Introduction to Logic Programming in Prolog
Outline

Programming paradigms

Logic programming basics
   Introduction to Prolog
   Predicates, queries, and rules

Understanding the query engine
   Goal search and unification
   Structuring recursive rules

Complex terms, numbers, and lists

Cuts and negation
What is a programming paradigm?

**Paradigm**  
adapted from Oxford American  
A conceptual model underlying the theories and practice of a scientific subject

**scientific subject = programming**

**Programming paradigm**  
A conceptual model underlying the theories and practice of programming
Imperative paradigm

**Imperative model**

<table>
<thead>
<tr>
<th>data</th>
<th>set of state variables</th>
<th>type State = [(Name, Val)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>computation</td>
<td>transformation of state</td>
<td>State -&gt; State</td>
</tr>
</tbody>
</table>

Needs two sub-languages:

- **expressions** to describe values to store in variables (Expr)
- **statements** to describe state changes and control flow (Stmt)

Semantic functions:

- \( \text{semE} :: \text{Expr} \rightarrow \text{State} \rightarrow \text{Val} \)
- \( \text{semS} :: \text{Stmt} \rightarrow \text{State} \rightarrow \text{State} \)
Object-oriented paradigm

An extension/refinement of the imperative paradigm

Object-oriented model

<table>
<thead>
<tr>
<th>data</th>
<th>set of objects with state</th>
</tr>
</thead>
<tbody>
<tr>
<td>type Object</td>
<td>(State, [Method])</td>
</tr>
<tr>
<td>type Method</td>
<td>(Name, State -&gt; State)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>computation</th>
<th>evolution of objects</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Object] -&gt; [Object]</td>
<td></td>
</tr>
</tbody>
</table>

Needs **expression** and **statement** sub-languages, but extend statements with:

- constructs to **create objects** and **invoke methods**

New statement semantic function:

- \( \text{semS} :: \text{Stmt} \to [\text{Object}] \to [\text{Object}] \)
Functional paradigm

**Functional model**

<table>
<thead>
<tr>
<th>data</th>
<th>structured values</th>
<th>data Val = ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>computation</td>
<td>functions over values</td>
<td>Val -&gt; Val</td>
</tr>
</tbody>
</table>

In general, just one language (e.g. lambda calculus)

- **expressions** describe functions and values (Expr)

Semantic function:

- \( \text{sem} :: \text{Expr} \rightarrow \text{Val} \)
Logic paradigm

<table>
<thead>
<tr>
<th>Logical model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>data</strong></td>
</tr>
<tr>
<td>set of values and relations</td>
</tr>
<tr>
<td><strong>type</strong>Known = [(Val,...,Val)]</td>
</tr>
<tr>
<td><strong>computation</strong></td>
</tr>
<tr>
<td>query over relations</td>
</tr>
<tr>
<td><strong>type</strong> Query = Known -&gt; Known</td>
</tr>
</tbody>
</table>

In general, just one language:

- **relations** describe both knowledge and queries (**Rel**)

Semantic function:

- **sem :: Rel -> Query**
Comparison of programming paradigms

<table>
<thead>
<tr>
<th>Paradigm</th>
<th>View of computation</th>
</tr>
</thead>
<tbody>
<tr>
<td>imperative</td>
<td>sequence of state transformations</td>
</tr>
<tr>
<td>object-oriented</td>
<td>simulation of interacting objects</td>
</tr>
<tr>
<td>functional</td>
<td>function mapping input to output</td>
</tr>
<tr>
<td>logic</td>
<td>queries over logical relations</td>
</tr>
</tbody>
</table>
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What is Prolog?

• an untyped logic programming language

• programs are rules that define relations on values

• run a program by formulating a goal or query

• result of a program: a true/false answer and a binding of free variables
Logic: a tool for reasoning

**Syllogism** (logical argument) – Aristotle, 350 BCE

*Every human is mortal.*
*Socrates is human.*
*Therefore, Socrates is mortal.*

**First-order logic** – Gottlob Frege, 1879 *Begriffsschrift*

\[ \forall x. \text{Human}(x) \rightarrow \text{Mortal}(x) \]
*Human(Socrates)*
*\therefore \text{Mortal}(Socrates)*
Logic and programming

Rule: \( \forall x. \text{Human}(x) \rightarrow \text{Mortal}(x) \)

Fact: \( \text{Human}(\text{Socrates}) \)

Goal/query: \( \therefore \text{Mortal}(\text{Socrates}) \)

\begin{align*}
\text{Prolog program} \\
\text{mortal}(X) :- \text{human}(X). \\
\text{human}(\text{socrates}).
\end{align*}

\begin{align*}
\text{Prolog query (interactive)} \\
\text{?- mortal(socrates).} \\
\text{true.}
\end{align*}
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## SWI-Prolog logistics

<table>
<thead>
<tr>
<th>Predicate</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>[myfile].</td>
<td>load definitions from “myfile.pl”</td>
</tr>
<tr>
<td>listing(P).</td>
<td>lists facts and rules related to predicate P</td>
</tr>
<tr>
<td>trace.</td>
<td>turn on tracing</td>
</tr>
<tr>
<td>nodebug.</td>
<td>turn off tracing</td>
</tr>
<tr>
<td>help.</td>
<td>open help window (requires X11 on Mac)</td>
</tr>
<tr>
<td>halt.</td>
<td>quit</td>
</tr>
</tbody>
</table>

All of these except `help` also work in GNU Prolog!
Atoms

An **atom** is just a primitive value

- string of characters, numbers, underscores starting with a **lowercase letter**:
  - hello, socrates, sUp3r_At0m

- any single quoted string of characters:
  - 'Hello world!', 'Socrates'

- numeric literals: 123, -345

- empty list: []
A **variable** can be used in rules and queries

- string of characters, numbers, underscores starting with an **uppercase letter** or an **underscore**
  - X, SomeHuman, _g_123

- special variable: _ (just an underscore)
  - unifies with anything – “don’t care”
Predicates

Basic entity in Prolog is a **predicate**  

- **Unary predicate**
  - hobbit(bilbo).
  - hobbit(frodo).
  - hobbit(sam).

- **Binary predicate**
  - likes(bilbo, frodo).
  - likes(frodo, bilbo).
  - likes(sam, frodo).
  - likes(frodo, ring).

\[ \text{hobbit} = \{\text{bilbo, frodo, sam}\} \]

\[ \text{likes} = \{ (\text{bilbo, frodo}), (\text{frodo, bilbo}), (\text{sam, frodo}), (\text{frodo, ring}) \} \]
Simple goals and queries

Predicates are:

- **defined** in a file  *the program*
- **queried** in the REPL  *running the program*

Response to a query is a **true/false** answer  *(or yes/no)* when **true**, provides a **binding** for each variable in the query

Is sam a hobbit?

?- hobbit(sam).
true.

Is gimli a hobbit?

?- hobbit(gimli).
false.

Who is a hobbit?

?- hobbit(X).
X = bilbo ;
X = frodo ;
X = sam .
Querying relations

You can query **any argument** of a predicate

- this is fundamentally different from passing arguments to functions!

```
? - likes(frodo,Y).
Y = bilbo ;
Y = ring .

? - likes(X,frodo).
X = bilbo ;
X = sam .

? - likes(X,Y).
X = bilbo,
Y = frodo ;
X = sam,
Y = frodo ;
X = frodo,
Y = ring .
```

**Definition**

- `likes(bilbo, frodo).`
- `likes(frodo, bilbo).`
- `likes(sam, frodo).`
- `likes(frodo, ring).`
Overloading predicates

Predicates with the **same name** but **different arities** are **different predicates**!

**hobbit/1**
- hobbit(bilbo).
- hobbit(frodo).
- hobbit(sam).

**hobbit/2**
- hobbit(bilbo, rivendell).
- hobbit(frodo, hobbiton).
- hobbit(sam, hobbiton).
- hobbit(merry, buckland).
- hobbit(pippin, tookland).

?- hobbit(X).
X = bilbo ;
X = frodo ;
X = sam.

?- hobbit(X,_).
...  
X = merry ;
X = pippin .
Conjunction

Comma (,) denotes **logical and** of two predicates

Do **sam** and **frodo** like each other?

?- likes(sam,frodo), likes(frodo,sam).
true.

Do **merry** and **pippin** live in the same place?

?- hobbit(merry,X), hobbit(pippin,X).
false.

Do any hobbits live in the same place?

?- hobbit(H1,X), hobbit(H2,X), H1 \= H2.
H1 = frodo, X = hobbiton, H2 = sam.

H1 and H2 must be different!
Rule:  \textbf{head} :- \textbf{body}

The \textbf{head} is true \textbf{if} the \textbf{body} is true

Examples

\begin{verbatim}
likes(X,beer) :- hobbit(X,\_).
likes(X,boats) :- hobbit(X,buckland).
danger(X) :- likes(X,ring).
danger(X) :- likes(X,boats), likes(X,beer).
\end{verbatim}

Note that \textit{disjunction} is described by multiple rules
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How does prolog solve queries?

Basic algorithm for solving a (sub)goal

1. Linearly **search** database for candidate facts/rules
2. Attempt to **unify** candidate with goal
   - If unification is **successful**:
     - if a **fact** – we’re done with this goal!
     - if a **rule** – add body of rule as new subgoals
   - If unification is **unsuccessful**: keep searching
3. Backtrack if we reach the end of the database
1. Search the database for candidate matches

What is a candidate fact/rule?

- **fact**: predicate matches the goal
- **rule**: predicate of its **head** matches the goal

Example goal: `likes(merry,Y)`

**Candidates**

- `likes(sam,frodo).
- `likes(merry,pippin).
- `likes(X,beer) :- hobbit(X).`

**Not candidates**

- `hobbit(merry,buckland).
- `danger(X) :- likes(X,ring).
- `likes(merry,pippin,mushrooms).`
2. Attempt to unify candidate and goal

Unification
Find an assignment of variables that makes its arguments syntactically equal
Prolog: $A = B$ means attempt to unify $A$ and $B$

?- likes(merry,Y) = likes(sam,frodo).
false.

?- likes(merry,Y) = likes(merry,pippin).
Y = pippin .

?- likes(merry,Y) = likes(X,beer).
X = merry ; Y = beer .

2a. if fail, try next candidate
2b. if success, add new subgoals
Tracking subgoals

Deriving solutions through rules

1. Maintain a list of goals that need to be solved
   - when this list is empty, we’re done!
2. If current goal unifies with a rule head, add body as subgoals to the list
3. After each unification, substitute variables in all goals in the list!

Database

<table>
<thead>
<tr>
<th></th>
<th>lt(one,two).</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>lt(two,three).</td>
</tr>
<tr>
<td>3</td>
<td>lt(three,four).</td>
</tr>
<tr>
<td>4</td>
<td>lt(X,Z) :- lt(X,Y), lt(Y,Z).</td>
</tr>
</tbody>
</table>

Sequence of goals for \( \text{lt(one,four)} \)

<table>
<thead>
<tr>
<th></th>
<th>( \text{lt(one,four)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>( X=\text{one}, Z=\text{four} )</td>
</tr>
<tr>
<td>1</td>
<td>( Y1=\text{two} )</td>
</tr>
<tr>
<td>4</td>
<td>( X=\text{two}, Z=\text{four} )</td>
</tr>
<tr>
<td>2</td>
<td>( Y2=\text{three} )</td>
</tr>
<tr>
<td>3</td>
<td>( \text{true} )</td>
</tr>
</tbody>
</table>

\( \text{done!} \)
3. Backtracking

For each subgoal, Prolog maintains:

- the **search state** (goals + assignments) before it was produced
- a **pointer** to the rule that produced it

When a **subgoal fails**:

- **restore** the previous state
- **resume** search for previous goal from the pointer

When the **initial goal fails**: return **false**
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Potential for infinite search

Why care about how goal search works?

One reason: to write **recursive rules** that don’t loop!

**Bad example: symmetry**

married(abe, mona).
married(clancy, jackie).
marrried(homer, marge).
marrried(X, Y) :- married(Y, X).

?- married(jackie, abe).
**ERROR: Out of local stack**

**Bad example: transitivity**

lt(one, two).
l(two, three).
l(tthree, four).
l(X, Z) :- lt(X, Y), lt(Y, Z).

?- lt(three, one).
**ERROR: Out of local stack**
Strategies for writing recursive rules

How to avoid infinite search

1. Always list **non-recursive cases first** (in database and rule bodies)
2. Use helper predicates to **enforce progress** during search

**Example: symmetry**

marriedP(abe, mona).
mappedP(clancy, jackie).
mappedP(homer, marge).
mapped(X, Y) :- mappedP(X,Y).
mapped(X, Y) :- mappedP(Y,X).

?- mapped(jackie, abe).
false.

**Example 2: transitivity**

ltP(one, two).
ltP(two, three).
ltP(three, four).
lt(X, Y) :- ltP(X,Y).
lt(X, Z) :- ltP(X,Y), lt(Y,Z).

?- lt(three, one).
false.
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Representing structured data

Can represent structured data by **nested predicates**

**Example database**

<table>
<thead>
<tr>
<th>drives(bart, skateboard(green))</th>
<th>drives(bart, bike(blue))</th>
</tr>
</thead>
<tbody>
<tr>
<td>drives(lisa, bike(pink))</td>
<td>drives(homer, car(pink))</td>
</tr>
</tbody>
</table>

?- drives(lisa, X).
X = bike(pink).

?- drives(X, bike(Y)).
X = bart, Y = blue;
X = lisa, Y = pink.

Variables can’t be used for predicates:
?- drives(X, Y(pink)). ← illegal!
Relationship to Haskell data types

Haskell data type

```
data Expr = Lit Int
  | Neg Expr
  | Add Expr Expr
  | Mul Expr Expr
```

Add (Neg (Lit 3))
(Mul (Lit 4) (Lit 5))

• build values w/ data constructors
• data types statically define valid combinations

Prolog predicate

```
expr(N) :- number(N).
expr(neg(E)) :- expr(E).
expr(add(L,R)) :- expr(L), expr(R).
expr(mul(L,R)) :- expr(L), expr(R).
```

add(neg(3), mul(4,5))

• build values w/ predicates
• use rules to dynamically identify or enumerate valid combinations
Lists are structured data with special syntax

- similar basic structure to Haskell
- but can be heterogeneous

\[ [3, 4, 5] \equiv .(3, .(4, .5(, []))) \]

\[ ["hi", [atom, p(x)], 6] \]

Complex terms, numbers, and lists
List patterns

Database

\texttt{story([3,little,pigs]).}

?- \texttt{story([X,Y,Z|V]).}
X = 3,
Y = little,
Z = pigs,
V = [].

?- \texttt{story([X,Y,Z]).}
X = 3,
Y = little,
Z = pigs.

?- \texttt{story([X,Y,Z,V]).}
false.
Arithmetic in Prolog

**Arithmetic expressions** are also structured data (nested predicates)

- special syntax: can be written infix, standard operator precedence
- can be evaluated:
  
  \[
  \begin{align*}
  \text{X is } & \text{ expr } \quad \text{evaluate expr and bind to } \text{X} \\
  \text{expr } &=:= \text{ expr } \quad \text{evaluate expressions and check if equal}
  \end{align*}
  \]

\[
3*4+5*6 \equiv +(*(3, 4), *(5, 6))
\]

?- X is 3*4+5*6.
X = 42.

?- 8 is X*2.
ERROR: is/2: Arguments are not sufficiently instantiated

**Arithmetic operations**

- \(+\)
- \(-\)
- \(*\)
- \(/\)
- \(\mod\)

**Comparison operations**

- \(<\)
- \(>\)
- \(<=\)
- \(>=\)
- \(=:=\)
- \(!=\)
Using arithmetic in rules

Example database

```
fac(1,1).
fac(N,M) :- K is N-1, fac(K,L), M is L*N.
```

?- fac(5,M).
M = 120.

?- fac(N,6).
ERROR: fac/2: Arguments are not sufficiently instantiated
Unification vs. arithmetic equality

**Unification:** $A = B$
Find an assignment of variables that makes its arguments syntactically equal

**Arithmetic equality:** $A =:= B$
Evaluate terms as arithmetic expressions and check if numerically equal

?- $X = 3*5.$  
$X = 3*5.$

?- 8 = X*2.  
false.

?- 4*2 = X*2.  
$X = 4.$

?- $X$ is 3*5.  
$X = 8.$

?- 8 is X*2.  
ERROR: is/2: Arguments are not sufficiently instantiated
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How cut works

**Cut** is a special atom used to **prevent backtracking**

When encountered as a subgoal it:
- always succeeds
- commits the search to the matches and assignments made so far

---

**Database without cut**

```prolog
foo(1). foo(2).
bar(X,Y) :- foo(X), foo(Y).
bar(3,3).
```

?- bar(A,B).

A = 1, B = 1 ; A = 1, B = 2 ;
A = 2, B = 1 ; A = 2, B = 2 ;
A = 3, B = 3.

**Database with cut**

```prolog
foo(1). foo(2).
bar(X,Y) :- foo(X), !, foo(Y).
bar(3,3).
```

?- bar(A,B).

A = 1, B = 1 ; A = 1, B = 2.

A = 1, B = 1 ;
A = 1, B = 2.
Green cuts vs. red cuts

A **green cut**: doesn’t affect the members of a predicate
- only cuts paths that would have failed anyway
- the cut is used purely for efficiency

A **red cut**: any cut that isn’t green
- if removed, meaning of the predicate changes
- the cut is part of the “logic” of the predicate

```prolog
max(X, Y, Y) :- X < Y, !.
max(X, Y, X) :- X >= Y.
```

```prolog
find(X, [X|_]) :- !.
find(X, [L|_]) :- find(X, L).
```
Negation as failure

Negation predicate

\[
\text{not}(P) :- P, !, \text{fail.} \\
\text{not}(P).
\]

if \( P \) is true, commit that \( \text{not}(P) \) is false
otherwise, \( \text{not}(P) \) is true

Database

\[
\text{hobbit}(frodo). \\
\text{hobbit}(bilbo). \\
\text{likes}(X, \text{beer}) :- \text{hobbit}(X).
\]

?- \text{not}(\text{likes}(frodo, \text{beer})). \\
false.

?- \text{not}(\text{likes}(\text{gimli}, \text{beer})). \\
true.

?- \text{not}(\text{likes}(\text{bilbo}, X)). \\
false.

?- \text{not}(\text{likes}(X, \text{pizza})). \\
true.

“frodo doesn’t like beer”

“gimli doesn’t like beer”

“bilbo doesn’t like anything”

“nobody likes pizza”