SECTION 6: INTEGRATED CIRCUIT BUILDING BLOCKS

ECE 322 – Electronics I

Integrated Circuit Design Principles

- On-chip real estate is at a premium
  - Chip area is costly
- High-value resistors consume too much area and are avoided
  - Use transistor current sources for loads and biasing
- High-value capacitors consume too much area
  - Amplifier stages typically directly coupled
- Must account for transistor output resistance
  - Device dimensions are smaller
- Do not have precise control over device parameters
  - But, precise matching of devices is attainable
- Designers have control over transistor dimensions
  - Gate/channel aspect ratio: W, L for CMOS devices
  - Emitter area: W, L for BJTs

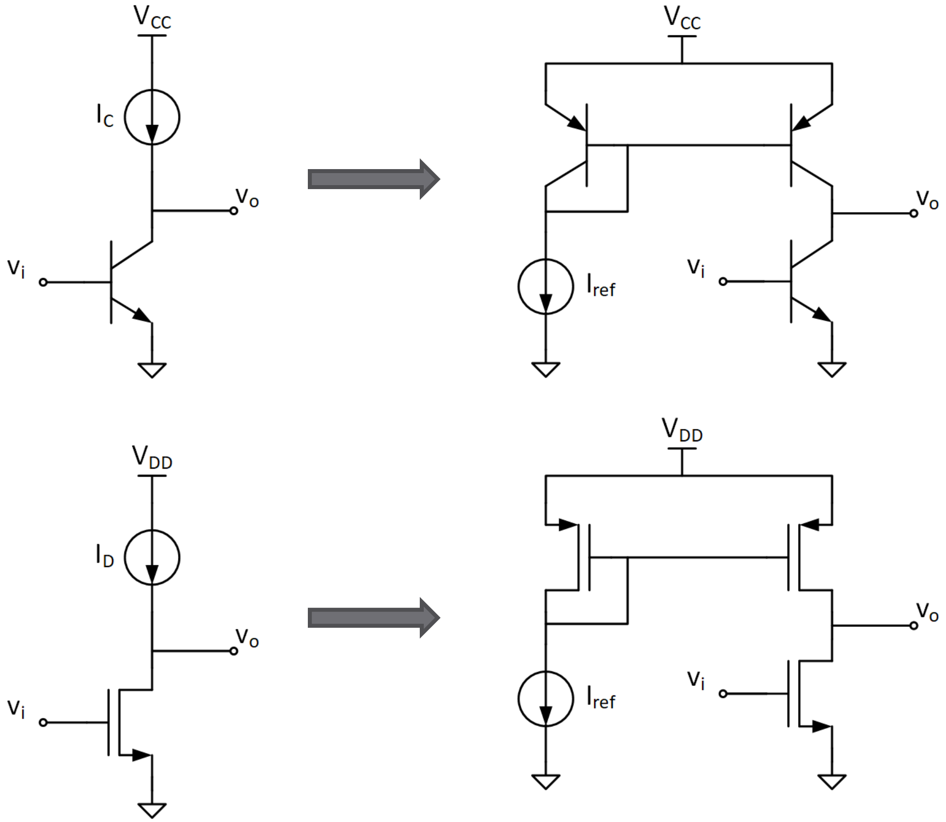
Building Blocks of Integrated Circuits

- Analog integrated circuits make use of several very common building blocks:
- ***Amplifiers***
  - The amplifiers we have already learned about, with some modifications
  - ***Cascode amplifiers*** employ current buffers between transconductance devices and loads
- ***Current sources/current mirrors***
  - Active loads for amplifiers
  - Biasing for amplifiers and other circuits

Current Mirrors

Active Loads

- Transistor current sources are typically used as amplifier loads instead of resistors
  - Higher resistance, higher gain
  - Smaller area

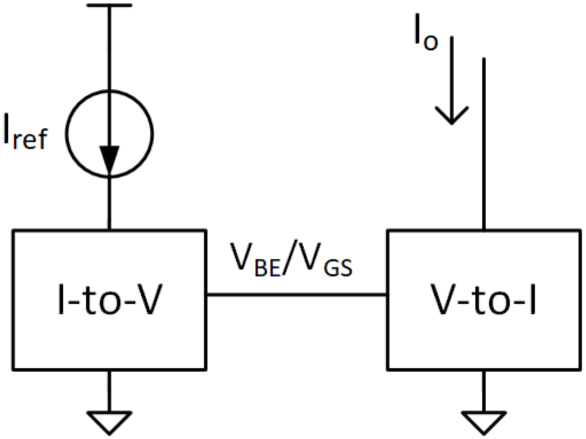


Current Mirrors

- Current sources are used everywhere on ICs
  - Amplifier loads
  - Transistor biasing
- Single circuit used to generate a reference current
  - Often a single bias generator circuit on a chip
- Reference current is replicated, and scaled, as needed at various circuits across the chip
  - ***Current steering***
- Replication of current accomplished by ***current mirrors***

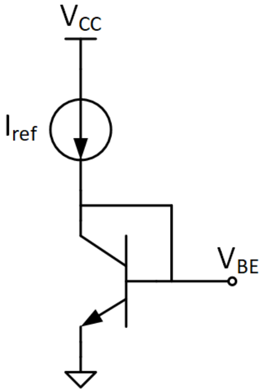
Current Mirror – Basic Principles

- Current mirrors have two components:
- ***Transresistance*** stage
  - Current-to-voltage conversion
  - Low input resistance
  - Generates a voltage (V_{BE} or V_{GS}) proportional to the input current (I_C or I_D)
- ***Transconductance*** stage
  - Voltage-to-current conversion
  - High output resistance (a current source)
  - Generates a current (I_C or I_D) proportional to the input voltage (V_{BE} or V_{GS})

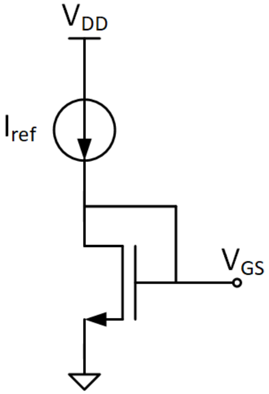


Current Mirror – Transresistance Stage

- Current-to-voltage conversion
- ***Diode-tied transistor***
  - Base/collector or gate/drain connected together
  - Looks like a diode
- Reference current applied to collector or drain
- Base-emitter or gate-source voltage generated proportional to collector or drain current



$$V_{BE} = V_{th} \ln\left(\frac{I_C}{I_S}\right)$$

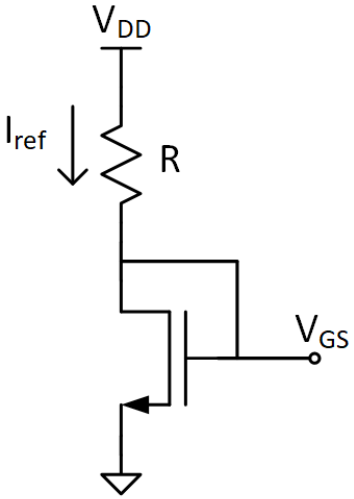


$$V_{GS} = \sqrt{\frac{2I_D}{k_n'\left(\frac{W}{L}\right)}} + V_t$$

Simple MOS Transresistance Stage

- Drain current

$$I_{ref} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_{DD} - I_D R - V_t)^2$$



- A quadratic equation for I_{ref}

$$R^2 I_{ref}^2 - \left[2(V_{DD} - V_t)R + \frac{2}{k_n'\left(\frac{W}{L}\right)}\right] I_{ref} + (V_{DD} - V_t)^2 = 0$$

  - Solve to determine I_{ref}

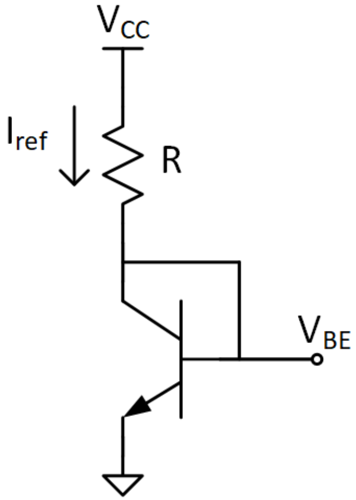
- Or, to determine R for desired I_{ref}:
  - Calculate overdrive voltage, V_{OV}
  - Determine gate voltage
  - Apply Ohm's law:

$$R = \frac{V_{DD} - V_G}{I_{ref}}$$

Simple BJT Transresistance Stage

- Resistor in series with a diode-tied transistor
- Current through the resistor

$$I_{ref} = \frac{V_{CC} - V_{BE}}{R}$$



- Using the large-signal BJT model, where $V_{BE} = 700\,mV$:

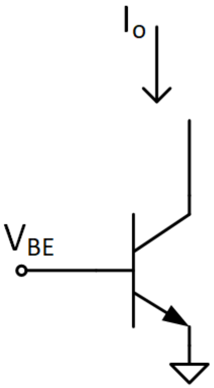
$$I_{ref} = \frac{V_{CC} - V_{BE}}{R}$$

$$I_{ref} = \frac{V_{CC} - 700\,mV}{R}$$

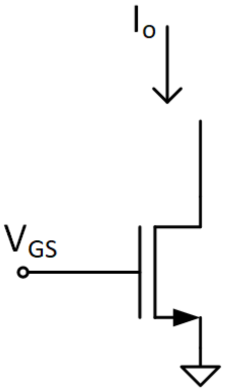
- V_{BE} is inversely proportional to temperature
  - I_{ref} increases with increasing temperature
  - A proportional-to-absolute-temperature (PTAT) current

Current Mirror – Transconductance Stage

- Voltage-to-current conversion
- Transresistance device's output voltage applied to the base or gate of the transconductance device
- Transistor must remain in the forward active (BJT) or saturation region (MOS)
- Output current proportional to the applied voltage
- High output resistance



$$I_o = I_s e^{\frac{V_{BE}}{V_{th}}}$$



$$I_o = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (V_{GS} - V_t)^2$$

Simple MOS Current Mirror

- Gate-source voltages:

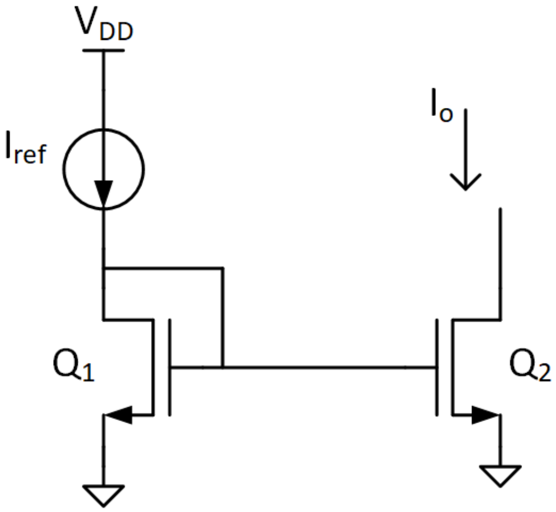
$$V_{GS} = \sqrt{\frac{2I_{ref}}{k_n'\left(\frac{W}{L}\right)_1}} + V_t$$

- Output current:

$$I_o = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_2 (V_{GS} - V_t)^2$$

$$I_o = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_2 \frac{2I_{ref}}{k_n'\left(\frac{W}{L}\right)_1}$$

$$\boxed{I_o = I_{ref}\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}$$



- Output current scaled by the device aspect ratios
  - The ***current transfer ratio*** or current gain of the mirror

Simple BJT Current Mirror

- First, assume $\beta \approx \infty$
- Base-emitter voltages:

$$V_{BE} = V_{th}\ln\left(\frac{I_{ref}}{I_{S_1}}\right)$$

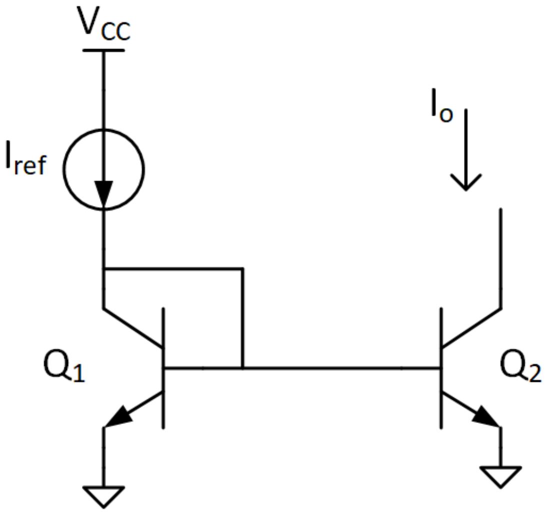
- Output current:

$$I_o = I_{S_2}e^{\frac{V_{BE}}{V_{th}}} = I_{S_2}e^{\ln\left(\frac{I_{ref}}{I_{S_1}}\right)}$$

$$I_o = I_{ref}\frac{I_{S_2}}{I_{S_1}} = I_{ref}\frac{A_{E_2}}{A_{E_1}}$$

- Output current is the reference current scaled by the emitter area ratios
  - The current transfer ratio:

$$\boxed{\frac{I_o}{I_{ref}} = \frac{A_{E_2}}{A_{E_1}} = m}$$



Simple BJT Current Mirror

- Now, accounting for finite β
- Collector currents still scale with emitter area

$$I_o = I_{C_2} = I_{C_1}\frac{A_{E_2}}{A_{E_1}} = m \cdot I_{C_1}$$

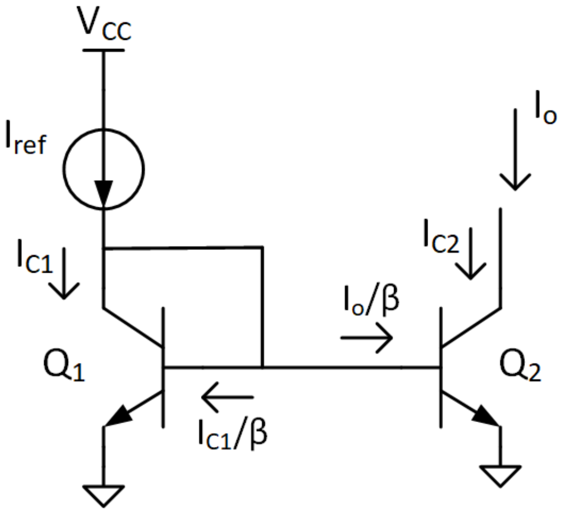
- The current transfer ratio:

$$I_{ref} = I_{C_1} + \frac{I_{C1}}{\beta} + \frac{I_o}{\beta} = \frac{I_o}{m} + \frac{I_o}{m\beta} + \frac{I_o}{\beta}$$

$$I_{ref} = I_o\left(\frac{1}{m} + \frac{1}{m\beta} + \frac{1}{\beta}\right)$$

$$I_{ref} = I_o\left(\frac{\beta + 1 + m}{m\beta}\right)$$

$$\boxed{\frac{I_o}{I_{ref}} = \frac{m}{1 + \frac{m+1}{\beta}}}$$



BJT Current Mirror With β Compensation

- Addition of a transistor can reduce the effect of base current on the current transfer ratio
- KCL at the collector of Q_1:

$$I_{ref} = I_{C1} + I_{B3} = \frac{I_o}{m} + \frac{I_{C3}}{\beta}$$

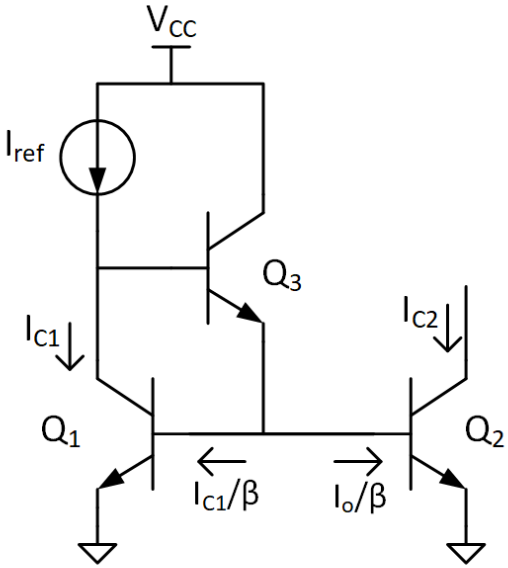
$$I_{ref} = \frac{I_o}{m} + \frac{\beta}{(\beta+1)}\frac{I_{E3}}{\beta}$$

where

$$I_{E3} = \frac{I_{C1}}{\beta} + \frac{I_{C2}}{\beta} = \frac{I_o}{m\beta} + \frac{I_o}{\beta}$$

Substituting into the expression for I_{ref}:

$$I_{ref} = \frac{I_o}{m} + \frac{I_o}{\beta(\beta+1)}\left(\frac{1}{m} + 1\right)$$



BJT Current Mirror With β Compensation

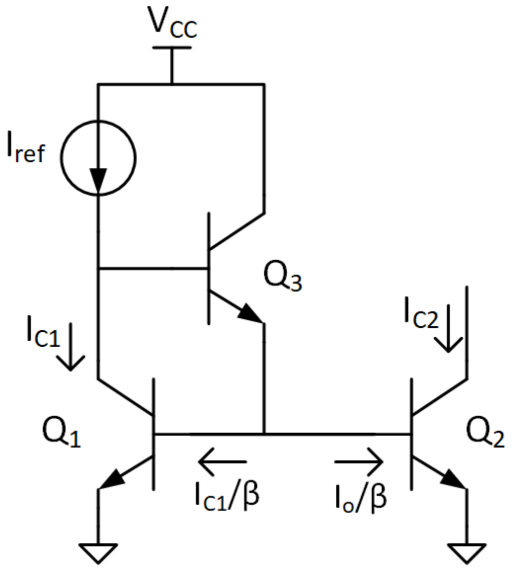
$$I_{ref} = \frac{I_o}{m} + \frac{I_o}{\beta(\beta+1)}\left(\frac{1}{m}+1\right)$$

$$I_{ref} = I_o\left[\frac{1}{m} + \frac{m+1}{m\beta(\beta+1)}\right]$$

- The current transfer ratio:

$$\frac{I_o}{I_{ref}} = \frac{1}{\frac{1}{m} + \frac{m+1}{m\beta(\beta+1)}}$$

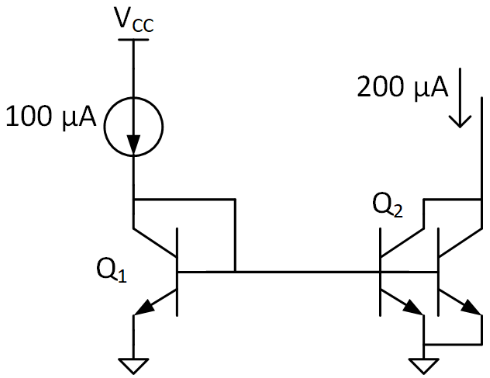
$$\boxed{\frac{I_o}{I_{ref}} = \frac{m}{1 + \frac{m+1}{\beta(\beta+1)}}}$$

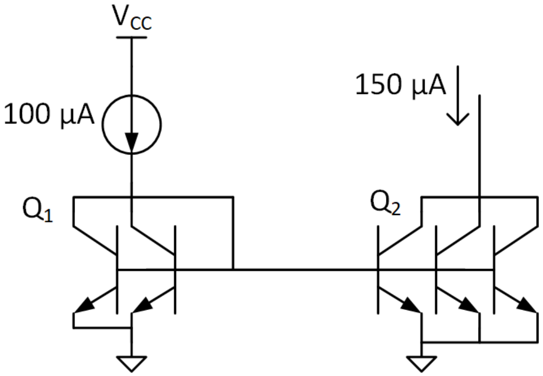


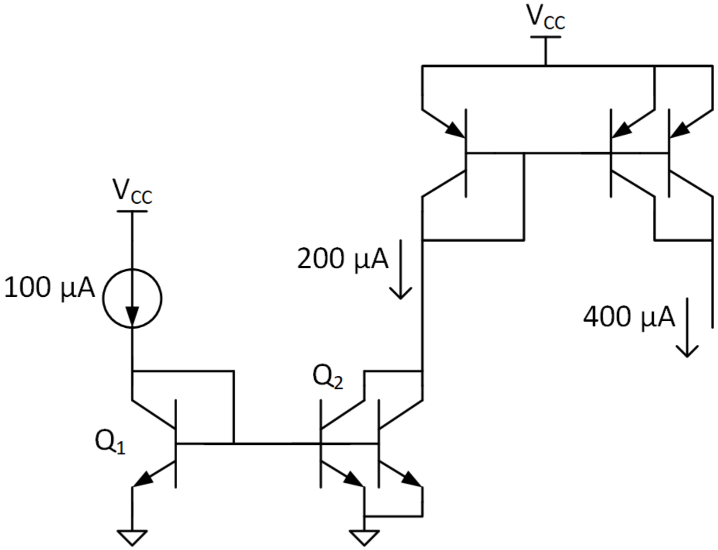
- The error term has been improved by a factor of $(\beta+1)$

BJT Current Scaling

- For BJTs emitter area scaling is typically accomplished by connecting multiple devices in parallel
  - For example ($\beta = \infty$):

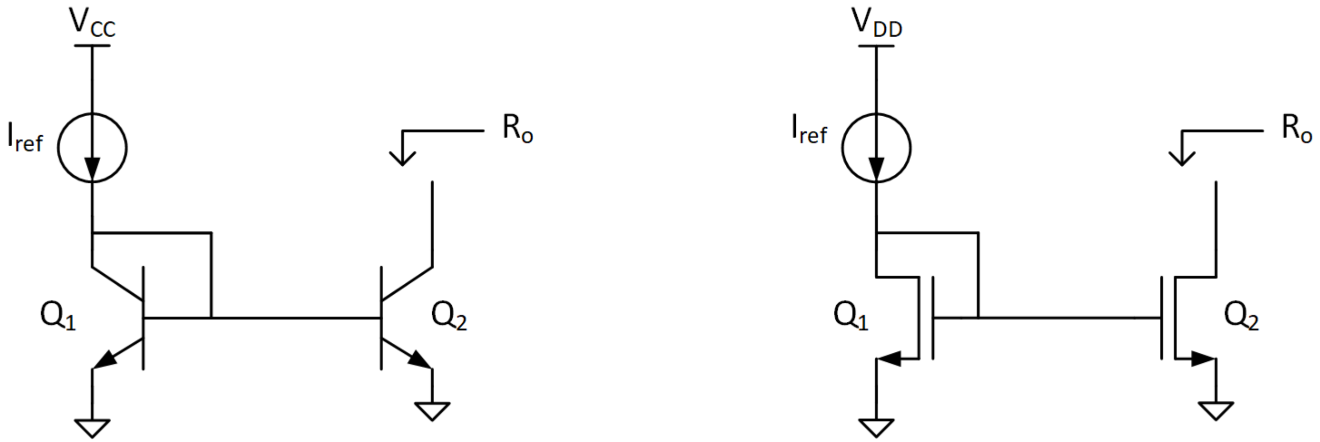






Current Mirrors – Output Resistance

- An important figure of merit for any current source is its output resistance
  - For an ideal current source, $R_o = \infty$
- For simple BJT and MOS current mirrors the output resistance is r_o of the transconductance transistor



$$R_o = r_o = \frac{V_A}{I_C}$$

$$R_o = r_o = \frac{1}{\lambda I_D} = \frac{V_A}{I_D}$$

Current Mirror – Example

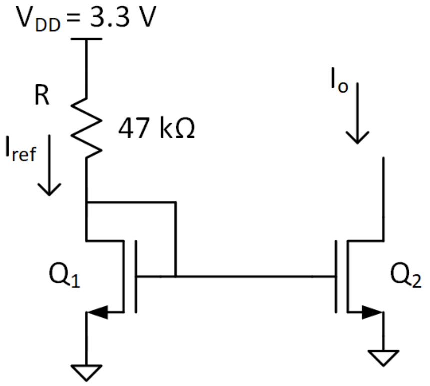
- Determine the output current, I_o
- Solve the quadratic for I_{ref}:

$$R^2 I_{ref}^2 - \left[2(V_{DD} - V_t)R + \frac{2}{k_n'\left(\frac{W}{L}\right)}\right] I_{ref} + (V_{DD} - V_t)^2 = 0$$

$$(47\ k\Omega)^2 I_{ref}^2 - \left[2(2.6\ V)47\ k\Omega + \frac{2}{1.9\frac{mA}{V^2}}\right] I_{ref} + (2.6\ V)^2 = 0$$

$$2.248e9 \cdot I_{ref}^2 - 247{,}593 \cdot I_{ref} + 6.76 = 0$$

$$I_{ref} = 50.4\ \mu A$$



$$\mu_n C_{ox} = 190\frac{\mu A}{V^2}$$

$$V_t = 700\ mV$$

$$\left(\frac{W}{L}\right)_1 = 10$$

$$\left(\frac{W}{L}\right)_2 = 15$$

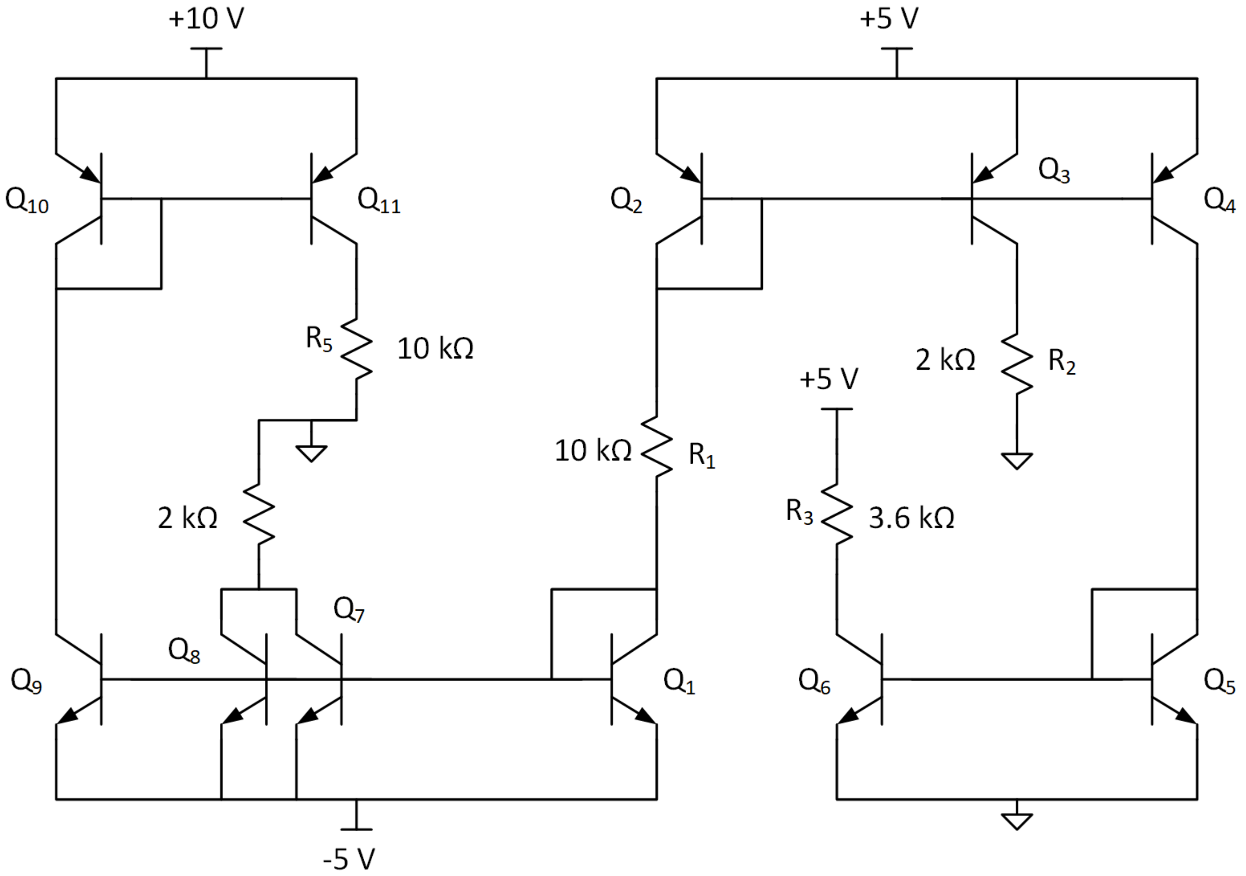
- Output current scales with transistor aspect ratio

$$I_o = I_{ref}\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1} = 50.4\ \mu A \cdot \frac{15}{10}$$

$$\boxed{I_o = 75.6\ \mu A}$$

Current Steering – Example

- Determine all collector currents and node voltages
  - Assume $\beta = \infty$ and $V_{BE} = 700\ mv$



Current Steering – Example

$$I_{C1} = I_{C2} = \frac{(+5\,V - V_{BE}) - (-5\,V + V_{BE})}{R_1} = \frac{8.6\,V}{10\,k\Omega} = 860\,\mu A$$

$$I_{C3} = I_{C2} = 860\,\mu A$$

$$V_{C3} = I_{C3}R_2 = 860\,\mu A \cdot 2\,k\Omega = 1.72\text{ V}$$

$$I_{C4} = I_{C3} = 860\,\mu A = I_{C5} = I_{C6}$$

$$V_{C6} = +5\,V - I_{C6}R_3 = +5\,V - 860\,\mu A \cdot 3.6\,k\Omega = 1.904\text{ V}$$

$$I_{C7} = I_{C8} = I_{C9} = I_{C1} = 860\,\mu A$$

$$V_{C7,8} = -(I_{C7} + I_{C8}) \cdot 2\,k\Omega = -1.72\,mA \cdot 2\,k\Omega = -3.44\text{ V}$$

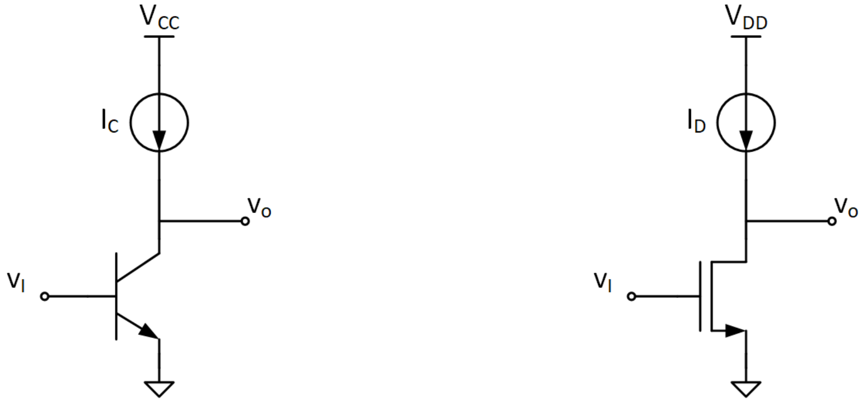
$$I_{C10} = I_{C11} = I_{C9} = 860\,\mu\text{A}$$

$$V_{C11} = I_{C11} \cdot 10\,k\Omega = 860\,\mu A \cdot 10\,k\Omega = 8.6\,V$$

The Basic Gain Cell

Basic Gain Cell

- The basic gain cell used in integrated circuit (IC) amplifiers
  - Common-emitter or common-source amplifier
  - Load resistance replaced with a transistor current source – an ***active load***



- Benefits of an active load
  - Provides both a high-resistance load and bias current
  - Consumes less chip area than a resistor
  - Enables higher gain

Amplifier gain

- To understand why active loads can provide high gain, consider a resistively-loaded CS amplifier:
- Amplifier gain:

$$|A_v| = g_m R_D$$

where

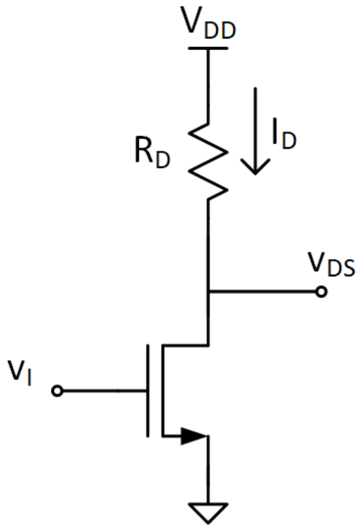
$$g_m = k_n' \left(\frac{W}{L}\right) V_{OV}$$

And, since

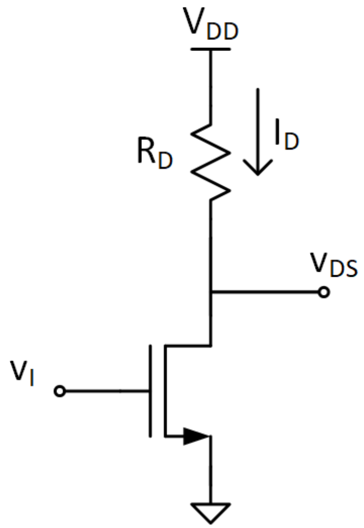
$$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right) V_{OV}^2$$

we can express g_m as

$$g_m = \frac{I_D}{\frac{1}{2} V_{OV}}$$



Amplifier gain



- The amplifier gain becomes:

$$|A_v| = g_m R_D = \frac{I_D R_D}{\frac{1}{2} V_{OV}}$$

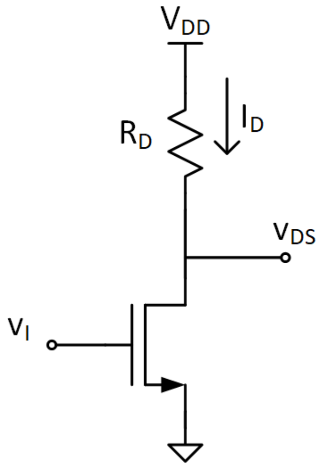
- The numerator represents the voltage drop across R_D:

$$|A_v| = \frac{V_{DD} - V_{DS}}{\frac{1}{2} V_{OV}}$$

- For active region operation, $V_{DS} \geq V_{OV}$, so the maximum gain is

$$|A_v|_{max} = \frac{V_{DD} - V_{OV}}{\frac{1}{2} V_{OV}}$$

Amplifier gain

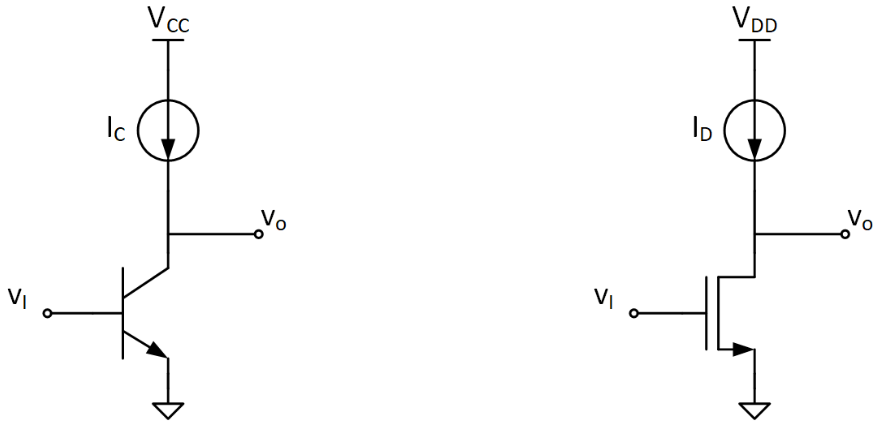


- For an IC amplifier with $V_{DD} = 3.3\ V$ and $V_{OV} = 200\ mV$ max gain is only

$$|A_v|_{max} = \frac{1.1\ V}{100\ mV} = 11$$

- Note that at this bias point (i.e., the edge of triode, $V_{DS} = V_{OV}$), the amplifier's dynamic range is zero
  - An absolute limit, not a usable gain
- Active loads allow for higher gain, because:
  - voltage drop across the load is decoupled from current and from high small-signal resistance
  - They are not resistors, so are not constrained by Ohm's law

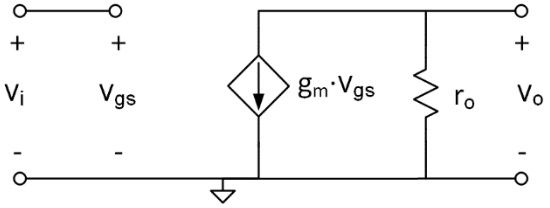
Active Loads and Biasing

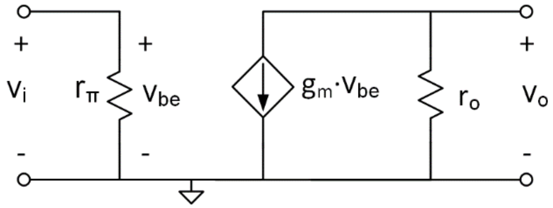


- Note that, in order to remain in the active region, I_D or I_C set by V_{GS} or V_{BE} must exactly match that of the current-source load
- You are only getting part of the story here
  - In practice, additional circuitry, including negative feedback stabilizes the bias point
- For now, just assume that the DC component of the input is such that its bias current matches the current source

Basic Gain Cell - Gain

- Small-signal models for the amplifiers:





- Treat the current sources as ideal
  - Infinite resistance – open circuit
  - In practice it would be a transistor output resistance
- Transistor r_o is the only load
- For both amplifiers, the gain is

$$A_v = -g_m r_o$$

- This is the maximum possible CS or CE gain
  - The ***intrinsic gain***, A_0

Intrinsic Gain – MOSFET

$$A_0 = -g_m r_o$$

- For the MOSFET,

$$g_m = \frac{I_D}{V_{OV}/2}$$

$$r_o = \frac{V_A}{I_D}$$

- So, the intrinsic gain is

$$A_0 = -\frac{V_A}{V_{OV}/2}$$

  - Proportional to the Early voltage
  - Inversely proportional to channel length, L
  - Inversely proportional to the overdrive voltage, V_{OV}
  - Inversely proportional to drain current, I_D

Intrinsic Gain – BJT

$$A_0 = -g_m r_o$$

- For the BJT,

$$g_m = \frac{I_C}{V_t}$$

$$r_o = \frac{V_A}{I_C}$$

- So, the intrinsic gain is

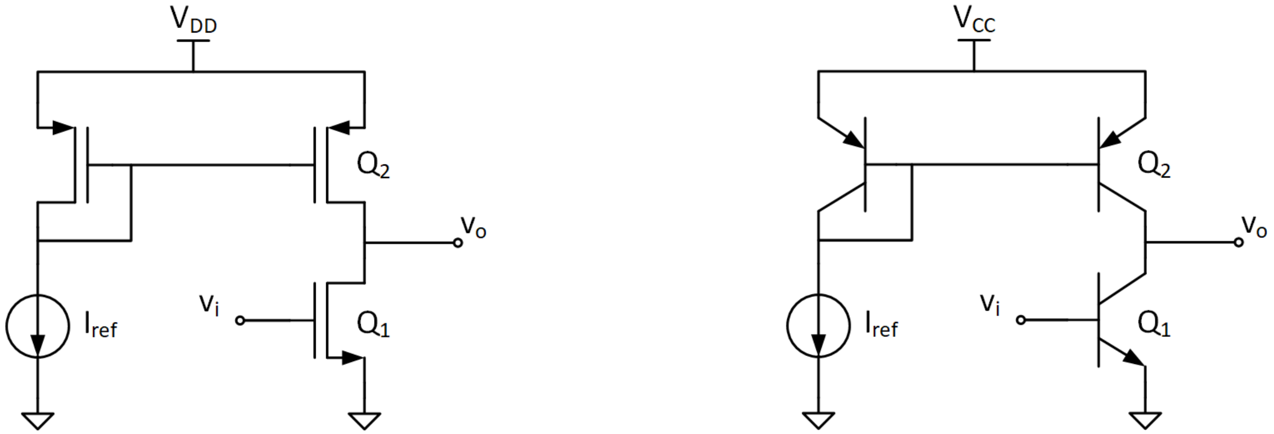
$$A_0 = -\frac{V_A}{V_t}$$

  - Proportional to the Early voltage
  - Inversely proportional to collector current, I_C
- BJTs vs. MOSFETs
  - V_t is much smaller than typical overdrive voltages (e.g. $150-300\ mV$)
  - V_A is typically larger for BJTs
  - In comparable modern processes, BJT intrinsic gain is an order of magnitude higher than that of MOSFETs

Finite Current-Source Resistance

- In practice, active loads are not ideal current sources
  - Transistor current sources
  - Resistance is the r_o of that transistor
  - Appears in parallel with the amplifier transistor's r_o
  - Gain will be significantly lower than the intrinsic gain

$$A_v = -g_m r_{01} || r_{o2}$$

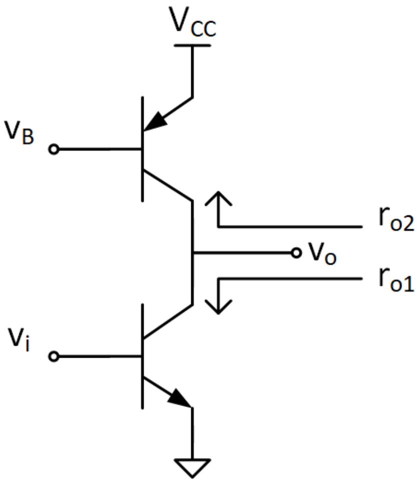


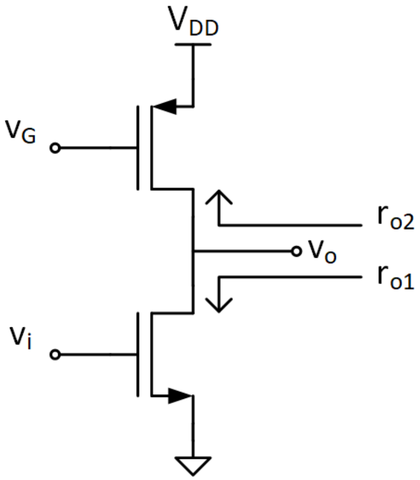
- Next, we'll see how we can increase gain by increasing the output resistance, $r_{o1}||r_{o2}$

CG/CB Amplifiers as Current Buffers

Increasing the Gain

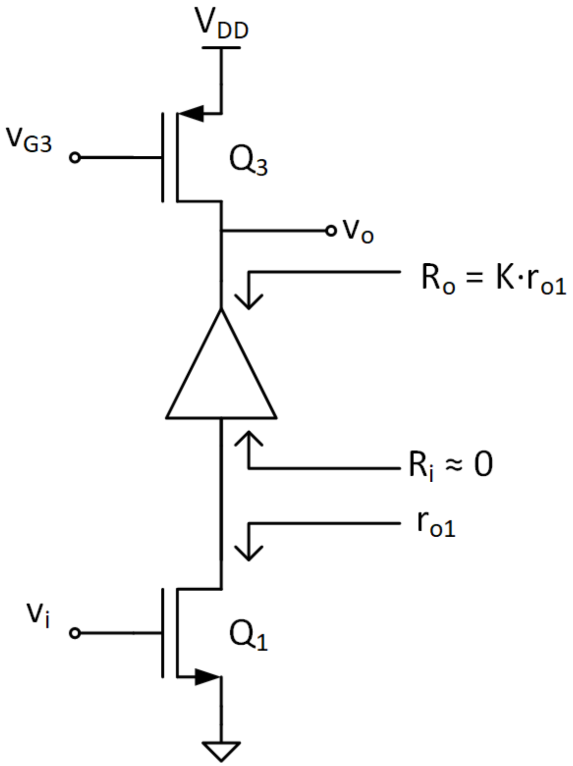
- If we can increase $r_{o1}||r_{o2}$, we can increase gain
- For now, we'll focus on increasing r_{o1}
  - I.e., the resistance seen looking back toward the amplifier transistor





Current Buffers

- The amplifier transistor’s output current is applied to the active load transistor, where it is converted to a voltage
- Want a device that will:
  - Replicate that current
  - Apply it to the active load
  - Do so with a higher r_o
- A ***current buffer***
  - Low input resistance
    - Input current unaffected
  - High output resistance
    - Approximating an ideal current source
    - Gain will be increased



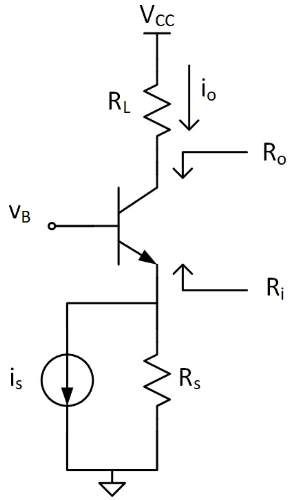
Current Buffers

- We have already seen transistor circuits that fit this description of a current buffer
  - ***Common-base*** amplifier
  - ***Common-gate*** amplifier

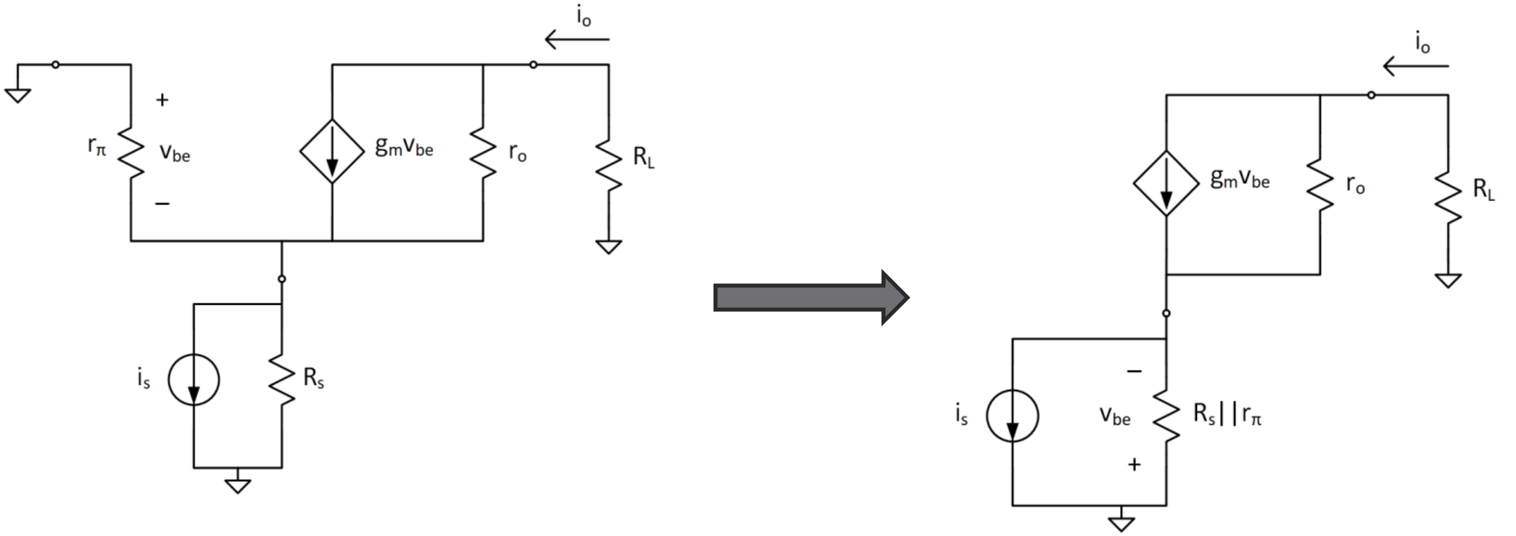
- We now revisit these circuits, looking more closely at the characteristics that make them suitable current buffers:
  - Current gain
  - Input resistance
  - Output resistance

Common-Base – Small-Signal Circuit

- Consider the following CB amplifier
  - Current source input with finite source resistance, R_S
  - Load resistance, R_L



- The small-signal equivalent circuit:

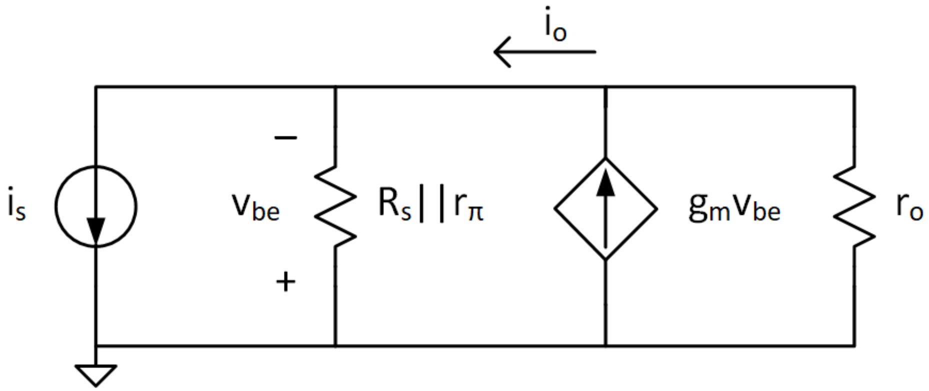


Common-Base – Current Gain

- First, we will determine the ***short-circuit current gain*** for the CB amplifier

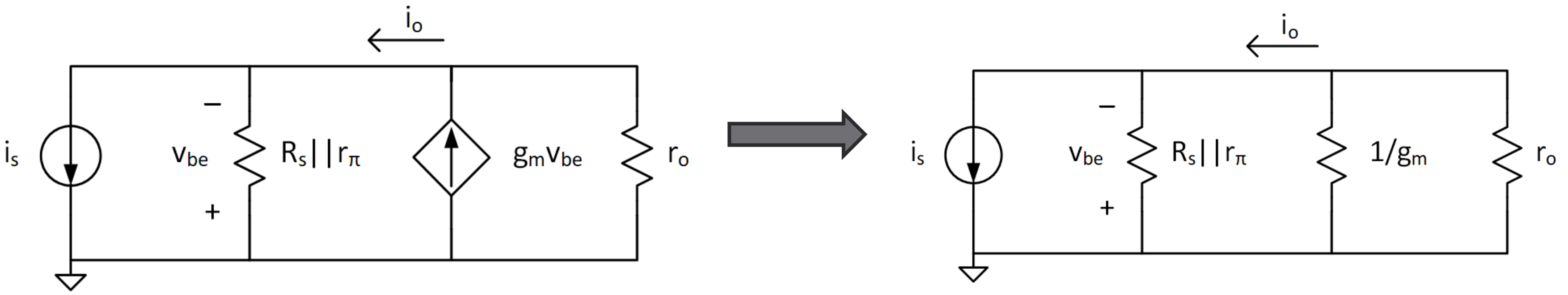
$$A_{is} = \frac{i_o}{i_s}$$

- For the short circuit-current gain, the output is shorted to ground
- The small-signal circuit simplifies to



Common-Base – Current Gain

- Note that the voltage across the VCCS is the source's controlling voltage, v_{be}
  - The VCCS, therefore, can be replaced by a resistance:



- Output current is given by current division:

$$i_o = i_s \frac{g_m + \frac{1}{r_o}}{g_m + \frac{1}{r_o} + \frac{1}{R_S||r_\pi}}$$

Common-Base – Current Gain

- The short-circuit current gain is

$$\boxed{A_{is} = \frac{i_o}{i_s} = \frac{g_m + \frac{1}{r_o}}{g_m + \frac{1}{r_o} + \frac{1}{R_S||r_\pi}}}$$

- If $r_o \gg 1/g_m$ and $R_S||r_\pi \gg 1/g_m$, then the current gain is approximately unity

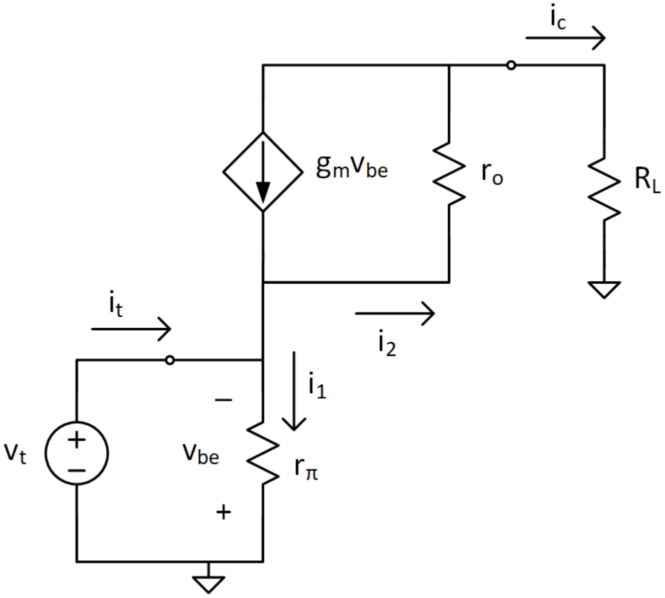
$$\boxed{A_{is} \approx 1}$$

  - As would be desired for a current buffer

Common-Base – Input Resistance

- To determine the input resistance, apply a test voltage source, v_t, and determine the resulting current, i_t
- KVL around the loop:

$$v_t - i_2 r_o - i_c R_L = 0 \qquad (1)$$



- KCL at the input node:

$$i_t + g_m v_{be} - i_1 - i_2 = 0$$

$$i_t + g_m v_{be} - \frac{v_t}{r_\pi} - i_2 = 0$$

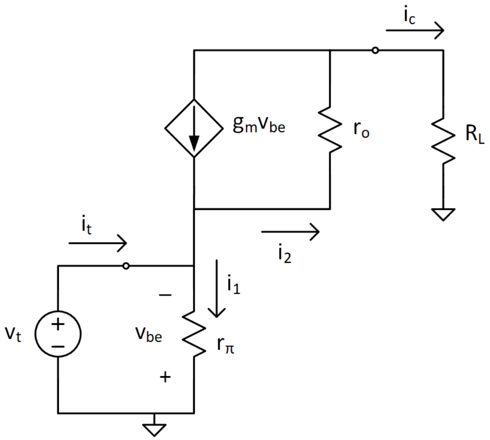
- Note that $v_{be} = -v_t$, so

$$i_t - g_m v_t - \frac{v_t}{r_\pi} - i_2 = 0$$

$$i_2 = i_t - g_m v_t - \frac{v_t}{r_\pi} \qquad (2)$$

Common-Base – Input Resistance

- KCL at the collector gives the collector current



$$i_c = i_2 - g_m v_{be} = i_2 + g_m v_t$$

$$i_c = i_t - g_m v_t - \frac{v_t}{r_\pi} + g_m v_t$$

$$i_c = i_t - \frac{v_t}{r_\pi} \qquad (3)$$

- Substituting (2) and (3) into (1):

$$v_t - i_t r_o + g_m r_o v_t + \frac{v_t}{r_\pi} r_o - i_t R_L + \frac{v_t R_L}{r_\pi} = 0$$

$$v_t \left(1 + g_m r_o + \frac{r_o}{r_\pi} + \frac{R_L}{r_\pi}\right) = i_t (r_o + R_L) \qquad (4)$$

Common-Base – Input Resistance

- The input resistance is

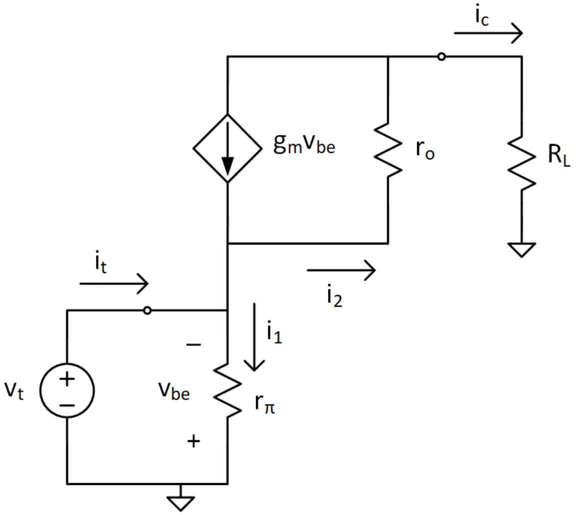
$$\boxed{R_i = \frac{v_t}{i_t} = \frac{r_o + R_L}{1 + g_m r_o + \frac{r_o}{r_\pi} + \frac{R_L}{r_\pi}}} \qquad (5)$$

- We can simplify (5) by noting the following

$$g_m r_o \gg 1$$

$$\frac{r_o}{r_\pi} = \frac{g_m r_o}{\beta} \ll g_m r_o$$

$$\frac{R_L}{r_\pi} = \frac{g_m R_L}{\beta} \ll g_m r_o$$



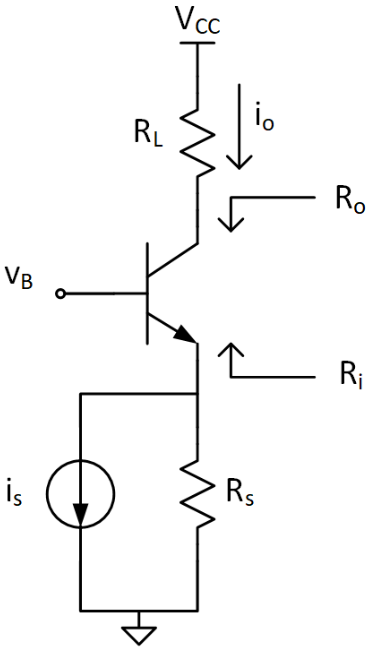
- The input resistance simplifies to

$$R_i \approx \frac{r_o + R_L}{g_m r_o} = \frac{1}{g_m} + \frac{R_L}{g_m r_o} = r_e + \frac{R_L}{g_m r_o} \qquad (6)$$

Common-Base – Input Resistance

- Common-base input resistance:

$$\boxed{R_i \approx \frac{1}{g_m} + \frac{R_L}{g_m r_o} = r_e + \frac{R_L}{g_m r_o}} \qquad (6)$$



- Two components to R_i:
  - Emitter resistance, r_e
    - Typically small
  - Load resistance, R_L, ***reduced by the transistor's intrinsic gain***
    - Effect of R_L reduced
- R_i is typically small, as desired from a current buffer

Common-Base – Output Resistance

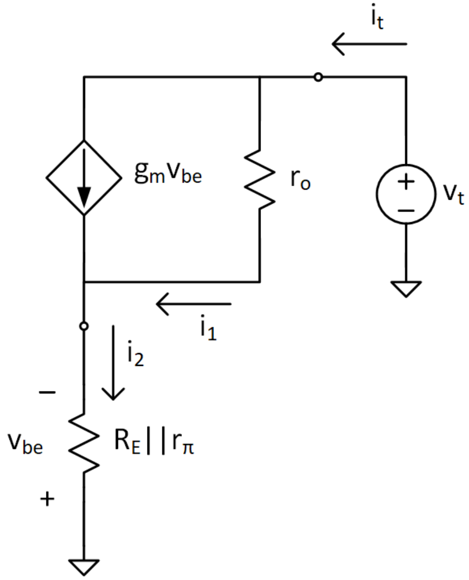
- To determine output resistance, R_o, apply a test voltage to the output
- Applying KVL

$$v_t - i_1 r_o - i_2(R_E||r_\pi) = 0 \tag{7}$$

- Applying KCL gives

$$i_1 = i_t - g_m v_{be} \tag{8}$$

$$i_2 = i_1 + g_m v_{be} = i_t \tag{9}$$



- The base-emitter voltage is

$$v_{be} = -i_2(R_E||r_\pi) = -i_t(R_E||r_\pi) \tag{10}$$

- Substituting (8), (9), and (10) into (7) gives

$$v_t - i_t r_o - g_m i_t(R_E||r_\pi) r_o - i_t(R_E||r_\pi) = 0 \tag{11}$$

Common-Base – Output Resistance

- Rearranging (11) gives the output resistance

$$R_o = \frac{v_t}{i_t} = r_o + (R_E||r_\pi) + g_m r_o (R_E||r_\pi)$$

$$\boxed{R_o = r_o + (1 + g_m r_o)(R_E||r_\pi)} \qquad (12)$$

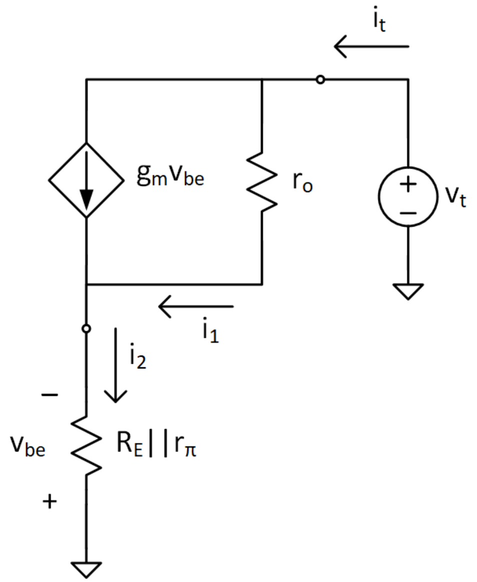
- We can simplify, assuming $g_m r_o \gg 1$

$$\boxed{R_o \approx r_o + g_m r_o (R_E||r_\pi)} \qquad (13)$$

- And, if $g_m (R_E||r_\pi) \gg 1$, then

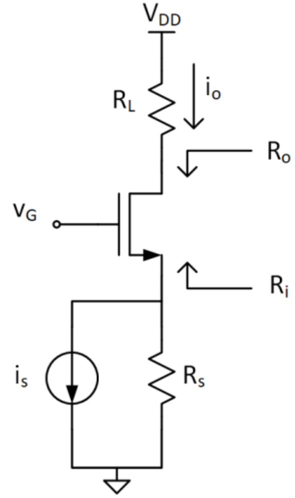
$$\boxed{R_o \approx g_m r_o (R_E||r_\pi)}$$

- Here, the resistance at the input is ***increased by the transistor's intrinsic gain***
- R_o will typically be a large resistance, as would be desired

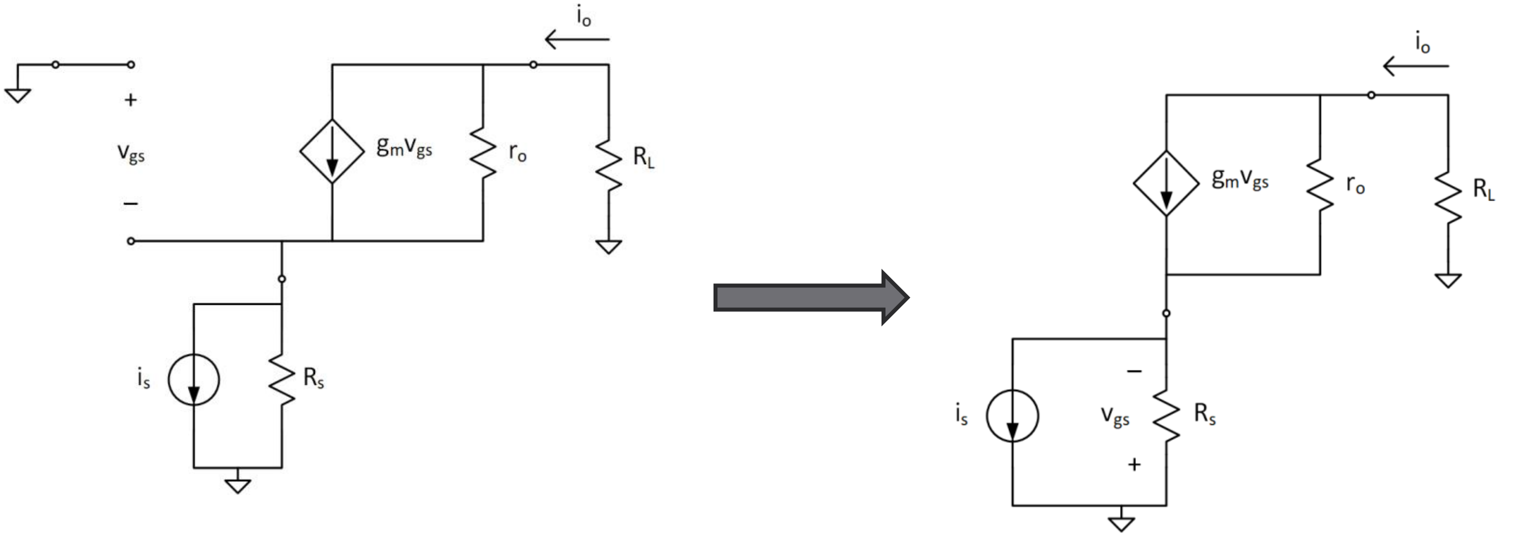


Common-Gate – Small-Signal Circuit

- Consider the following CG amplifier
  - Current source input with finite source resistance, R_S
  - Load resistance, R_L

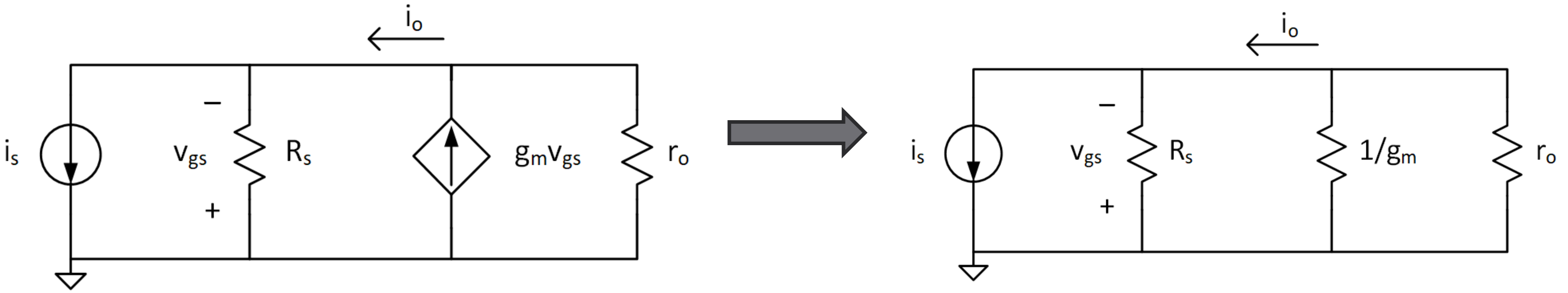


- The small-signal equivalent circuit:



Common-Gate – Current Gain

- To determine short-circuit current gain, A_{is}, we can simplify the small-signal circuit just as we did for the CB circuit:



- Output current is given by current division:

$$i_o = i_s \frac{g_m + \frac{1}{r_o}}{g_m + \frac{1}{r_o} + \frac{1}{R_S}}$$

Common-Gate – Current Gain

- The short-circuit current gain is

$$\boxed{A_{is} = \frac{i_o}{i_s} = \frac{g_m + \frac{1}{r_o}}{g_m + \frac{1}{r_o} + \frac{1}{R_S}}}$$

- If $r_o \gg 1/g_m$ and $R_S \gg 1/g_m$, then the current gain is approximately unity

$$\boxed{A_{is} \approx 1}$$

  - As desired, and similar to the CB amplifier

Common-Gate – Input Resistance

- To determine the input resistance, apply a test voltage source, v_t, and determine the resulting current, i_t
- KVL around the loop:

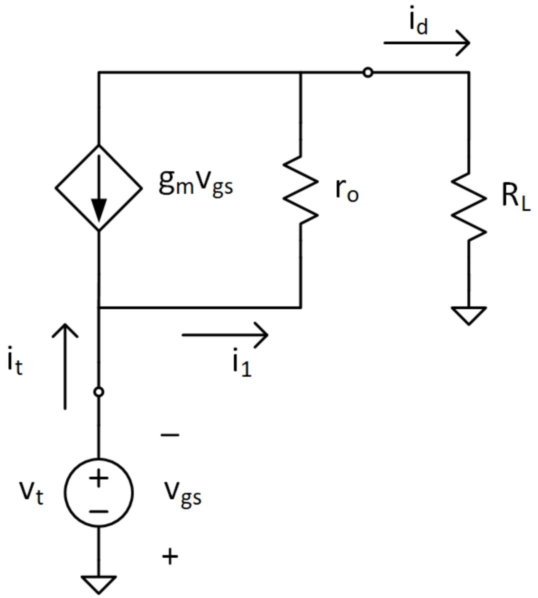
$$v_t - i_1 r_o - i_d R_L = 0 \qquad (14)$$

- KCL at the input node:

$$i_1 = i_t + g_m v_{gs} = 0$$

- Note that $v_t = -v_{gs}$, so

$$i_1 = i_t - g_m v_t \qquad (15)$$



- Substituting (15) into (14) and noting that drain and source currents are equal

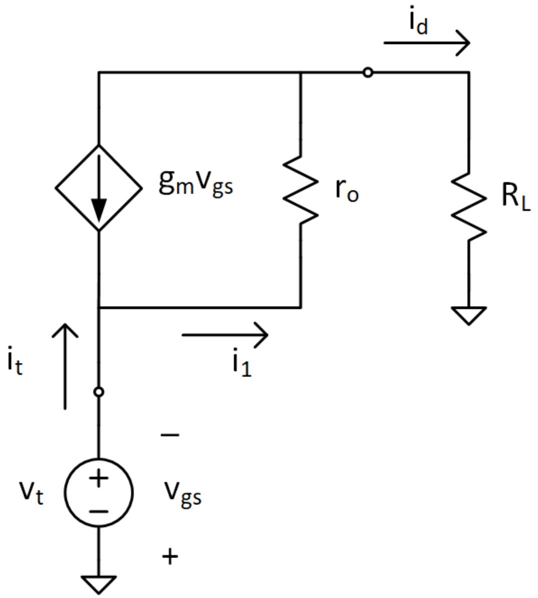
$$v_t - i_t r_o + g_m r_o v_t - i_t R_L = 0$$

$$v_t(1 + g_m r_o) = i_t(r_o + R_L) \qquad (16)$$

Common-Gate – Input Resistance

- Solving (16) gives the input resistance:

$$\boxed{R_i = \frac{v_t}{i_t} = \frac{r_o+R_L}{1+g_m r_o}} \qquad (17)$$



- For $g_m r_o \gg 1$, R_i simplifies to

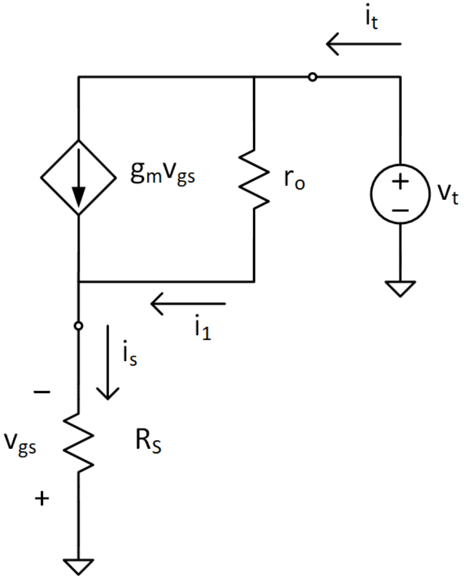
$$\boxed{R_i \approx \frac{1}{g_m} + \frac{R_L}{g_m r_o}} \qquad (18)$$

- Two components to R_i
  - The input resistance when neglecting r_o
  - The load resistance ***reduced by the intrinsic gain***

Common-Gate – Output Resistance

- To determine output resistance, R_o, apply a test voltage to the output
- Applying KVL

$$v_t - i_1 r_o - i_s R_S = 0 \tag{19}$$



- Applying KCL gives

$$i_1 = i_t - g_m v_{gs} \tag{20}$$

- The gate-source voltage is

$$v_{gs} = -i_s R_S = -i_t R_S \tag{21}$$

- Substituting (20) and (21) into (19) gives

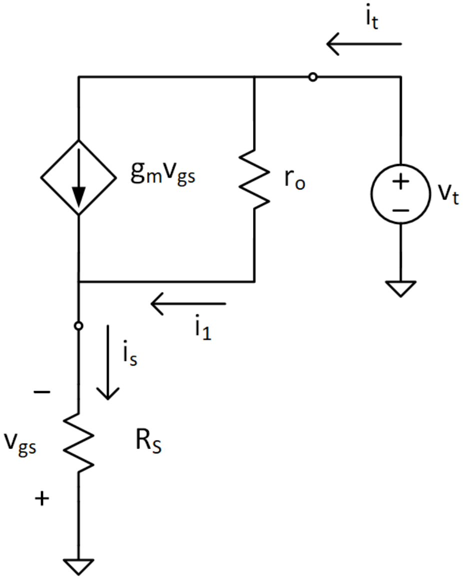
$$v_t - i_t r_o - g_m i_t R_S r_o - i_t R_S = 0 \tag{22}$$

Common-Gate – Output Resistance

- Rearranging (22) gives the output resistance

$$R_o = \frac{v_t}{i_t} = r_o + R_S + g_m r_o R_S$$

$$\boxed{R_o = r_o + (1 + g_m r_o) R_S} \qquad (23)$$



- We can simplify, assuming $g_m r_o \gg 1$

$$\boxed{R_o \approx r_o + g_m r_o R_S} \qquad (24)$$

- If we assume $g_m r_o \gg 1$, we can simplify further

$$\boxed{R_o \approx g_m r_o R_S}$$

- Here, the resistance at the input is ***increased by the transistor's intrinsic gain***
- R_o will typically be a relatively large resistance

Common-Base/Common-Gate - Summary

$$A_{is} = \frac{g_m + \frac{1}{r_o}}{g_m + \frac{1}{r_o} + \frac{1}{R_S||r_\pi}} \approx 1$$

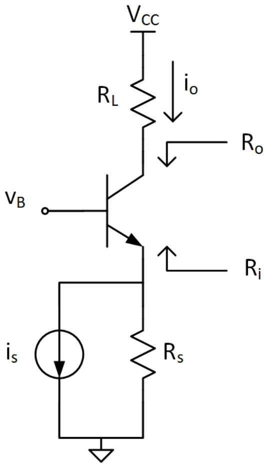
$$R_i = \frac{r_o + R_L}{1 + g_m r_o + \frac{r_o}{r_\pi} + \frac{R_L}{r_\pi}}$$

$$R_i \approx \frac{1}{g_m} + \frac{R_L}{g_m r_o} = r_e + \frac{R_L}{g_m r_o}$$

$$R_o = r_o + (1 + g_m r_o)(R_E||r_\pi)$$

$$R_o \approx \; r_o + g_m r_o (R_E||r_\pi)$$

$$R_o \approx \; g_m r_o (R_E||r_\pi)$$



$$A_{is} = \frac{g_m + \frac{1}{r_o}}{g_m + \frac{1}{r_o} + \frac{1}{R_S}} \approx 1$$

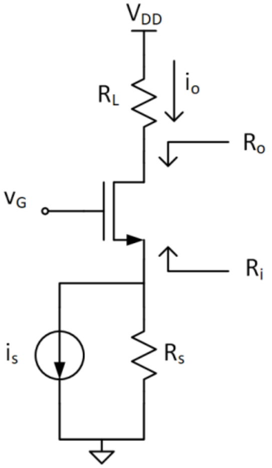
$$R_i = \frac{r_o + R_L}{1 + g_m r_o}$$

$$R_i \approx \frac{1}{g_m} + \frac{R_L}{g_m r_o}$$

$$R_o = r_o + (1 + g_m r_o) R_S$$

$$R_o \approx \; r_o + g_m r_o R_S$$

$$R_o \approx \; g_m r_o R_S$$



Cascode Amplifiers

Cascode Amplifier – MOSFET

- Recall the gain of a basic gain cell with an active load

$$A_v = -g_{m1}R_o = -g_{m1}(r_{o1}||r_{02})$$

- We can increase this gain by increasing r_{o1} or r_{o2}
- First, we'll focus on r_{o1} by adding a ***cascode*** device
- Now, R_o becomes

$$R_o = R_{on}||R_{op} = R_{on}||r_{o3}$$

where

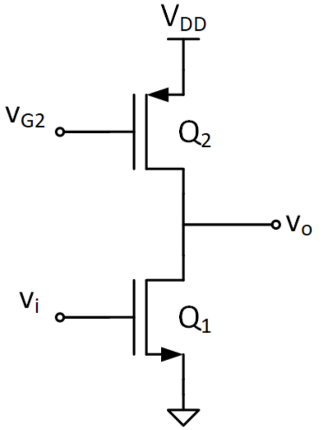
$$R_{on} = g_{m2}r_{o2}(r_{o1})$$

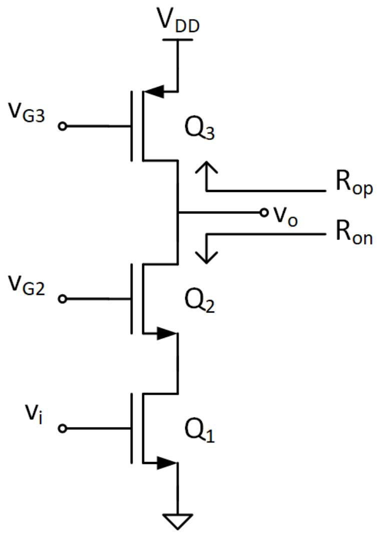
- The gain is then

$$A_v = -g_{m1}(R_{on}||r_{o3})$$

$$A_v = -g_{m1}[(g_{m2}r_{o2}r_{o1})||r_{o3}]$$

- r_{o1} has been increased by a factor of $g_{m2}r_{o2}$
  - Gain is increased significantly





Cascode Amplifier – BJT

- For the BJT cascode, R_{on} is

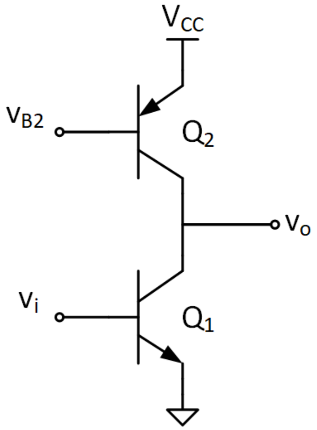
$$R_{on} = g_{m2} r_{o2} (r_{o1} || r_{\pi 2})$$

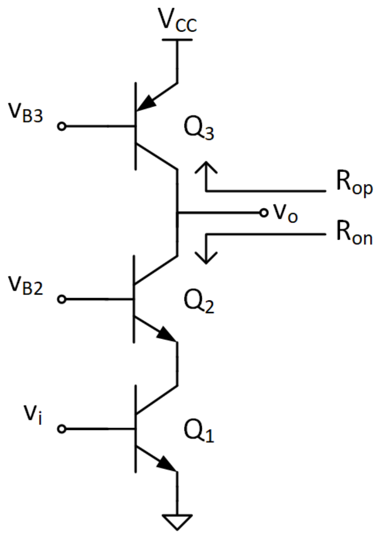
- And the gain is

$$A_v = -g_{m1} (R_{on} || r_{o3})$$

$$A_v = -g_{m1} \left[\left(g_{m2} r_{o2} (r_{o1} || r_{\pi 2}) \right) || r_{o3} \right]$$

- A bit more complicated result than for the MOS circuit
  - Gain still significantly increased
- Note that in both cases, we have only improved one component of R_o
  - We can further increase gain by increasing r_{o3} as well





Cascode Current Source – MOSFET

- We can further increase R_o and gain by adding a cascode device to the active load transistor
  - Increases r_{o3} and R_{op}
- Now, R_{op} is

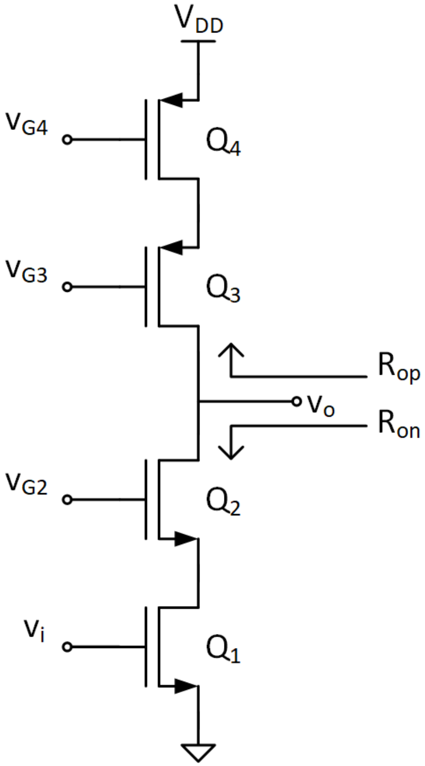
$$R_{op} = g_{m3} r_{o3} (r_{o4})$$

- The gain is

$$A_v = -g_{m1}\left(R_{on} || R_{op}\right)$$

$$A_v = -g_{m1}[(g_{m2} r_{o2} r_{o1}) || (g_{m3} r_{o3} r_{o4})]$$

- Now r_{o4} has been increased by a factor of $g_{m3} r_{o3}$
  - Another significant gain increase



Cascode Current Source – BJT

- For the BJT cascode amplifier, the cascode device on the active load increases R_{op} similarly

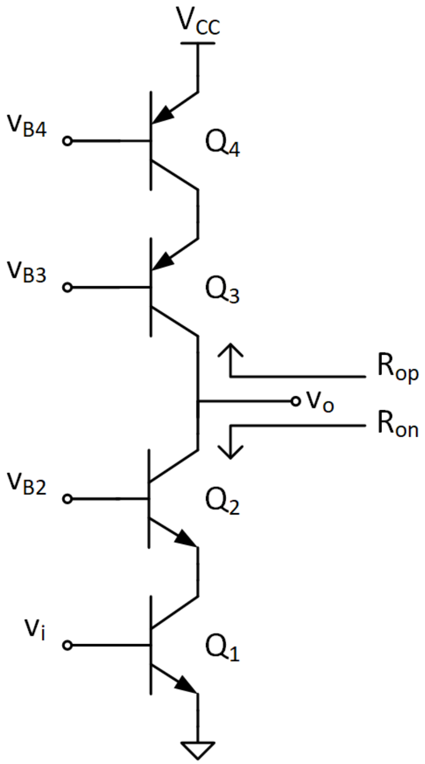
$$R_{op} = g_{m3} r_{o3} (r_{o4} || r_{\pi 3})$$

- The gain is

$$A_v = -g_{m1} (R_{on} || R_{op})$$

$$A_v = -g_{m1} \{[g_{m2} r_{o2} (r_{o1} || r_{\pi 2})] || [g_{m3} r_{o3} (r_{o4} || r_{\pi 3})]\}$$

- Again, the result is a bit more complicated than for the MOS circuit
  - Resulting increase of gain is similar



Cascode Amplifier – Example 1

- Find the gain of the basic gain cell in a 0.18 μm CMOS process with the given parameters
- Gain is given by

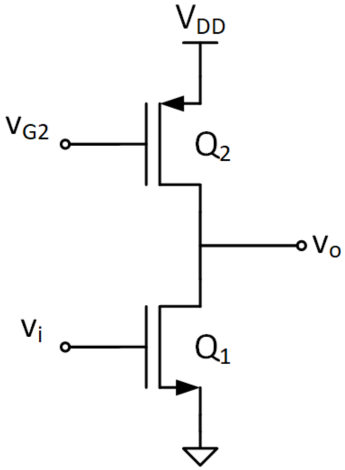
$$A_v = -g_{m1}R_o = -g_{m1}r_{o1}||r_{o2}$$

- The transistor transconductances:

$$g_{m1} = \sqrt{2k_n'\left(\frac{W}{L}\right)_n I_D}$$

$$g_{m1} = \sqrt{2 \cdot 387\,\frac{\mu A}{V^2} \cdot 10 \cdot 100\;\mu A}$$

$$g_{m1} = 880\;\mu S$$



$I_D = 100\;\mu A$

$\mu_n C_{ox} = 387\frac{\mu A}{V^2}$

$\mu_p C_{ox} = 86\frac{\mu A}{V^2}$

$\left(\frac{W}{L}\right)_n = \frac{10\;\mu m}{1\;\mu m}$

$\left(\frac{W}{L}\right)_p = \frac{50\;\mu m}{1\;\mu m}$

$V_{An} = 5\;V$

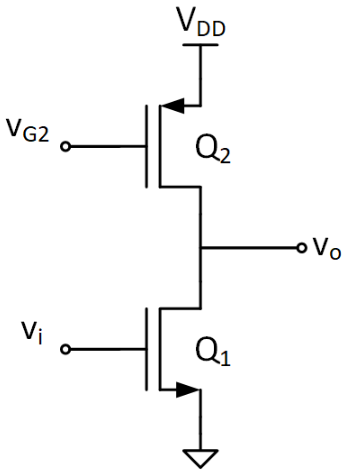
$V_{Ap} = 6\;V$

Cascode Amplifier – Example 1

- The transistor output resistances:

$$r_{o1} = \frac{V_{An}}{I_D} = \frac{5\ V}{100\ \mu A} = 50\ k\Omega$$

$$r_{o2} = \frac{V_{Ap}}{I_D} = \frac{6\ V}{100\ \mu A} = 60\ k\Omega$$



$I_D = 100\ \mu A$

$\mu_n C_{ox} = 387 \frac{\mu A}{V^2}$

$\mu_p C_{ox} = 86 \frac{\mu A}{V^2}$

$\left(\frac{W}{L}\right)_n = \frac{10\ \mu m}{1\ \mu m}$

$\left(\frac{W}{L}\right)_p = \frac{50\ \mu m}{1\ \mu m}$

$V_{An} = 5\ V$

$V_{Ap} = 6\ V$

- The gain:

$$A_v = -g_{m1} r_{o1} || r_{o2}$$

$$A_v = -880\ \mu S \cdot 50\ k\Omega || 60\ k\Omega$$

$$A_v = -880\ \mu S \cdot 27.3\ k\Omega$$

$$\boxed{A_v = -24}$$

Cascode Amplifier – Example 2

- Next, add a cascode device and determine the resulting gain
- Now, the gain is given by

$$A_v = -g_{m1}R_o = -g_{m1}R_{on}||r_{o3}$$

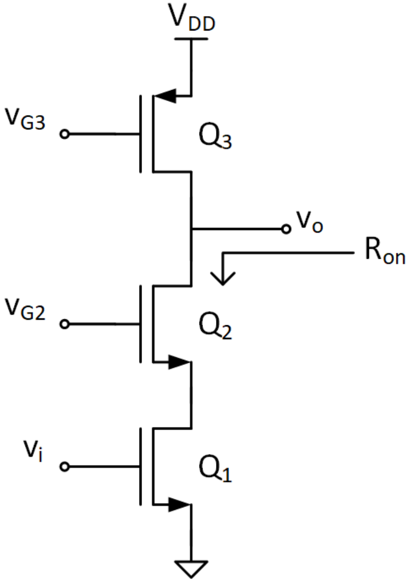
- Transconductance of the NMOS devices is unchanged:

$$g_{m1} = g_{m2} = 880\ \mu S$$

- As are all output resistances

$$r_{o1} = r_{o2} = 50\ k\Omega$$

$$r_{o3} = 60\ k\Omega$$



$I_D = 100\ \mu A$

$\mu_n C_{ox} = 387\frac{\mu A}{V^2}$

$\mu_p C_{ox} = 86\frac{\mu A}{V^2}$

$\left(\frac{W}{L}\right)_n = \frac{10\ \mu m}{1\ \mu m}$

$\left(\frac{W}{L}\right)_p = \frac{50\ \mu m}{1\ \mu m}$

$V_{An} = 5\ V$

$V_{Ap} = 6\ V$

Cascode Amplifier – Example 2

- The resistance looking into the drain of Q_2 is

$$R_{on} = g_{m2}r_{o2} \cdot r_{o1}$$

$$R_{on} = 880\ \mu S \cdot 50\ k\Omega \cdot 50\ k\Omega$$

$$R_{on} = 44 \cdot 50\ k\Omega = 2.2\ M\Omega$$

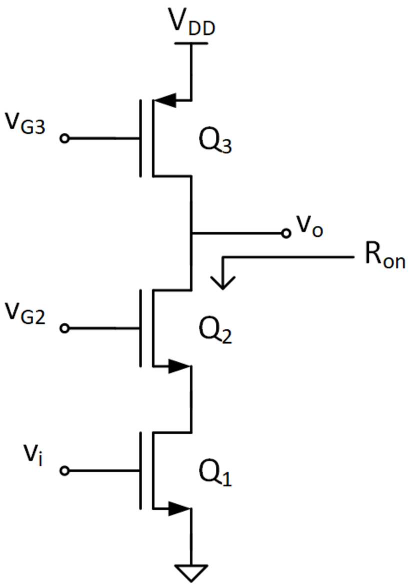
- The gain:

$$A_v = -g_{m1}R_{on}||r_{o3}$$

$$A_v = -880\ \mu S \cdot 2.2\ M\Omega||60\ k\Omega$$

$$A_v = -880\ \mu S \cdot 58.4\ k\Omega$$

$$\boxed{A_v = -51.4}$$



$I_D = 100\ \mu A$

$\mu_n C_{ox} = 387 \frac{\mu A}{V^2}$

$\mu_p C_{ox} = 86 \frac{\mu A}{V^2}$

$\left(\frac{W}{L}\right)_n = \frac{10\ \mu m}{1\ \mu m}$

$\left(\frac{W}{L}\right)_p = \frac{50\ \mu m}{1\ \mu m}$

$V_{An} = 5\ V$

$V_{Ap} = 6\ V$

- The cascode transistor increased the gain from 24 to 51.4
  - A relatively small increase compared to the 44x increase in R_{on}
  - R_o dominated by r_{o3}

Cascode Amplifier – Example 3

- Finally, add a cascode device to the current-source load and determine the gain
- Now, the gain is given by

$$A_v = -g_{m1}R_o = -g_{m1}R_{on}||R_{op}$$

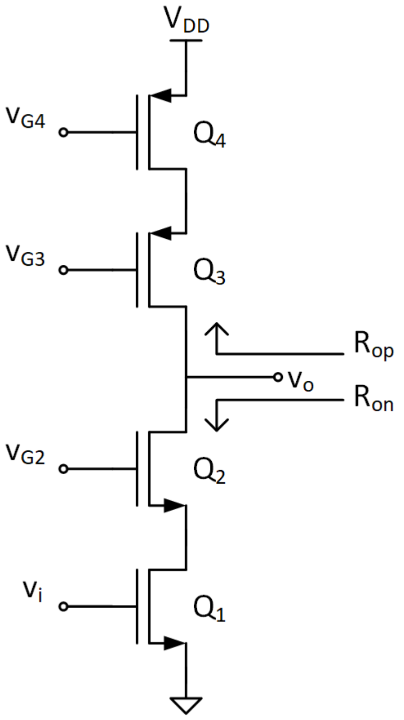
- Transconductance of the PMOS devices:

$$g_{m3} = g_{m4} = \sqrt{2k_p'\left(\frac{W}{L}\right)_p I_D}$$

$$g_{m3} = \sqrt{2 \cdot 86\frac{\mu A}{V^2} \cdot 50 \cdot 100\ \mu A}$$

$$g_{m3} = 927\ \mu S$$

- All other quantities are the same



$I_D = 100\ \mu A$

$\mu_n C_{ox} = 387\frac{\mu A}{V^2}$

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$\left(\frac{W}{L}\right)_n = \frac{10\ \mu m}{1\ \mu m}$

$\left(\frac{W}{L}\right)_p = \frac{50\ \mu m}{1\ \mu m}$

$V_{An} = 5\ V$

$V_{Ap} = 6\ V$

Cascode Amplifier – Example 3

- The resistance looking into the drain of Q_3 is

$$R_{op} = g_{m3} r_{o3} \cdot r_{o4}$$

$$R_{op} = 927\ \mu S \cdot 60\ k\Omega \cdot 60\ k\Omega$$

$$R_{op} = 55.6 \cdot 60\ k\Omega = 3.34\ M\Omega$$

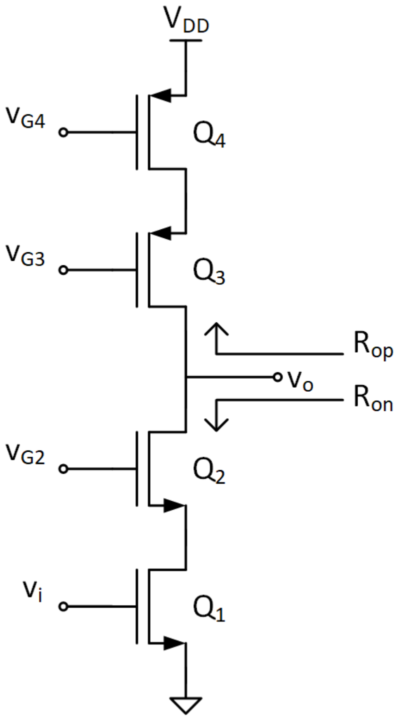
- The gain:

$$A_v = -g_{m1} R_{on} || R_{op}$$

$$A_v = -880\ \mu S \cdot 2.2\ M\Omega || 3.34\ M\Omega$$

$$A_v = -880\ \mu S \cdot 1.33\ M\Omega$$

$$\boxed{A_v = -1167}$$



$$I_D = 100\ \mu A$$

$$\mu_n C_{ox} = 387 \frac{\mu A}{V^2}$$

$$\mu_p C_{ox} = 86 \frac{\mu A}{V^2}$$

$$\left(\frac{W}{L}\right)_n = \frac{10\ \mu m}{1\ \mu m}$$

$$\left(\frac{W}{L}\right)_p = \frac{50\ \mu m}{1\ \mu m}$$

$$V_{An} = 5\ V$$

$$V_{Ap} = 6\ V$$

- Cascode current-source load results in a significant gain increase
  - Both components of R_o were increased by a factor of $g_m r_o$

Improved Current Mirrors

Current Mirror Performance

- There are three main current-mirror figures of merit:
- ***Current transfer ratio***
  - How closely does the output current match the input/reference current?
  - A function of the Early effect and of β (BJTs)
- ***Output resistance***
  - Enables higher amplifier gain if used as an active load
  - Also affects current transfer ratio
    - How much is output current affected by output voltage?
- ***Overhead voltage***
  - What is the minimum voltage required at the output such that the transistors remain in the active region?
- We will now look at a few circuits that aim to improve on one or more of these characteristics

Cascode Current Mirror – MOSFET

- As we have seen, a cascode device can improve output resistance
- A MOSFET cascode current mirror:
  - Assume matched devices (i.e., $m = 1$)
- Output resistance:

$$R_o = g_{m3} r_{o3} \cdot r_{o2}$$

- Voltage at the gate of Q_3/Q_4:

$$V_{G3} = 2(V_t + V_{OV})$$

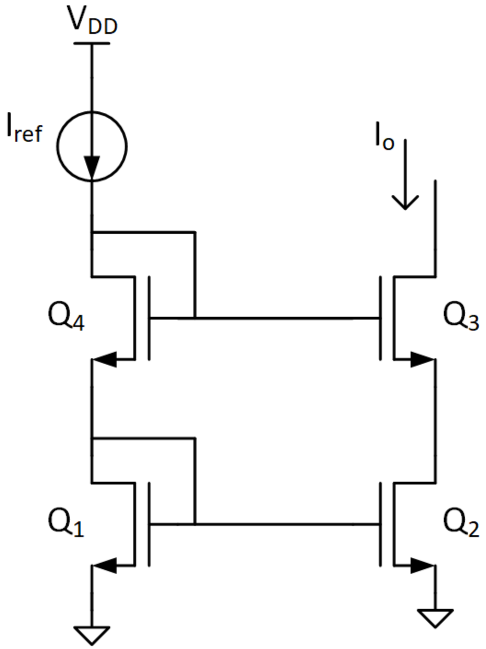
- To remain in the active region:

$$V_{D3} > V_t + 2V_{OV}$$

  - If, for example, $V_t = 500\ mV$ and $V_{OV} = 200\ mV$

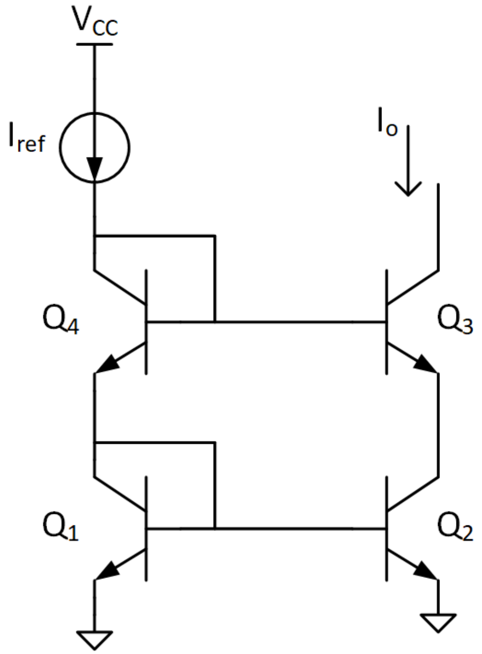
$$V_{D3} > 900\ mV$$

  - A significant portion of the total supply voltage



Cascode Current Mirror – BJT

- The BJT cascode current mirror:



- Output resistance:

$$R_o = g_{m3} r_{o3} (r_{o2} || r_{\pi 3})$$

- Voltage at the base of Q_3/Q_4:

$$V_{B3} = 2V_{BE}$$

- To remain in the active region:

$$V_{C3} > V_{B3} = 2V_{BE}$$

$$V_{C3} > \sim 1.4\ V$$

  - Again, a significant portion of the total supply voltage

- Cascode does not affect the current transfer ratio:

$$\frac{I_o}{I_{ref}} = \frac{1}{1 + \frac{2}{\beta}}$$

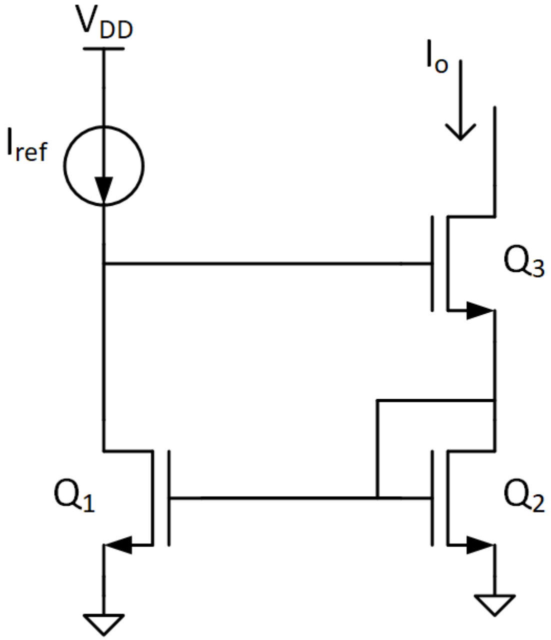
Wilson Current Mirror – MOSFET

- The Wilson current mirror uses ***negative feedback*** to increase R_o
  - Output current, I_o, is applied to the current mirror formed by Q_2 and Q_1
  - Mirrored I_o is fed back to the gate of Q_3
  - $I_{G3} = 0$, so I_o must be equal to I_{ref}
  - If $I_o > \mathrm{I_{ref}}$, V_{G3} will drop, decreasing I_o
  - If $I_o < \mathrm{I_{ref}}$, V_{G3} will rise, increasing I_o
- Output resistance:

$$R_o = g_{m3} r_{o3} \cdot r_{o1} + r_{o3} + \frac{1}{g_{m2}}$$

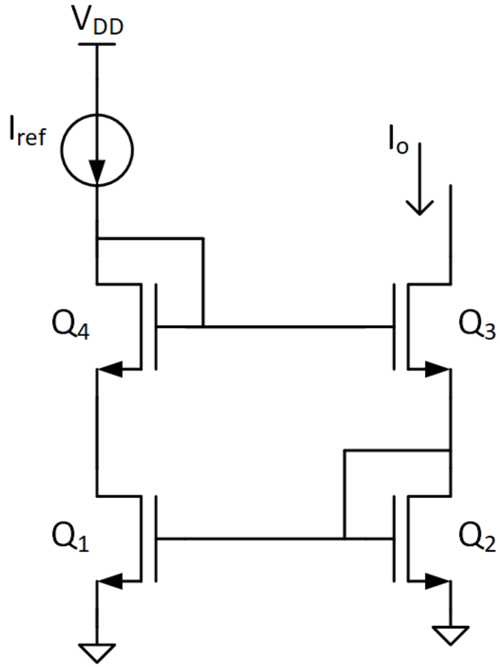
$$\boxed{R_o \approx g_{m3} r_{o3} \cdot r_{o1}}$$

- Same R_o as for the cascode current mirror



Improved Wilson Mirror – MOSFET

- Wilson mirror can be improved by adding a fourth transistor
  - Q_4 helps to balance the drain-source voltages for Q_1 and Q_2
  - Reduces current-transfer-ratio errors



Wilson Current Mirror – BJT

- Benefits of the Wilson mirror are greater for BJTs than for MOSFETs
  - β-related current error reduced
- Current transfer ratio:

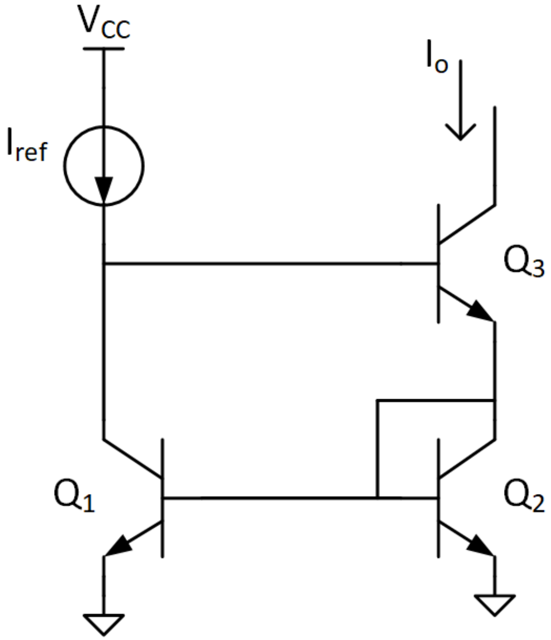
$$\frac{I_o}{I_{ref}} = \frac{1}{1+\frac{2}{\beta(\beta+2)}}$$

$$\boxed{\frac{I_o}{I_{ref}} \approx \frac{1}{1+\frac{2}{\beta^2}}}$$

  - Error significantly reduced compared to the simple mirror and cascode mirror
- Output resistance:

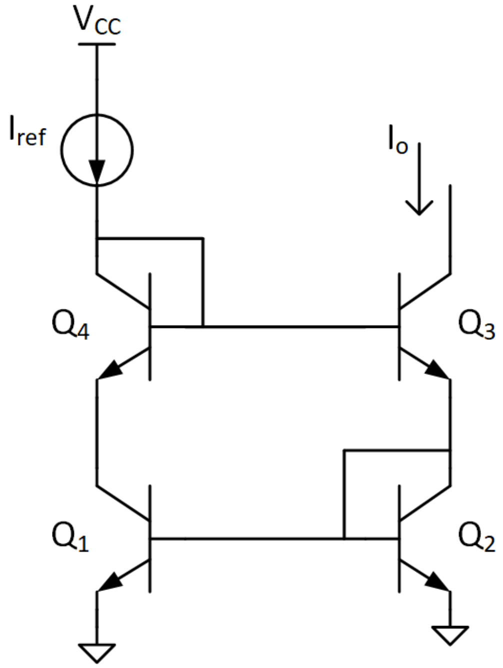
$$R_o = \left(\frac{\beta_3}{2}+1\right) r_{o3} + \frac{1}{2g_m 2}$$

$$\boxed{R_o \approx \frac{\beta_3}{2} r_{o3}}$$



Improved Wilson Mirror – BJT

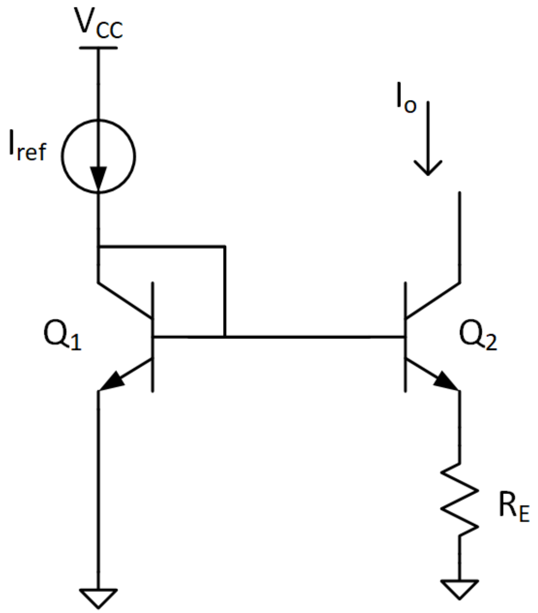
- Similar to the MOSFET circuit, current-transfer-ratio error can be reduced by adding a fourth transistor to balance the Q_1 and Q_2 collector voltages



Widlar Current Source – BJT

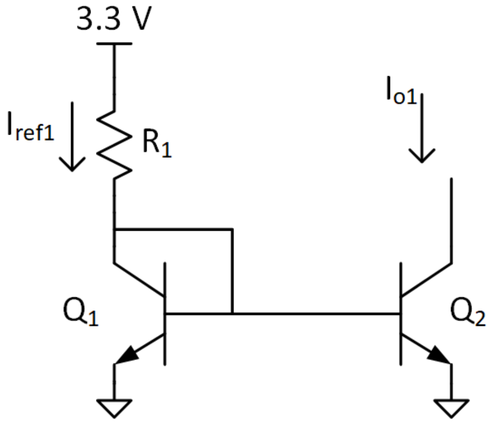
- Widlar current source adds an emitter resistor on the output transistor
- More of a *source* than a *mirror*
  - Current not replicated from one branch to the other
- Can generate small currents without the need for very large resistors
- Output resistance:

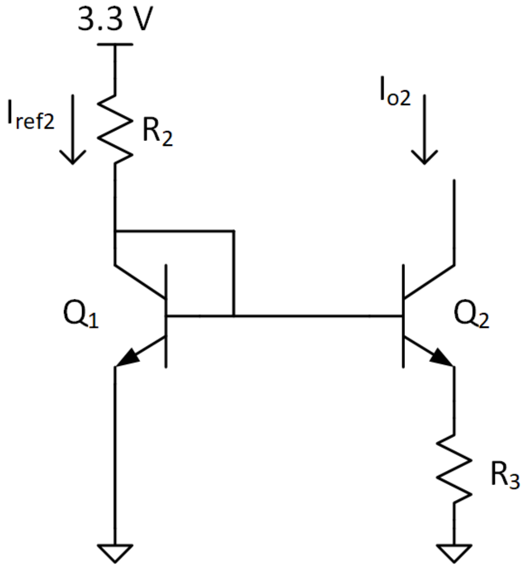
$$\boxed{R_o \approx [1 + g_{m2}(R_{E2}||r_{\pi 2})]r_{o2}}$$



Widlar Current Source – Example

- We can illustrate the function and benefit of a Widlar current source through an example
  - Compare the Widlar current source to a basic current mirror
- Design each source below for $I_o = 10\ \mu A$
  - Emitter areas are equal
  - $I_s = 2 \times 10^{15}\ A = 2\ fA$





Widlar Current Source – Example

- First, design the basic current mirror
- The reference and output currents are equal

$$I_{ref1} = I_o = 10\ \mu A$$

- Use I_{ref1} to determine V_{BE}

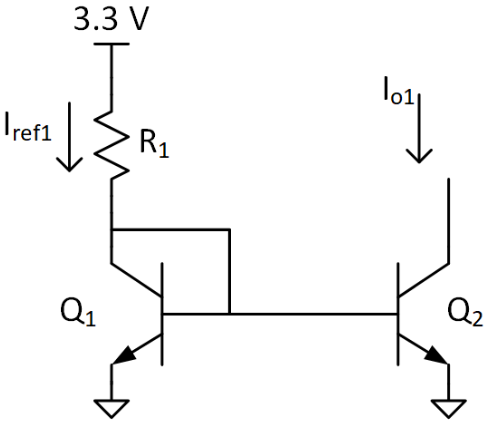
$$V_{BE} = V_T \ln\left(\frac{I_{ref1}}{I_s}\right) = 26\ mV \ln\left(\frac{10\ \mu A}{2\ fA}\right)$$

$$V_{BE} = 581\ mV$$

- Determine the value of R_1

$$R_1 = \frac{V_{CC} - V_{BE1}}{I_{ref1}} = \frac{3.3\ V - 581\ mV}{10\ \mu A}$$

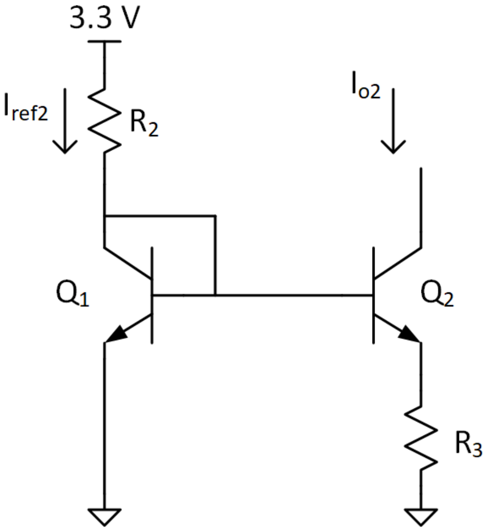
$$\boxed{R_1 = 272\ k\Omega}$$



Widlar Current Source – Example

- Next, design the Widlar source
- Here, we can choose the reference current

$$I_{ref2} = 1\ mA$$



- Use I_{ref2} to determine V_{BE1}

$$V_{BE1} = V_T \ln\left(\frac{I_{ref2}}{I_s}\right) = 26\ mV \ln\left(\frac{1\ mA}{2\ fA}\right)$$

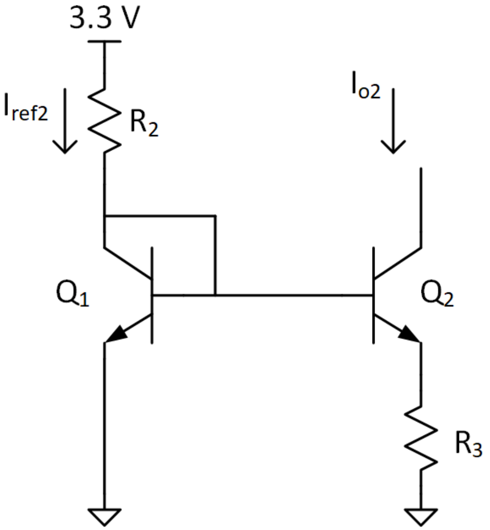
$$V_{BE1} = 700\ mV$$

- Determine the value of R_2

$$R_2 = \frac{V_{CC} - V_{BE1}}{I_{ref1}} = \frac{3.3\ V - 700\ mV}{1\ mA}$$

$$R_2 = 2.6\ k\Omega$$

Widlar Current Source – Example



- Next, determine R_3
  - KVL around the B-E loop gives

$$V_{BE1} - V_{BE2} - I_{O2}R_3 = 0$$

$$R_3 = \frac{V_{BE1} - V_{BE2}}{I_{O2}}$$

- We already determined V_{BE2}

$$V_{BE2} = 26\,mV \ln\left(\frac{10\,\mu A}{2\,fA}\right) = 581\,mV$$

- So, R_3 is

$$R_3 = \frac{700\,mV - 581\,mV}{10\,\mu A} = 11.9\,k\Omega$$

$$\boxed{R_2 = 2.6\,k\Omega \quad \text{and} \quad R_3 = 11.9\,k\Omega}$$

- Both much smaller than required for the basic mirror ($272\,k\Omega$)