SECTION 5: MOSFET AMPLIFIERS

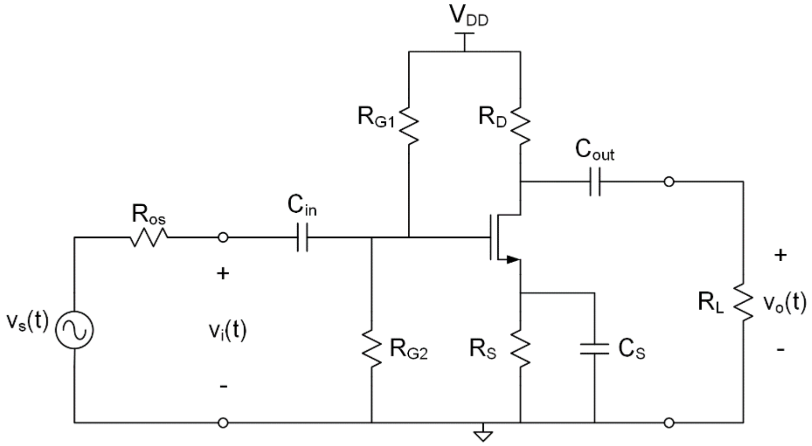
ECE 322 – Electronics I

MOSFET Amplifier Circuits

MOSFET Amplifier Circuits – Preview

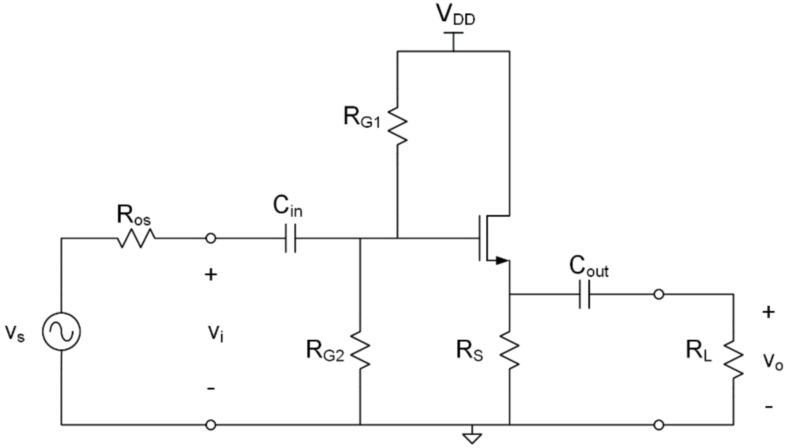
- In this section of the course, we will look at three MOSFET amplifiers, with a focus on the following two circuits:

Common-Source Amplifier:



- High voltage gain
- An amplifier

Source-Follower Amplifier:



- Near unity gain
- A buffer

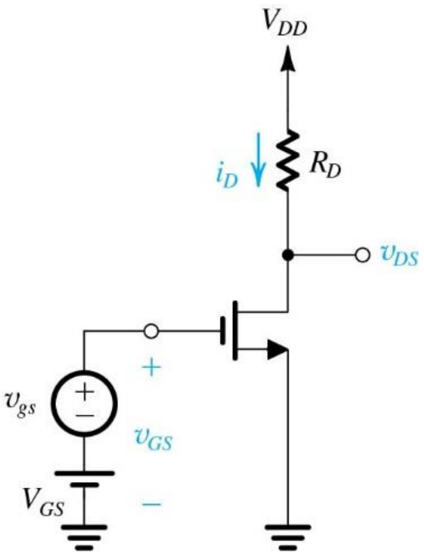
MOSFET Amplifier Biasing

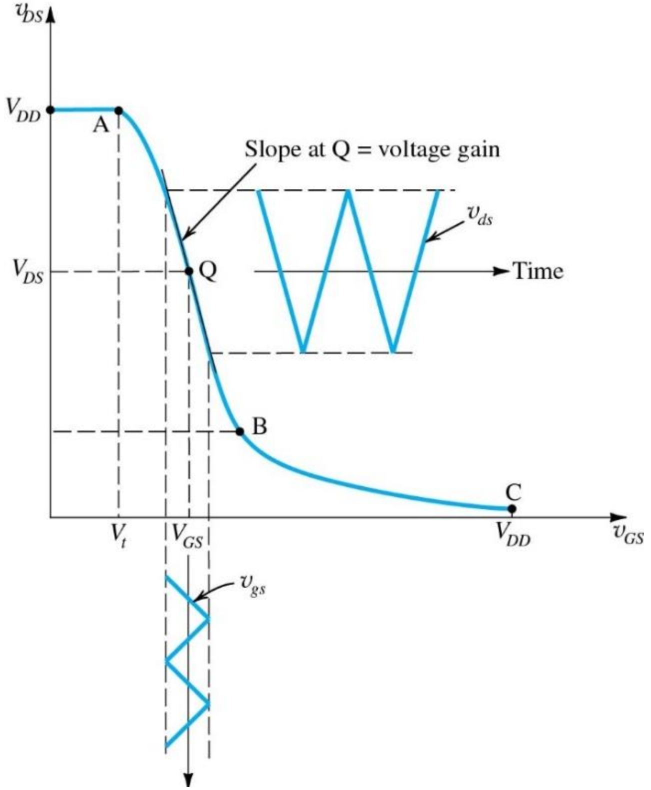
MOSFET Amplifier Biasing

- To function as an amplifier, a MOSFET must be biased in the ***saturation region***
- DC operating point set by the ***bias network***
  - Resistors and power supply voltages
  - Sets the transistor's ***DC terminal voltages and currents*** – its DC bias
- How a transistor is ***biased*** determines:
  - Small-signal characteristics
  - Small-signal model parameters
  - How it will behave as an amplifier

Voltage Transfer Characteristic

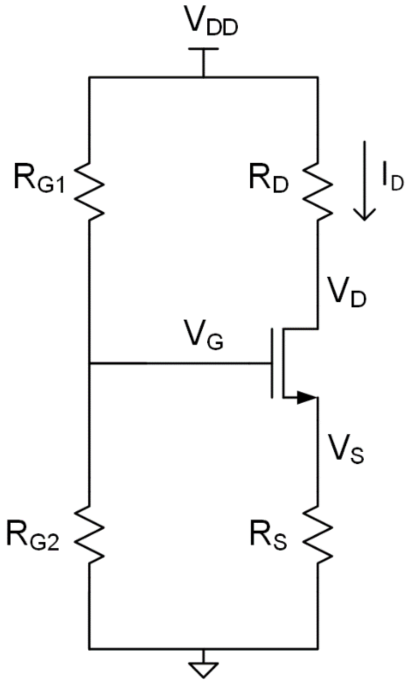
- MOSFET amplifier biased in the middle of its saturation region
- Slope of the large-signal transfer characteristic gives the amplifier gain
  - Negative slope – gain is inverting
  - Small input signals yield larger output signals
  - Slope is nearly linear in this region





MOSFET Biasing – Four-Resistor Bias Circuit

- We can use a similar four-resistor bias network for MOSFET amplifiers
- Commonly-used for both ***common-source*** amplifiers and ***source-followers***
  - Single power supply or bipolar supply
- Stable biasing over device parameter variations
  - Insensitive to variations in $V_t, k_n', \frac{W}{L}$



Analysis of the Four-Resistor Bias Circuit

- Since $I_G = 0$, gate voltage is simply set by the voltage divider

$$V_G = V_{DD}\frac{R_{G2}}{R_{G1} + R_{G2}}$$

- Drain current is given by

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2 = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$$

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_G - V_S - V_t)^2 = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_G - I_D R_S - V_t)^2$$

- After some rearranging, we arrive at a quadratic equation, which we can solve for I_D:

$$R_S^2 I_D^2 - \left[2R_S(V_G - V_t) + \frac{1}{\frac{1}{2}k_n'\left(\frac{W}{L}\right)}\right]I_D + (V_G - V_t)^2 = 0$$

Four-Resistor Bias Circuit – Example

- Determine terminal voltages and drain current for the following circuit
- Gate voltage:

$$V_G = 12\ V \cdot \frac{30\ k\Omega}{50\ k\Omega + 30\ k\Omega} = 4.5\ V$$

- Drain current:

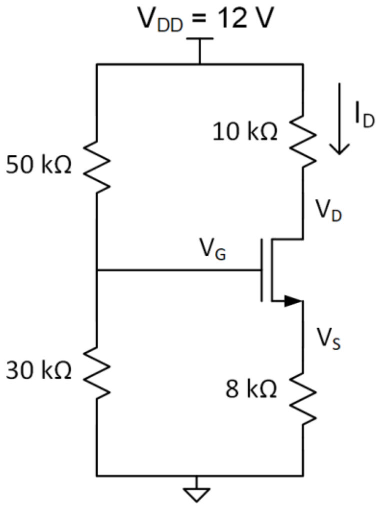
$$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right)(V_G - V_S - V_t)^2$$

$$I_D = 1 \frac{mA}{V^2}(4.5\ V - I_D \cdot 8k\Omega - 700\ mV)^2$$

$$I_D = 1 \frac{mA}{V^2}(-8\ k\Omega \cdot I_D + 3.8\ V)^2$$

$$1 \frac{mA}{V^2}(64e6 \cdot I_D^2 - 60.8e3 \cdot I_D + 14.44) - I_D = 0$$

$$64e6 \cdot I_D^2 - 61.8e3 \cdot I_D + 14.44 = 0$$



$$V_t = 700\ mV$$

$$k_n' \left(\frac{W}{L}\right) = 2 \frac{mA}{V^2}$$

Four-Resistor Bias Circuit – Example

$$64e6 \cdot I_D^2 - 61.8e3 \cdot I_D + 14.44 = 0$$

- Solving the quadratic equation for I_D gives

$$I_D = 569\ \mu A \text{ or } I_D = 396\ \mu A$$

- For $I_D = 569\ \mu A$

$$V_S = I_D R_S = 569\ \mu A \cdot 8\ k\Omega = 4.55\ V$$

$$V_{GS} = -50\ mV < V_t$$

  - The transistor would be cut-off, so this is not a valid solution

- DC operating point:

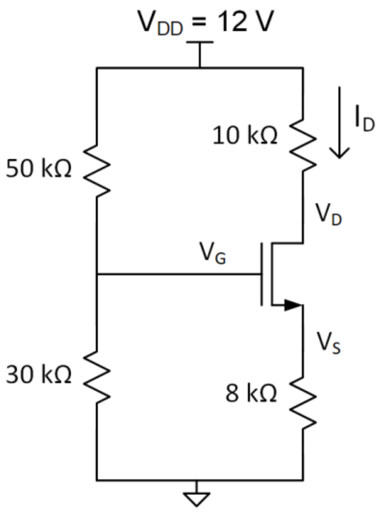
$$I_D = 396\ \mu A$$

$$V_S = 396\ \mu A \cdot 8\ k\Omega = 3.17\ V$$

$$V_{GS} = 1.33\ V$$

$$V_{OV} = 630\ mV$$

$$V_D = V_{DD} - I_D R_D = 8.04\ V$$



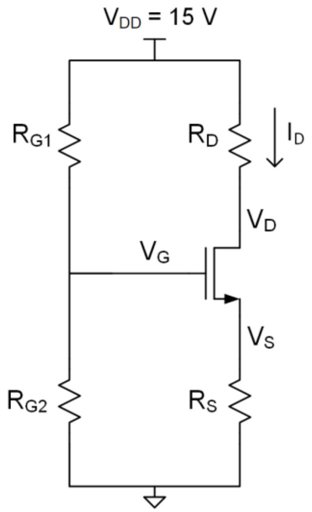
$$V_t = 700\ mV$$

$$k_n'\left(\frac{W}{L}\right) = 2\frac{mA}{V^2}$$

Design of the Four-Resistor Bias Circuit

- To design a bias network to provide a desired drain current:
  - Select R_D and R_S to each drop approximately one third of the supply voltage
    - That will leave approximately one third of the supply voltage across V_{DS}
  - Calculate the required V_{OV}, V_{GS}, and V_G
  - Select the voltage divider resistors at the gate to provide the required gate voltage

Bias Circuit Design - Example



$V_t = 800\ mV$

$k_n'\left(\frac{W}{L}\right) = 1\frac{mA}{V^2}$

- Design the bias network to provide $I_D = 800\ \mu A$
- Calculate R_D and R_S to each drop $V_{DD}/3$

$$R_D = R_S = \frac{V_{DD}/3}{I_D} = \frac{5\ V}{800\ \mu A} = 6.25\ k\Omega$$

- The required overdrive voltage is

$$V_{OV} = \sqrt{\frac{2I_D}{k_n'\left(\frac{W}{L}\right)}} = \sqrt{\frac{1.6\ mA}{1\frac{mA}{V^2}}} = 1.26\ V$$

- The gate-source voltage

$$V_{GS} = V_{OV} + V_t = 1.26\ V + 800\ mV$$

$$V_{GS} = 2.06\ V$$

Bias Circuit Design - Example

- Determine the required gate voltage

$$V_G = V_S + V_{GS} = I_D R_S + V_{GS}$$

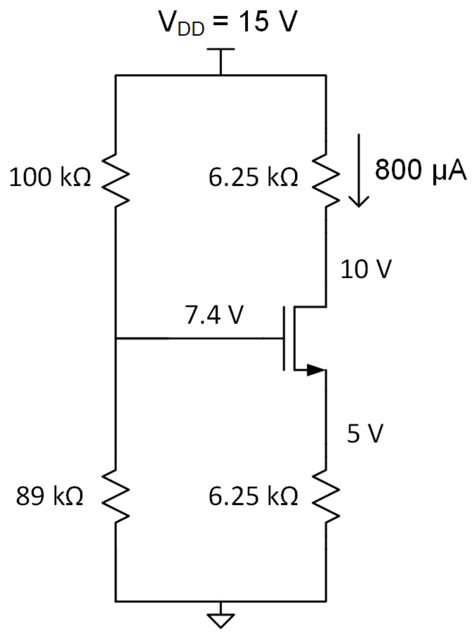
$$V_G = 800\ \mu A \cdot 6.25\ k\Omega + 2.06\ V$$

$$V_G = 7.06\ V$$

- Finally, select R_{G1} and R_{G2} to provide the required V_G

$$R_{G1} = 100\ k\Omega$$

$$R_{G2} = 89\ k\Omega$$



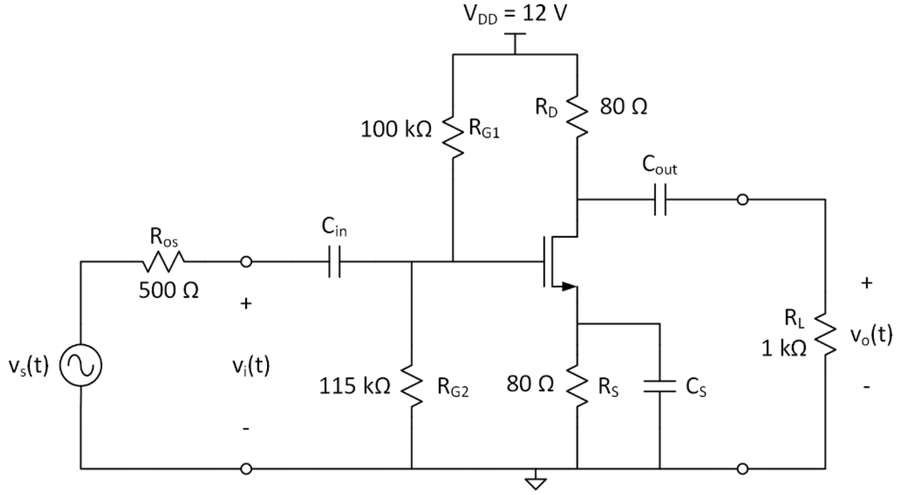
$$V_t = 800\ mV$$

$$k_n'\left(\frac{W}{L}\right) = 1\frac{mA}{V^2}$$

Common-Source Amplifier

Common-Source Amplifier

- Common-source amplifier
- All capacitors are ***AC-coupling/DC blocking capacitors***
  - Open at DC
  - Shorts at signal frequencies
  - Isolate transistor bias from source/load



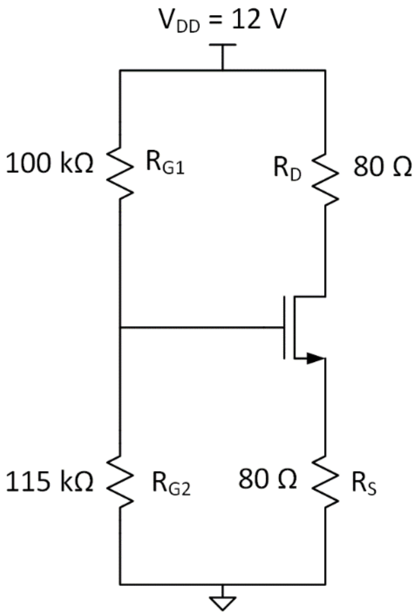
$V_t = 1.6\ V$ $\quad k_n'\left(\frac{W}{L}\right) = 170\frac{mA}{V^2}$

- Called *common*-source, because source is connected to common – i.e., ground or a power supply
  - C_S is a small-signal short to ground
  - Source is at small-signal ground

Common-Source Amplifier

- Analyze the amplifier to find:
  - DC operating point
  - Small-signal voltage gain

- DC operating point:
  - The gate voltage is given by



$$V_G = V_{DD}\frac{R_{G2}}{R_{G1}+R_{G2}}$$

$$V_G = 12\ V\frac{115\ k\Omega}{100\ k\Omega + 115\ k\Omega}$$

$$V_G = 6.4\ V$$

C-S Amplifier – Large-Signal Analysis

- Drain current is given by

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2 = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_G - I_D R_S - V_t)^2$$

- As we have seen, solving for I_D results in the following quadratic

$$R_S^2 I_D^2 - \left[2R_S(V_G - V_t) + \frac{1}{\frac{1}{2}k_n'\left(\frac{W}{L}\right)}\right]I_D + (V_G - V_t)^2 = 0$$

$$6.4e3 \cdot I_D^2 - 779.8 \cdot I_D + 23.0 = 0$$

- This has two solutions

$$I_D = 72\,mA \quad \text{or} \quad I_D = 51\,mA$$

- The first solution would put the transistor in cutoff, so $I_D = 51\,mA$

C-S Amplifier – Large-Signal Analysis

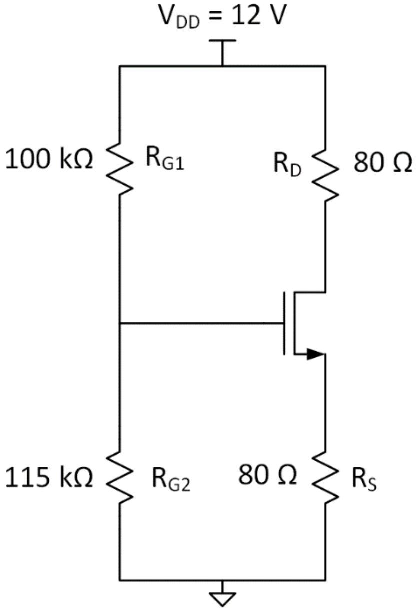
- Use the drain current to determine terminal voltages

$$V_D = V_{DD} - I_D R_D$$

$$V_D = 12\,V - 51\,mA \cdot 80\,\Omega = 7.95\,V$$

$$V_S = I_D R_S = 51\,mA \cdot 80\,\Omega$$

$$V_S = 4.05\,V$$



- The complete DC operating point:

$$V_G = 6.42\,V \qquad I_D = 51\,mA$$

$$V_{GS} = 2.37\,V \qquad V_D = 7.95\,V$$

$$V_{OV} = 0.77\,V \qquad V_S = 4.05\,V$$

C-S Amplifier – Small-Signal Analysis

- The DC operating point allows us to determine the transconductance for the transistor's small-signal model

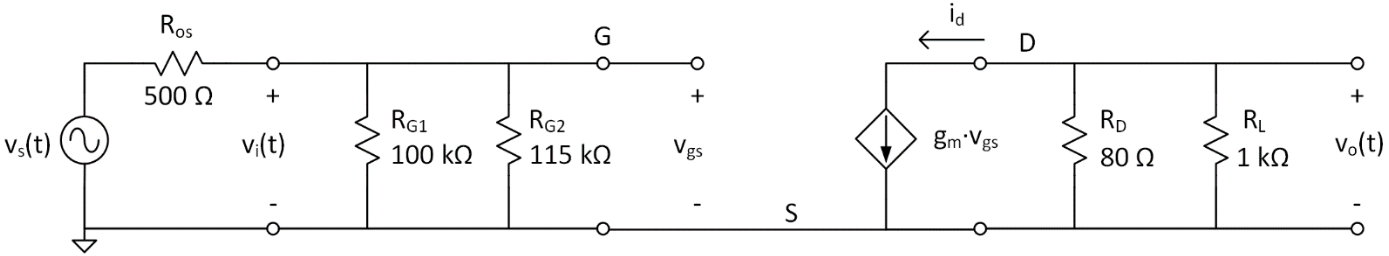
$$g_m = k_n'\left(\frac{W}{L}\right)V_{OV} = 170\,\frac{mA}{V^2}\cdot 0.77\,V = 131\,mS$$

- Next, create the ***small-signal equivalent circuit*** for the amplifier and perform a ***small-signal analysis:***

1. Replace all AC coupling capacitors with shorts
   - Large enough to look like shorts at signal frequencies
2. Connect all DC supply voltages to ground
   - From a small-signal perspective these are all constant voltages
   - Small-signal ground
3. Replace the transistor with its small-signal model

C-S Amplifier – Small-Signal Analysis

- Small-signal equivalent circuit
  - Use to determine small-signal voltage gain



- Source is connected to small signal ground through C_S
- R_{G1} and R_{G2} appear in parallel at the gate

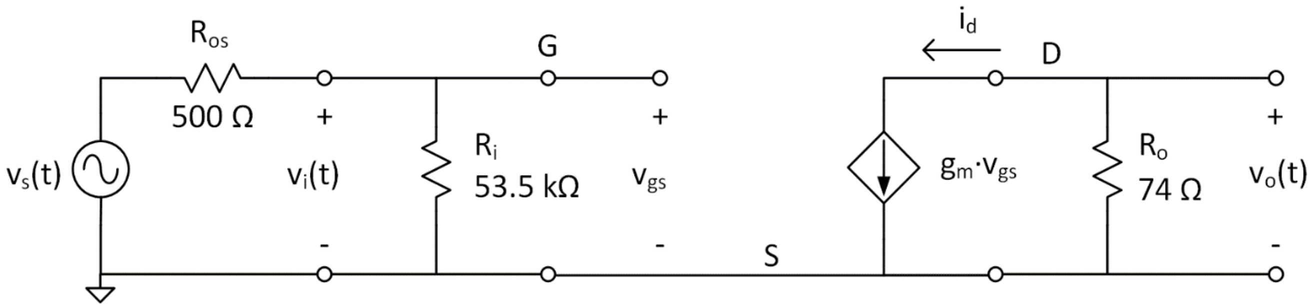
$$R_i = R_{G1} || R_{G2} = 53.5\ k\Omega$$

- R_D and R_L are in parallel at the output

$$R_o = R_D || R_L = 74\ \Omega$$

- Input voltage, $v_i(t)$, is the gate-source voltage, v_{gs}

C-S Amplifier – Small-Signal Analysis



- Determine the small-signal voltage gain:

$$A_v = \frac{v_o}{v_i} \tag{1}$$

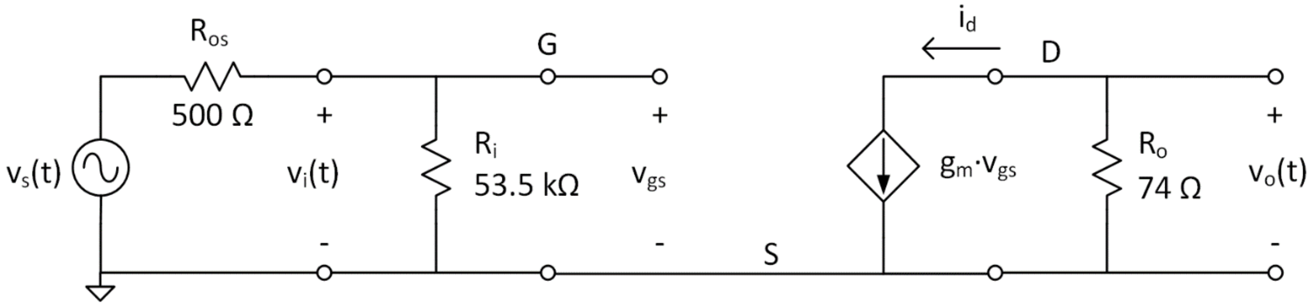
- The input is applied across the G-S junction, so

$$v_i = v_{gs} \tag{2}$$

- The output is the drain current applied across the output resistance

$$v_o = -i_d R_o = -g_m v_{gs} R_o \tag{3}$$

C-S Amplifier – Small-Signal Analysis



- Substituting (3) and (2) into (1) gives the gain:

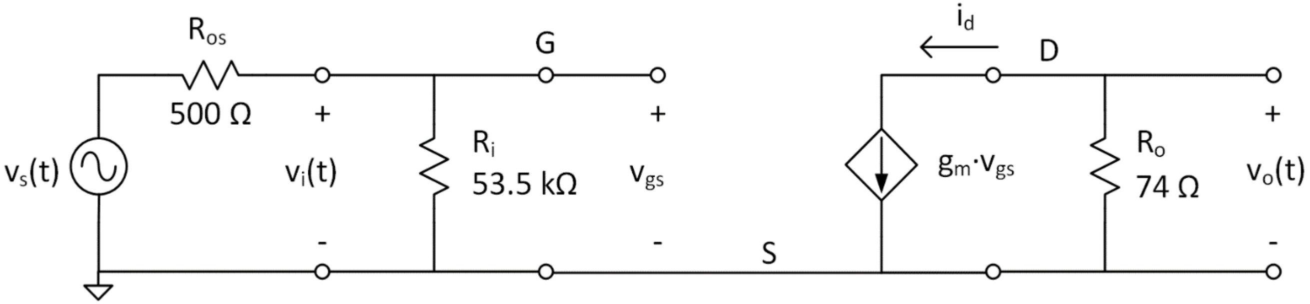
$$A_v = \frac{v_o}{v_i} = -\frac{g_m v_{gs} R_o}{v_{gs}} = -g_m R_o$$

- This is the gain for *any* common-source amplifier

$$\boxed{A_v = -g_m R_o}$$

- The negative sign indicates that the amplifier has ***inverting*** gain

C-S Amplifier – Small-Signal Analysis



- For this circuit, the gain (from v_i to v_o) is

$$A_v = \frac{v_o}{v_i} = -131\ mS \cdot 74\ \Omega = -9.7$$

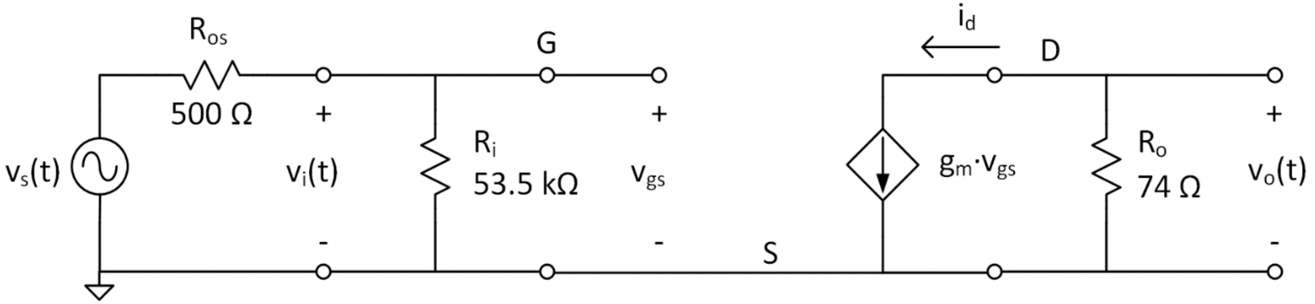
- For the gain from v_s to v_o, account for attenuation due to source loading

$$A_v = \frac{v_o}{v_s} = \frac{v_i}{v_s} \cdot \frac{v_o}{v_i} = \frac{R_i}{R_s + R_i} \cdot (-g_m R_o)$$

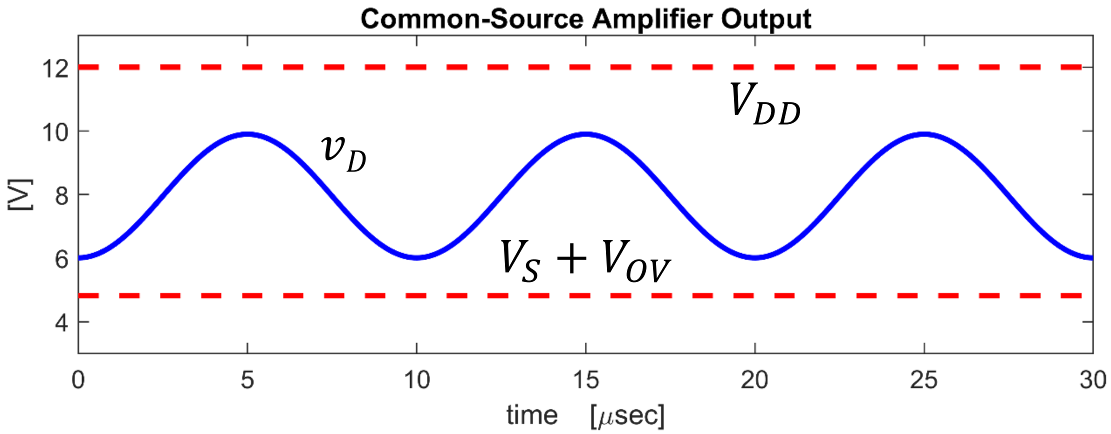
- Here,

$$A_v = \frac{v_o}{v_s} = \frac{53.5\ k\Omega}{500\ \Omega + 53.5\ k\Omega} \cdot (-9.7) = -9.6$$

C-S Amplifier – Small-Signal Analysis

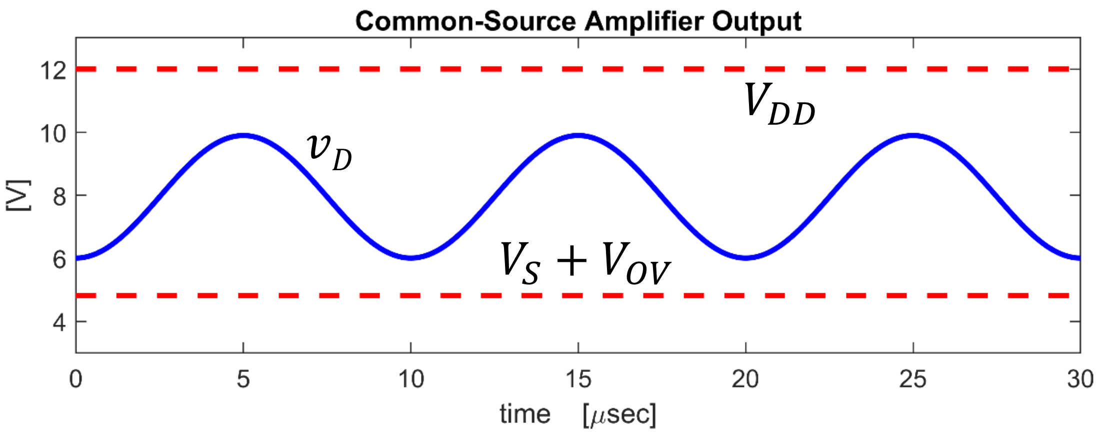


- The output for a $200\ mV_{pp}$, $100\ kHz$ input:



C-S Amplifier – Dynamic Range

- ***Dynamic range***
  - Range of input or output signal for which the transistor remains in the ***saturation region***
  - The amplifier's ***linear range***



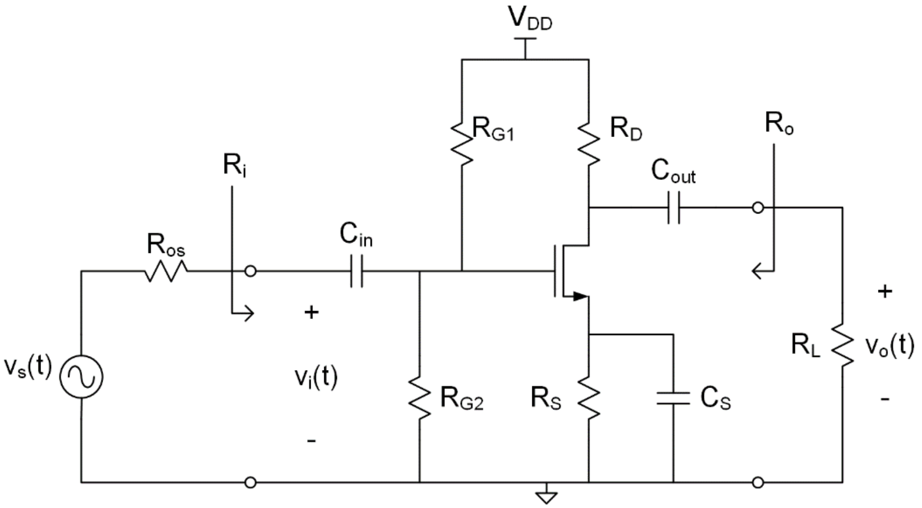
- For saturation bias:
  - D-S voltage must remain greater than the overdrive voltage

$$v_{DS} > V_{OV}$$

  - G-S voltage must remain greater than the threshold voltage

$$v_{GS} > V_t$$

C-S Amplifier – Input & Output Resistance



- Gate resistance is infinite, so amplifier input resistance is

$$R_i = R_{G1}||R_{G2}$$

- Output resistance is the drain resistance:

$$R_o = R_D$$

  - Or, if accounting for channel-length modulation:

$$R_o = R_D||r_o$$

C-S Amplifier – Gain

$$A_v = -g_m R_o$$

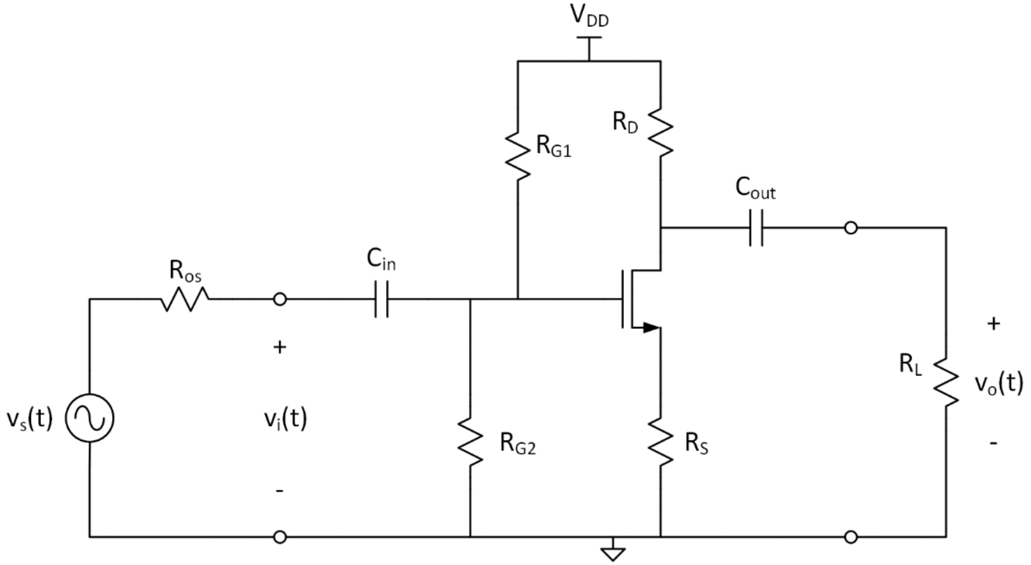
- C-S gain is ***determined by g_m and R_o***
  - Select R_o (R_D) and set g_m for desired gain
  - Transconductance is proportional to the square root of bias current

$$g_m = \sqrt{k_n' \left(\frac{W}{L}\right) I_D}$$

- Therefore, ***gain is proportional to the square root of bias current***

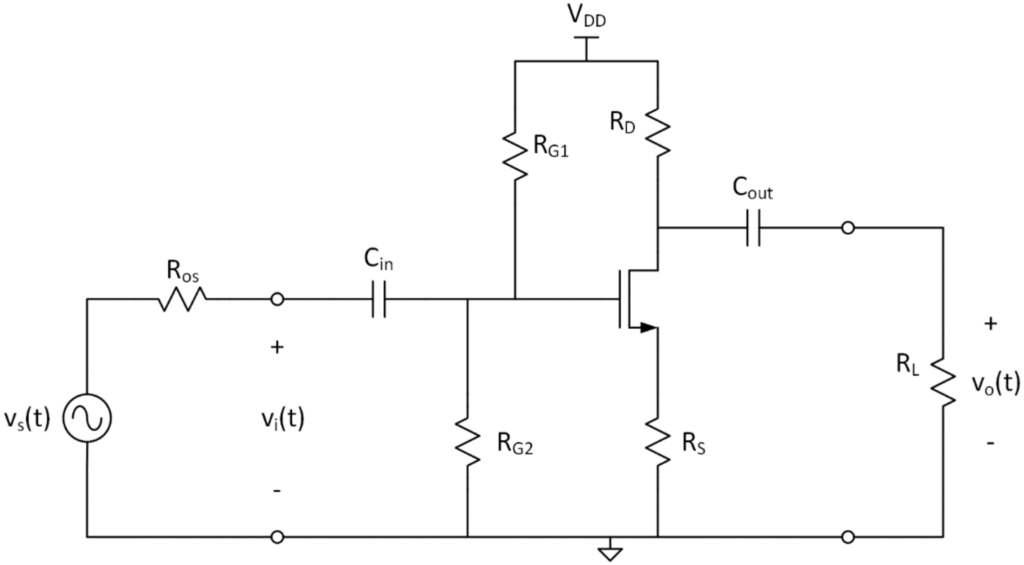
Source Degeneration

C-S Amplifier – Source Degeneration



- The C-S amplifier we have looked at so far had its source grounded (small-signal ground)
  - Due to bypass capacitor, C_S, around R_S
- What if we remove C_S?
  - Or add another source resistor not bypassed by C_S
  - ***Source degeneration***

C-S Amplifier – Source Degeneration



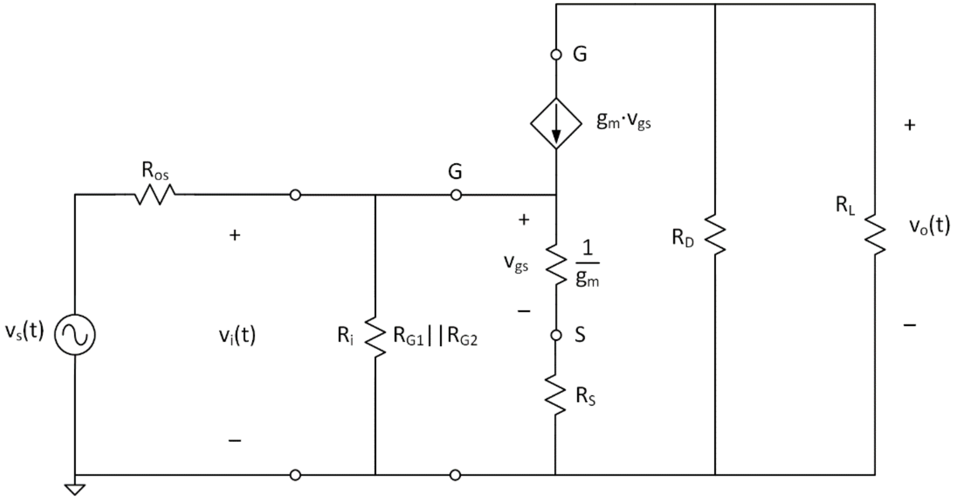
- Now, R_S is included in the small signal equivalent circuit
  - Source is no longer connected to small-signal ground
- Analysis will be simplified if we use the T-model
  - Usually the case whenever we have source resistance
  - R_S will be in series with resistance in the model

C-S Amplifier – Source Degeneration

- The output is still given by

$$v_o = -i_d R_o = -g_m v_{gs} R_o$$

- But, now, v_{gs} is the portion of v_i that appears across the $1/g_m$ resistance



$$v_{gs} = v_i \frac{1/g_m}{1/g_m + R_S}$$

$$v_{gs} = v_i \frac{1}{1 + g_m R_S}$$

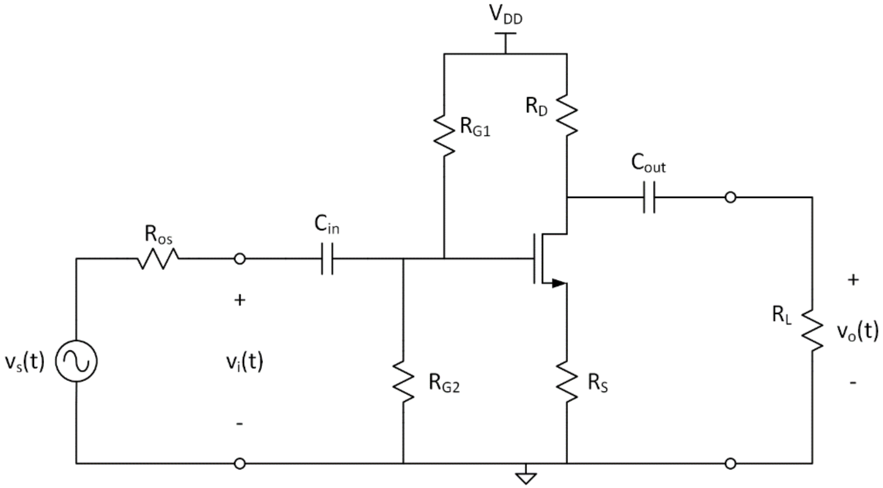
- The output is

$$v_o = v_i \left(-g_m R_o \frac{1}{1 + g_m R_S} \right)$$

Source Degeneration – Gain

- Rearranging the expression for the output gives the gain

$$\boxed{A_v = -\frac{g_m R_o}{1 + g_m R_S}}$$

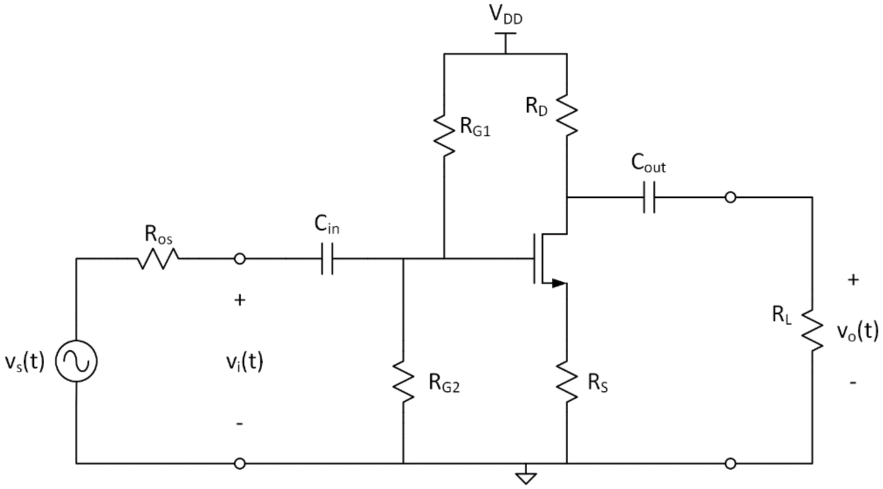


- ***Source degeneration reduces the gain by a factor of*** $(1 + g_m R_S)$
- If $R_S \gg 1/g_m$, then $g_m R_S \gg 1$, and

$$\boxed{A_v = -\frac{R_o}{R_S}}$$

Source Degeneration – Transconductance

$$A_v = -\frac{g_m R_o}{1 + g_m R_S}$$



- We can rewrite the gain as

$$\boxed{A_v = -G_m R_o}$$

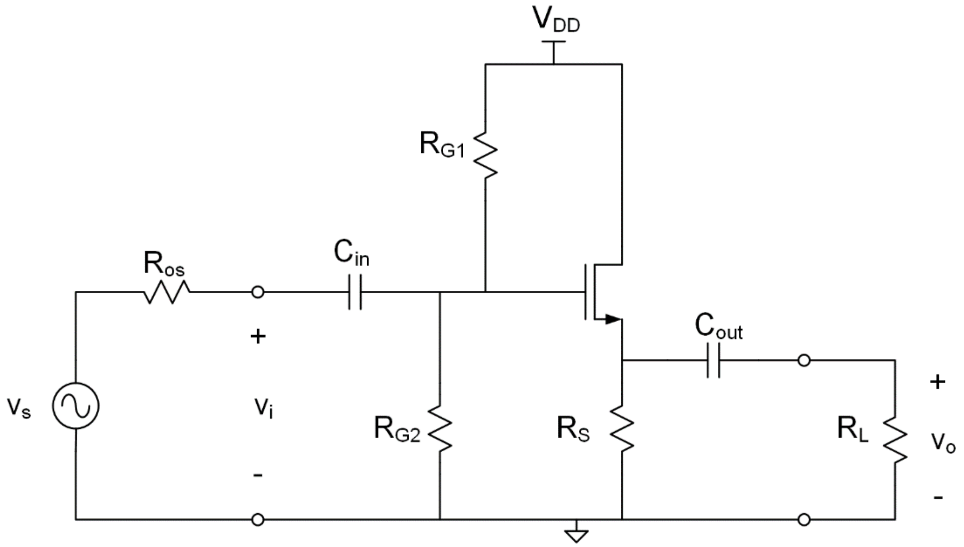
- G_m is the ***effective transconductance of the amplifier***

$$\boxed{G_m = \frac{g_m}{1 + g_m R_S}}$$

- ***Source degeneration reduces the transconductance by a factor of*** $(1 + g_m R_S)$
  - This is why we see a reduction in gain by the same factor

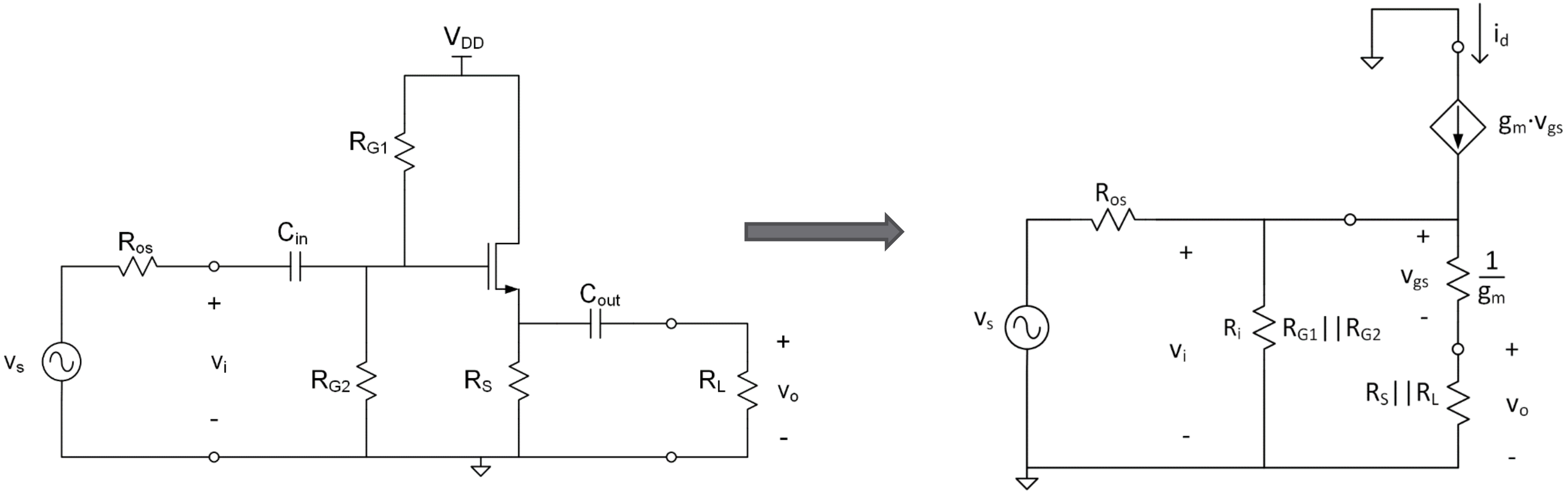
Source Follower

Source-Follower



- ***Source-follower amplifier***
  - Input applied to the gate
  - Output at the source
  - Source *follows* the gate
- Also called a ***common-drain*** amplifier (CD)
  - Drain is connected to small-signal ground

Source-Follower – Small-Signal Analysis



- Replace the MOSFET with small-signal model
  - Source resistance, so use T-model
  - Short coupling caps
  - DC voltages connect to ground
  - Simplify parallel resistances

Source-Follower – Small-Signal Analysis

- Determine the gain from v_i to v_o

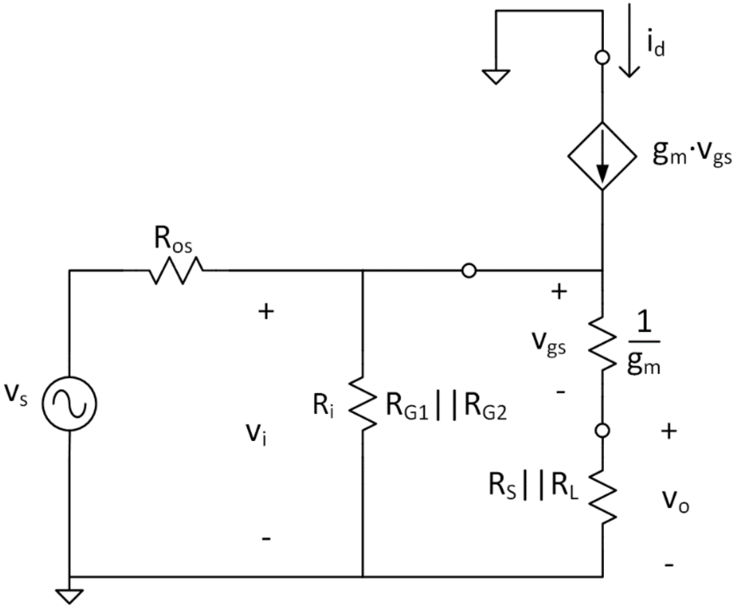
$$A_v = \frac{v_o}{v_i}$$

Applying voltage division gives the output

$$v_o = v_i \frac{R_S||R_L}{\left(R_S||R_L + \frac{1}{g_m}\right)}$$

Rearrange to get the gain

$$A_v = \frac{R_S||R_L}{\left(R_S||R_L + \frac{1}{g_m}\right)}$$



  - Clearly, $A_v < 1$
  - But, for $R_S||R_L \gg 1/g_m$, $A_v \approx 1$

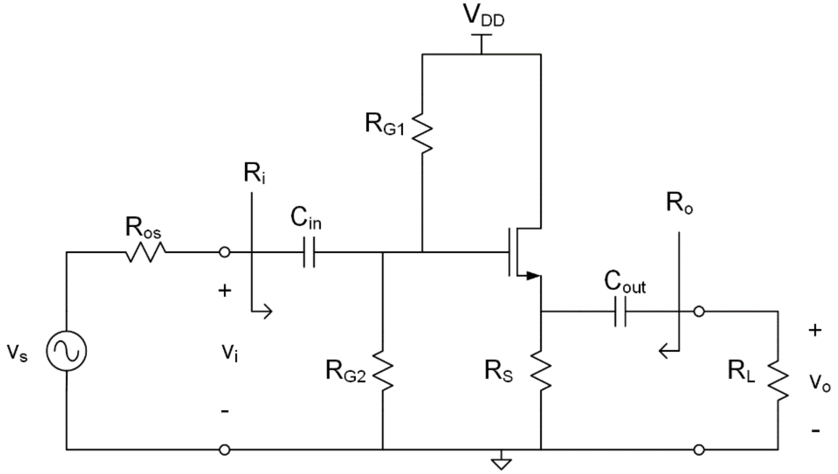
Source Follower – Input & Output Resistance

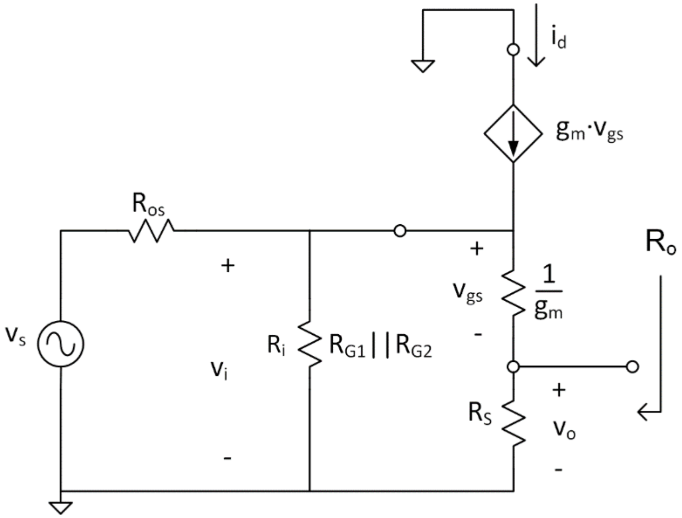
- Gate resistance is infinite, so amplifier input resistance is

$$R_i = R_{G1}||R_{G2}$$

- The output resistance is the source resistance in parallel with $1/g_m$:

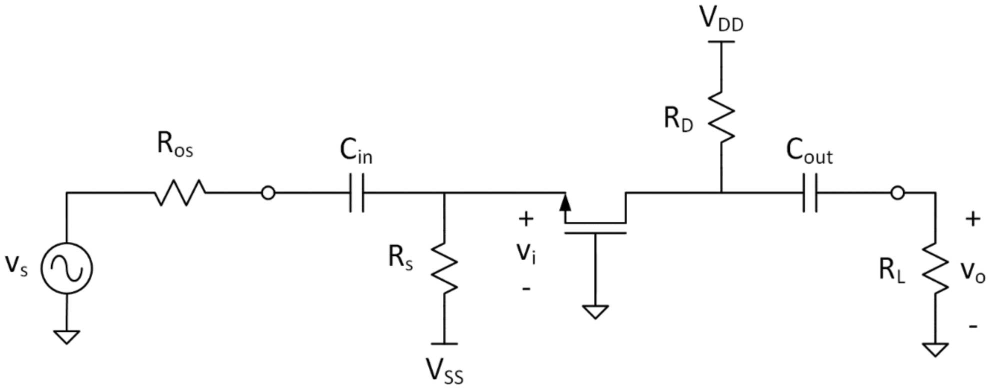
$$R_o = R_S||\frac{1}{g_m}$$





Common-Gate Amplifier

Common-Gate Amplifier



- The third MOSFET amplifier configuration we will look at is the ***common-gate amplifier***
  - Input applied to the source
  - Output taken from the drain
  - Gate is connected to small-signal ground
  - The least common of the three amplifiers

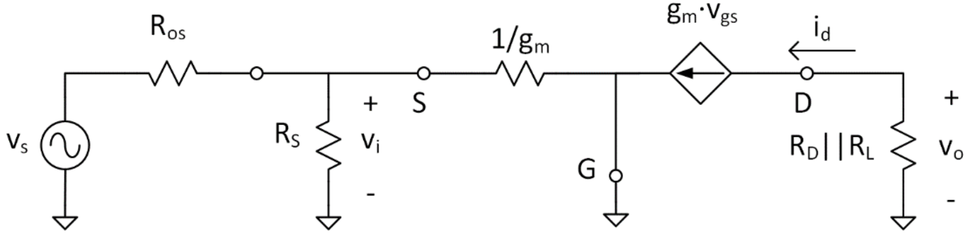
Common-Gate Amplifier – Gain

- There is source resistance, so use the T-model for small-signal analysis
- The output is given by

$$v_o = -i_d R_D||R_L$$

$$v_o = -g_m v_{gs} R_D||R_L$$

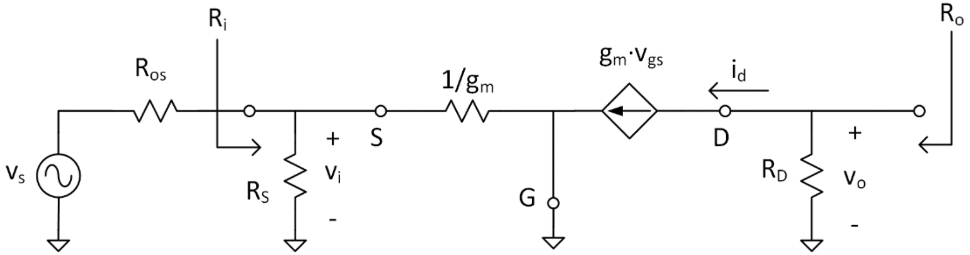
$$v_o = g_m v_i R_D||R_L$$



- Common-gate voltage gain is

$$\boxed{A_v = g_m R_D||R_L}$$

Common-Gate – Input Resistance



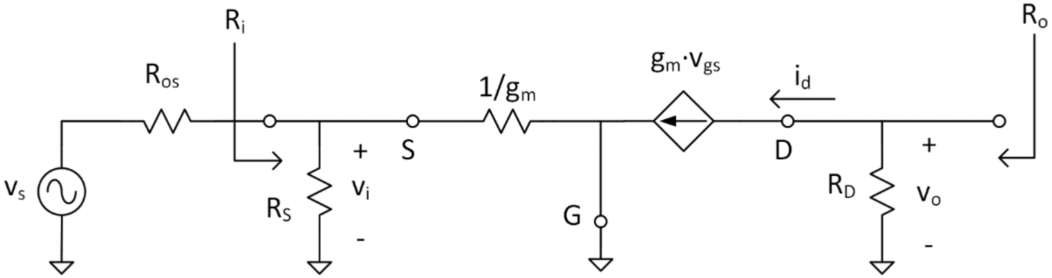
- R_i is the parallel combination of the resistance connected to the source and the resistance looking into the source

$$\boxed{R_i = R_S || \frac{1}{g_m}}$$

- If $1/g_m \ll R_S$, then

$$\boxed{R_i \approx \frac{1}{g_m}}$$

Common-Gate – Output Resistance



- If we neglect the transistor's output resistance, r_o, the common-gate output resistance is

$$\boxed{R_o = R_D}$$

  - Entirely determined by the drain resistor

Common-Gate Amplifier

- Low input resistance

$$R_i \approx \frac{1}{g_m}$$

  - For $R_{os} \gg 1/g_m$, there will be significant attenuation from v_s to v_i

$$v_i \ll v_s$$

  - The overall gain from v_s to v_o may be small

- Like the common-base amplifier, useful in specific applications:
  - Low source resistance
    - E.g., amplifiers driven by cables
    - R_i matched to Z_0 (e.g. 50 Ω or 75 Ω) to avoid reflections
  - Current buffers
    - E.g., in ***cascode*** amplifiers

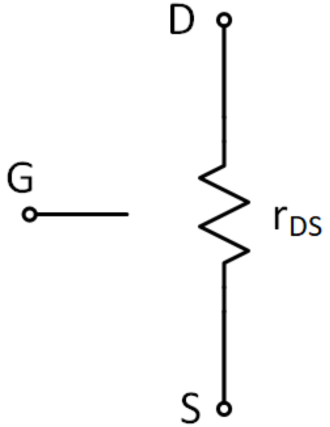
MOSFETs as Switches

MOSFETs as Switches

- MOSFETs used as ***switches*** operate alternately in the ***triode*** (closed) and ***cutoff*** (open) regions
- Equivalent circuit models:

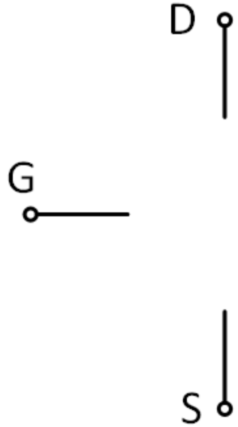
Triode Region (ON):

- $V_{GS} > V_t$
  - $V_{GS} = V_{DD}$
- Switch is on
- $I_D \geq 0$
- $V_{DS} = I_D r_{DS} < V_{OV}$



Cutoff Region (OFF):

- $V_{GS} < V_t$
  - $V_{GS} = 0$
- Switch is off
- $I_D = 0$
- $V_{DS} = V_{DD}$



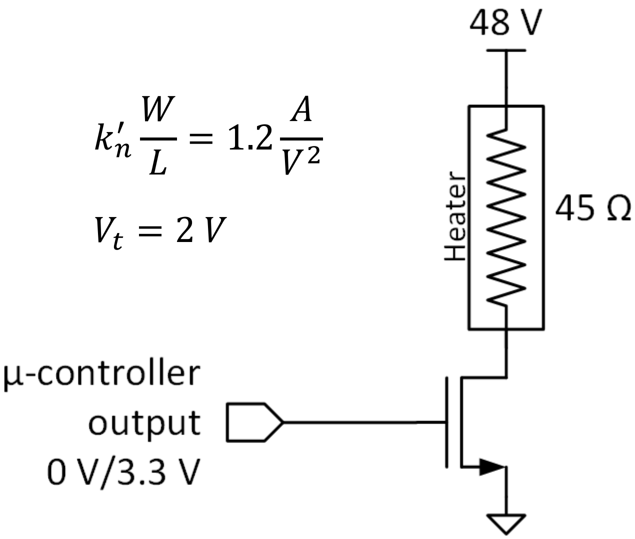
Using a MOSFET as a Switch - Example

- Turn resistance heater on and off using a microcontroller
- Heater may require amperes of current
- Microcontroller output may be limited to tens of mA

- Control a MOSFET switch with the microcontroller output
  - Low-current control signal from the microcontroller
    - Gate draws no DC current
  - MOSFET switches the large current required by the heater

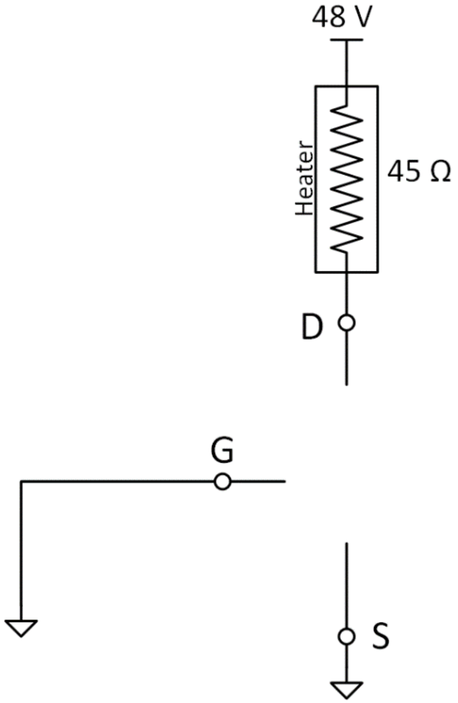
$$k_n' \frac{W}{L} = 1.2 \frac{A}{V^2}$$

$$V_t = 2\ V$$



Using a MOSFET as a Switch - Example

- When the μ-controller's output is low (0 V)
  - $V_{GS} = 0\ V$
  - Transistor is in the cutoff region
  - Switch is off
  - No current flows
  - The heater is off



Using a MOSFET as a Switch - Example

- When the μ-controller's output is high (3.3 V)
  - $V_{GS} = 3.3\,V, V_{OV} = 1.3\,V$
  - Transistor is in triode
  - Switch and heater are on

- Drain current in triode is:

$$I_D = k_n' \frac{W}{L}\left[V_{OV} - \frac{1}{2}V_{DS}\right]V_{DS}$$

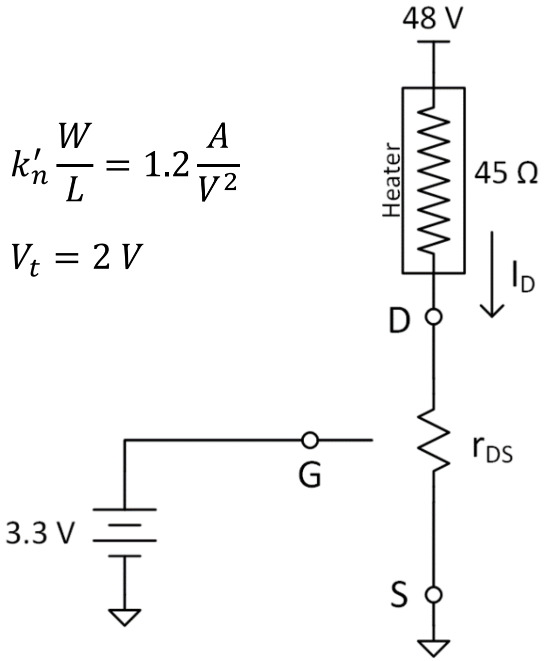
$$I_D = k_n' \frac{W}{L}\left[V_{OV} - \frac{1}{2}(48\,V - I_D R_h)\right](48\,V - I_D R_h)$$

- Can solve the above quadratic, or, assuming V_{DS} is small, approximate switch on-resistance as:

$$r_{DS} \approx \frac{1}{k_n' \frac{W}{L} V_{OV}} = \frac{1}{1.2\frac{A}{V^2} \cdot 1.3\,V} = 641\,m\Omega$$

$$k_n' \frac{W}{L} = 1.2\frac{A}{V^2}$$

$$V_t = 2\,V$$



Using a MOSFET as a Switch - Example

- Voltage division gives approximate drain voltage

$$V_D = 48\ V \cdot \frac{r_{DS}}{R_h + r_{DS}} = 48\ V \cdot \frac{641\ m\Omega}{45\ \Omega + 641\ m\Omega}$$

$$V_D = 674\ mV$$

- Drain current is approximately

$$I_D = \frac{48\ V}{R_h + r_{DS}} = \frac{48\ V}{45\ \Omega + 641\ m\Omega}$$

$$I_D = 1.05\ A$$

- Heater power is

$$P_h = I_D^2 R_h = (1.05\ A)^2 \cdot 45\ \Omega$$

$$P_h = 49.75\ W$$

$$k_n' \frac{W}{L} = 1.2 \frac{A}{V^2}$$

$$V_t = 2\ V$$