} \]
\[ I_3 = \frac{V_A - V_B}{8} \]
\[ I_4 = \frac{V_B + (5V_0)}{1} \]

KCL at \( V_A \):
\[-I_1 - I_2 - I_3 - 3 = 0\]
\[-\left(\frac{V_A - 12}{2}\right) - \frac{V_A}{4} - \left(\frac{V_A - V_B}{8}\right) = 3\]
\[-4V_A + 48 - 2V_A - V_A + V_B = 24\]
\[-7V_A + V_B = -24\]
\[\Rightarrow V_B = 7V_A - 24\]

KCL at \( V_B \):
\[I_3 + 3 - I_4 = 0\]
\[\frac{V_A - V_B}{8} - \frac{(V_B + 5V_0)}{1} = -3\]
\[V_A - V_B - 8V_B - 40V_0 = -24\]

By obsv., \[-12 + V_0 + V_A = 0 \quad (KVL)\]
\[V_0 = 12 - V_A\]
\[V_A - V_B - 8V_B - 40(12 - V_A) = -24\]
\[V_A - V_B - 8V_B - 480 + 40V_A = -24\]
\[41V_A - 9V_B = 480 - 24\]
\[41V_A - 7V_B = 456\]

\[41V_A - 9(7V_A - 24) = 456\]
\[41V_A - 63V_A + 216 = 456\]
\[22V_A = 240\]
\[V_A = -10.9\]
\[V_B = -100.34\]
**Supernode Example**

- **Voltage source** is between 2 non-reference nodes.
- Supernode encloses voltage source.

**Constraining equation from inside supernode:**

\[ I_1 = \frac{V_A}{6000} \quad I_2 = \frac{V_B}{12000} \]

\[ \begin{align*}
72 - 2V_A - V_B &= 48 = 0 \\
-2V_A - V_B &= -24 \\
-2(V_B + 6) - V_B &= -24 \\
-2V_B - 12 - V_B &= -24 \\
-3V_B &= -12 \\
V_B &= 4 \quad V_A = 10
\end{align*} \]

- **KVL method to get constraining eqn.**

\[ -V_B + 6 + V_A = 0 \]

\[ V_A = V_B + 6 \]
Rearrange \( V_0 \):

\[
I_1 - I_2 - I_3 = 0
\]

\[
30 - V_0 - \frac{V_0}{2} - \left( \frac{V_0 - V_b - 2V_0}{4} \right) = 0
\]

\[
120 - 4V_0 - 2V_0 - V_0 + V_b + 2V_0 = 0
\]

\[\boxed{-5V_0 + V_b = -120} \]

Let's substitute

\[
I_3 - I_4 + 3 = 0
\]

\[
\frac{V_0 - V_b - 2V_0}{4} - \frac{16}{10} + 3 = 0
\]

\[
4V_0 - 4V_b - 8V_0 - V_b = -48
\]

\[\boxed{-4V_0 - 5V_b = -48} \]

From (A)

\[V_0 = 5V_0 - 120 \]

\[\boxed{4V_0 - 5(V_0 - 120) = -48 \quad -4V_0 - 25V_0 + 600 = -48 \quad -29V_0 = -648 \quad V_0 = 22.345V} \]
**KVL i_1:**
\[-12 + 2(i_2 - i_3) + 4(i_1 - i_3) = 0\]
\[-12 + 2i_1 - 2i_2 + 4i_4 - 4i_3 = 0\]

(A) \[6i_4 - 2i_2 - 9i_3 = 12\]

**KVL i_3:**
\[4(i_3 - i_4) + 2(i_2 - i_3) + 8i_2 + 2V_0 = 0\]
\[4i_3 - 4i_4 + 2i_1 - 2i_2 + 8i_2 + 2V_0 = 0\]
\[-6i_4 + 10i_2 + 4i_3 + 2V_0 = 0\]

By observation, \(V_0 = 2(i_1 - i_2)\) so

\[-6i_1 + 10i_2 + 4i_3 + 4(i_1 - i_2) = 0\]
\[-6i_1 + 10i_2 + 4i_3 + 4i_4 - 4i_2 = 0\]

(B) \[-2i_4 + 6i_2 + 4i_3 = 0\]

**Constraining relationship:**
\[i_3 - i_2 = 3\]
\[\text{sso into both above}\]
\[i_2 = 3 + i_2\]

(A') \[6i_2 = 2i_2 - 4(3 + i_2) = 12\]
\[6i_4 - 2i_2 - 12 - 4i_2 = 12\]
\[6i_4 - 6i_2 = 24\]
\[i_2 - i_2 = 4\]
\[4i_4 = 4 + i_2\]

(B') \[-2i_4 + 6i_2 + 4(3 + i_2) = 0\]
\[-2i_4 + 6i_2 + 12 + 4i_2 = 0\]
\[-2i_4 + 10i_2 + 12 = 0\]
\[-2i_4 + 10i_2 + 12 - 12 = 0\]
\[-2i_4 + 10i_2 = -12\]
\[-i_2 + 5i_2 = -6\]
\[-4 + i_2 + 5i_2 = -6\]
\[-4 + 2i_2 = -6\]
\[4i_2 = -2\]
\[i_2 = -0.5\]