Introduction to Functional Programming in Haskell
Outline

Why learn functional programming?

The essence of functional programming
   What is a function?
   Equational reasoning
   First-order vs. higher-order functions
   Lazy evaluation

How to functional program
   Haskell style
   Functional programming workflow
   Data types
   Type-directed programming

Refactoring and reuse
   Refactoring
   Type classes

Type inference
Outline

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Refactoring and reuse

Type inference
Why learn (pure) functional programming?

1. This course: strong correspondence of core concepts to PL theory
   - *abstract syntax* can be represented by *algebraic data types*
   - *denotational semantics* can be represented by *functions*

2. It will make you a better (imperative) programmer
   - forces you to think *recursively* and *compositionally*
   - forces you to *minimize use of state*

   …essential skills for solving *big* problems

3. It is the future!
   - more scalable and parallelizable (MapReduce)
   - functional features have been added to most mainstream languages
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What is a (pure) function?

A function is pure if:

- it always returns the same output for the same inputs
- it doesn’t do anything else — no “side effects”

In Haskell: whenever we say “function” we mean a pure function!
What are and aren’t functions?

Always functions:

• mathematical functions  \( f(x) = x^2 + 2x + 3 \)
• encryption and compression algorithms

Usually not functions:

• C, Python, JavaScript, … "functions" (procedures)
• Java, C#, Ruby, … methods

Haskell only allows you to write (pure) functions!
Why procedures/methods aren’t functions

- output depends on environment
- may perform arbitrary side effects
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Getting into the Haskell mindset

In Haskell, “=” means is not change to!

Haskell

```haskell
sum :: [Int] -> Int
sum [] = 0
sum (x:xs) = x + sum xs
```

Java

```java
int sum(List<Int> xs) {
    int s = 0;
    for (int x : xs) {
        s = s + x;
    }
    return s;
}
```
Getting into the Haskell mindset

Quicksort in Haskell

```haskell
qsort :: Ord a => [a] -> [a]
qsort [] = []
qsort (x:xs) = qsort (filter (<= x) xs) ++ x : qsort (filter (> x) xs)
```

Quicksort in C

```c
void qsort(int low, int high) {
    int i = low, j = high;
    int pivot = numbers[low + (high-low)/2];

    while (i <= j) {
        while (numbers[i] < pivot) {
            i++;
        }
        while (numbers[j] > pivot) {
            j--;
        }
        if (i <= j) {
            swap(i, j);
            i++;
            j--;
        }
    }
    if (low < j)
        qsort(low, j);
    if (i < high)
        qsort(i, high);
}
void swap(int i, int j) {
    int temp = numbers[i];
    numbers[i] = numbers[j];
    numbers[j] = temp;
}
```
Referential transparency

An expression can be replaced by its value without changing the overall program behavior

\[ \text{length } [1,2,3] + 4 \]
\[ \Rightarrow 3 + 4 \]

what if \text{length} was a Java method?

**Corollary**: an expression can be replaced by any expression with the same value without changing program behavior

Supports \text{equational reasoning}
Equational reasoning

Computation is just substitution!

```
sum :: [Int] -> Int
sum [] = 0
sum (x:xs) = x + sum xs
```

| sum [2,3,4] | \[\Rightarrow\] | sum (2:(3:(4:[]))) |
| 2 + sum (3:(4:[])) | \[\Rightarrow\] | 2 + 3 + sum (4:[]) |
| 2 + 3 + 4 + sum [] | \[\Rightarrow\] | 2 + 3 + 4 + 0 |
| \[\Rightarrow\] | | 9 |

The essence of functional programming
Describing computations

**Function definition**: a list of **equations** that relate inputs to output
- matched top-to-bottom
- applied left-to-right

**Example: reversing a list**

**imperative view**: how do I rearrange the elements in the list? \( \times \)

**functional view**: how is a list related to its reversal? \( \checkmark \)

```haskell
reverse :: [a] -> [a]
reverse [] = []
reverse (x:xs) = reverse xs ++ [x]
```

**Exercise**: Use equational reasoning to compute the reverse of the list \([2,3,4,5]\)
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First-order functions

Examples

- \text{cos} :: \text{Float} \to \text{Float}
- \text{even} :: \text{Int} \to \text{Bool}
- \text{length} :: [\text{a}] \to \text{Int}
Higher-order functions

Examples

- `map :: (a -> b) -> [a] -> [b]`
- `filter :: (a -> Bool) -> [a] -> [a]`
- `(.) :: (b -> c) -> (a -> b) -> a -> c`
Higher-order functions as control structures

**map**: loop for doing something to each element in a list

map :: (a -> b) -> [a] -> [b]
map f [] = []
map f (x:xs) = f x : map f xs

map f [2,3,4,5] = [f 2, f 3, f 4, f 5]
map even [2,3,4,5] = [even 2, even 3, even 4, even 5] = [True, False, True, False]

**fold**: loop for aggregating elements in a list

foldr :: (a->b->b) -> b -> [a] -> b
foldr f y [] = y
foldr f y (x:xs) = f x (foldr f y xs)

foldr f y [2,3,4] = f 2 (f 3 (f 4 y))
foldl (+) 0 [2,3,4] = (+) 2 ((+) 3 ((+) 4 0)) = 2 + (3 + (4 + 0)) = 9
Function composition

Can create new functions by composing existing functions

- apply the second function, then apply the first

Function composition

\[(\cdot) : (b \to c) \to (a \to b) \to a \to c\]

\[f \cdot g = \lambda x \rightarrow f (g x)\]

Types of existing functions

- \text{not} :: \text{Bool} \to \text{Bool}
- \text{succ} :: \text{Int} \to \text{Int}
- \text{even} :: \text{Int} \to \text{Bool}
- \text{head} :: \text{[a]} \to \text{a}
- \text{tail} :: \text{[a]} \to \text{[a]}

Definitions of new functions

- \text{plus2} = \text{succ} \cdot \text{succ}
- \text{odd} = \text{not} \cdot \text{even}
- \text{second} = \text{head} \cdot \text{tail}
- \text{drop2} = \text{tail} \cdot \text{tail}
Currying / partial application

In Haskell, functions that take multiple arguments are implicitly higher order

\[ \text{plus} :: \text{Int} \to \text{Int} \to \text{Int} \]

Curried
\[ \text{plus} \ 2 \ 3 \]

plus :: Int -> Int -> Int

Uncurried
\[ \text{plus} \ (2,3) \]

plus :: (Int,Int) -> Int

a pair of ints
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Lazy evaluation

In Haskell, expressions are reduced:

- only when needed
- at most once

Supports:

- infinite data structures
- separation of concerns

```
nats :: [Int]
nats = 1 : map (+1) nats

fact :: Int -> Int
fact n = product (take n nats)

min3 :: [Int] -> [Int]
min3 = take 3 . sort
```

What is the running time of this function?
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Type inference
Good Haskell style

Why it matters:

- layout is significant!
- eliminate misconceptions
- we care about elegance

Easy stuff:

- **use spaces!** (tabs cause layout errors)
- align patterns and guards

See style guides on course web page
Formatting function applications

Function application:
- is *just a space*
- associates to the left
- binds most strongly

\[
\begin{align*}
\text{f(x)} & \quad \text{f x} \\
(f \ x) \ y & \quad f \ x \ y \\
(f \ x) + (g \ y) & \quad f \ x + g \ y
\end{align*}
\]

Use parentheses only to *override* this behavior:
- f (g x)
- f (x + y)
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Refactoring and reuse

Type inference
FP workflow (simple)

Refactor

Define functions

Identify/define types

“obsessive compulsive refactoring disorder”
FP workflow (detailed)

Norman Ramsey, On Teaching “How to Design Programs”, ICFP’14
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Type inference
Algebraic data types

Data type definition
- introduces new type of value
- enumerates ways to construct values of this type

Definitions consists of …
- a type name
- a list of data constructors with argument types

Some example data types

```
data Bool = True | False

data Nat = Zero | Succ Nat

data Tree = Node Int Tree Tree | Leaf Int
```

Definition is inductive
- the arguments may recursively include the type being defined
- the constructors are the only way to build values of this type
Anatomy of a data type definition

```
data Expr = Lit Int
           | Plus Expr Expr
```

Example: $2 + 3 + 4 \quad \text{Plus (Lit 2) (Plus (Lit 3) (Lit 4))}$
FP data types vs. OO classes

Haskell

```
data Tree = Node Int Tree Tree
         | Leaf
```

- separation of type- and value-level
- set of cases closed
- set of operations open

Java

```
abstract class Tree { ... }
class Node extends Tree {
    int label;
    Tree left, right;
    ...
}
class Leaf extends Tree { ... }
```

- merger of type- and value-level
- set of cases open
- set of operations closed

Extensibility of cases vs. operations = the “expression problem”
Type parameters

Like generics in Java

data List a = Nil
    | Cons a (List a)

reference to type parameter
recursive reference to type

Specialized lists

type IntList = List Int
type CharList = List Char
type RaggedMatrix a = List (List a)
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Type inference
Tools for defining functions

Recursion and other functions

```haskell
sum :: [Int] -> Int
sum xs = if null xs then 0
         else head xs + sum (tail xs)
```

(1) case analysis

Pattern matching

```haskell
sum :: [Int] -> Int
sum []    = 0
sum (x:xs) = x + sum xs
```

(2) decomposition

Higher-order functions

```haskell
sum :: [Int] -> Int
sum = foldr (+) 0
```

no recursion or variables needed!
What is type-directed programming?

Use the type of a function to help implement it.
Type-directed programming

Basic goal: transform values of **argument types** into **result type**

If argument type is …
- **atomic type** (e.g. `Int`, `Char`)
  - apply functions to it
- **algebraic data type**
  - use pattern matching
    - case analysis
    - decompose into parts
- **function type**
  - apply it to something

If result type is …
- **atomic type**
  - output of another function
- **algebraic data type**
  - build with data constructor
- **function type**
  - function composition or partial application
  - build with lambda abstraction
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Refactoring and reuse
  Refactoring
  Type classes

Type inference
Refactoring in the FP workflow

Motivations:

- separate concerns
- promote reuse
- promote understandability
- gain insights

"obsessive compulsive refactoring disorder"
Semantics-preserving **laws** can prove with equational reasoning + induction

- Eta reduction:
  \[ x \rightarrow f \ x \equiv f \]
- Map–map fusion:
  \[ \text{map } f \ . \ \text{map } g \equiv \text{map } (f \ . \ g) \]
- Fold–map fusion:
  \[ \text{foldr } f \ b \ . \ \text{map } g \equiv \text{foldr } (f \ . \ g) \ b \]

“Algebra of computer programs”

John Backus, *Can Programming be Liberated from the von Neumann Style?*, ACM Turing Award Lecture, 1978
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Type inference
What is a type class?

1. an **interface** that is supported by many different types
2. a **set of types** that have a common behavior

```haskell
class Eq a where
  (==) :: a -> a -> Bool

class Show a where
  show :: a -> String

class Num a where
  (+) :: a -> a -> a
  (*) :: a -> a -> a
  negate :: a -> a
  ...
```

types whose values can be compared for equality

types whose values can be shown as strings

types whose values can be manipulated like numbers
Type constraints

- List elements can be of any type
  - `length :: [a] -> Int`
  - `length [] = 0`
  - `length (_:xs) = 1 + length xs`

- List elements must support equality!
  - `elem :: Eq a => a -> [a] -> Bool`
  - `elem _ [] = False`
  - `elem y (x:xs) = x == y || elem y xs`

*use method ⇒ add type class constraint*
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How to perform type inference

If a literal, data constructor, or named function: write down the type – you’re done!

Otherwise:

1. identify the top-level application $e_1 e_2$
2. recursively infer their types $e_1 : T_1$ and $e_2 : T_2$
3. $T_1$ should be a function type $T_1 = T_{\text{arg}} \rightarrow T_{\text{res}}$
4. unify $T_{\text{arg}} = ? T_2$, yielding type variable assignment $\sigma$
5. return $e_1 e_2 : \sigma T_{\text{res}}$ ($T_{\text{res}}$ with type variables substituted)

If any of these steps fails, it is a type error!

Example: `map even`
**Exercises**

**Given**

```haskell
data Maybe a = Nothing | Just a

gt :: Int -> Int -> Bool

not :: Bool -> Bool

map :: (a -> b) -> [a] -> [b]

even :: Int -> Bool

(.) :: (b -> c) -> (a -> b) -> a -> c
```

1. Just
2. not even 3
3. not (even 3)
4. not . even
5. even . not
6. map (Just . even)