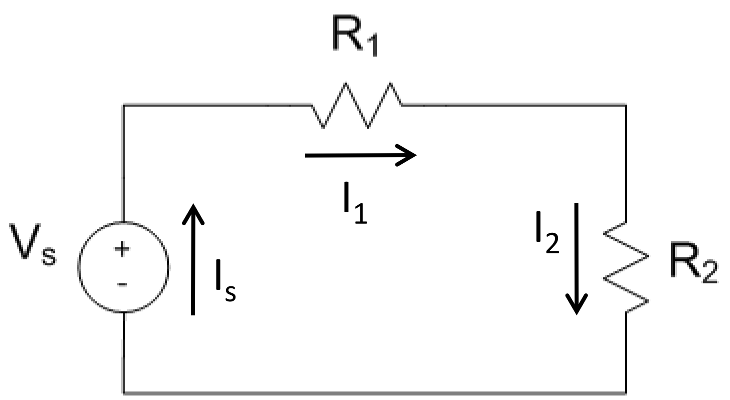
CLASS 5: VOLTAGE & CURRENT DIVIDERS

ENGR 102 – Introduction to Engineering

Series & Parallel Circuits

Series Circuits

- Series-connected components
  - Share **one common node**
    - Nothing else connected to that node
  - Connected end-to-end
  - **Equal current** through each component

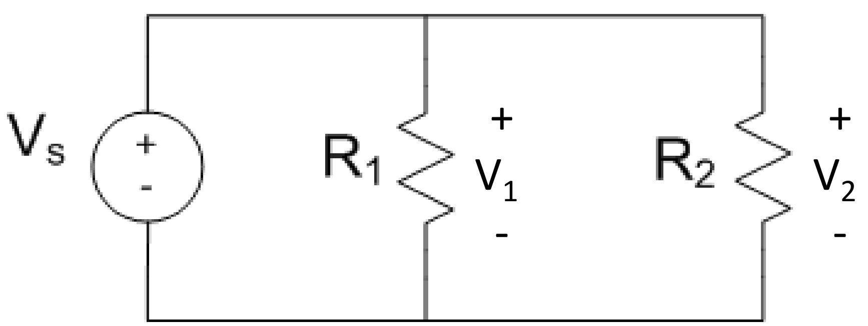


Resistors, R_1 and R_2, and voltage source, V_s, are all connected in **series**

$$I_s = I_1 = I_2$$

Parallel Circuits

- Components in **parallel**
  - Share **two common nodes**
  - Connected side-by-side
  - **Equal voltage** across each component

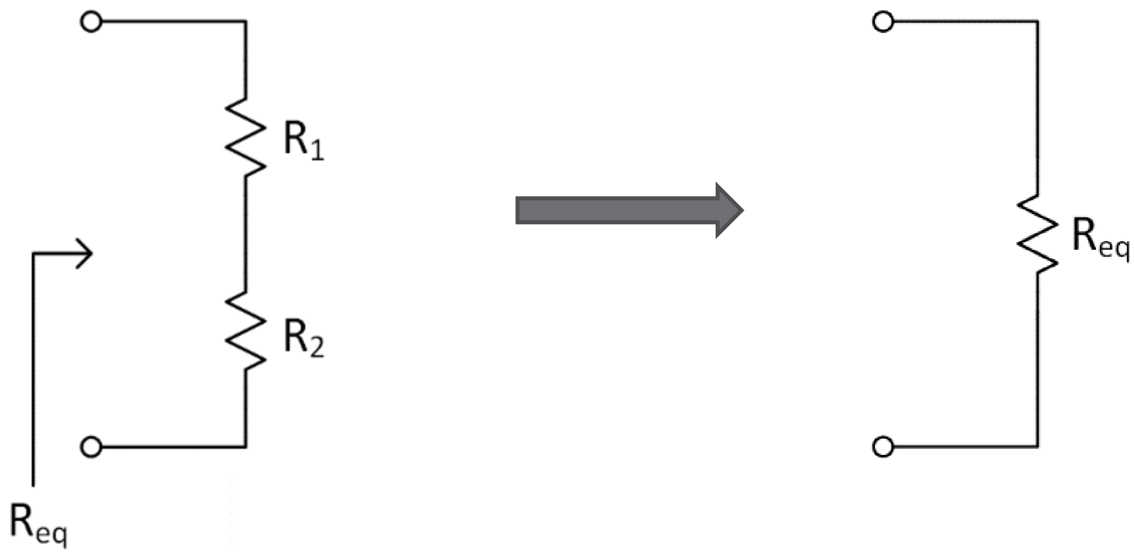


Resistors, R_1 and R_2, and voltage source, V_s, are all connected in **parallel**

$$\mathbf{V_s = V_1 = V_2}$$

Series Resistance

- ***Resistances in series add***



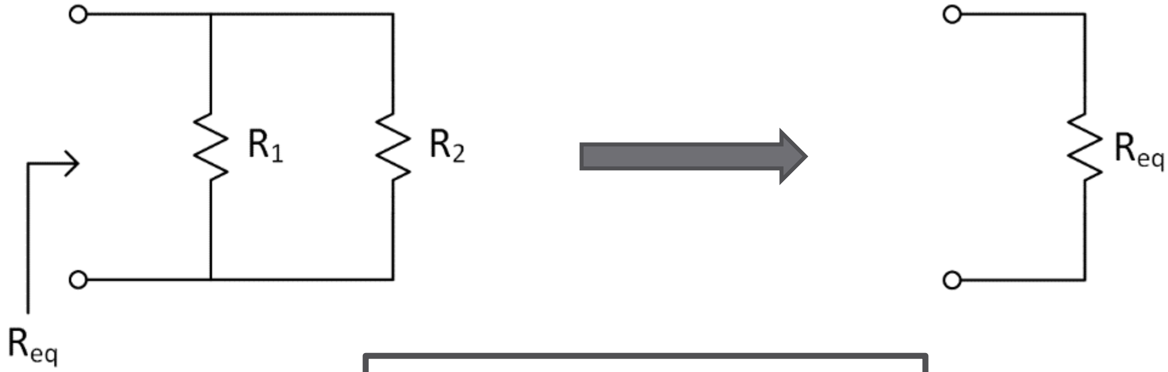
$$R_{eq} = R_1 + R_2$$

- In general,

$$\boxed{R_{eq} = \sum R_i}$$

Parallel Resistance

- ***Conductances in parallel add***



$$R_{eq} = \left(\sum \frac{1}{R_i}\right)^{-1}$$

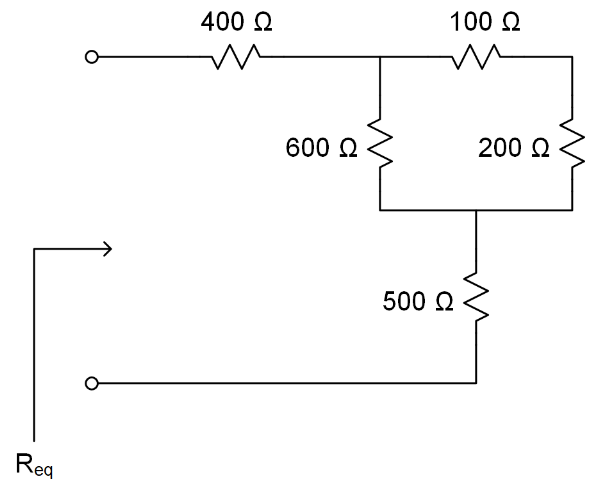
- For ***two*** parallel resistors (only):

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$

Equivalent Resistance

Exercise

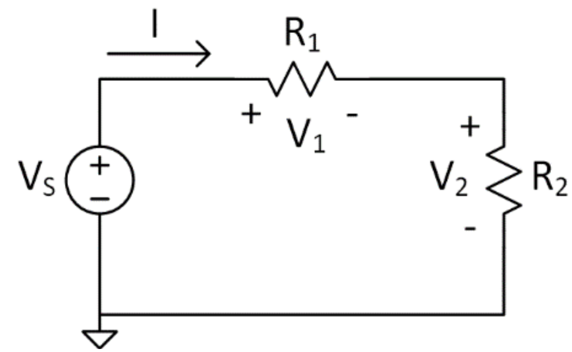
- Determine the equivalent resistance seen looking into the terminals of the following network



Voltage & Current Dividers

Voltage Dividers

- Voltage across series resistors divides proportional to resistance
- Consider two series resistors:
  - Current through the resistors



$$I = \frac{V_s}{R_1 + R_2}$$

  - Ohm's law gives the voltage across either resistor

$$V_n = I R_n$$

$$V_1 = \frac{V_s}{R_1 + R_2} R_1 = V_s \frac{R_1}{R_1 + R_2}$$

$$V_2 = \frac{V_s}{R_1 + R_2} R_2 = V_s \frac{R_2}{R_1 + R_2}$$

Voltage Dividers

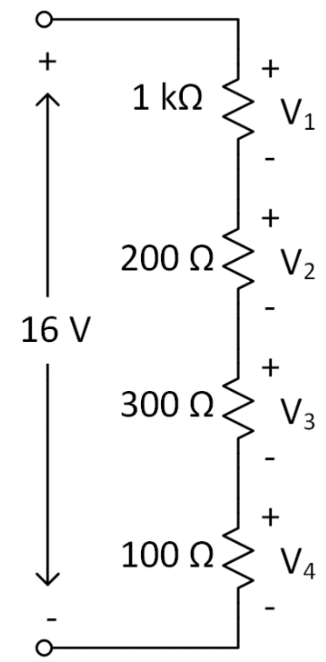
- In general, the voltage across one in a series of resistors is given by

$$V_n = V_{total} \cdot \frac{R_n}{\Sigma R_i}$$

  - For example:

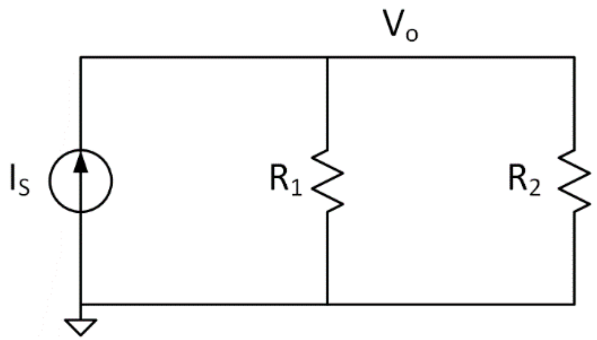
$$V_3 = 16\ V \frac{300\ \Omega}{1\ k\Omega + 200\ \Omega + 300\ \Omega + 100\ \Omega}$$

$$V_3 = 16\ V \frac{300\ \Omega}{1.6\ k\Omega} = 3\ V$$



Current Dividers

- Current through parallel-connected resistances divides proportional to conductance
- Consider two parallel resistors:
  - Voltage across the resistors



$$V_o = \frac{I_S}{G_1 + G_2} = \frac{I_S}{\frac{1}{R_1} + \frac{1}{R_2}} = I_S \frac{R_1 R_2}{R_1 + R_2}$$

  - Ohm's law gives the current through either resistor

$$I_n = \frac{V_o}{R_n}$$

$$I_1 = \frac{I_S}{R_1} \frac{R_1 R_2}{R_1 + R_2} = I_s \frac{R_2}{R_1 + R_2}$$

$$I_2 = \frac{I_S}{R_2} \frac{R_1 R_2}{R_1 + R_2} = I_s \frac{R_1}{R_1 + R_2}$$

Current Dividers

- Current through one of ***two*** parallel resistors is given by

$$I_1 = I_{total} \cdot \frac{R_2}{R_1 + R_2}$$

$$I_2 = I_{total} \cdot \frac{R_1}{R_1 + R_2}$$

  - One of the two resistors may be a parallel combination of multiple resistors

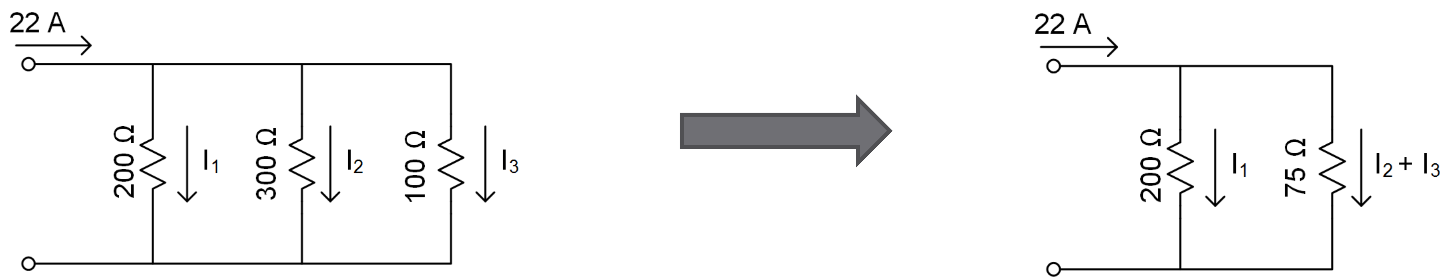
- More generally, expressed in terms of ***conductance***
  - Applies to any number of parallel resistances

$$I_n = I_{total} \cdot \frac{G_n}{\Sigma G_i}$$

Current Dividers

- For example, determine I_1
- First, combine the 300 Ω and 100 Ω resistors in parallel

$$R_{eq} = \left(\frac{1}{300\ \Omega} + \frac{1}{100\ \Omega}\right)^{-1} = 75\ \Omega$$



- Next, apply the current divider equation:

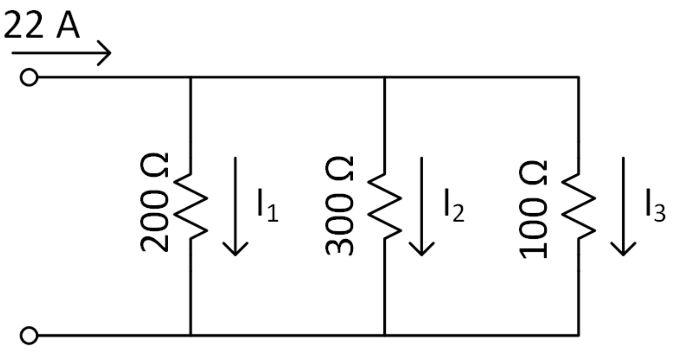
$$I_1 = I_{total}\frac{R_2}{R_1 + R_2}$$

$$I_1 = 22\ A\frac{75\ \Omega}{200\ \Omega + 75\ \Omega}$$

$$I_1 = 6\ A$$

Current Dividers

- Or, using conductances:



$$I_1 = 22\ A \frac{\frac{1}{200\ \Omega}}{\frac{1}{200\ \Omega} + \frac{1}{300\ \Omega} + \frac{1}{100\ \Omega}}$$

$$I_1 = 22\ A \cdot \frac{5\ mS}{5\ mS + 3.33\ mS + 10\ mS}$$

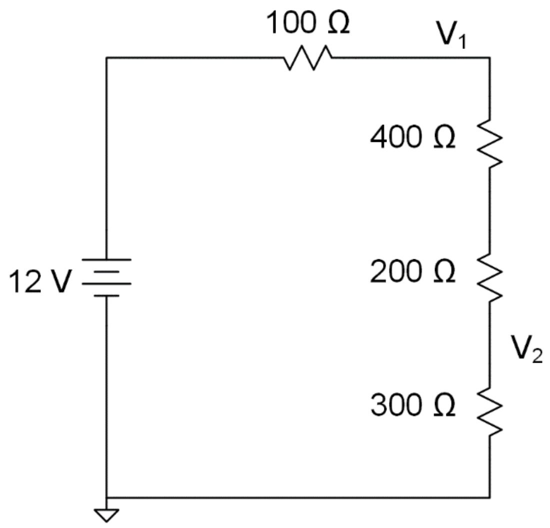
$$I_1 = 22\ A \cdot 0.2727$$

$$I_1 = 6\ A$$

Voltage Division

Exercise

- Apply the principle of voltage division to determine V_1 and V_2 in the circuit below.



Current Division

Exercise

- Apply the principle of current division to determine V_o in the circuit below.