SECTION 4: ALGORITHMIC THINKING

ENGR 103 – Introduction to Engineering Computing

Algorithmic Thinking

Algorithmic Thinking

- ***Algorithmic thinking***:
  - The ability to identify and analyze problems, and to develop and refine algorithms for the solution of those problems
- ***Algorithm***:
  - Detailed step-by-step procedure for the performance of a task
- Learning to program is about developing algorithmic thinking skills, *not* about learning a programming language

Algorithms

- Ultimately, algorithms will be implemented by writing code in a particular programming language
- Algorithm design is (mostly) language-independent
  - A procedure that can be implemented in any language
- Universal algorithm representations:
  - Flowcharts
    - Graphical representation
  - Pseudocode
    - Natural language
    - Not necessarily language-independent

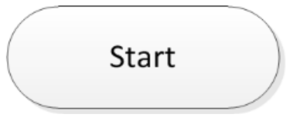
Flowcharts

Flow Charts

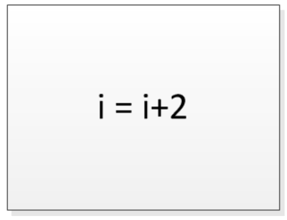
- ***Flowcharts*** are graphical representations of algorithms
- Interconnection of different types of blocks
  - Start/End
  - Process
  - Conditional
  - Input/Output
- Connection paths indicate flow from one step in the procedure to the next
- Well-constructed flowcharts are easily translated into code later

Flowchart Blocks

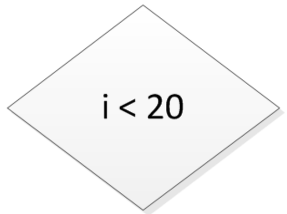
- **Start/End**
  - Always indicate the start and end of any flowchart



- **Process**
  - Indicates the performance of some action



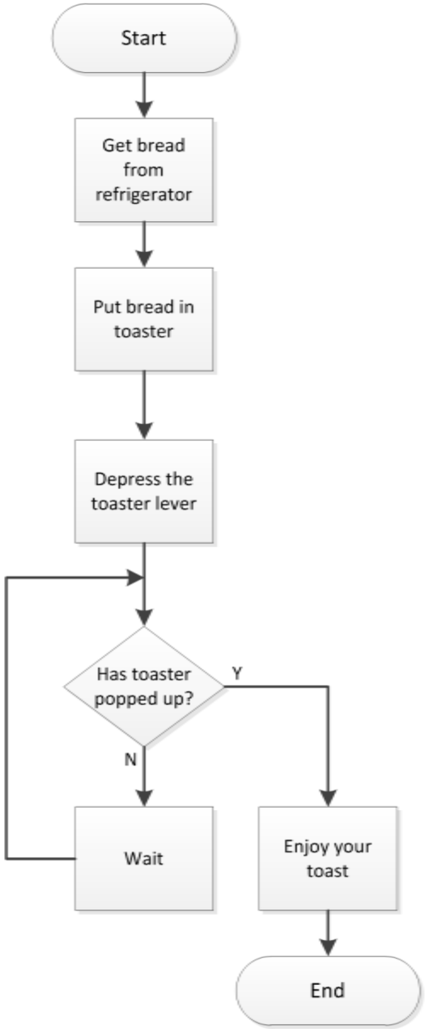
- **Conditional**
  - Performs a check and makes a decision
  - Binary result: True/False, Yes/No, 1/0
  - Algorithm flow branches depending on result



- **Input/Output**
  - Input or output of variables or data

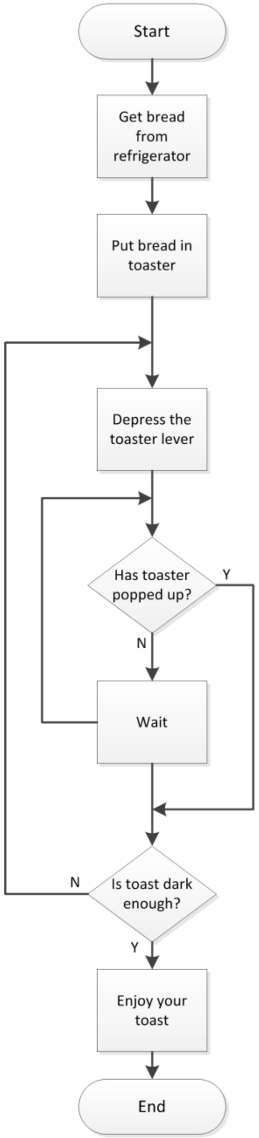
Flowchart – Example

- Consider the very simple example of making toast
- Process flows from Start to the End through the process and conditional blocks
  - Arrows indicate flow
  - Conditional blocks control flow branching
- Note the loop defining the waiting process
  - *Wait* block is unnecessary



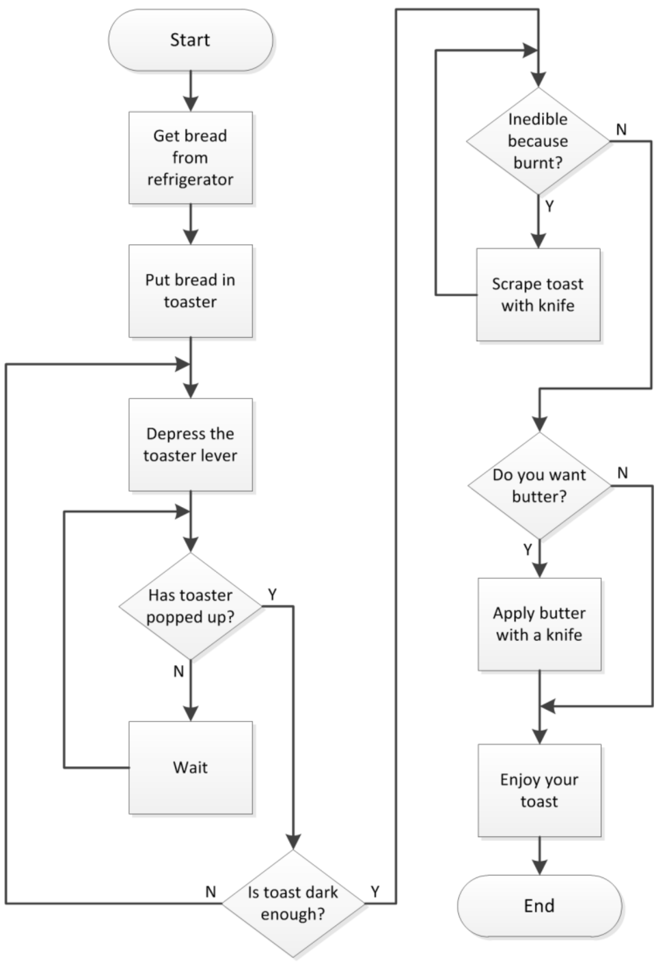
Flowchart – Example

- Flowchart for a given procedure is not unique
  - Varying levels of complexity and detail are always possible
- Often important to think about and account for various possible outcomes and cases
  - For example, is your toast always done after it first pops up?
  - Here, part of the procedure is repeated if necessary



Flowchart – Example

- Taking this example further, consider the possibility of burnt toast or the desire for butter
  - Another loop added for continued scraping until edible
  - Also possible to bypass portions of the procedure – e.g., the scraping of the toast or the application of butter
- Can imagine significantly more complex flow chart for the same simple procedure …



Common Flowchart Structures

Common Flowchart Structures

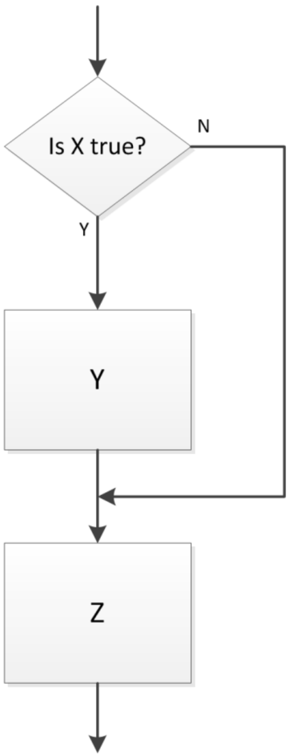
- Several basic structures occur frequently in many different types of flowcharts
  - Recurrent basic structures in many algorithms
- Ultimately translate to recurrent code structures
- Two primary categories
  - ***Conditional statements***
  - ***Loops***
- In this section of notes, we'll gain an understanding of flowchart structures that fall into these two categories
- In the next section of notes we'll learn how to implement these structures in code

Conditional Statements

- `if` statements
- Logical and relational operators
- `if…else` statements

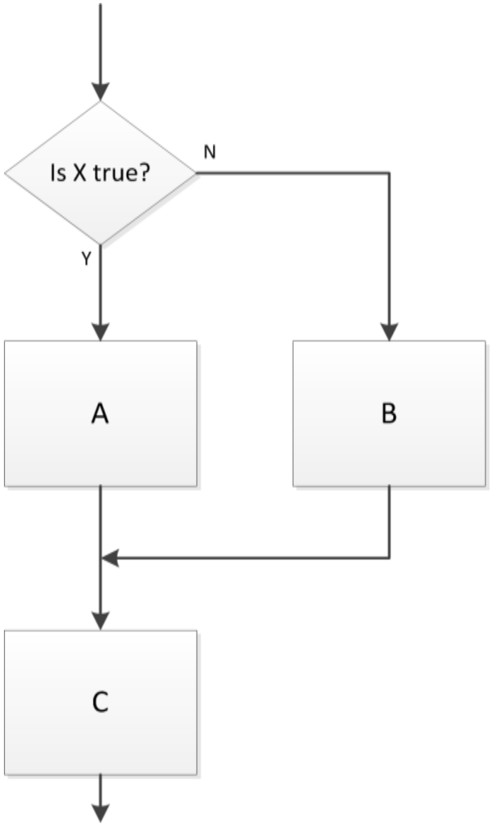
Conditional Statements – *if*

- Flowcharts represent a set of instructions
  - Blocks and block structures can be thought of as ***statements***
- Simplest ***conditional statement*** is a single ***conditional block***
  - An ***if structure***
  - If X is true, then do Y, if not, don't do Y
  - In either case, then proceed to do Z
  - Y and Z could be any type of process or action
    - E.g. add two numbers, turn on a motor, butter the toast, etc.
  - X is a ***logical expression*** or ***Boolean expression***
    - Evaluates to either true (1) or false (0)



Conditional Statements – *if … else*

- Can instead specify an action to perform if X is not true
  - An ***if … else structure***
  - If X is true, then do A, else do B
  - Then, move on to do C
- Here, a different process is performed depending on the value of X (1/0, T/F, Y/N)

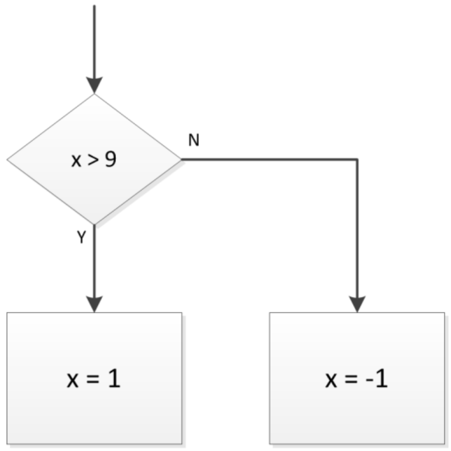


Conditional Statements – *if … else*

- Logical expression with a single ***relational operato***r

$$x > 9$$

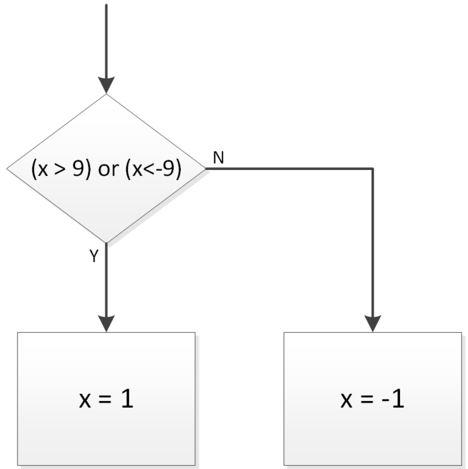
  - Either ***true*** (Y) or ***false*** (N)
  - If true, $x = 1$
  - If false, $x = -1$



- Logical expression may also include a ***logical operator***

$$(x > 9) \text{ or } (x < -9)$$

  - Again, statement is either ***true*** or ***false***
  - Next process step dependent on value of the conditional logical expression



Logical or Relational Expressions

- Logical expressions use ***logical*** and ***relational operators***

Operator	Relationship or Logical Operation	Example
==	Equal to	x == b
!=	Not equal to	k != 0
<	Less than	t < 12
>	Greater than	a > -5
<=	Less than or equal to	7 <= f
>=	Greater than or equal to	(4+r/6) >= 2
and	AND – ***both*** expressions must evaluate to true for result to be true	(t > 0) and (c == 5)
or	OR – ***either*** expression must evaluate to true for result to be true	(p > 1) or (m > 3)
not	NOT– negates the logical value of an expression	not (b < 4*g)

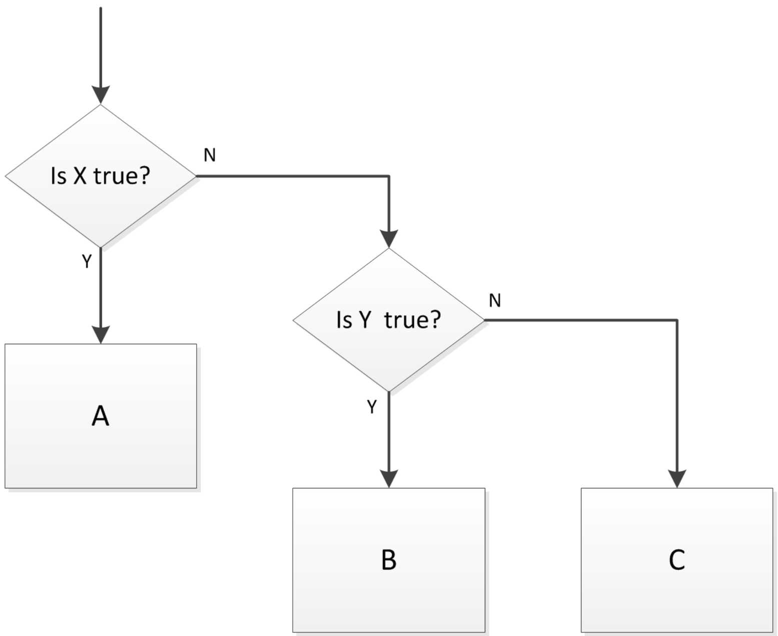
Logical Expressions – Examples

- Let $x = 12$ and $y = -3$
- Consider the following logical expressions:

Logical Expression	Value
$(x + y) == 15$	0
$(y == 2)$ or $(x > 8)$	1
not $(y < 0)$	0
$(y/2 + 1 < -1)$	0
$(x == 12)$ and not $(y \geq 5)$	1
$(y! = 2)$ or $(x < 10)$ or $(x < y)$	1
((x==2) and (y<0)) or ((x≥5) and (y!=8))	1

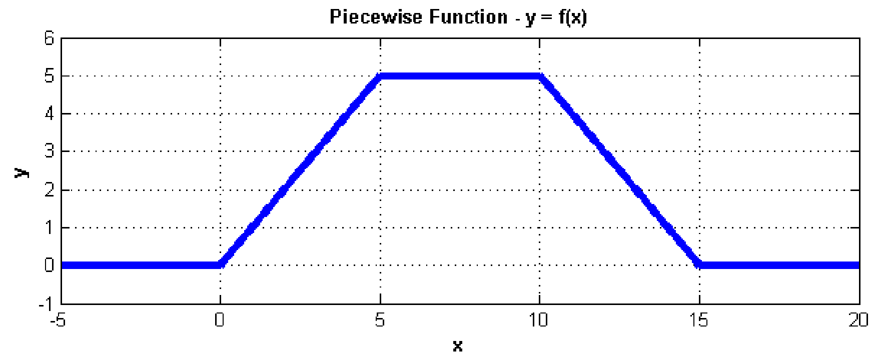
Conditional Statements – *if … elseif … else*

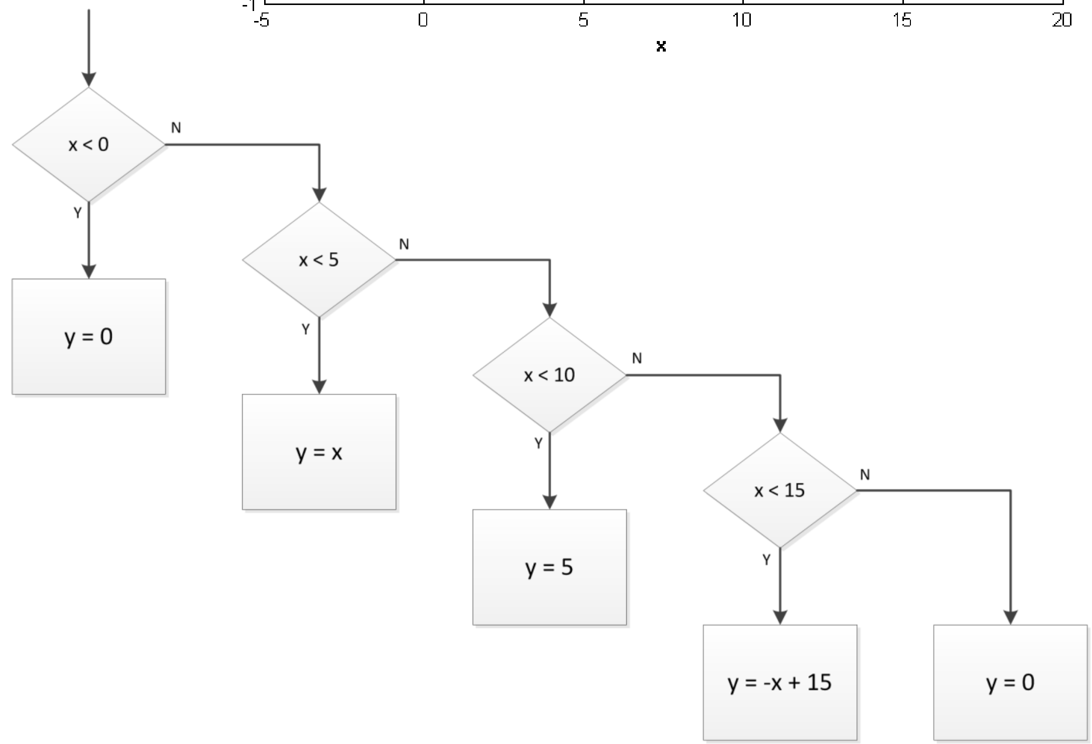
- Two conditional logical expressions
  - If the X is true, do A
  - If X is false, evaluate Y
    - If Y is true, do B
    - If Y is false, do C
- The ***if … elseif … else structure***
- Can include an arbitrary number of *elseif* statements
  - Successive logical statements evaluated only if preceding statement is false



if … elseif … else – Example

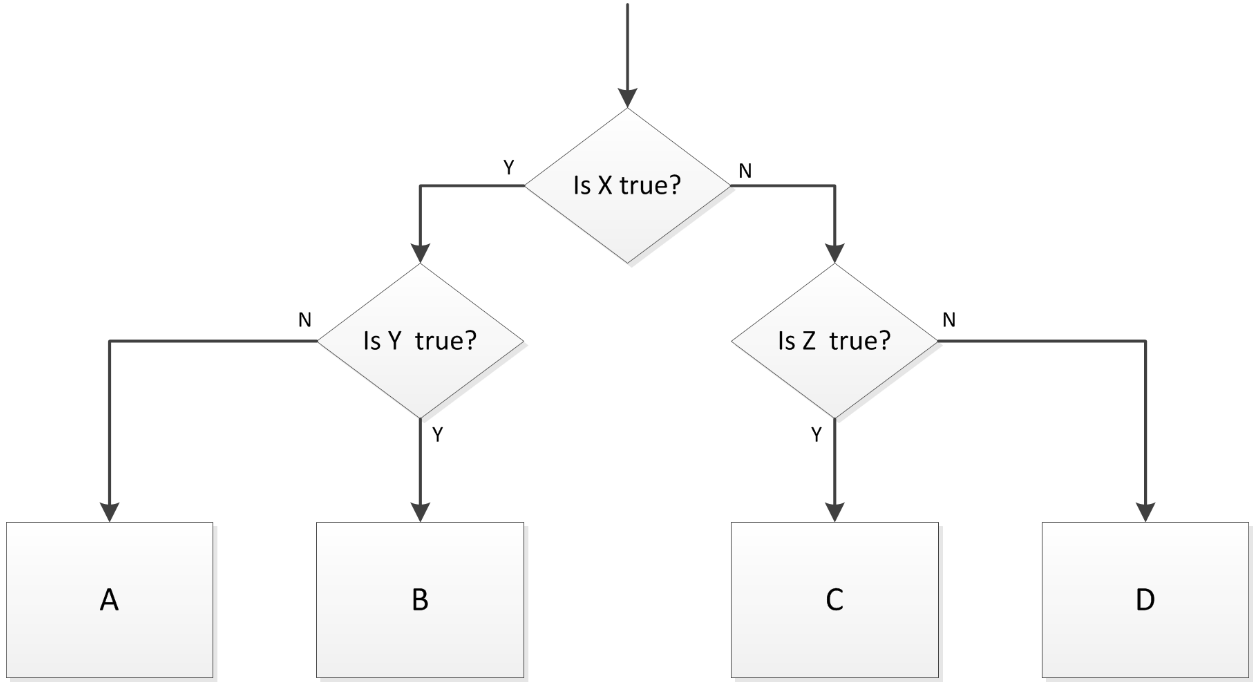
- Consider a ***piecewise linear function*** of x
  - $y = f(x)$ not defined by a single function
  - Function depends on the value of x
  - Can implement with an *if … elseif … else* structure





if Statements – Other Configurations

- In previous examples, successive logical statements only evaluated if preceding statement is false
- Result of a true logical expression can also be the evaluation of a second logical expression

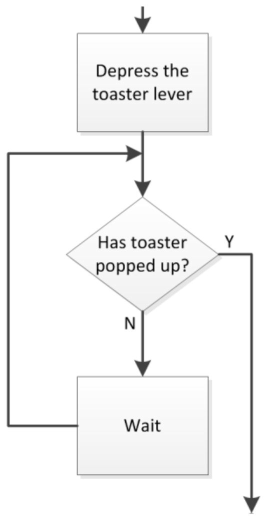


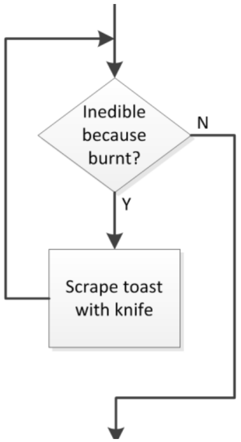
Loops

- *while* loops
- *for* loops

Loops

- We've already seen some examples of flow charts that contain ***loops***:





- Structures where the algorithmic flow loops back and repeats process steps
  - Repeats as long as a certain condition is met, e.g., toaster has not popped up, toast is inedible, etc.

Loops

- Algorithms employ two primary types of loops:
  - ***while loops***: loops that execute as long as a specified condition is met – loop executes as many times as is necessary
  - ***for loops***: loops that execute a specified exact number of times
- Similar looking flowchart structures
  - for loop can be thought of as a special case of a while loop
  - However, the distinction between the two is very important

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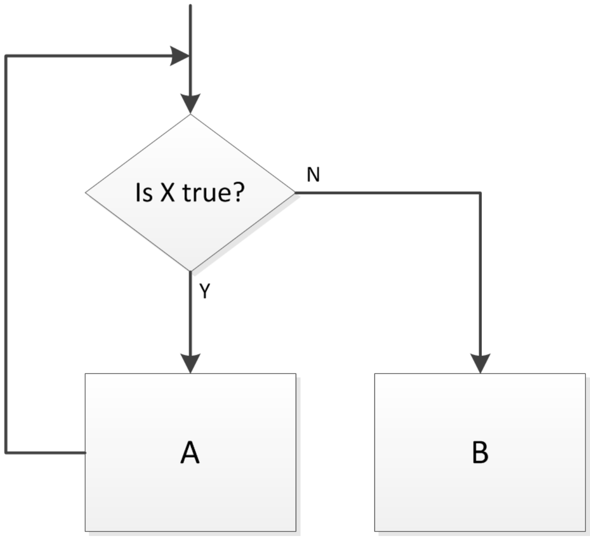
while Loop

while Loop

- Repeatedly execute an instruction or set of instructions as long as (*while*) a certain condition is met (is *true*)

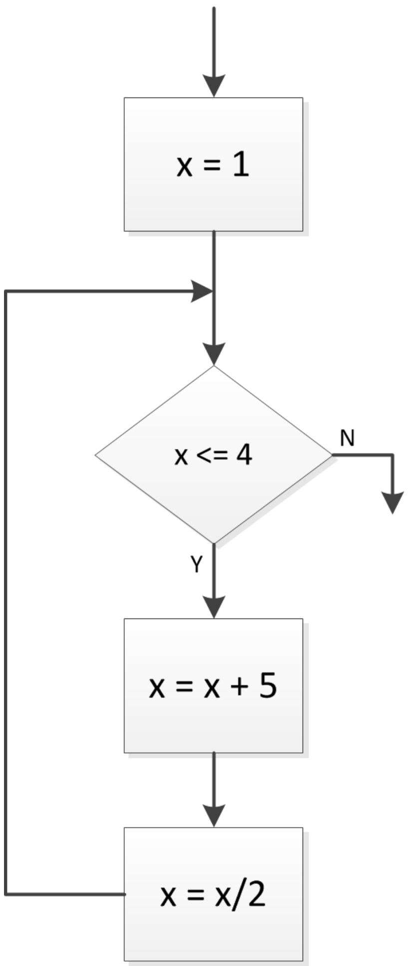
- Repeat A *while* X is true
  - As soon as X is no longer true, *break* out of the loop and continue on to B
  - A may never execute
  - A may execute only once
  - A may execute forever – an ***infinite loop***
    - If A never causes X to be false
    - *Usually* not intentional



while Loop

- Algorithm loops while $x \leq 4$
  - Loops three times:

Iteration	x
0	1
1	6 3
2	8 4
3	9 4.5

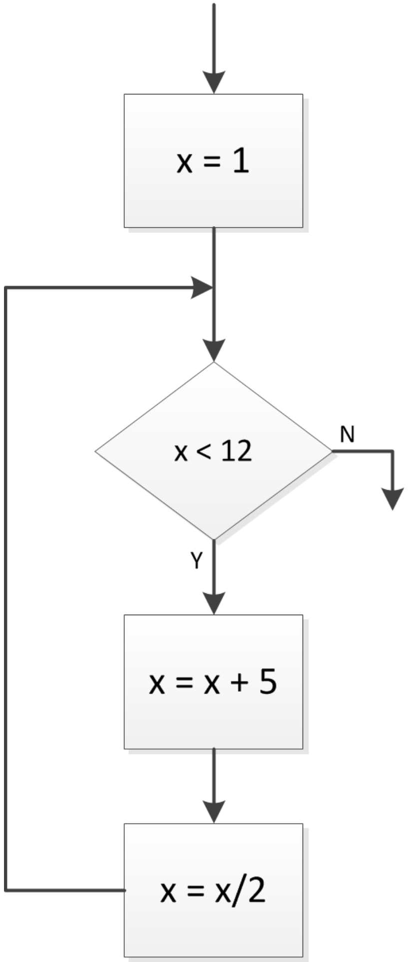


- Value of x exceeds 4 several times during execution
  - x value checked at the beginning of the loop
- Final value of x is greater than 4

while Loop – Infinite Loop

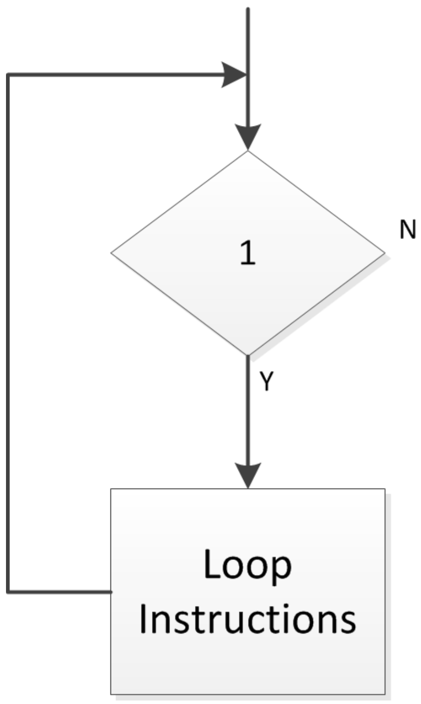
- Now looping continues as long as $x < 12$
  - x never exceeds 12
  - Loops forever – an ***infinite loop***

Iteration	x
0	1
1	6 3
2	8 4
3	9 4.5
4	9.5 4.75
5	9.75 4.875
6	9.875 4.9375
⋮	⋮



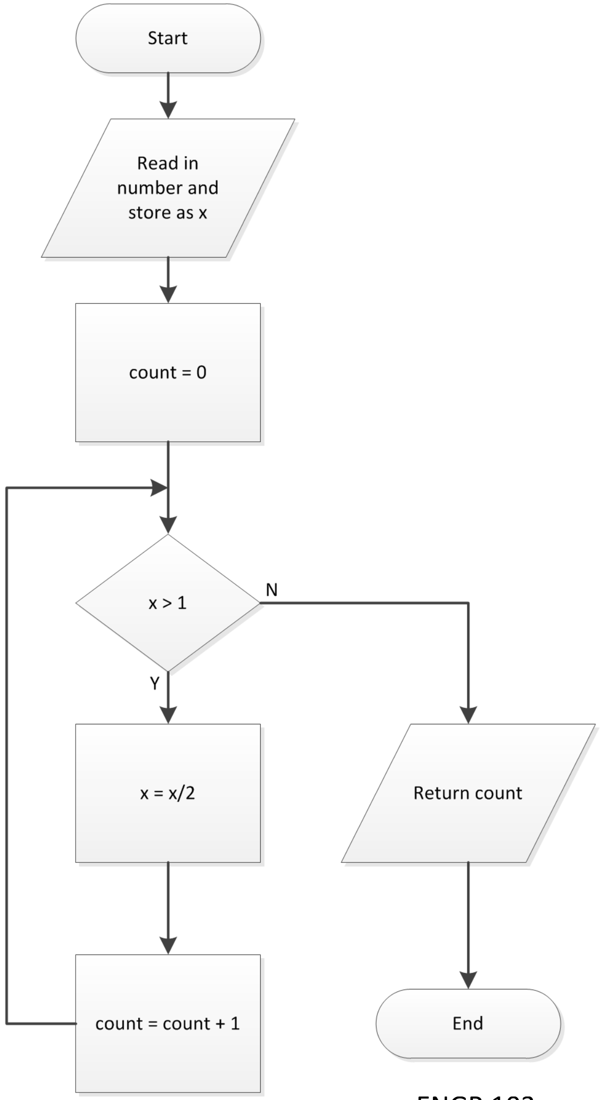
Infinite Loops

- Occasionally infinite loops are desirable
  - Consider for example microcontroller code for an environmental monitoring system
    - Continuously takes measurements and displays results while powered on
- Note the logical statement in the conditional block
  - Logical statements are either true (Y, 1) or false (N, 0)
  - 1 is the Boolean representation of true or Y



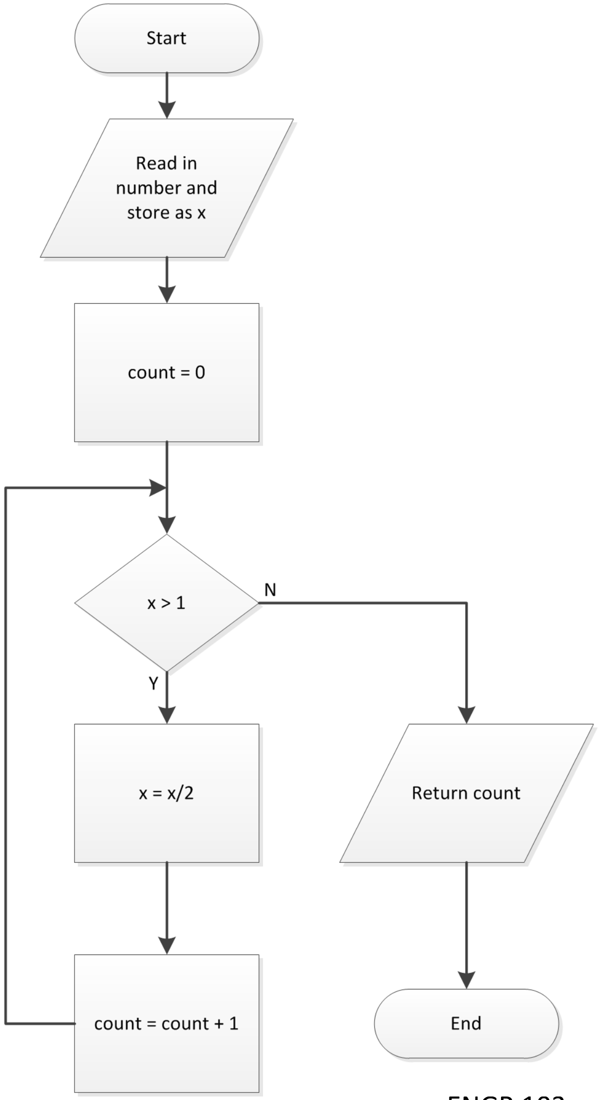
while Loop – Example 1

- Consider the following algorithm:
  - Read in a number (e.g. user input, from a file, etc.)
  - Determine the number of times that number can be successively divided by 2 before the result is ≤ 1
- Use a ***while loop***
  - Divide by 2 ***while*** number is > 1



while Loop – Example 1

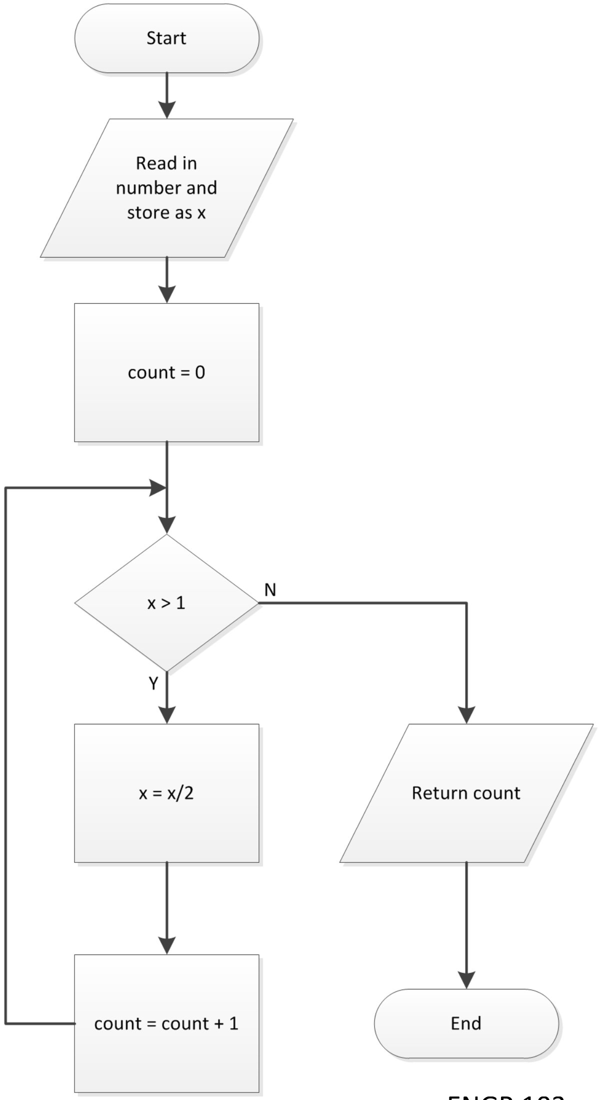
- Number of loop iterations depends on value of the input variable, x
  - Characteristic of while loops
    - # of iterations unknown a priori
  - If x $\leq$ 1 loop instructions never execute
- Note the data I/O blocks
  - Typical – many algorithms have ***inputs*** and ***outputs***



while Loop – Example 1

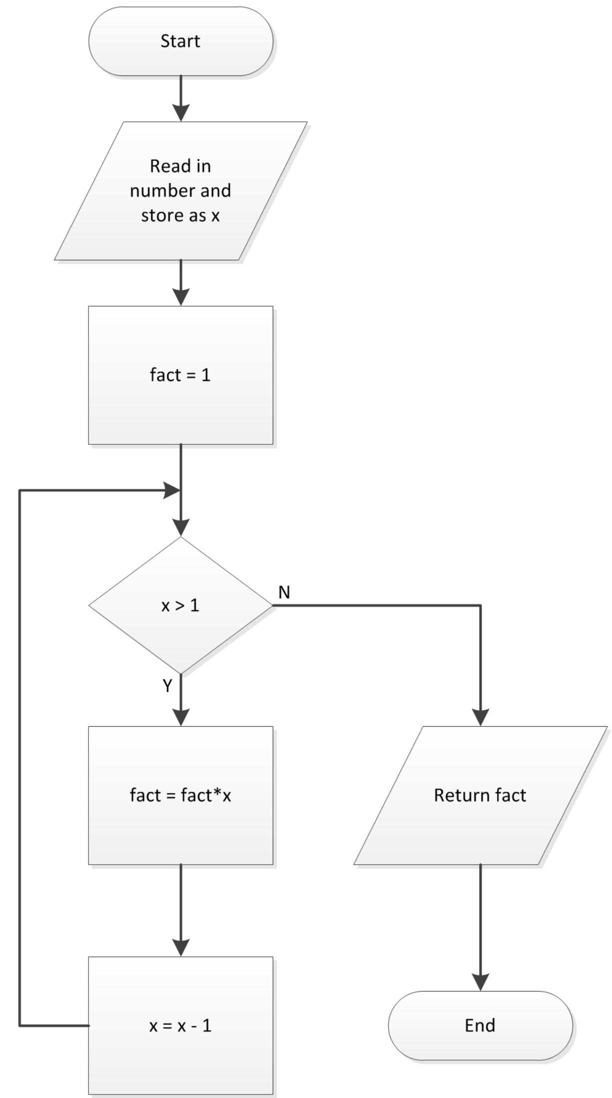
- Consider a few different input, x, values:

count	x		x		x
0	5		16		0.8
1	2.5		8		-
2	1.25		4		-
3	0.625		2		-
4	-		1		-
5	-		-		-



while Loop – Example 2

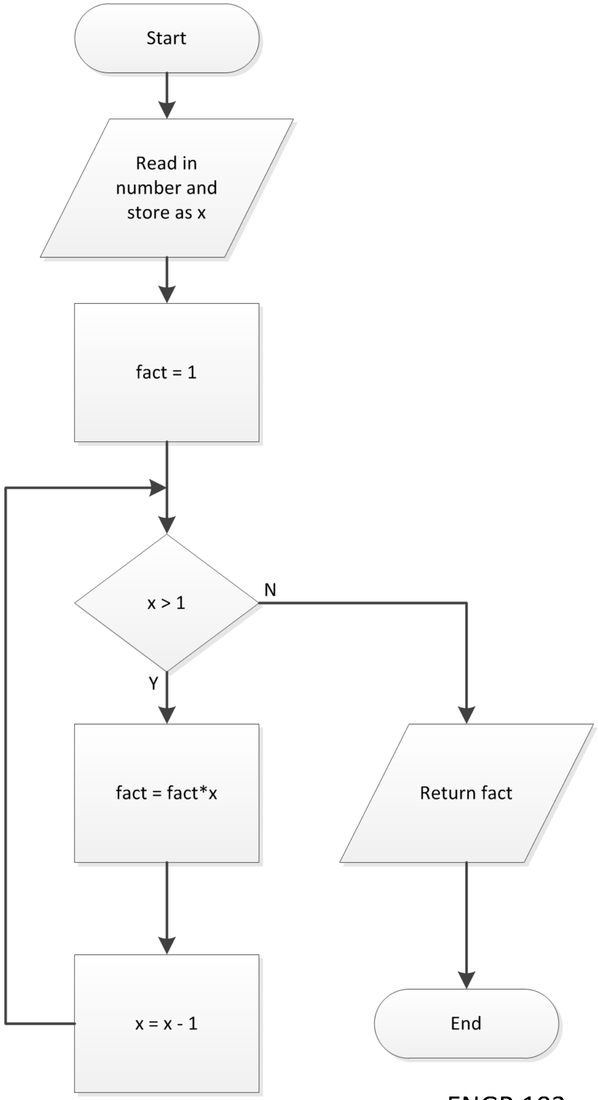
- Next, consider an algorithm to calculate x!, the ***factorial*** of x:
  - Read in a number, x
  - Compute the product of all integers between 1 and x
  - Initialize result, fact, to 1
  - Multiply fact by x
  - Decrement x by 1
- Use a ***while loop***
  - Multiply fact by x, then decrement x ***while*** $x > 1$



while Loop – Example 2

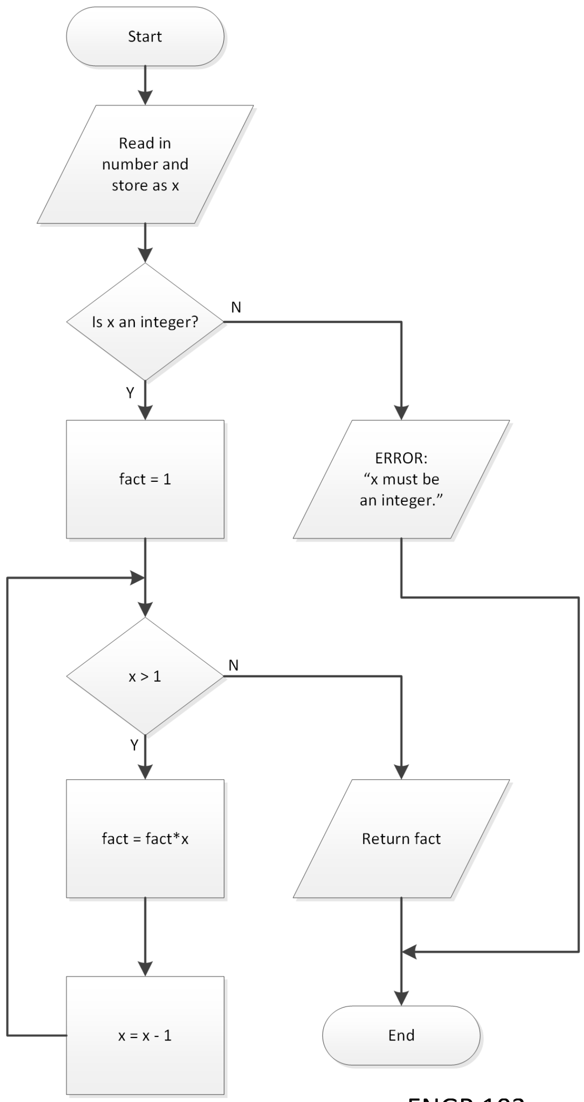
- Consider a few different input, x, values:

x	fact		x	fact		x	fact
5	1		4	1		0	1
5	5		4	4		-	-
4	20		3	12		-	-
3	60		2	24		-	-
2	120		1	24		-	-
1	120		-	-		-	-



while Loop – Example 2

- Let's say we want to define our factorial algorithm only for ***integer*** arguments
- Add ***error checking*** to the algorithm
  - After reading in a value for x, check if it is an integer
  - If not, generate an error message and exit
  - Could also imagine rounding x, generating a ***warning*** message and continuing



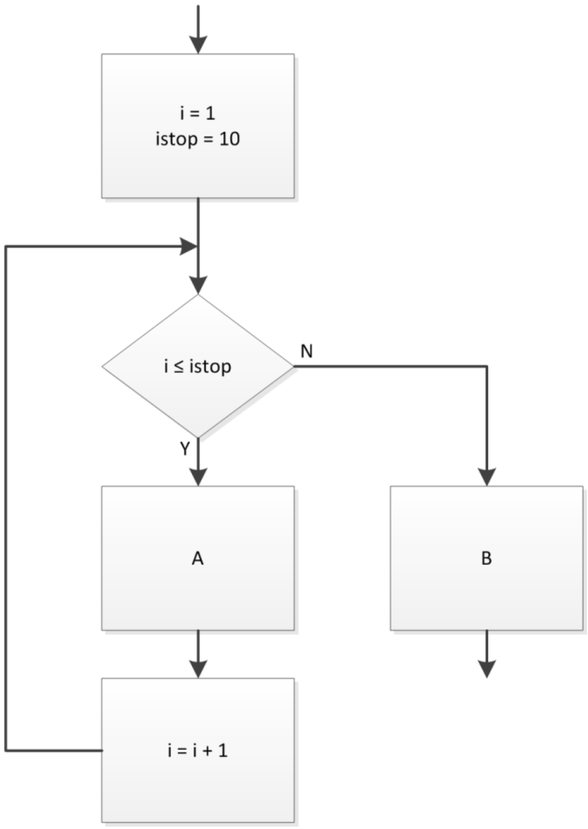
for Loop

for Loop

- We've seen that the number of while loop iterations is not known ahead of time
  - May depend on inputs, for example
- Sometimes we want a loop to execute an exact, specified number of times
- A ***for loop***
  - Utilize a ***loop counter***
  - Increment (or decrement) the counter on each iteration
  - Loop until the counter reaches a certain value
- Can be thought of as a while loop with the addition of a loop counter
  - But, a very distinct entity when implemented in code

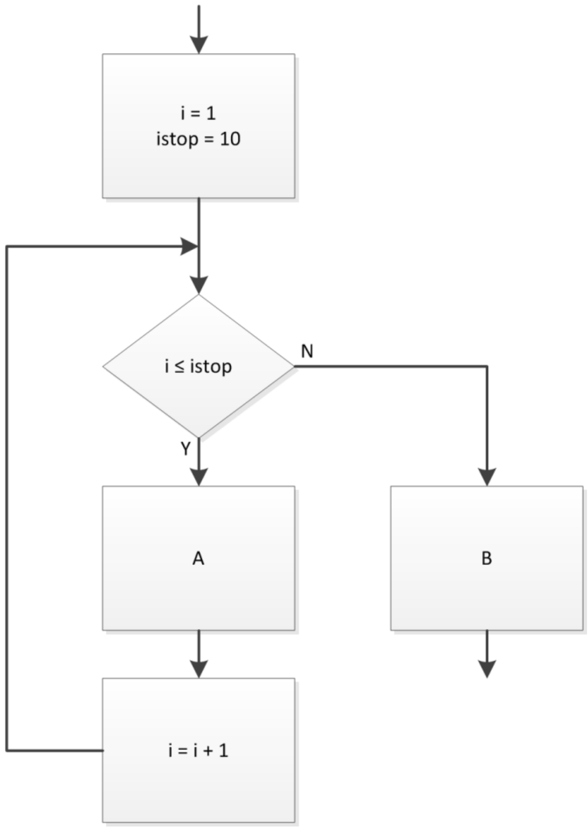
for Loop

- Initialize the loop counter
  - i, j, k are common, but name does not matter
- Set the range for i
  - Not necessary to define variable istop
- Execute loop instructions, A
- Increment loop counter, i
- Repeat until loop counter reaches its stopping value
- Continue on to B



for Loop

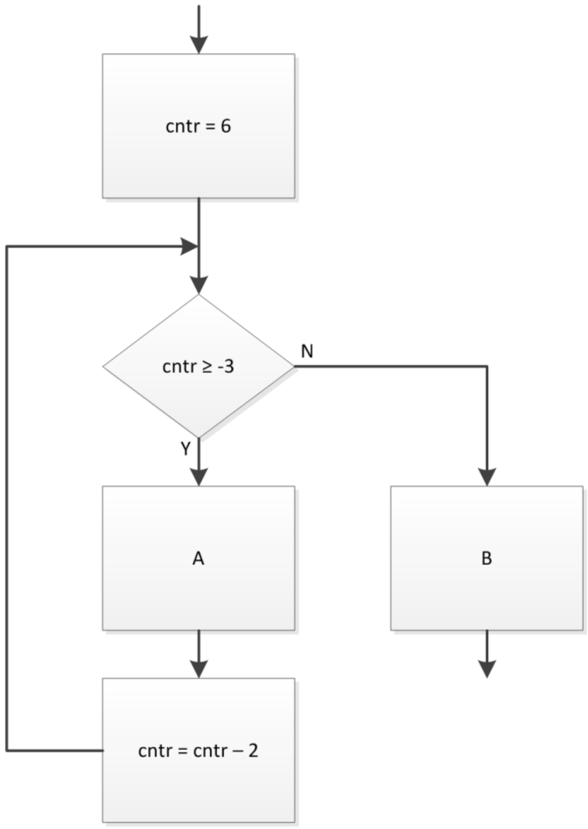
- for loops are ***counted loops***
- Number of loop iterations is known and is constant
  - Here loop executes 10 times
- Stopping value not necessarily hard-coded
  - Could depend on an input or vector size, etc.



for Loop

- Loop counter may start at value other than 1
- Increment size may be a value other than 1
- Loop counter may count backwards

Iteration	cntr	Process
1	6	A
2	4	A
3	2	A
4	0	A
5	-2	A
6	-4	B

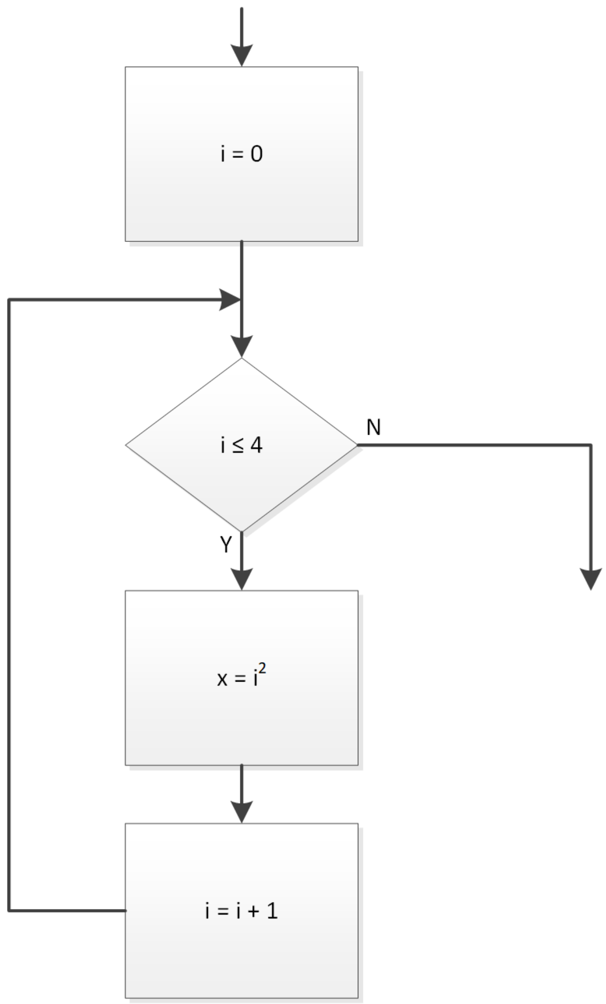


for Loop – Example 1

- Here, the loop counter, i, is used to update a variable, x, on each iteration

Iteration	i	x
1	0	0
2	1	1
3	2	4
4	3	9
5	4	16

- When loop terminates, and flow proceeds to the next process step, x = 16
  - A scalar
  - No record of previous values of x

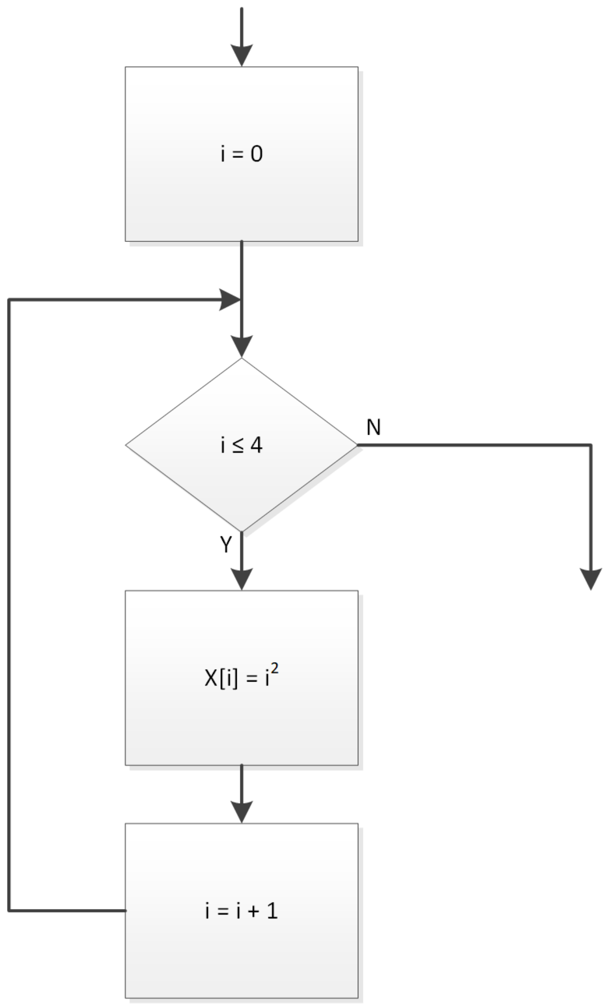


for Loop – Example 2

- Now, modify the loop process to store values of x as a ***vector***
  - Use loop counter to index the vector

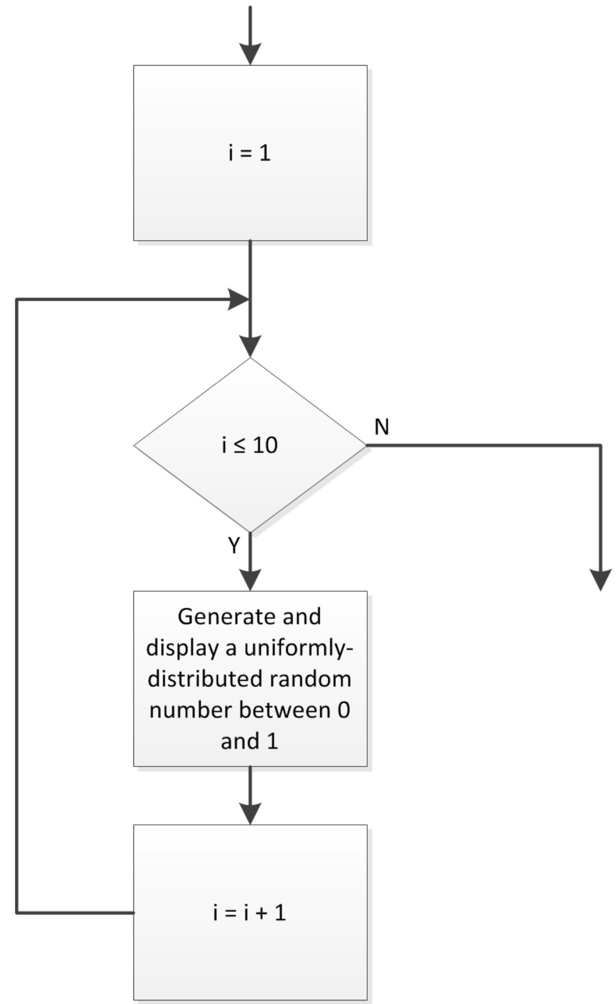
i	X[i]	x
0	0	[0]
1	1	[0, 1]
2	4	[0, 1, 4]
3	9	[0, 1, 4, 9]
4	16	[0, 1, 4, 9, 16]

- When loop terminates, x = [0, 1, 4, 9, 16]
  - A ***vector***
  - x grows with each iteration



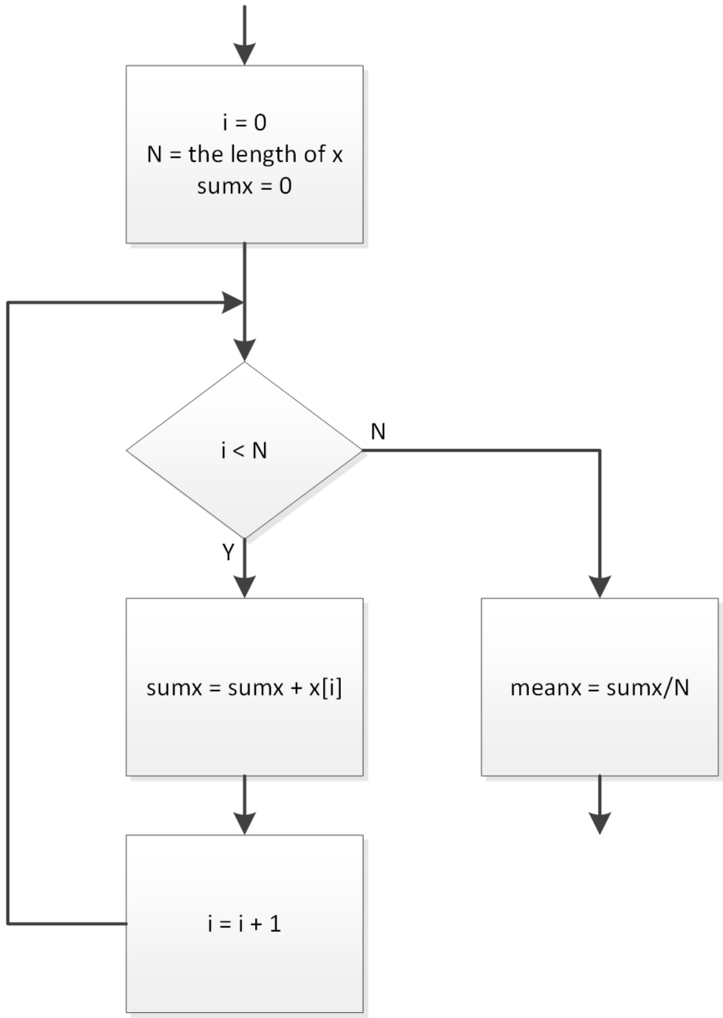
for Loop – Example 3

- The loop counter does not need to be used within the loop
  - Used as a counter *only*
- Here, a random number is generated and displayed each of the 10 times through the loop
  - Counter, i, has nothing to do with the values of the random numbers displayed



for Loop – Example 4

- Have a vector of values, x
- Find the ***mean*** of those values
  - Sum all values in x
    - A for loop
    - # of iterations equal to the length of x
    - Loop counter indexes x
  - Divide the sum by the number of elements in x
    - After exiting the loop



Nested Loops

Nested Loops

- A loop repeats some process some number of times
  - The repeated process can, itself, be a loop
  - A ***nested loop***
- Can have nested *for loops* or *while loops*
  - Can nest for loops within while loops and vice versa
- One application of a ***nested for loop*** is to step through every element in a matrix
  - Loop counter variables used as matrix indices
  - Outer loop steps through rows (or columns)
  - Inner loop steps through columns (or rows)

Nested for Loop – Example

- Recall how we index the elements within a matrix:
  - A_{ij} is the element on the i^{th} row and j^{th} column of the matrix A
  - Using Python syntax: A[i,j]
- Consider a 3×2 matrix

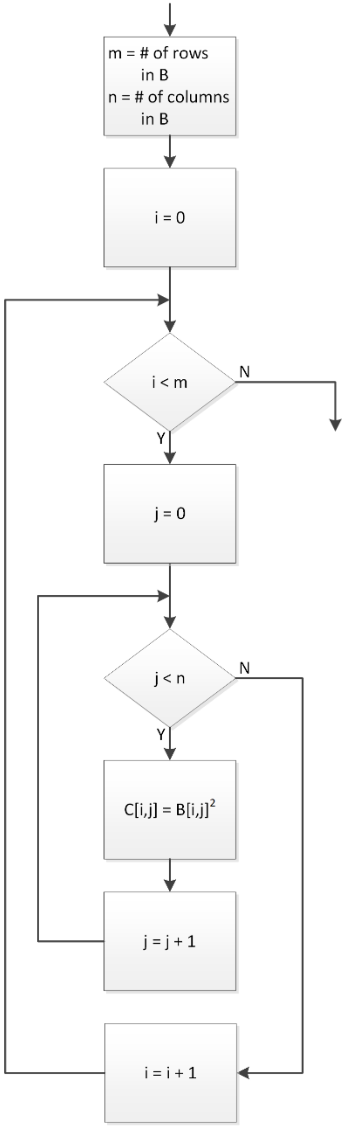
$$B = \begin{bmatrix} -2 & 1 \\ 0 & 8 \\ 7 & -3 \end{bmatrix}$$

- To access every element in B:
  - start on the first row and increment through all columns
  - Increment to the second row and increment through all columns
  - Continue through all rows
  - Two nested for loops

Nested for Loop – Example

$$B = \begin{bmatrix} -2 & 1 \\ 0 & 8 \\ 7 & -3 \end{bmatrix}$$

- Generate a matrix C whose entries are the squares of all of the elements in B
  - ***Nested for loop***
  - Outer loop steps through rows
    - Counter is row index
  - Inner loop steps through columns
    - Counter is column index



Pseudocode & Top-Down Design

Pseudocode

- Flowcharts provide a useful tool for designing algorithms
  - Allow for describing algorithmic structure
  - Ultimately used for generation of code
  - Details neglected in favor of concise structural and functional description
- ***Pseudocode*** provides a similar tool
  - One step closer to actual code
  - ***Textual*** description of an algorithm
  - ***Natural language*** mixed with language-specific syntax

Pseudocode – Example

- Consider an algorithm for determining the maximum of a vector of values
- Pseudocode might look like:

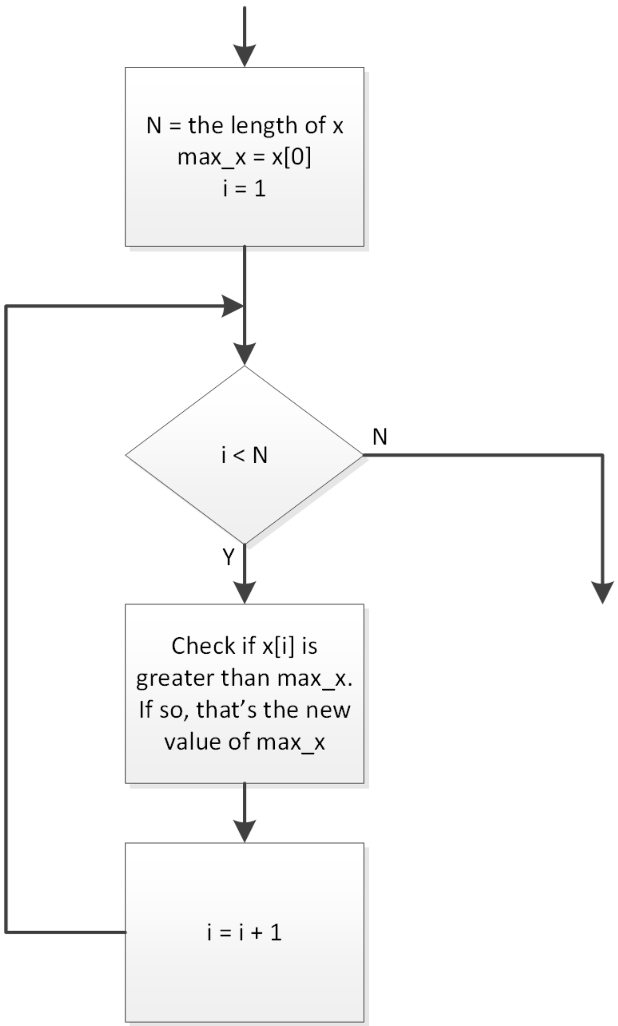
```
N = length of x

max_x = x[0]

for i = 1 through N-1

  if x[i] is greater than current
  max_x, then set max_x = x[i]
```

- We'll learn the Python-specific *for*-loop syntax in the following section of notes



Top-Down Design

- Flowcharts and pseudocode are useful tools for ***top-down design***
  - A good approach to any complex engineering design (and writing, as well)
  - First, define the overall system or algorithm at the top level (perhaps as a flowchart)
  - Then, fill in the details of individual functional blocks
- Top-level flowchart identifies individual functional blocks and shows how each fits into the algorithm
  - Each functional block may comprise its own flow chart or even multiple levels of flow charts
  - ***Hierarchical design***

Top-Down Design - Example

- Let's say you have deflection data from FEM analysis of a truss design
  - Data stored in text files
    - Deflection vs. location along truss
  - Parametric study
    - Three different component thicknesses
    - Two different materials
    - Six data sets
- Read in the data, calculate the max deflection and plot the deflection vs. position

Top-Down Design - Example