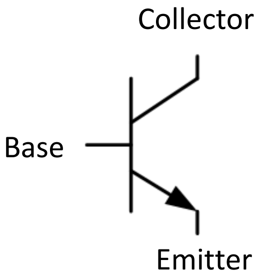
SECTION 2: BIPOLAR JUNCTION TRANSISTORS

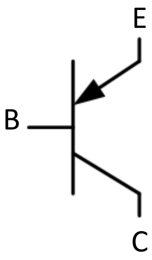
ECE 322 – Electronics I

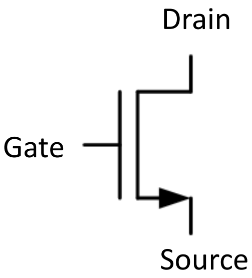
Introduction

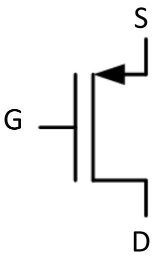
Transistors

- So far, we have only dealt with ***two-terminal*** Electrical components
- We now look at ***transistors***
  - ***Three-terminal*** components
  - Third terminal makes for very useful, though more complicated, device
- ***Semiconductor*** devices
  - Typically silicon, Si
  - Differently doped (N-type/P-type) Si at each terminal of the device







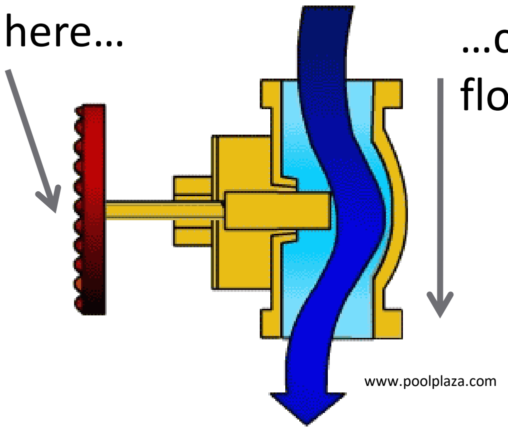


Transistors

- Three terminals:
  - Current or voltage (possibly small) at one terminal controls current (possibly large) flowing between the other two terminals
- Analogous to ***valves:***

Valve

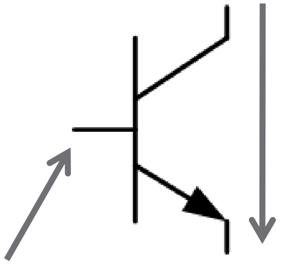
Turning the knob, here…



…controls the fluid flow, here

www.poolplaza.com

Transistor



…controls the current flow, here

The voltage or current, here…

- Useful as ***switches*** or ***amplifiers***

Types of Transistors

- Two primary classes of transistors:
  - ***Bipolar Junction Transistors*** – BJTs
  - ***Metal-Oxide-Semiconductor Field-Effect Transistors*** – MOSFETs

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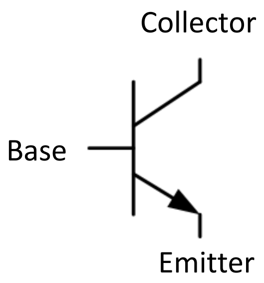
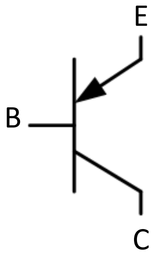
BJTs

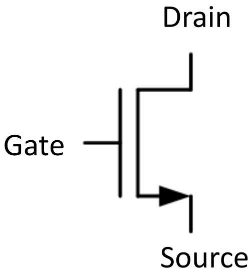


NPN

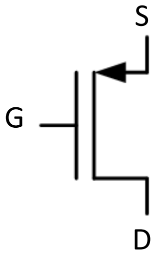


PNP

MOSFETs



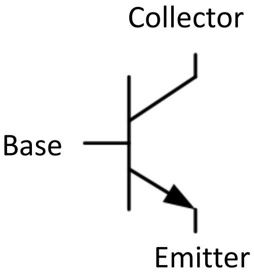
N-Channel (NMOS)



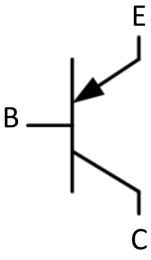
P-Channel (PMOS)

Types of Transistors

BJTs

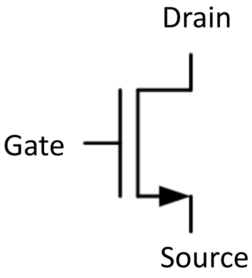


NPN

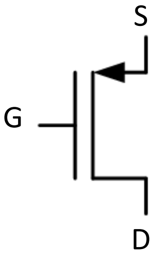


PNP

MOSFETs



N-Channel (NMOS)



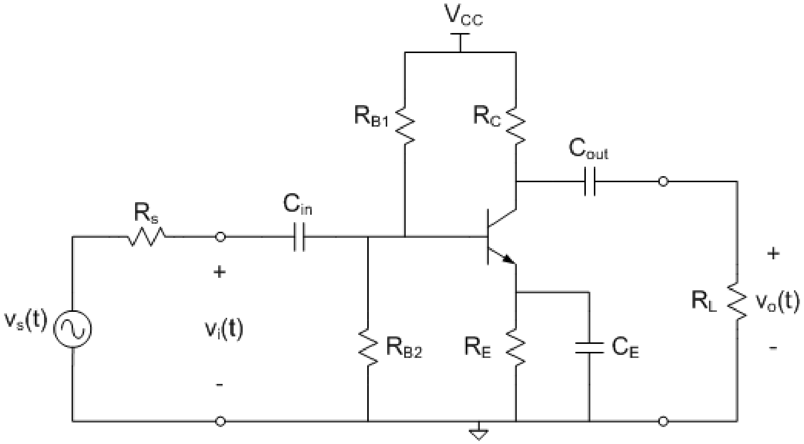
P-Channel (PMOS)

- Difference between NPN/PNP and NMOS/PMOS is the type of semiconductor (N-type/P-type) used at each terminal
- We will learn about both BJTs and MOSFETs:
  - Sections 2, 3: BJTs
  - Sections 4, 5: MOSFETs

Transistor Amplifier Circuits – Preview

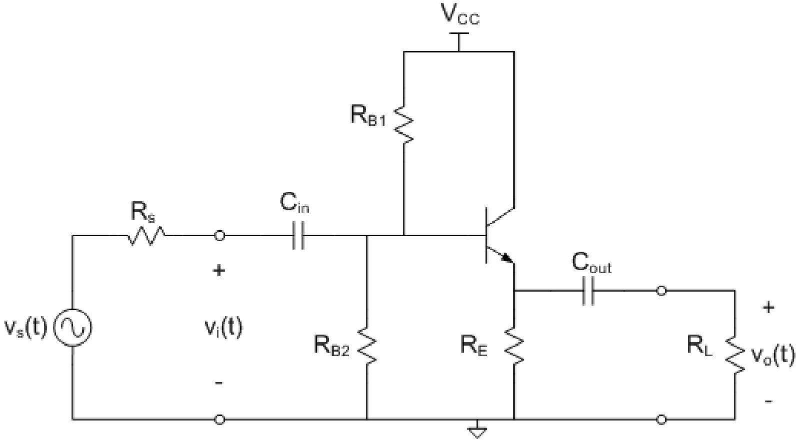
- Over the next two sections, you will learn to analyze and design circuits like the following

Common-Emitter Amplifier:



- High voltage gain
- An amplifier

Emitter-Follower Amplifier:



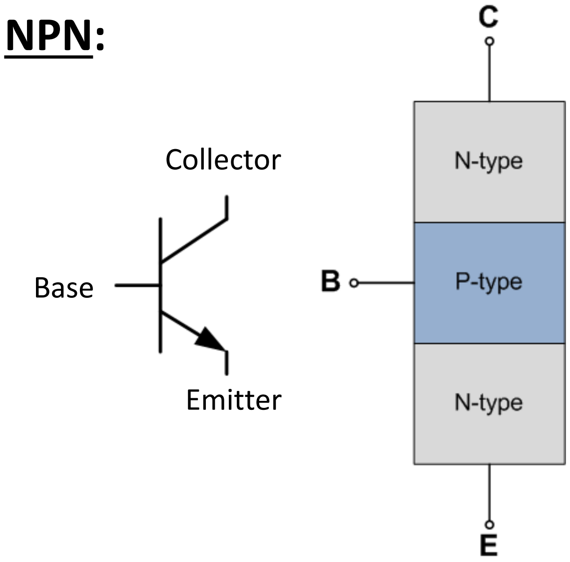
- Near unity gain
- A buffer

BJT Fundamentals

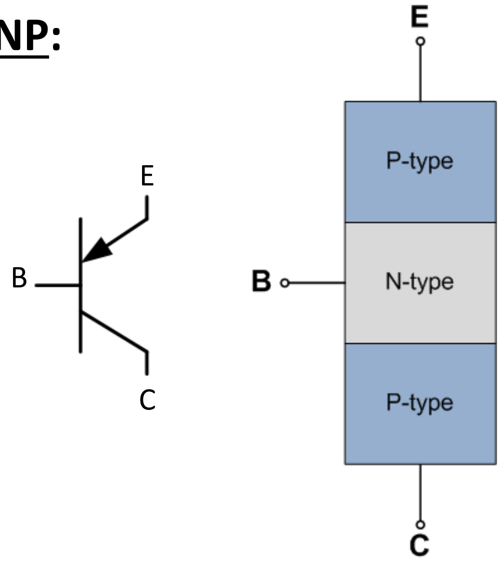
Bipolar Junction Transistors

- We have seen that diodes are constructed as PN-junctions
  - BJTs are essentially two back-to-back PN-junctions
- Two types: ***NPN*** and ***PNP***

NPN:



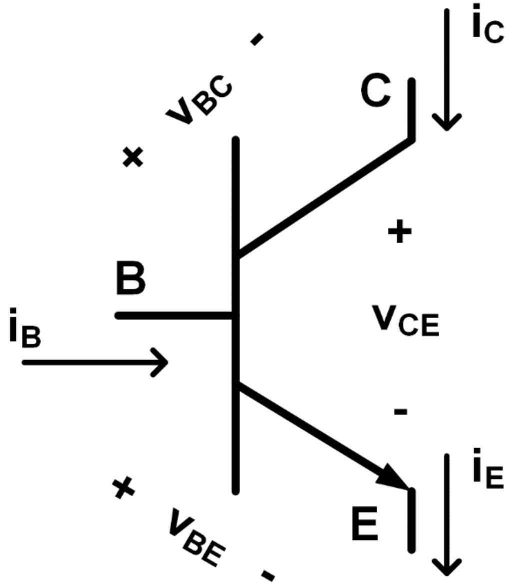
PNP:



- Symbol mnemonics:
  - ***NPN***: **N**ot **P**ointing i**N**
  - ***PNP***: **P**oint i**N** **P**lease

Terminal Voltages and Currents

- Terminal voltages and currents named as shown
- First letter in subscript is the *assumed* higher-potential terminal
    - E.g., $v_{CE} > 0$ if $v_C > v_E$
- Currents positive in direction shown, e.g.,
    - Positive collector current flows into the collector
    - Positive emitter current flows out of the emitter
- ***Lower-case*** v or i with ***upper-case subscript*** denotes ***both DC and AC*** signal components



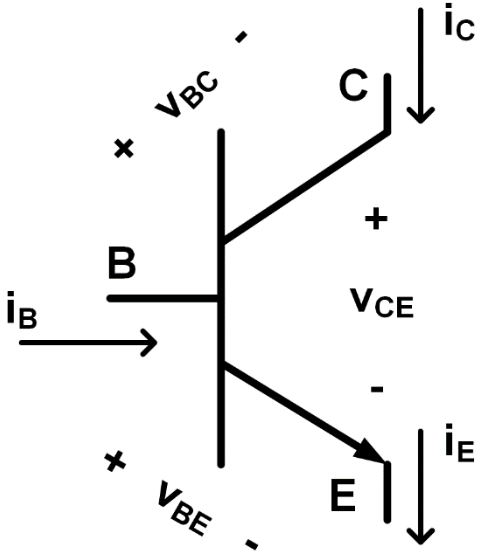
Kirchhoff’s Laws

- KVL and KCL apply to a transistor, just as they do to any electrical network
- ***KVL***
  - Voltages around the transistor sum to zero:

$$v_{BE} - v_{BC} - v_{CE} = 0$$

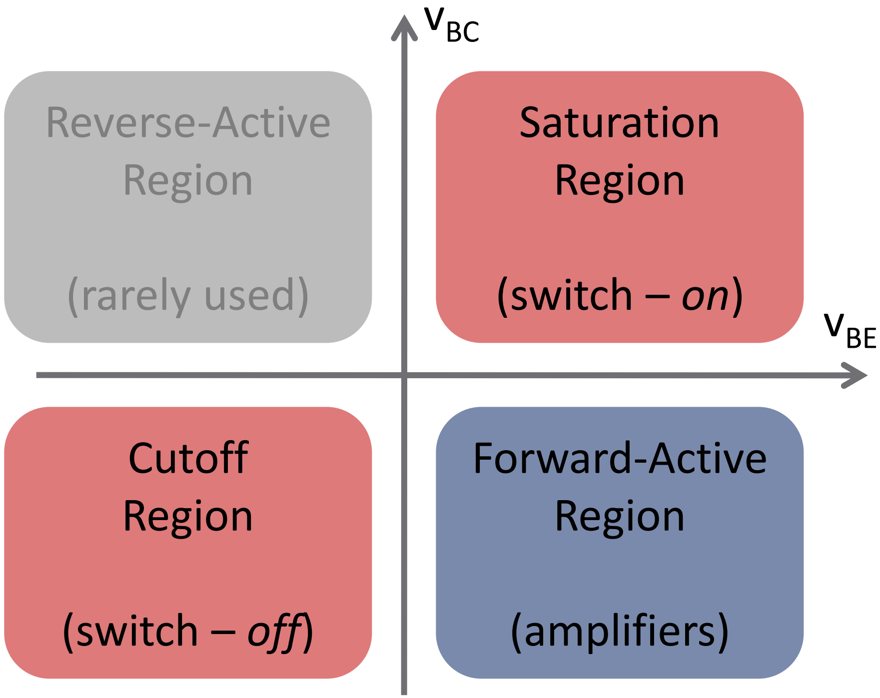
- ***KCL***
  - Transistor terminal currents sum to zero:

$$i_B + i_C - i_E = 0$$



BJT Operating Regions

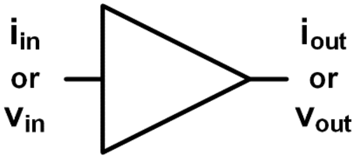
- Four operating regions
  - Defined by polarities of the junction voltages



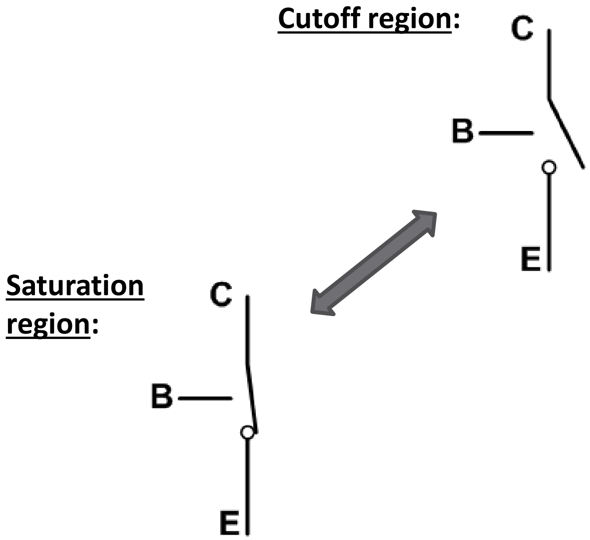
- ***Forward active region***
  - $v_{BC} < 0$
  - $v_{BE} > 0$
  - Linear region – ***amplifiers***
- ***Saturation region***
  - $v_{BC} > 0$
  - $v_{BE} > 0$
  - C-E looks like a ***closed switch***
- ***Cutoff region***
  - $v_{BC} < 0$
  - $v_{BE} < 0$
  - C-E looks like an ***open switch***

BJT Operating Regions

- Transistors used in electronic circuits fall into one of two categories:
- ***Linear amplifier***
    - Forward active region
    - Transistor provides ***linear gain***
    - Voltage or current gain
    - Greater than or less than unity
    - Type/value of gain depends on surrounding circuitry

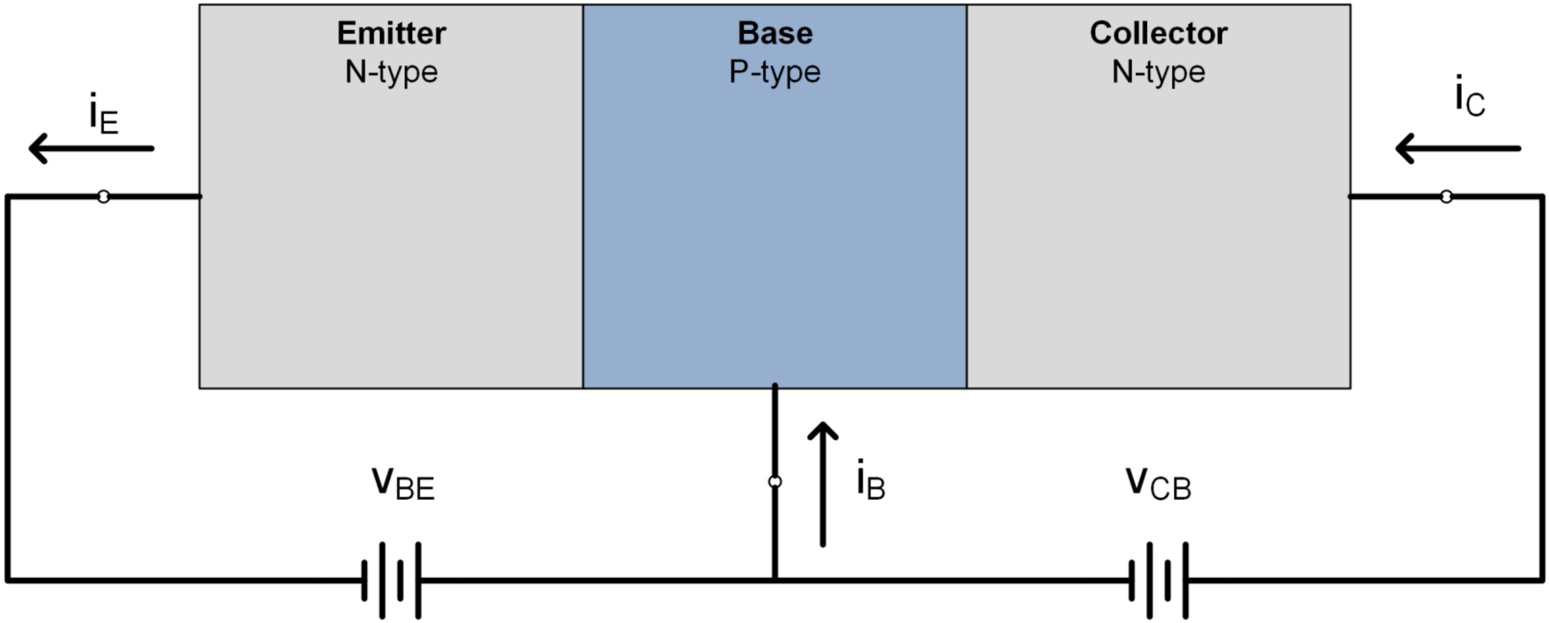


- ***Non-linear switch***
    - Open or closed switch between collector and emitter
    - Cutoff (open) and saturation (closed) regions
    - Switching large currents, e.g., controlling a fan
    - Digital logic



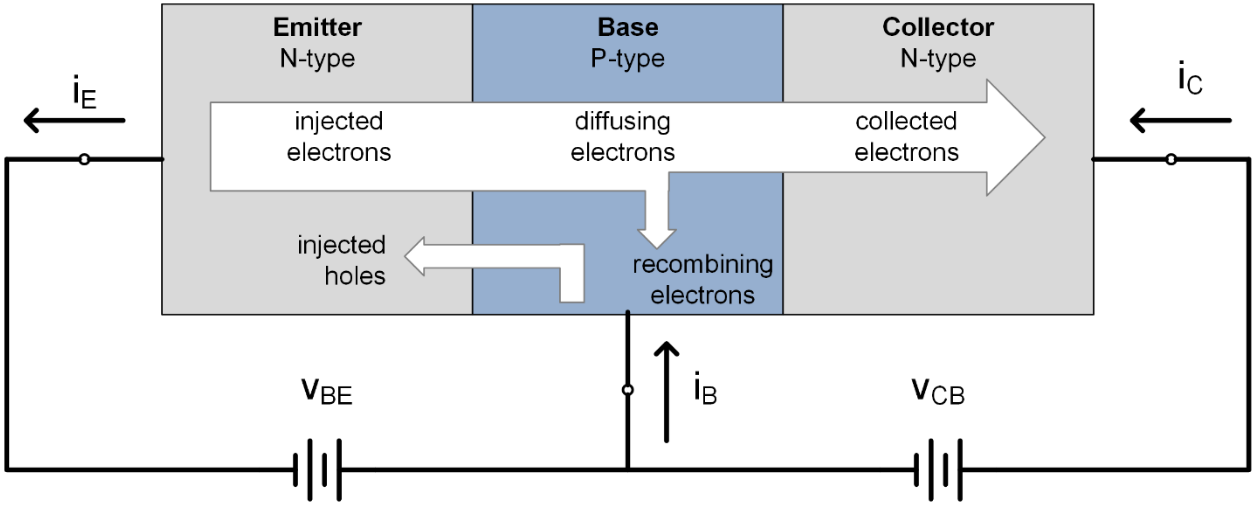
BJTs in the Forward-Active Region

Forward Active Region



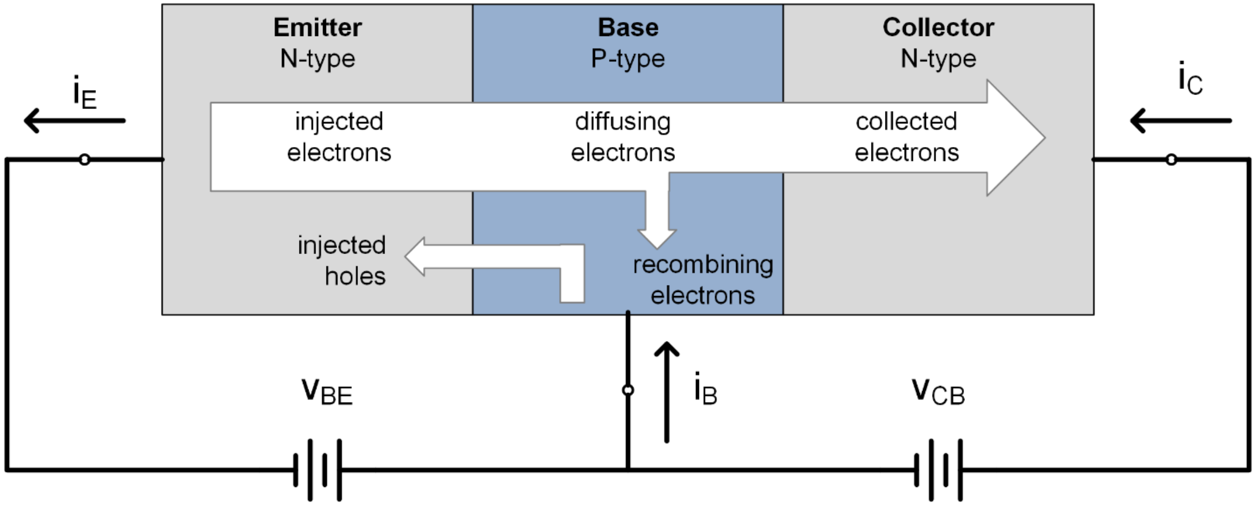
- B-E junction forward biased
  - Forward-biased P-N junction – current, i_E, will flow
- Reverse-biased C-B junction
  - Depletion region surrounding junction

Forward Active Region – Emitter Current



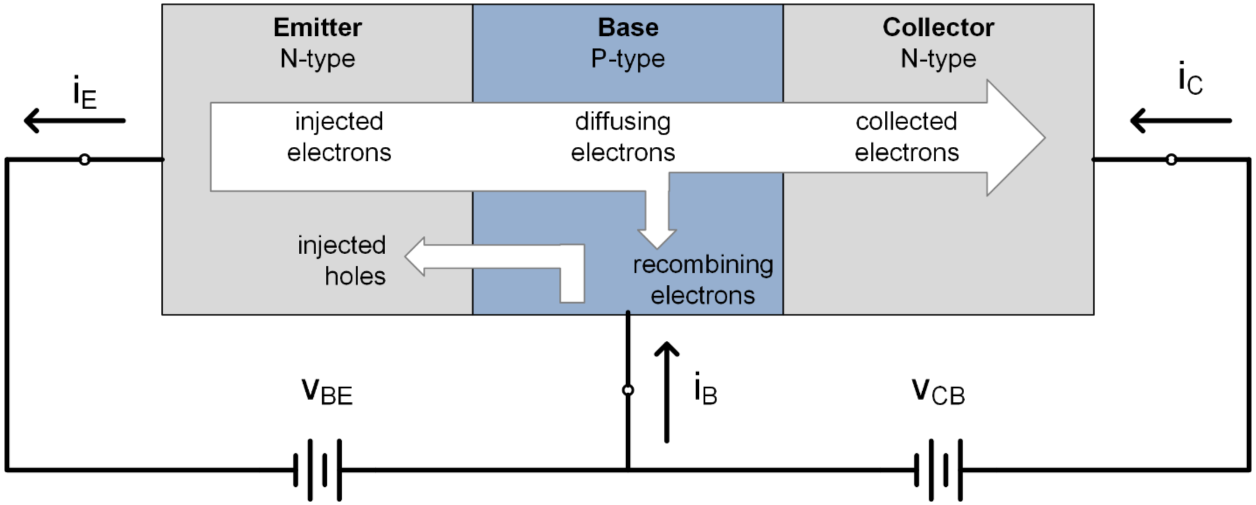
- Two components of emitter current, i_E:
  - Electrons injected (emitted) from the emitter into the base
    - Large – emitter is heavily-doped – large electron concentration
  - Holes injected from the base into the emitter
    - Small – base is lightly-doped – small hole concentration

Forward Active Region – Collector Current



- Electrons – minority carriers – injected from emitter into the base
  - Concentration gradient across base
    - Highest at emitter junction
    - Zero at the C-B depletion region
- Electrons ***diffuse*** from emitter to collector
  - Swept across C-B depletion region to be ***collected*** at the collector
  - This is ***collector current***, i_C

Forward Active Region – Base Current

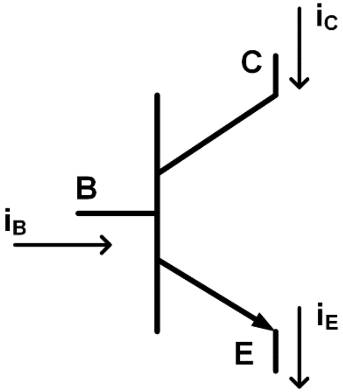


- Two components of base current, i_B
  - Holes injected from the base into the emitter
  - Holes recombining with diffusing electrons
- Both are small, due to lightly-doped base
  - ***Base current is relatively small***

Collector Current

- ***Collector current*** has an ***exponential*** dependence on base-emitter voltage:

$$\boxed{i_C = I_s e^{\frac{v_{BE}}{V_{th}}}}$$

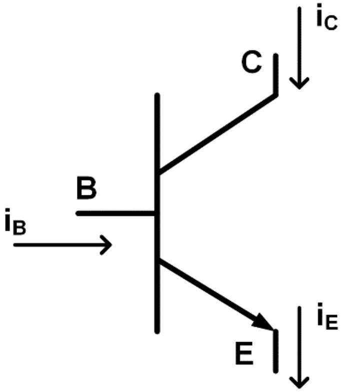


where:

  - I_s is the transistor's ***saturation*** or ***scale current***
  - $V_{th} = \frac{kT}{q}$ is the ***thermal voltage***

- Saturation current, I_s
  - ***Scales*** with emitter area
  - Strongly depends on temperature

Base Current



- ***Base current*** also has an ***exponential*** dependence on v_{BE}
- Much smaller than collector current
  - Base is lightly-doped
  - Hole currents are small
- Can express base current in terms of collector current:

$$\boxed{i_B = \frac{i_C}{\beta} = \frac{1}{\beta} I_s e^{\frac{v_{BE}}{V_{th}}}}$$

- β is the ***common-emitter current gain:***

$$\boxed{\beta = \frac{i_C}{i_B}}$$

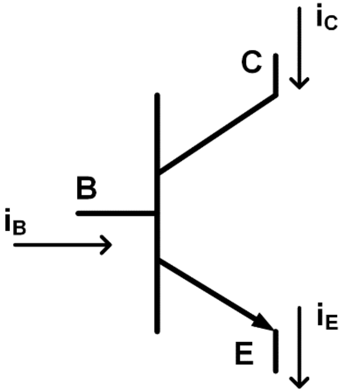
  - Typical β values: 50…200

Emitter Current

- Applying KCL gives the ***emitter current***:

$$i_E = i_C + i_B = i_C + \frac{1}{\beta} i_C$$

$$\boxed{\begin{aligned} i_E &= \frac{\beta + 1}{\beta} i_C = \frac{\beta + 1}{\beta} I_s e^{\frac{v_{BE}}{V_{th}}} \\ i_E &= \frac{1}{\alpha} i_C \end{aligned}}$$



- α is the ***common-base current gain:***

$$\boxed{\alpha = \frac{\beta}{\beta + 1} = \frac{i_C}{i_E}}$$

  - Typical α values: 0.98…0.998

BJT Current-Voltage Relationships

- A few key points:
- Base current is very small compared to collector and emitter currents

$$i_B \ll i_E,\ i_B \ll i_C$$

- Collector current and emitter current are approximately equal

$$i_C \approx i_E$$

- Small changes in i_B yield large changes in i_C

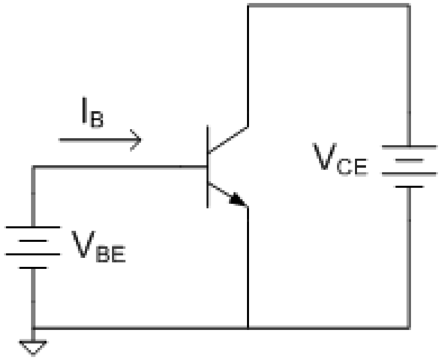
$$i_C = \beta i_B$$

- i_C is exponentially dependent on v_{BE}
  - Small changes in v_{BE} yield large changes in i_C
  - A potential (transconductance) amplifier
- Collector current is independent of collector voltage (v_{CE})

Plotting I-V Characteristics

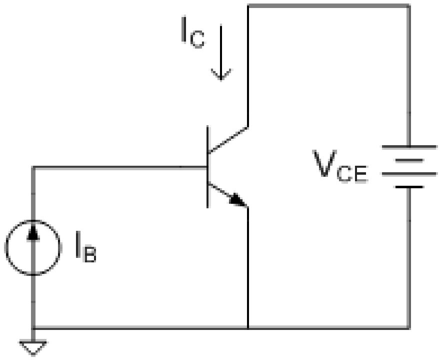
- BJTs are three-terminal devices
- Plot two different I-V characteristics:
  - ***Input I-V characteristic***
  - ***Output I-V characteristic***

Input I-V Characteristic:



- Single curve:
  - i_B (or i_C) as a function of v_{BE}

Output I-V Characteristic:



- Family of curves:
  - i_C as a function of v_{CE} ***parameterized*** by i_B (or v_{BE})

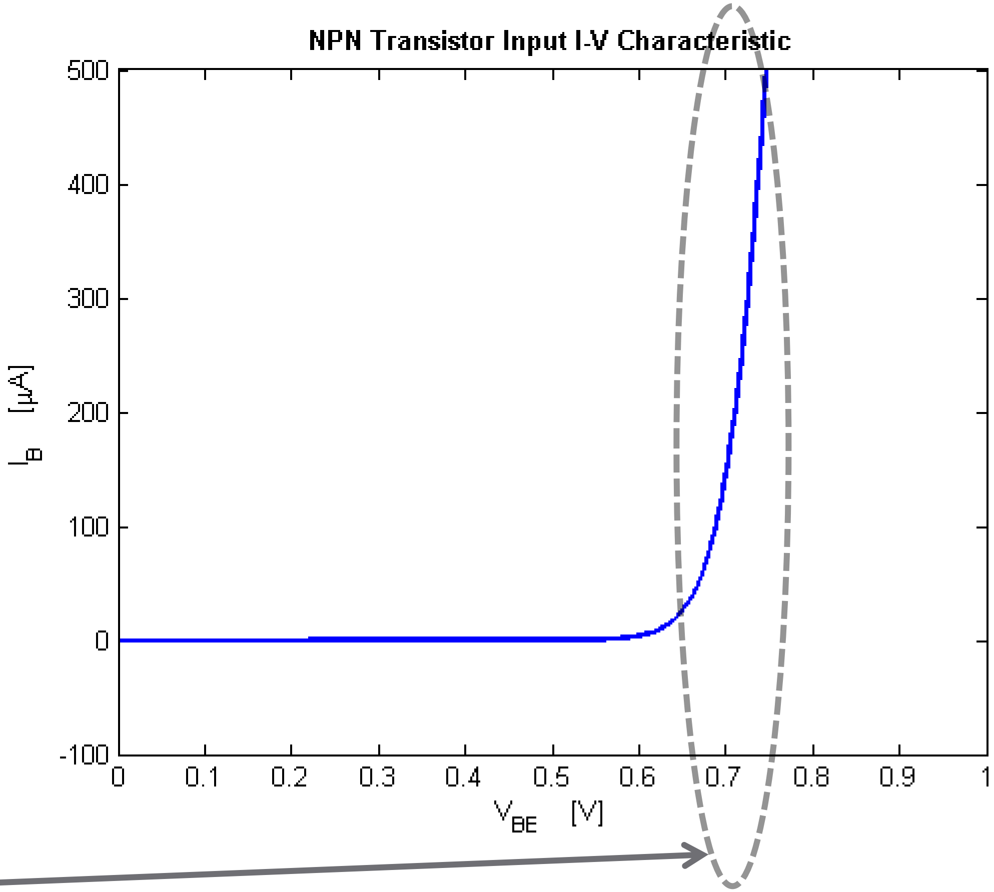
Input I-V Characteristic

- Governed by a form of the Shockley equation:

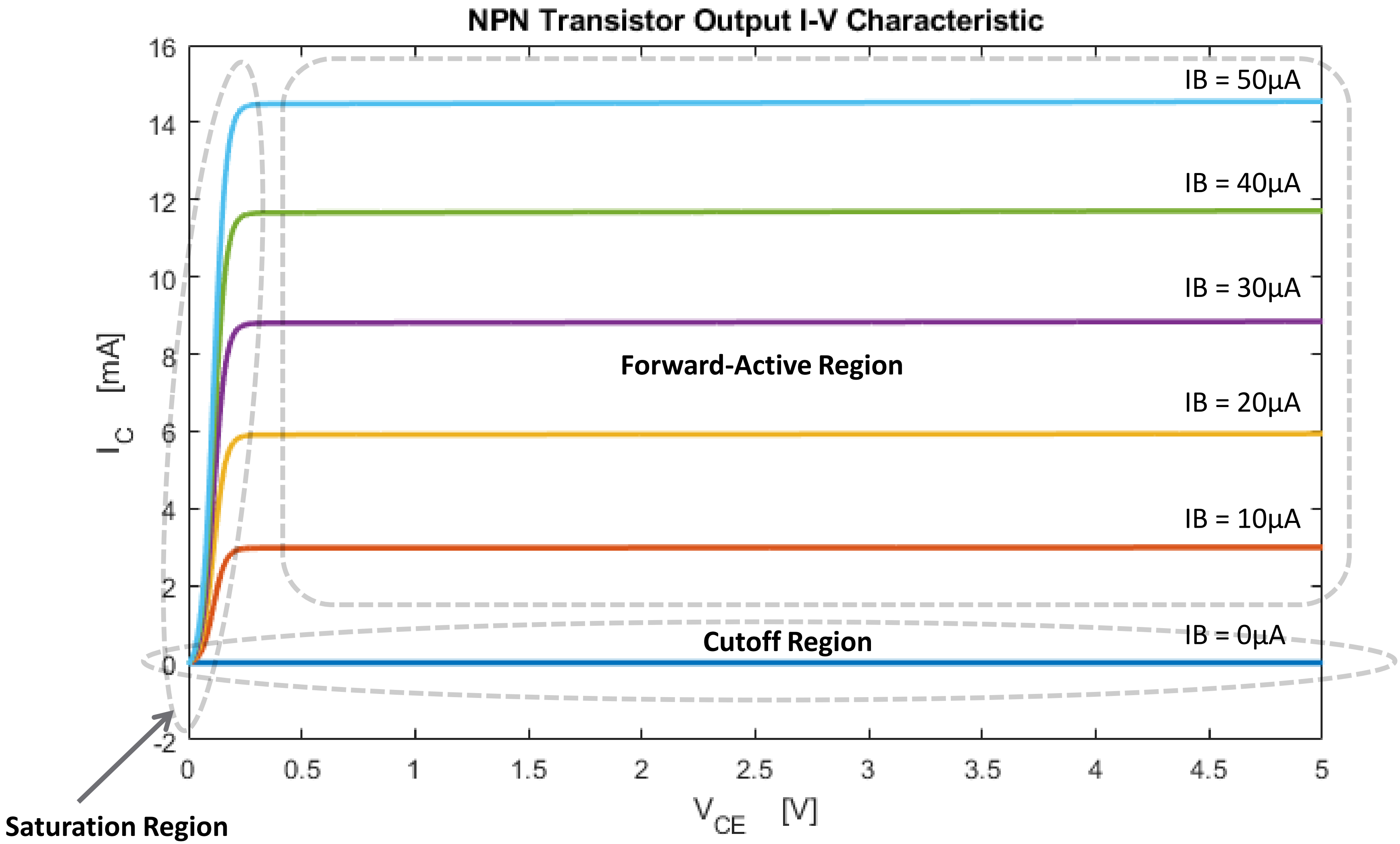
$$\boxed{i_B = \frac{1}{\beta} I_s e^{\frac{v_{BE}}{V_{th}}}}$$

- In the forward-active region:

$$V_{BE} \approx 700\ mV$$



BJT Output I-V Characteristic



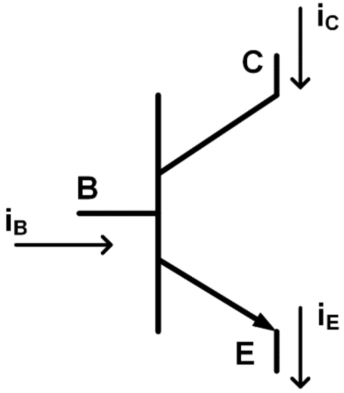
Forward-Active Region

Cutoff Region

Saturation Region

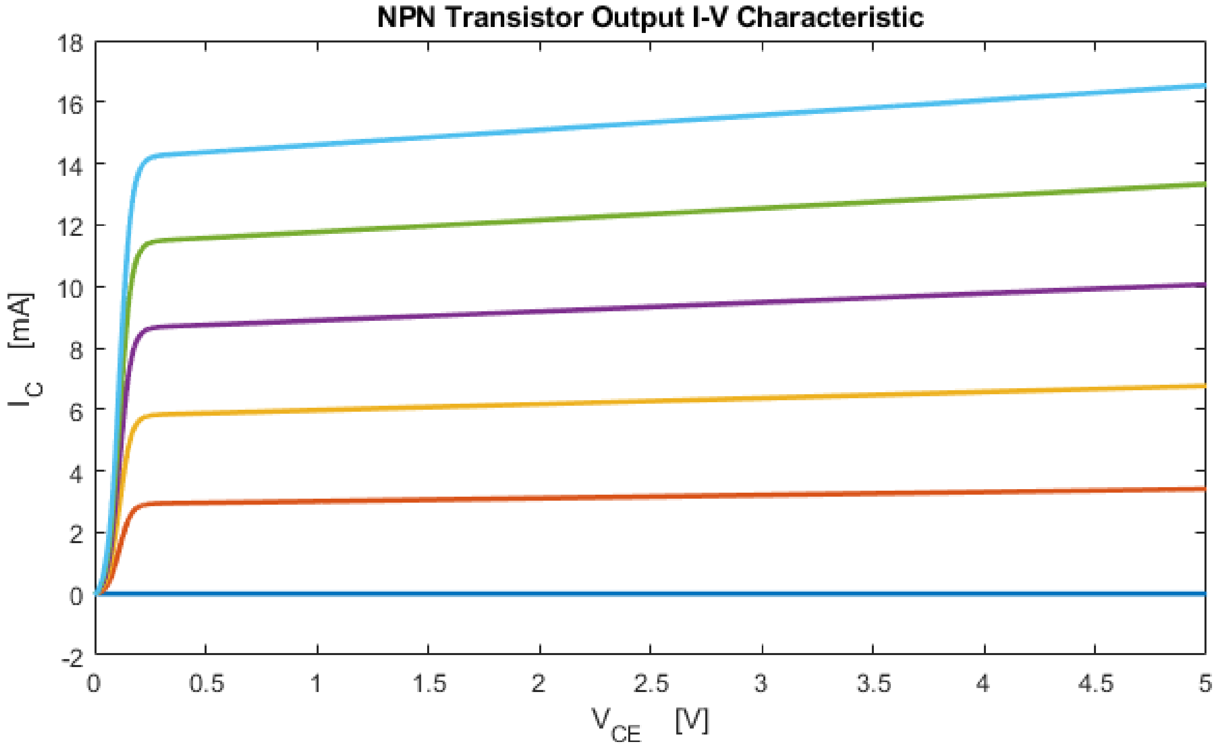
Active Region I-V Relationships – Summary

- $i_C = I_s e^{\frac{v_{BE}}{V_{th}}}$
- $i_B = \frac{i_C}{\beta} = \frac{1}{\beta} I_s e^{\frac{v_{BE}}{V_{th}}}$
- $i_E = \frac{1}{\alpha} i_C = \frac{1}{\alpha} I_s e^{\frac{v_{BE}}{V_{th}}}$
- $\beta = \frac{i_C}{i_B}, \quad \alpha = \frac{i_C}{i_E}$
- $\beta = \frac{\alpha}{1-\alpha}, \quad \alpha = \frac{\beta}{\beta+1}$



The Early Effect

- Our simple qualitative description and models for the BJT so far yield the I-V curves on the previous page
  - Collector current, i_C, is independent of v_{CE}
- Really, we observe something like this:

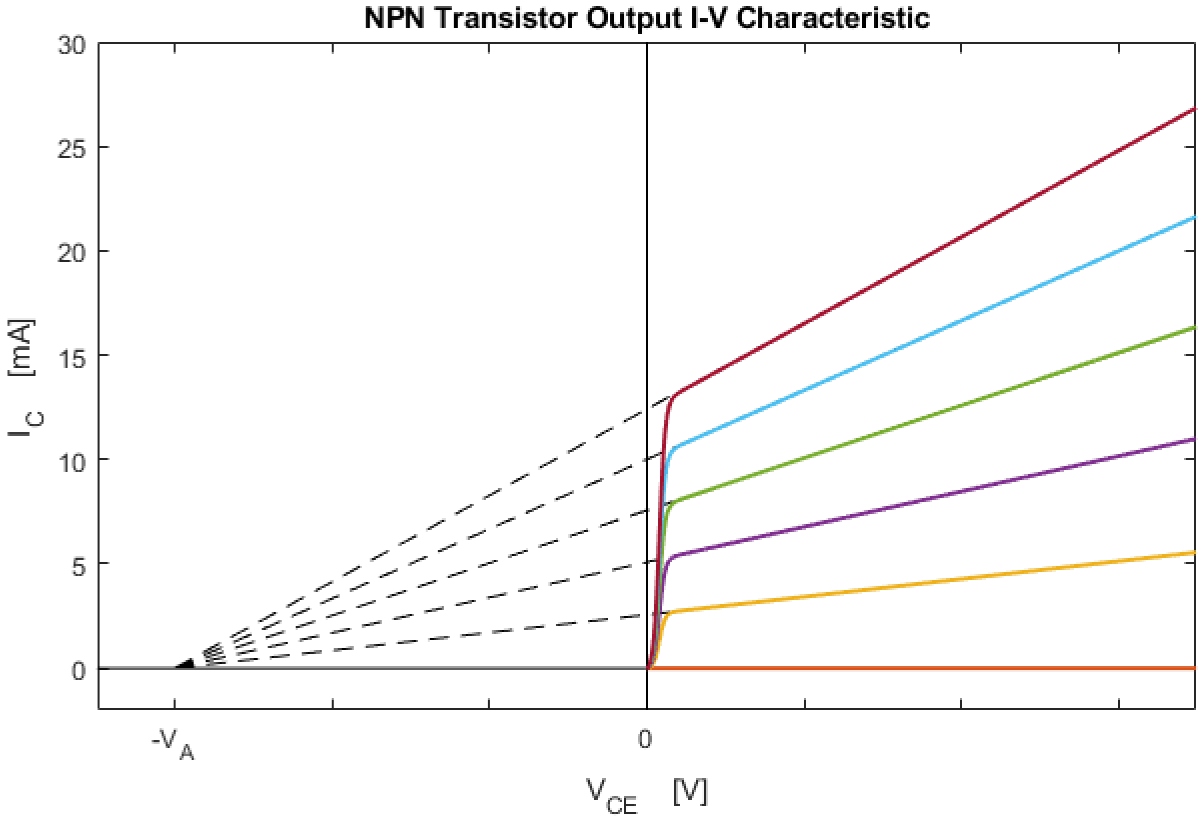


The Early Effect

- As v_{CE} increases:
  - C-B junction gets more reverse-biased
  - C-B depletion region increases
  - ***Effective base width shrinks***
- Saturation current, I_S, is inversely proportional to base width
- As base width shrinks:
  - Saturation current increases
  - Collector current, i_C, increases
- This is known as the ***Early effect*** or ***base-width modulation***

The Early Effect

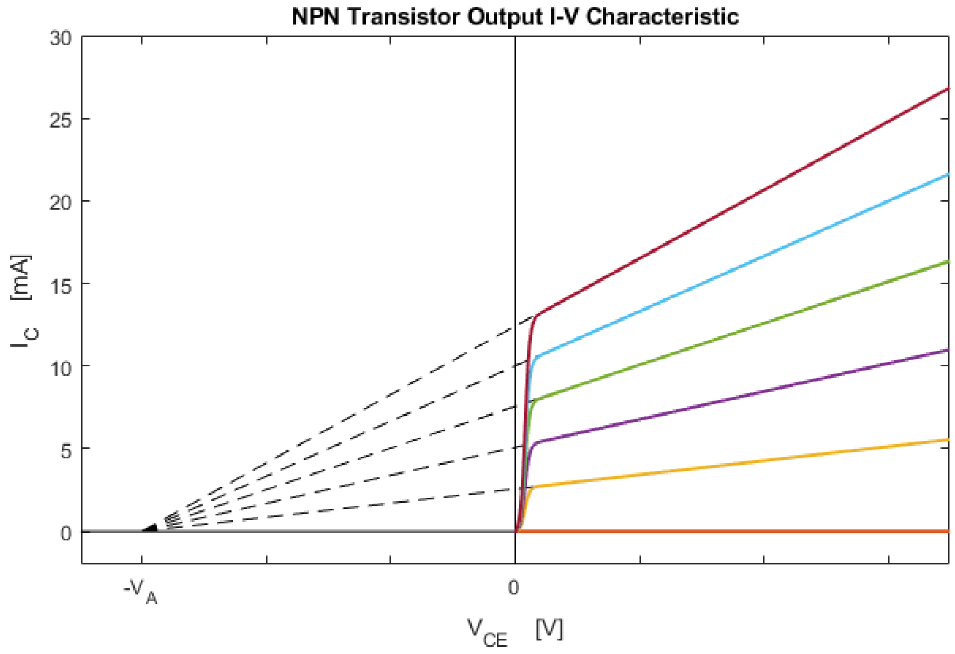
- Extrapolations of the forward-active regions of the I-V curves intersect at a single point on the V_{CE} axis
  - The ***Early voltage***, V_A



The Early Effect

- We can adjust our collector-current model to account for the Early effect:

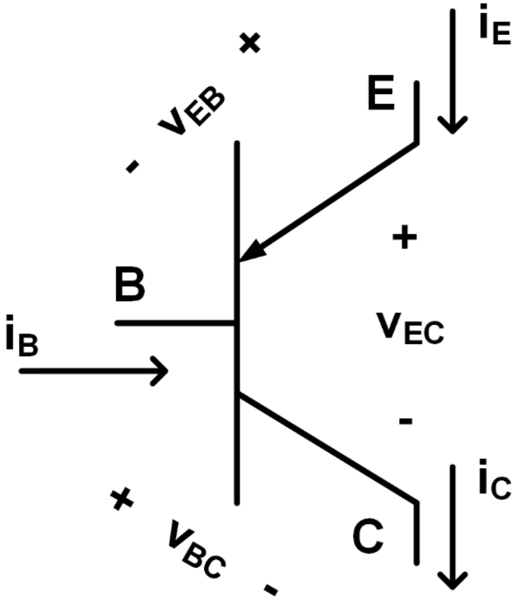
$$\boxed{i_C = I_s e^{\frac{v_{BE}}{V_{th}}}\left(1 + \frac{v_{CE}}{V_A}\right)}$$



PNP Transistors

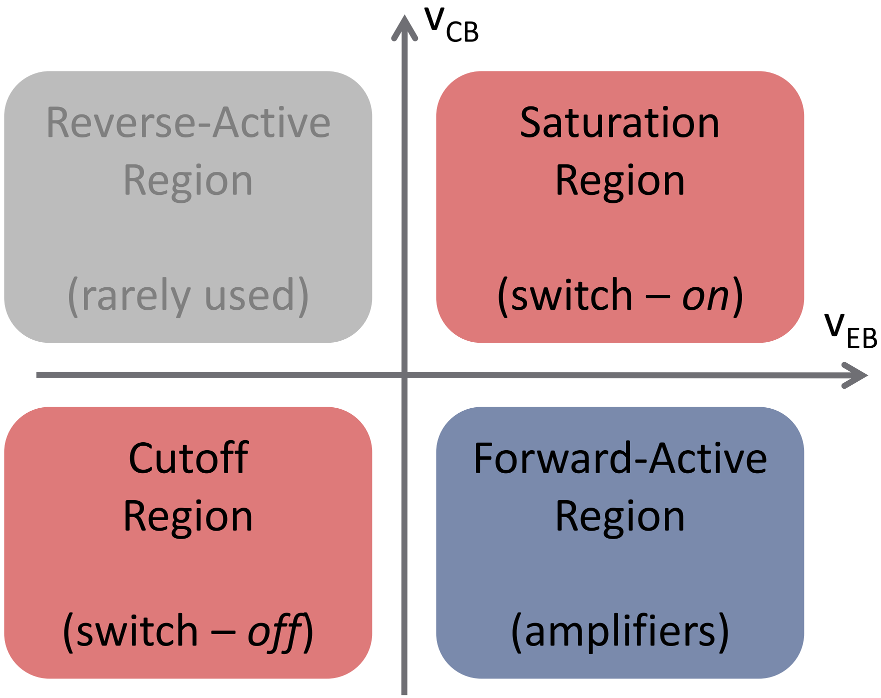
PNP Transistors

- Our focus has been on NPN transistors
- With the necessary adjustments, all descriptions and models apply equally to PNP transistors
- A PNP biased in the forward-active region:
  - E-B junction forward-biased
    - $v_{EB} > 0$
  - C-B junction reverse-biased
    - $v_{BC} > 0$



BJT Operating Regions – PNP

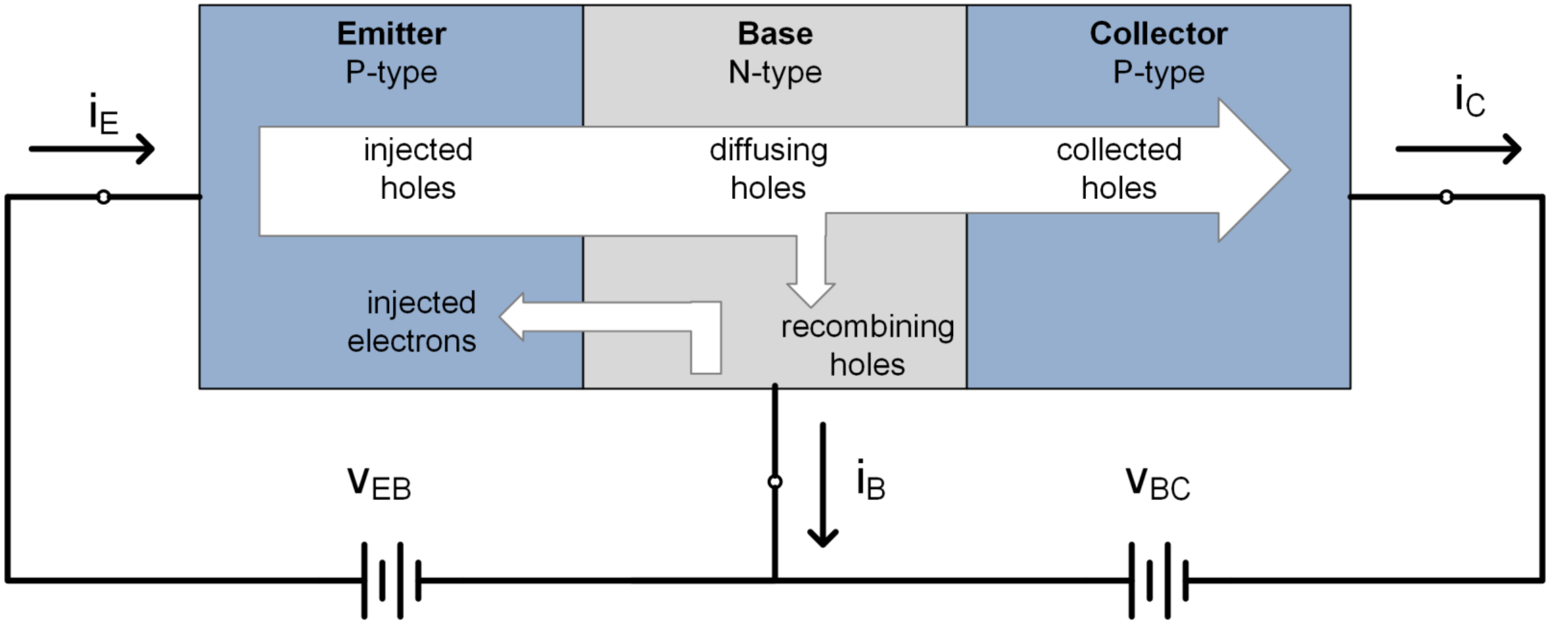
- Junction voltages change signs for PNPs
  - $v_{BC} \rightarrow v_{CB}$
  - $v_{BE} \rightarrow v_{EB}$



- ***Forward active region***
  - $v_{CB} < 0$
  - $v_{EB} > 0$
  - Linear region – ***amplifiers***
- ***Saturation region***
  - $v_{CB} > 0$
  - $v_{EB} > 0$
  - E-C looks like a ***closed switch***
- ***Cutoff region***
  - $v_{CB} < 0$
  - $v_{EB} < 0$
  - E-C looks like an ***open switch***

PNP Transistors

- NPN current primarily due to diffusion of ***minority carriers*** – ***electrons*** – across the base region
- Same is true in PNP transistors, but now the minority carriers are ***holes***



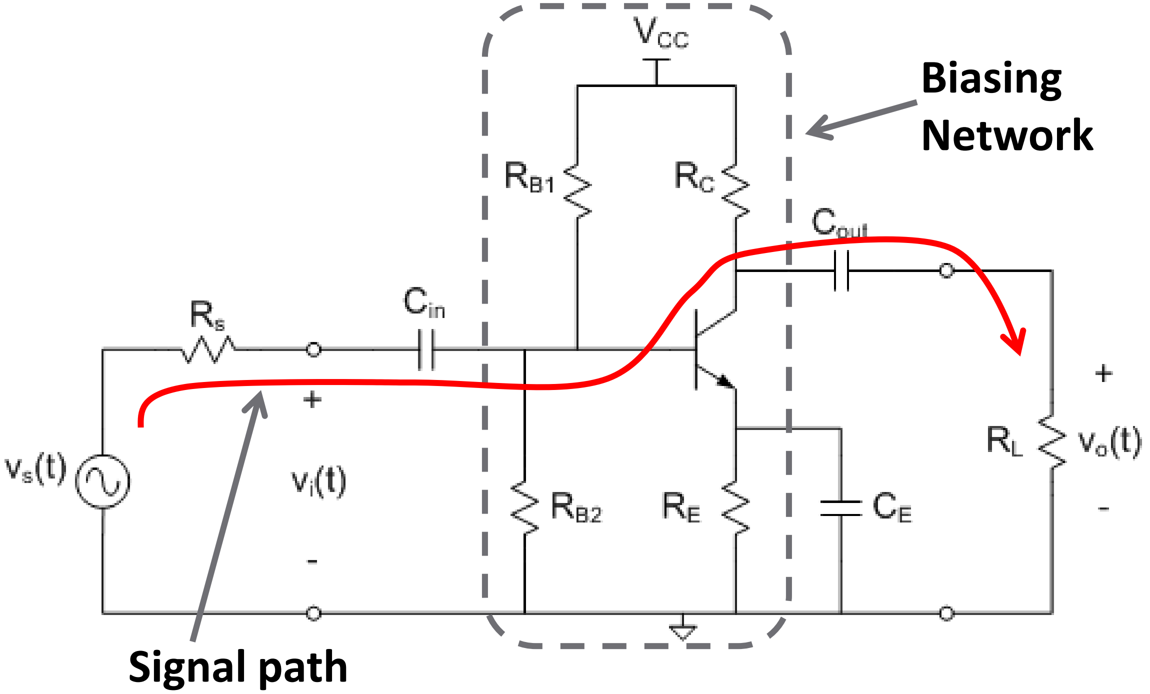
Equivalent Circuit Models

Equivalent Circuit Models

- We have seen that BJTs have an exponential current-voltage relationship
  - Small inputs, v_{BE}, can produce large outputs, i_C
  - BJTs are useful as ***amplifiers***
- Our goal is the analysis and design of ***transistor amplifier circuits***
- To do so, we first need ***equivalent circuit models*** for the transistors

BJT Amplifier Circuits

- Two functional pieces of a BJT amplifier:
  - ***Bias network***
    - Sets the DC operating point of the transistor
    - Ensures the BJT remains in the forward-active region
  - ***Signal path***
    - Sets the gain of the amplifier circuit
- Significant overlap between the two parts



Biasing Network

Signal path

Equivalent Circuit Models

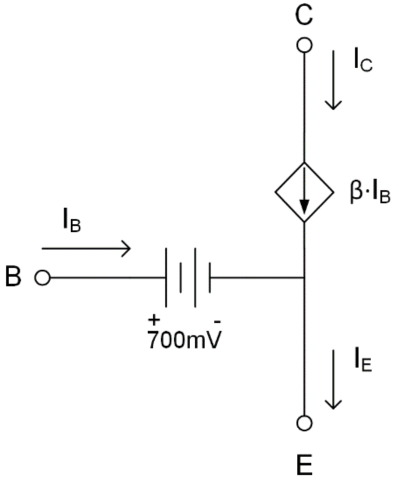
- We use two types of equivalent-circuit transistor models:
- ***Large-signal model***
  - Models the transistor’s behavior to DC signals
  - Used to determine the transistor’s DC operating point
- ***Small-signal model***
  - Models the behavior in response to small signals (w.r.t V_{th})
  - Describes the response to the AC signals to be amplified
  - Properties of the small-signal model determined by the DC operating point

Large-Signal Models

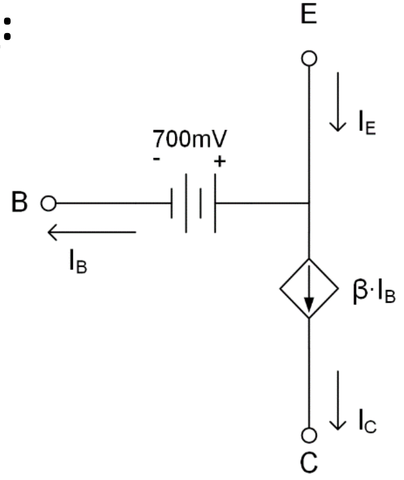
Large-Signal Model – Forward-Active

- Large-signal behavior in the forward-active region modeled by the following circuits:

NPN:



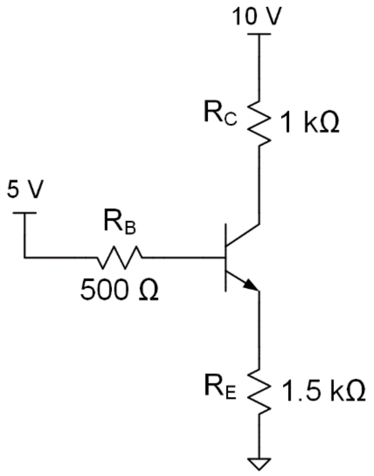
PNP:



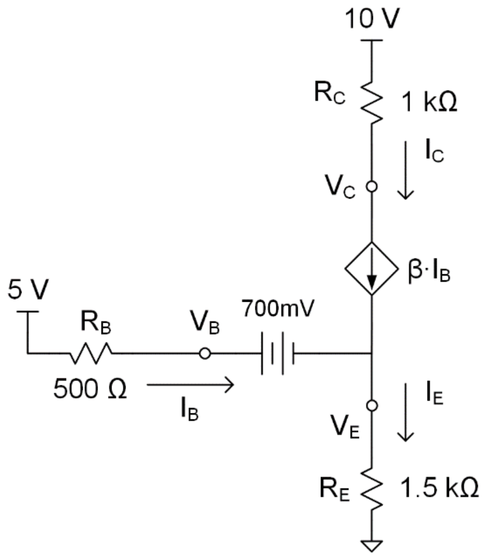
- Replace the transistor with the appropriate model to determine the DC operating point (Q-point)
- Forward-active-region bias assumed
  - If incorrect, model will say otherwise
- Note the use of upper-case I/V, and subscripts for DC signals

DC Operating Point – Example 1

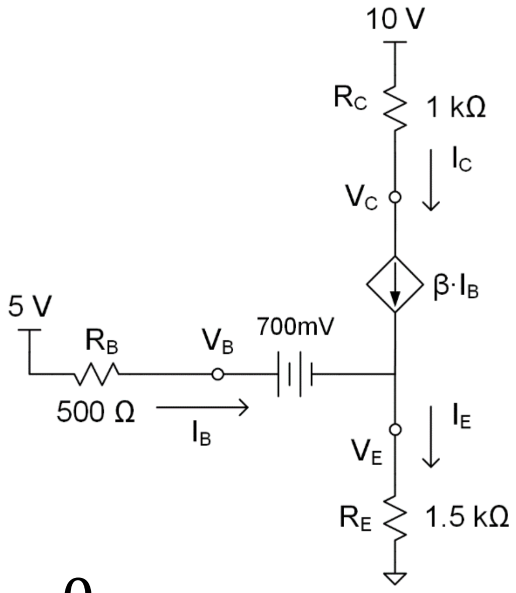
- Determine the DC operating point (i.e., all terminal voltages and currents) for the following transistor
  - Assume $\beta = 100$



- First, replace transistor with its large-signal equivalent-circuit model



DC Operating Point – Example 1



- Apply KVL around the B-E loop:

$$5\,V - I_B R_B - 700\,mV - I_E R_E = 0$$

- Express emitter current in terms of base current

$$I_E = (\beta + 1) I_B$$

$$5\,V - I_B R_B - 700\,mV - I_B(\beta + 1) R_E = 0$$

- Solve for base current

$$I_B[R_B + (\beta + 1)R_E] = 4.3\,V$$

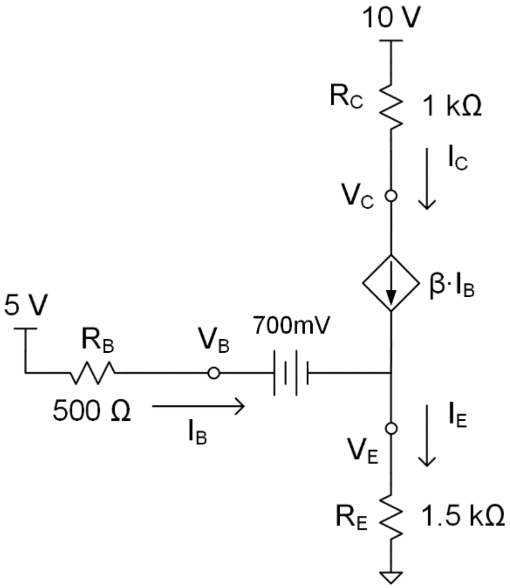
$$I_B = \frac{4.3\,V}{[R_B + (\beta + 1)R_E]}$$

DC Operating Point – Example 1

- The base current is

$$I_B = \frac{4.3\ V}{[R_B + (\beta + 1)R_E]}$$

$$I_B = \frac{4.3\ V}{500\Omega + 101 \cdot 1.5\ k\Omega} = 28.29\ \mu A$$



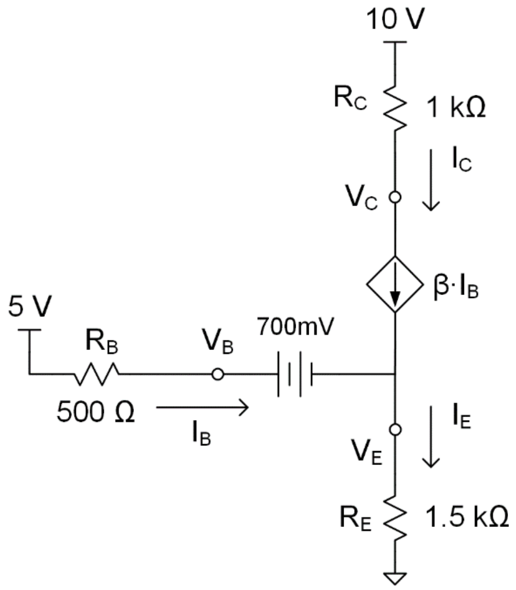
- Calculate I_C and I_E from I_B

$$I_C = \beta I_B = 100 \cdot 28.29\ \mu A = 2.829\ mA$$

$$I_E = (\beta + 1)I_B = 101 \cdot 28.29\ \mu A = 2.857\ mA$$

$$\boxed{I_B = 28.29\ \mu A,\ I_C = 2.829\ mA,\ I_E = 2.857\ mA}$$

DC Operating Point – Example 1



- Use terminal currents to calculate terminal voltages

$$V_B = 5\,V - I_B R_B$$

$$V_B = 5\,V - 28.29\,\mu A \cdot 500\,\Omega$$

$$\boxed{V_B = 4.986\,V}$$

- Collector voltage

$$V_C = 10\,V - I_C R_C = 10\,V - 2.829\,mA \cdot 1\,k\Omega$$

$$\boxed{V_C = 7.171\,V}$$

- Emitter voltage

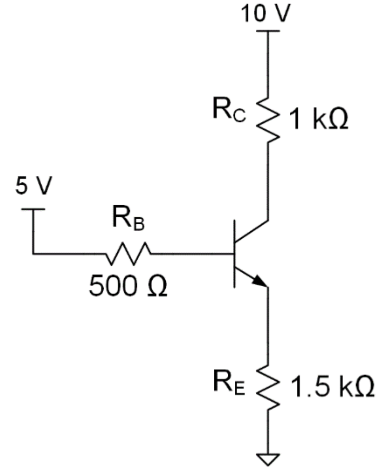
$$V_E = I_E R_E = 2.857\,mA \cdot 1.5\,k\Omega$$

$$\boxed{V_E = 4.286\,V}$$

DC Operating Point – Example 1

- We now know the complete DC operating point for the transistor
  - All voltages and currents

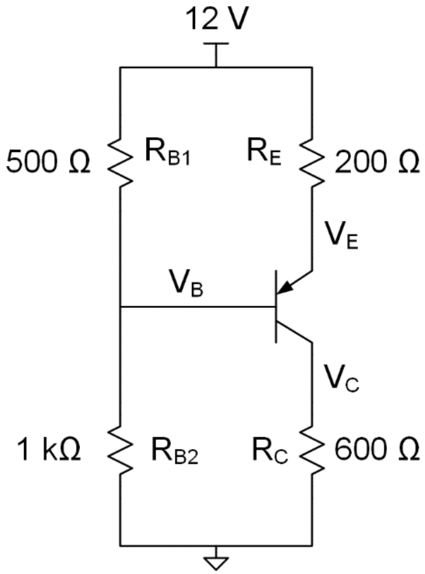
$$
\begin{aligned}
I_B &= 28.29\ \mu A \qquad & V_B &= 4.986\ V \\
I_C &= 2.829\ mA \qquad & V_C &= 7.171\ V \\
I_E &= 2.857\ mA \qquad & V_E &= 4.286\ V
\end{aligned}
$$

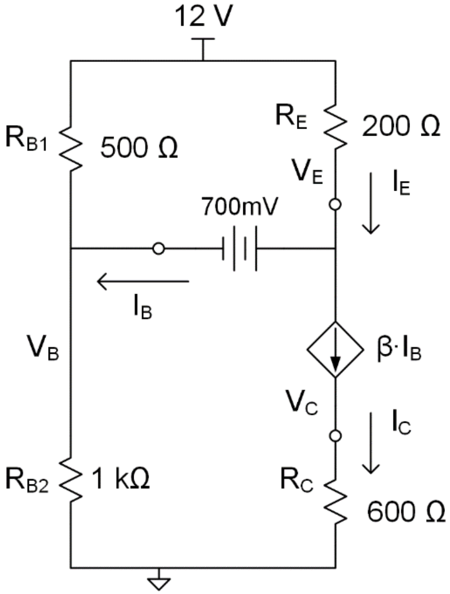


- Use of this model assumed forward-active region operation – verify
  - $V_B > V_E, V_{BE} > 0$
  - $V_C > V_B, V_{BC} < 0$
  - Transistor *is* biased in the forward active region

DC Operating Point – Example 2

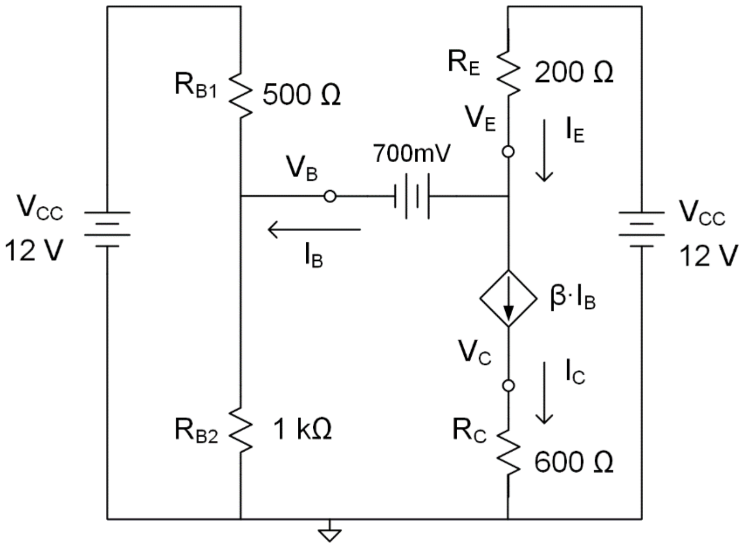
- Next, find the DC operating point of the following PNP transistor
- Again, begin by replacing the transistor with its large-signal equivalent circuit:

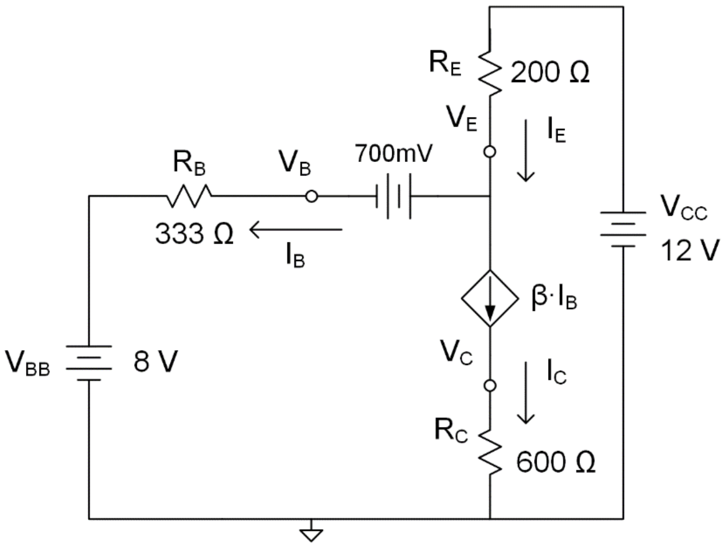




DC Operating Point – Example 2

- We can simplify the circuit by replacing the base bias circuit with its Thevenin equivalent





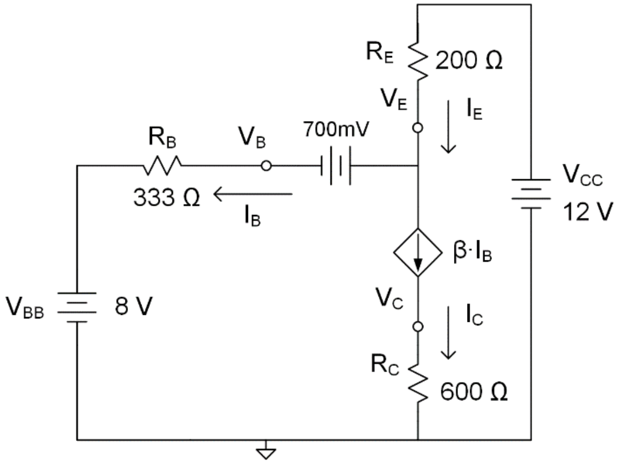
$$V_{BB} = V_{CC}\frac{R_{B2}}{R_{B1}+R_{B2}} = 12\,V\,\frac{1\,k\Omega}{500\,\Omega + 1\,k\Omega} = 8\,V$$

$$R_B = \frac{R_{B1}R_{B2}}{R_{B1}+R_{B2}} = \frac{500\,\Omega \cdot 1\,k\Omega}{500\,\Omega + 1\,k\Omega} = 333.3\,\Omega$$

DC Operating Point – Example 2

- Apply KVL around the B-E loop:

$$V_{BB} + I_B R_B + 700\,mV + I_E R_E - V_{CC} = 0$$



- Express emitter current in terms of base current

$$I_E = (\beta + 1) I_B$$

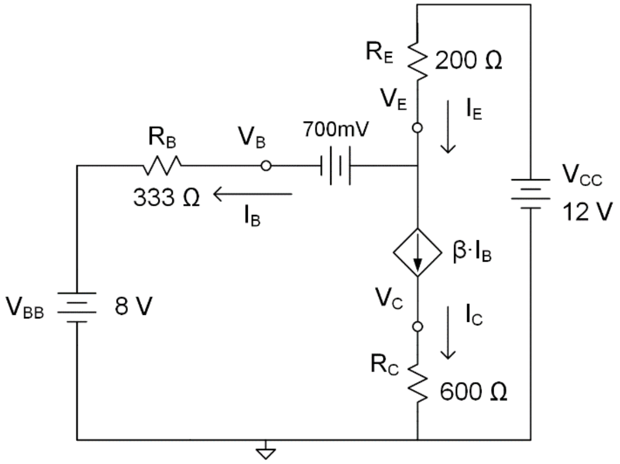
$$V_{BB} + I_B R_B + 700\,mV + I_B(\beta + 1) R_E - V_{CC} = 0$$

- Solve for base current

$$I_B[R_B + (\beta + 1) R_E] = V_{CC} - V_{BB} - 700\,mV$$

$$I_B = \frac{V_{CC} - V_{BB} - 700\,mV}{[R_B + (\beta + 1) R_E]}$$

DC Operating Point – Example 2



- The base current is

$$I_B = \frac{V_{CC} - V_{BB} - 700\ mV}{[R_B + (\beta + 1)R_E]}$$

$$I_B = \frac{3.3\ V}{333\Omega + 101 \cdot 200\ \Omega} = 160.7\ \mu A$$

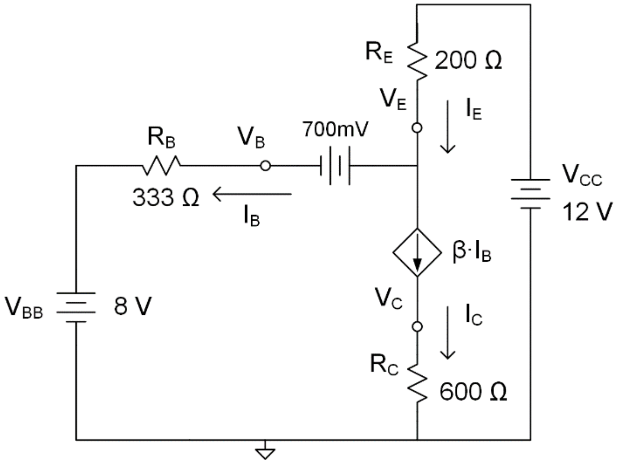
- Calculate I_C and I_E from I_B

$$I_C = \beta I_B = 100 \cdot 160.7\ \mu A = 16.07\ mA$$

$$I_E = (\beta + 1) I_B = 101 \cdot 160.7\ \mu A = 16.23\ mA$$

$$\boxed{I_B = 160.7\ \mu A,\ I_C = 16.07\ mA,\ I_E = 16.23\ mA}$$

DC Operating Point – Example 2



- Base voltage

$$V_B = V_{BB} + I_B R_B$$

$$V_B = 8\ V + 160.7\ \mu A \cdot 333\ \Omega$$

$$\boxed{V_B = 8.054\ V}$$

- Collector voltage

$$V_C = I_C R_C = 16.07\ mA \cdot 600\ \Omega$$

$$\boxed{V_C = 6.429\ V}$$

- Emitter voltage

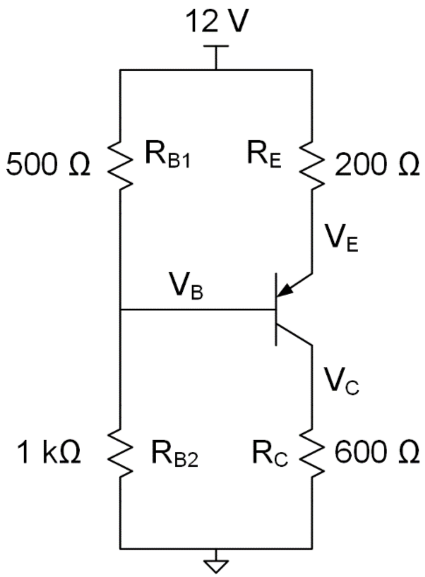
$$V_E = V_{CC} - I_E R_E = 12\ V - 16.23\ mA \cdot 200\ \Omega$$

$$\boxed{V_E = 8.754\ V}$$

DC Operating Point – Example 2

- The DC operating point:

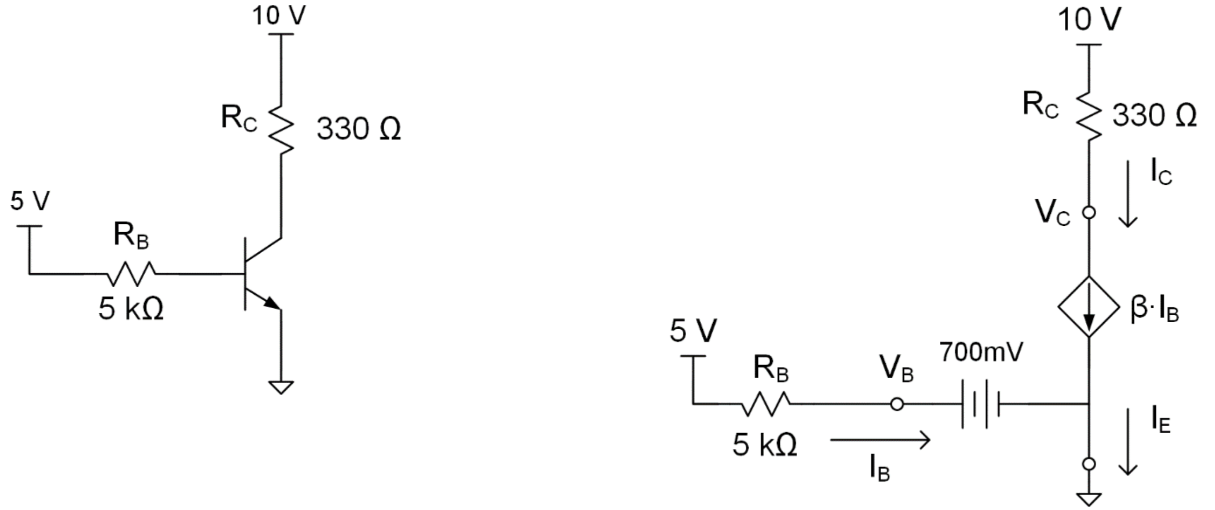
$$\boxed{\begin{array}{ll} I_B = 160.7\,\mu A & V_B = 8.054\,V \\ I_C = 16.07\,mA & V_C = 6.429\,V \\ I_E = 16.23\,mA & V_E = 8.754\,V \end{array}}$$



- Terminal voltages confirm that the transistor is biased in the forward active region:
  - $V_B < V_E, V_{EB} > 0$
  - $V_C < V_B, V_{CB} < 0$

DC Operating Point – Example 3

- Next, find the DC operating point for the following circuit



- Here, the emitter is grounded, so base current is given by

$$I_B = \frac{5\ V - 700\ mV}{R_B} = \frac{4.3\ V}{5\ k\Omega}$$

$$I_B = 860\ \mu A$$

DC Operating Point – Example 3

- Use β to get collector and emitter currents:

$$I_C = \beta I_B = 100 \cdot 860\ \mu A = 86\ mA$$

$$I_E = (\beta + 1) I_B = 101 \cdot 860\ \mu A = 86.9\ mA$$

- Base and emitter voltages are known:

$$V_E = 0\ V$$

$$V_B = 700\ mV$$

- The collector voltage:

$$V_C = 10\ V - I_C R_C = 10\ V - 86\ mA \cdot 330\ \Omega$$

$$V_C = -18.4\ V\ \text{(!)}$$

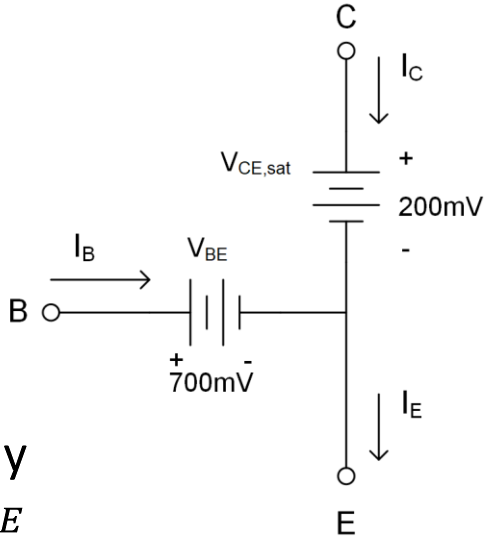
- B-C junction is not reverse biased
  - Transistor is not in the forward active region - ***saturated***
  - Inappropriate large-signal model used
  - Need to use a model for the saturation region

Saturation/Cutoff Region Models

- Transistors used as ***switches*** operate alternately in the ***saturation*** (closed) and ***cutoff*** (open) regions
- Equivalent circuit models:

Saturation Region:

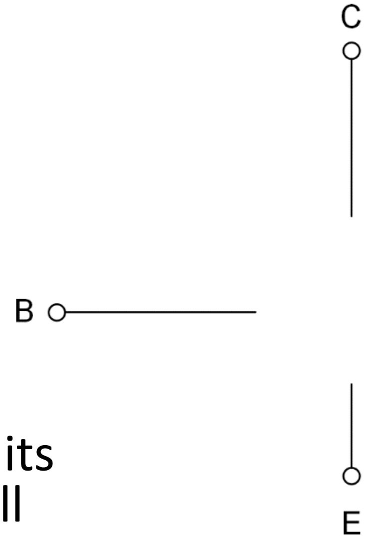
- Both junctions forward biased
- Small, nearly constant V_{CE}



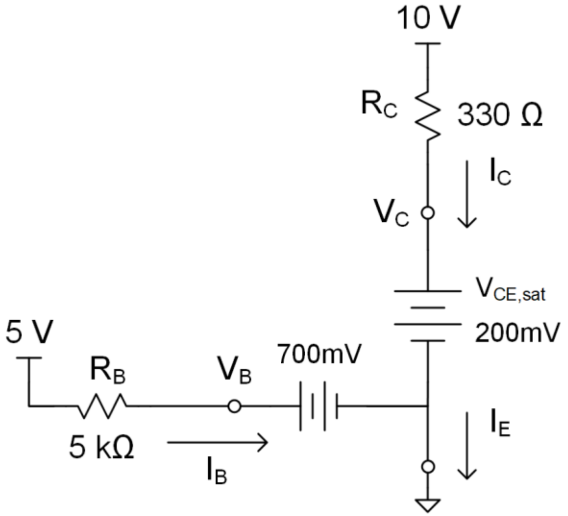
$$V_{CE,sat} \approx 200\ mV$$

Cutoff Region:

- Both junctions reverse biased
- Open circuits between all terminals



DC Operating Point – Example 3



- Returning to our example, now use the transistor model for saturation
- Base current does not change:

$$I_B = \frac{4.3\ V}{5\ k\Omega} = 860\ \mu A$$

- But, collector current is now:

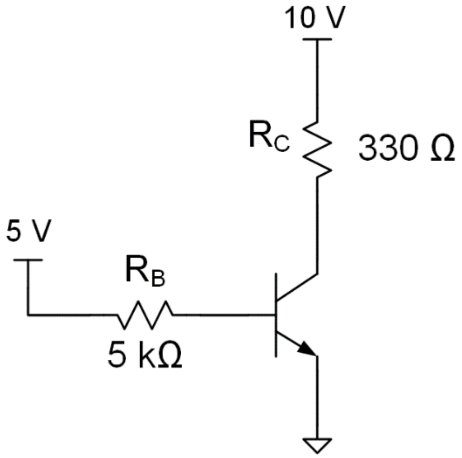
$$I_C = \frac{10\ V - V_{CE,sat}}{R_C} = \frac{9.8\ V}{330\ \Omega}$$

$$I_C = 29.7\ mA$$

  - Much less than would be predicted by β
  - β does not apply in the saturation region

DC Operating Point – Example 3

- The transistor is in the saturation region
  - Looks like a ***closed switch*** between collector and emitter



- Operating point:

$$I_B = 860\ \mu A \qquad V_B = 700\ mV$$
$$I_C = 29.7\ mA \qquad V_C = 200\ mV$$
$$I_E = 30.6\ mA \qquad V_E = 0\ V$$

Large-Signal Models – Early Effect

- We have seen that, due to ***base-width modulation***, or the ***Early effect***, I_C is dependent on V_{CE}
- We can modify the large-signal model to account for this
- Add ***output resistance***, r_o

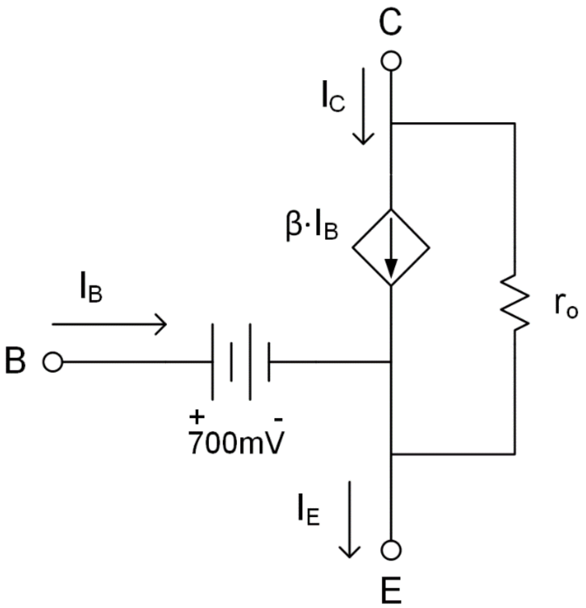
$$r_o = \frac{V_A}{I_C'}$$

where

  - V_A is the Early voltage
  - I_C' is the collector current with the Early effect neglected

$$I_C' = I_s e^{\frac{V_{BE}}{V_{th}}}$$

  - No longer an ideal current source

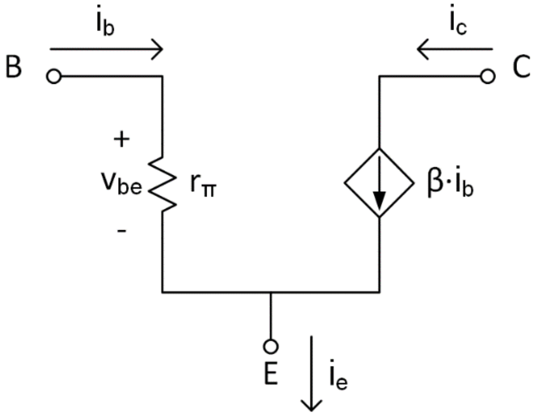


Small-Signal Models

BJT – Small-Signal Models

- Large-signal model allows us to determine the ***DC operating point***
  - DC terminal voltages and currents
- Remember, our objective is to ***amplify small signals***
  - Want to know how a transistor circuit responds to small AC signals
  - Need a ***small-signal model***
- ***Small-signal behavior is defined by the DC operating point***
  - Values of small-signal model parameters determined by the DC operating point
  - DC bias applied to the transistor serves to put the transistor in a state where it will behave as an amplifier
- We will look at two small-signal models:
  - Hybrid-π model
  - T model

Hybrid-π Model – β



- Two variations of the hybrid-π model
  - First one uses a ***CCCS***
- Same collector current relationship as in large-signal model:

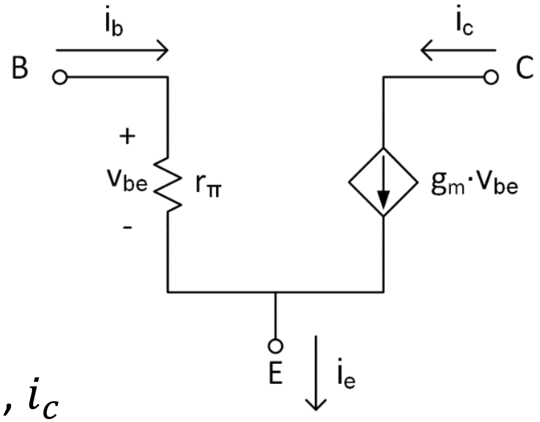
$$i_c = \beta i_b$$

- Base input resistance, r_π, set by the DC collector current:

$$r_\pi = \frac{\beta V_{th}}{I_C}$$

- Note that we now use ***small-signal notation***
  - Lower-case i and v and lower-case subscripts
  - These denote ***AC signals***

Hybrid-π Model – g_m



- Second hybrid-π model uses a ***VCCS***
- ***Transconductance*** parameter sets collector current as a function of v_{be}

$$i_C = g_m v_{be}$$

- ***Transconductance***, g_m
  - Voltage one place, v_{be}, causes current somewhere else, i_c
  - Determined by DC collector current

$$g_m = \frac{I_C}{V_{th}} = \frac{I_C q}{kT} \approx \frac{I_C}{26\ mV}$$

- g_m is related to β:

$$g_m = \frac{i_c}{v_{be}} = \frac{i_c}{i_b r_\pi} = \frac{\beta}{r_\pi}$$

- Note that, while g_m and r_π are dependent on bias, β is not
  - β is a device property

T-Model

- An alternative to the hybrid-π model is the T-model
  - Simplifies the analysis of some circuits
  - Again, either with a CCCS, using β, or a VCCS, using g_m
- Here, the resistance is the emitter resistance, r_e

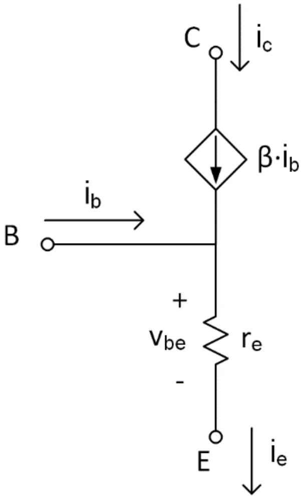
$$r_e = \frac{V_{th}}{I_E} = \frac{\alpha}{g_m} \approx \frac{1}{g_m}$$

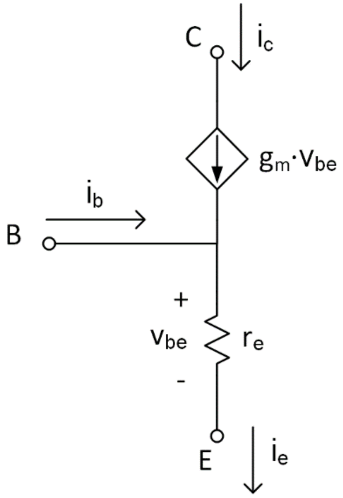
- Note the relationship to r_π

$$r_e = \frac{\alpha}{g_m} = \frac{\beta}{\beta+1} \cdot \frac{1}{g_m}$$

$$r_e = \frac{1}{\beta+1} \cdot \frac{\beta}{g_m} = \frac{r_\pi}{\beta+1}$$

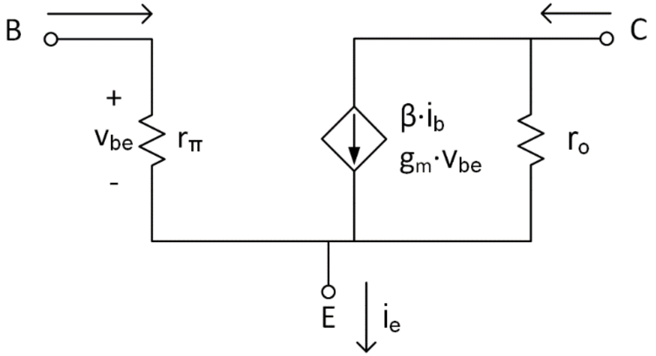
  - The same resistance referred to one terminal or the other

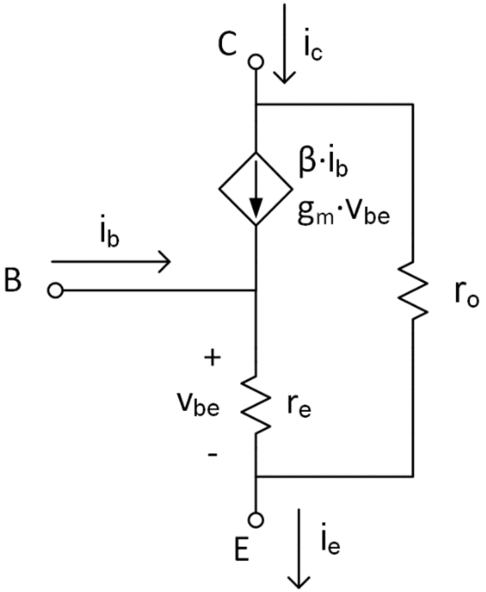




Small-Signal Models – Early Effect

- We have seen that we can add output resistance to the large-signal model to model the Early effect
  - Can do the same for the small-signal model
  - Will rarely, if ever, do this for the large signal model, but often will for the small-signal model

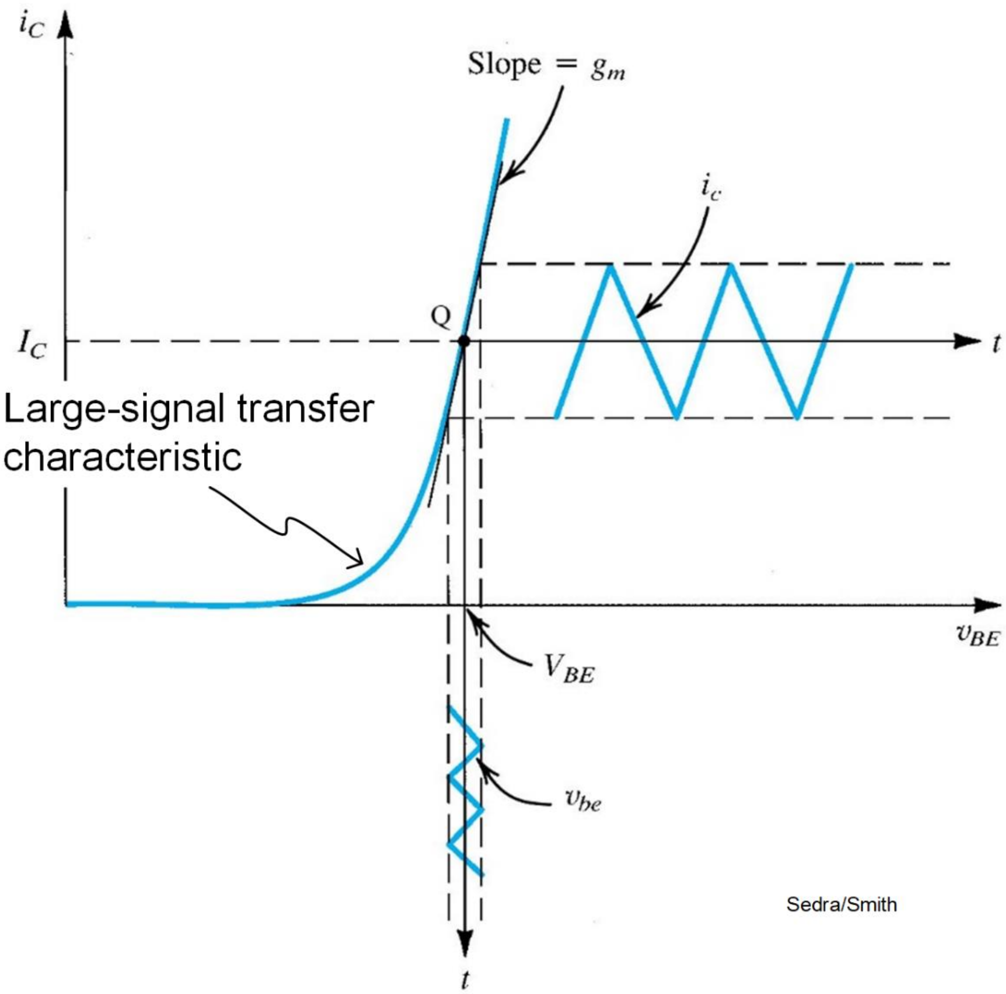




$$r_o = \frac{V_A}{I_C'} \approx \frac{V_A}{I_C}$$

BJT – Small-Signal Models

- ***Large-signal transfer characteristic***
  - Set of all possible Q-points
  - Very nonlinear
- As an amplifier, a transistor is operated ***near its Q-point***
  - Closer to linear
  - ***Small signal model*** describes behavior here
  - Valid for small signal excursions about the Q-point



Sedra/Smith

Using the BJT Models

- In the next section of the course, we will look at the analysis and design of transistor amplifiers
    - Bias network and DC operating point
    - Signal-path

- Much more detail in the next section, but our general procedure will be:
    - ***Large-signal analysis***
        - DC operating point
        - Small-signal model parameters
    - ***Small-signal analysis***
        - Circuit gain