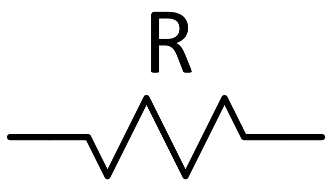
SECTION 2: RESISTIVE CIRCUIT ANALYSIS I

ENGR 201 – Electrical Fundamentals I

Resistance & Conductance

Resistance

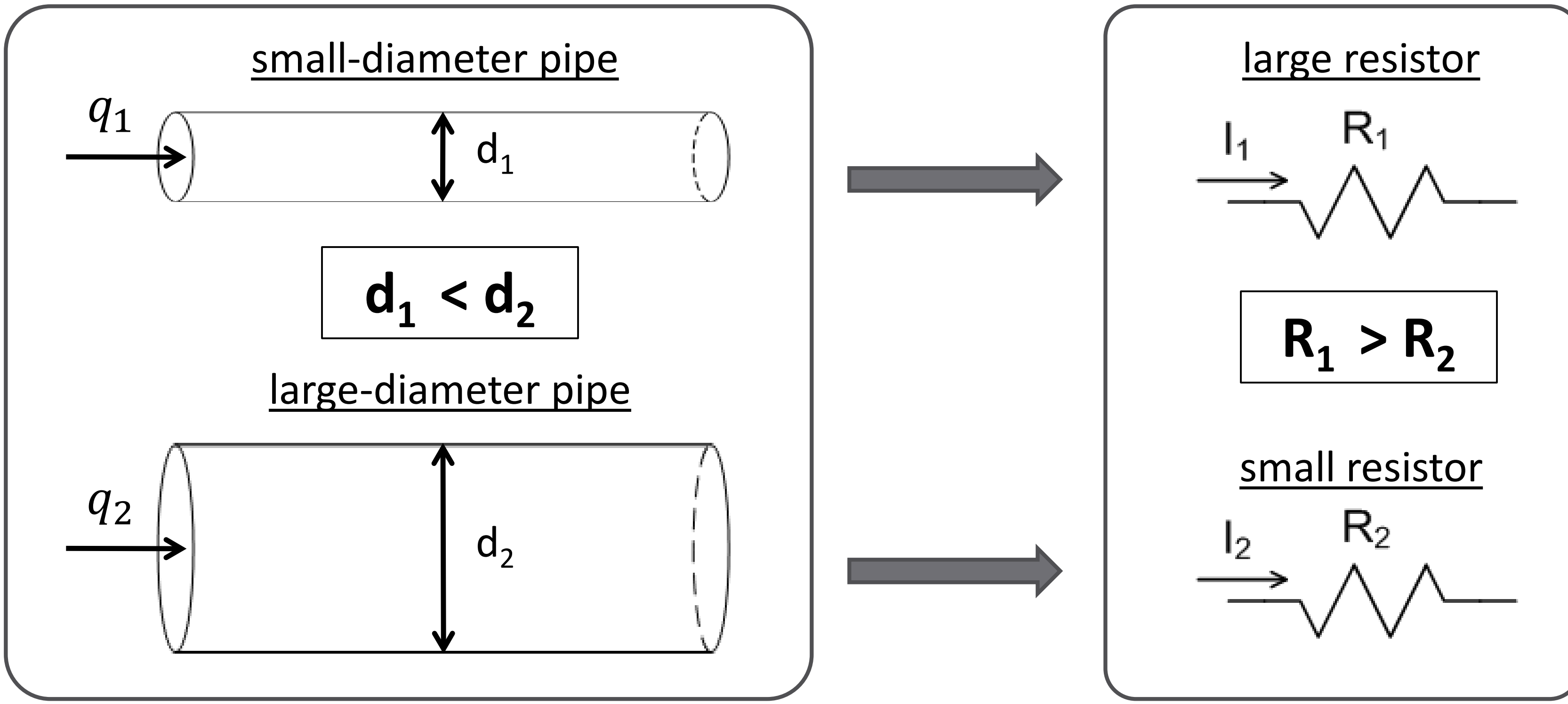
- ***Resistance***
  - The degree to which a circuit element opposes the flow of electrical current
- Schematic symbol:



- Units: ohms (Ω)
- May be discrete, intentional circuit components, or parasitic resistance of wires, cables, interconnects, etc.

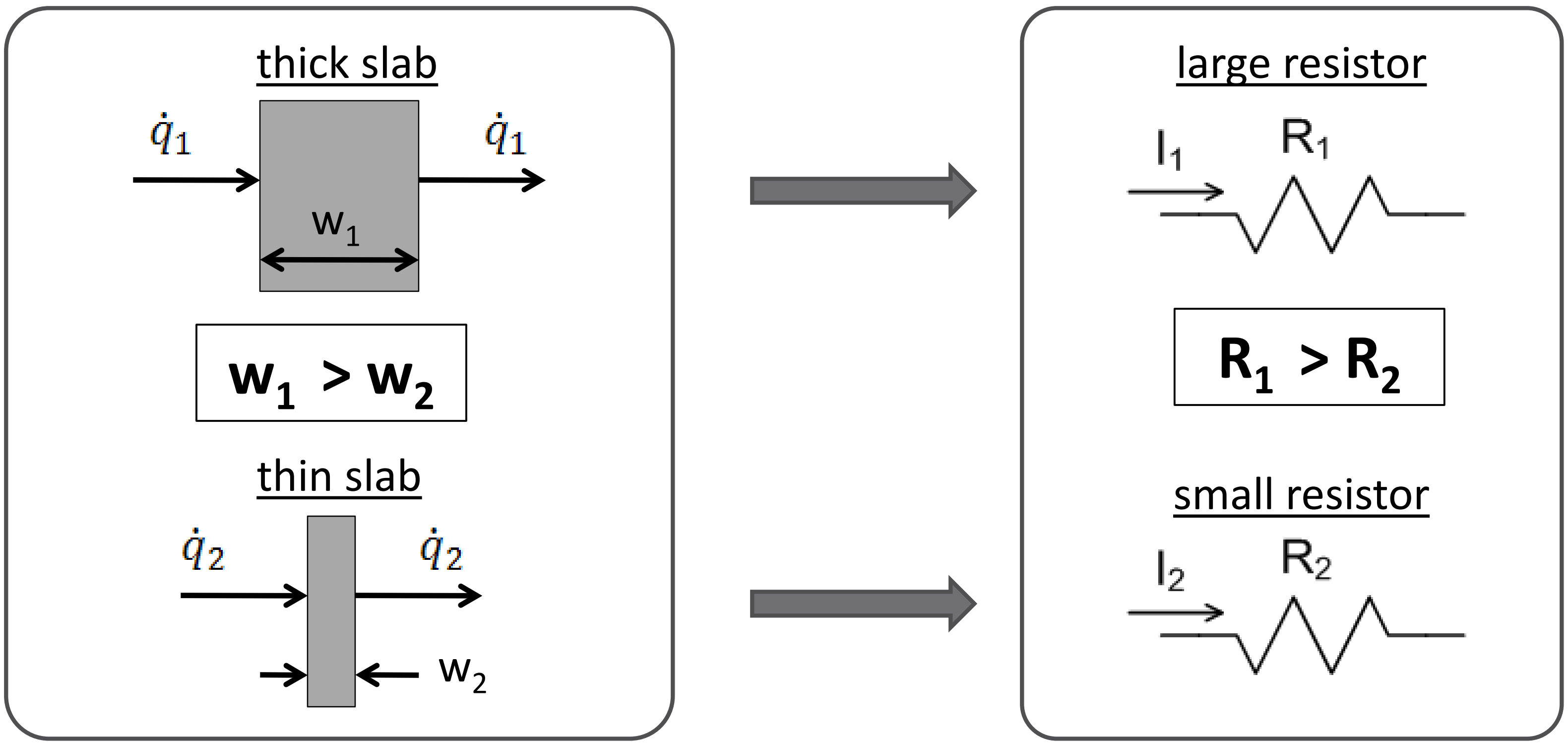
Resistance – Fluid Analogy

- Electrical resistance is analogous to the resistance of a pipe to fluid flow due to friction



Resistance – Thermal Analogy

- Electrical resistance is analogous to the resistance of heat conduction through a solid

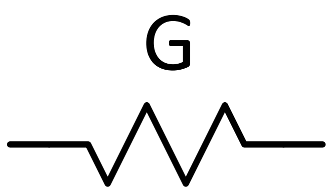


Conductance

- Electrical **conductance** is the degree to which a circuit element allows the flow of electrical current
- Conductance is the **inverse of resistance**

$$G = \frac{1}{R}$$

- Schematic symbol:



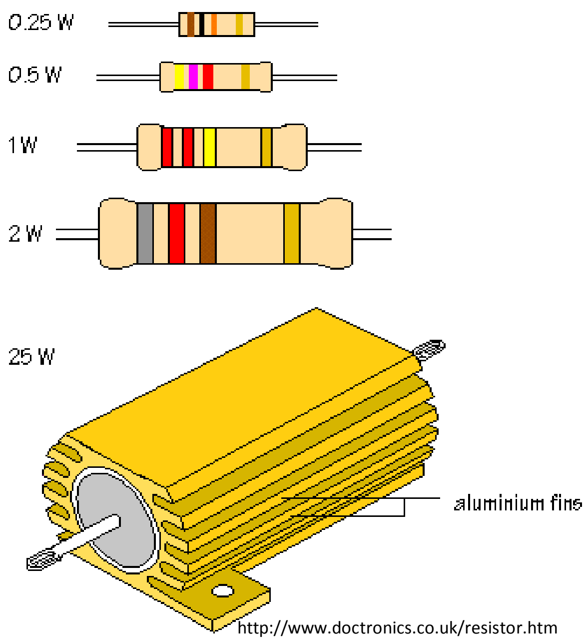
- Units: **siemens** or **mhos** (S or Ω^{-1})

Real Resistors

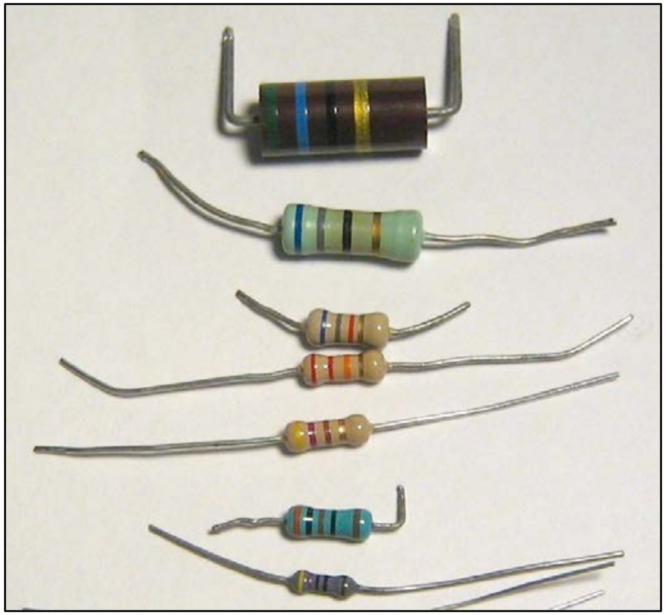
- Resistors for use in electronic circuits come in many shapes and sizes depending on their target application
- Size primarily determined by power handling capability
  - Larger resistors can dissipate more power
- Two primary form factors:
  - ***Axial lead*** resistors
  - ***Chip*** resistors

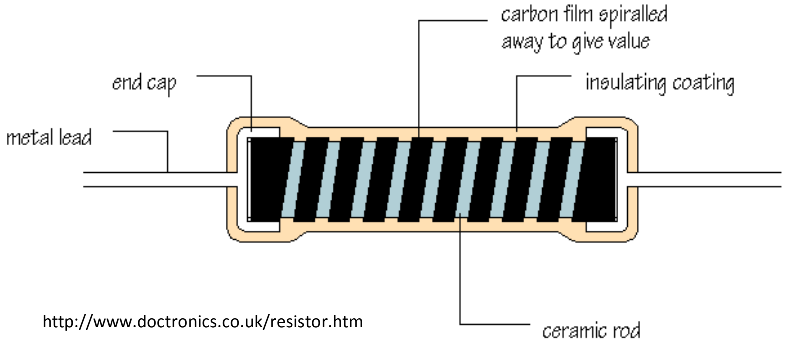
Axial Lead resistors

- Cylindrical resistive component with wire leads extending from each end
- Used with **through-hole technology** printed circuit boards (PCB's)
  - Useful for prototyping
  - Size varies with power handling capacity



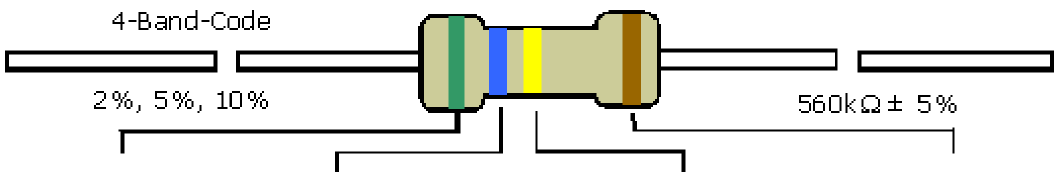
http://www.doctronics.co.uk/resistor.htm



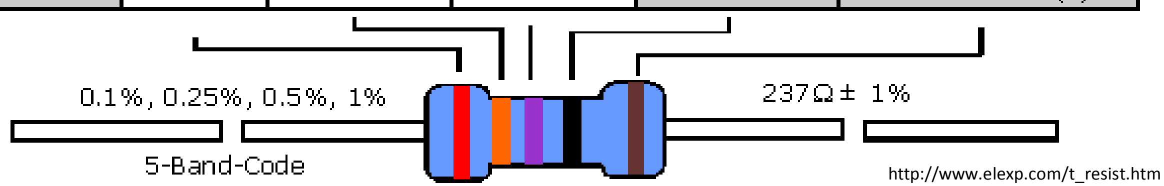


http://www.doctronics.co.uk/resistor.htm

Resistor Color Code



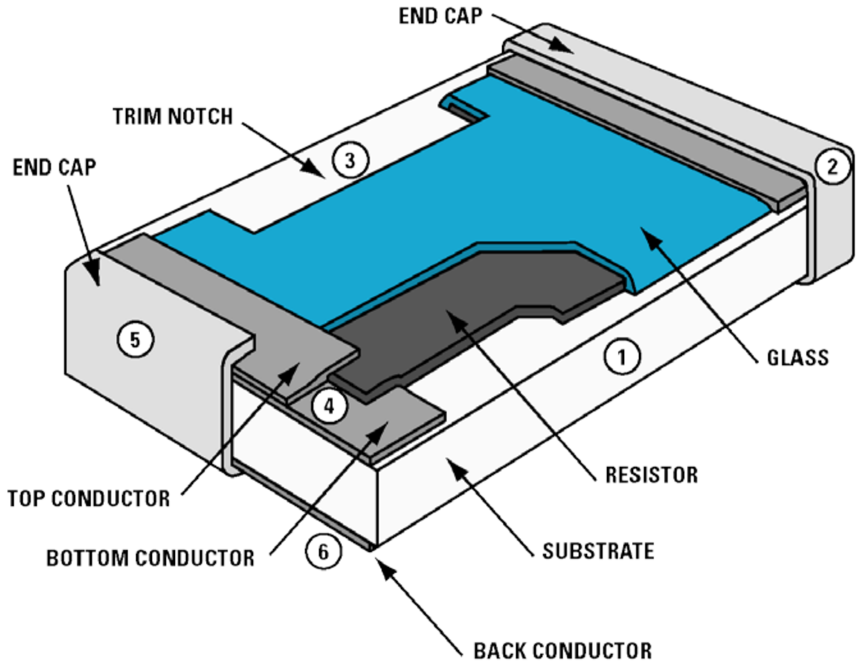
COLOR	1st BAND	2nd BAND	3rd BAND	MULTIPLIER	TOLERANCE
Black	0	0	0	1Ω	
Brown	1	1	1	10Ω	± 1% (F)
Red	2	2	2	100Ω	± 2% (G)
Orange	3	3	3	1KΩ	
Yellow	4	4	4	10KΩ	
Green	5	5	5	100KΩ	±0.5% (D)
Blue	6	6	6	1MΩ	±0.25% (C)
Violet	7	7	7	10MΩ	±0.10% (B)
Grey	8	8	8		±0.05%
White	9	9	9		
Gold				0.1	± 5% (J)
Silver				0.01	± 10% (K)



http://www.elexp.com/t_resist.htm

Chip Resistors

- Small rectangular footprint
  - 0805 – 0.080” x 0.050”
  - 0603 – 0.060” x 0.030”
  - 0402 – 0.040” x 0.020”
  - 0201 – 0.020” x 0.010”
- Used with **surface-mount technology** PCB’s
- More common than axial lead in modern electronics



http://www.mini-systemsinc.com/msithick/chipanat.asp

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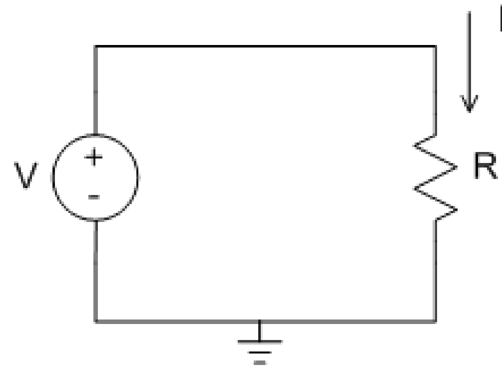
Ohm’s Law

Ohm's Law

Georg Simon Ohm, 1789 – 1854

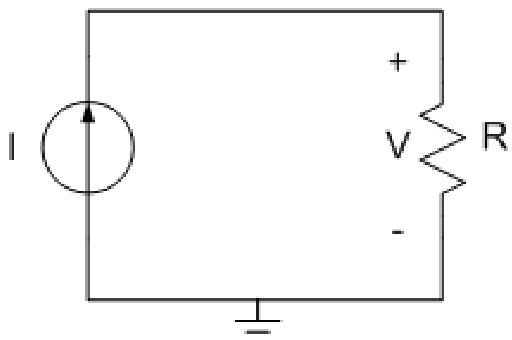
$$I = \frac{V}{R}$$



"The current through a resistor is proportional to the voltage across the resistor and inversely proportional to the resistance."

Ohm's Law – said differently

$$V = I \cdot R$$



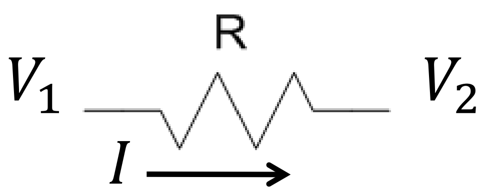
Georg Simon Ohm, 1789 – 1854

"The voltage across a resistor is proportional to the current through the resistor and proportional to the resistance."

Ohm's Law – fluid analogy

- ***Voltage*** is analogous to ***pressure***
  - Driving potentials
- Electrical ***current*** is analogous to ***flow rate***
- A pipe carrying fluid has some resistance determined by physical characteristics (length, diameter, roughness, etc.)

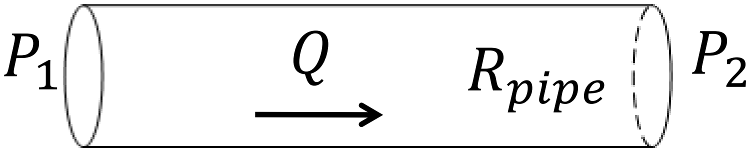
Resistor



$$I \propto (V_1 - V_2), \qquad I \propto \frac{1}{R}$$

$$(V_1 - V_2) \propto I, \qquad (V_1 - V_2) \propto R$$

Section of pipe



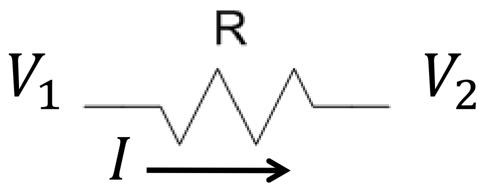
$$Q \propto (P_1 - P_2), \qquad Q \propto \frac{1}{R_{pipe}}$$

$$(P_1 - P_2) \propto Q, \qquad (P_1 - P_2) \propto R_{pipe}$$

Ohm's Law – thermal analogy

- **Voltage** is analogous to **temperature**
  - Driving potentials
- Electrical **current** is analogous to **heat flux**
- A solid slab or wall has some thermal resistance determined by physical characteristics (thickness, material properties, etc.)

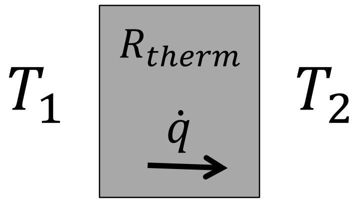
Resistor



$$I \propto (V_1 - V_2), \qquad I \propto \frac{1}{R}$$

$$(V_1 - V_2) \propto I, \qquad (V_1 - V_2) \propto R$$

Solid wall



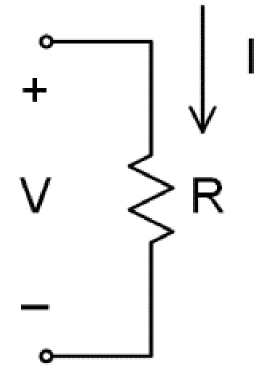
$$\dot{q} \propto (T_1 - T_2), \qquad \dot{q} \propto \frac{1}{R_{therm}}$$

$$(T_1 - T_2) \propto \dot{q}, \qquad (T_1 - T_2) \propto R_{therm}$$

Power in Resistors

- Resistors ***dissipate*** power
- Rate of power dissipation given by

$$P = V \cdot I$$



- According to Ohm's law

$$V = I \cdot R \quad \text{and} \quad I = V/R$$

- So ***for resistors*** (***only***), power is given by

$$\boxed{P = I^2 R}$$

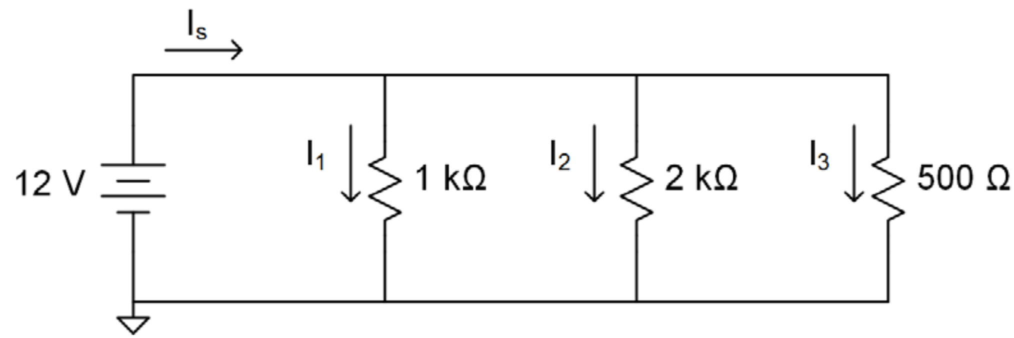
and

$$\boxed{P = \frac{V^2}{R}}$$

Example Problems

Find:

- I_1, I_2, I_3, and I_s.
- The power dissipated by each resistor three different ways.
- The power supplied by the source.



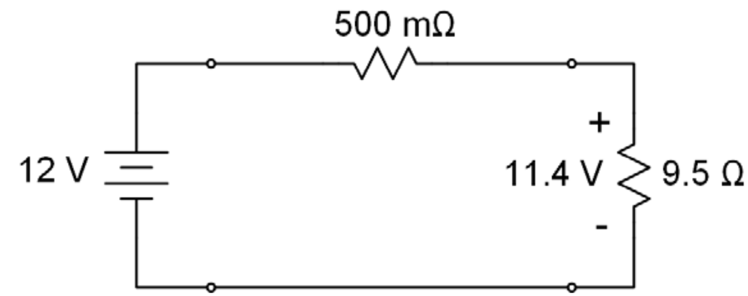
How much current does a 50 W incandescent lightbulb draw? What is its resistance?

The following circuit represents a battery connected to a load through a long wire.

How much current flows through the wire to the load?

How much power is delivered to the load?

How much power is lost in the wire?

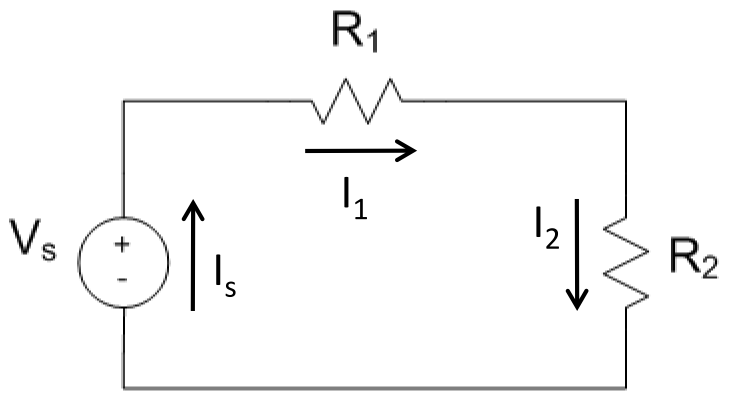


A 24 V source supplies 160 mA to a resistive load. How much power is delivered to the load? What is the equivalent resistance of the load?

Series & Parallel Circuits

Series Circuits

- Series-connected components
  - Share **one common node**
    - Nothing else connected to that node
  - Connected end-to-end
  - **Equal current** through each component

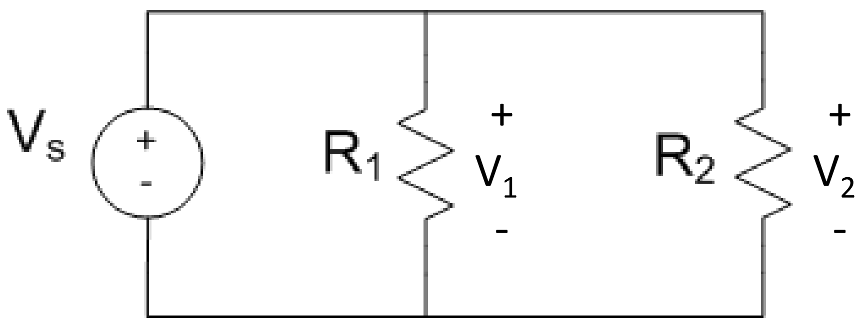


Resistors, R_1 and R_2, and voltage source, V_s, are all connected in **series**

$$\mathbf{I_s = I_1 = I_2}$$

Parallel Circuits

- Components in **parallel**
  - Share **two common nodes**
  - Connected side-by-side
  - **Equal voltage** across each component

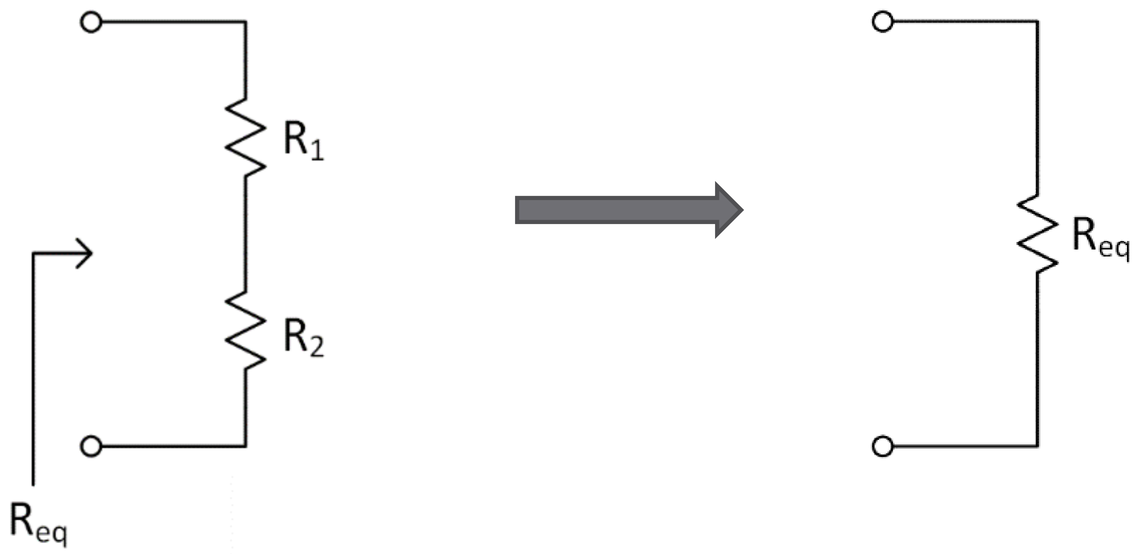


Resistors, R_1 and R_2, and voltage source, V_s, are all connected in **parallel**

$$V_s = V_1 = V_2$$

Series Resistance

- ***Resistances in series add***



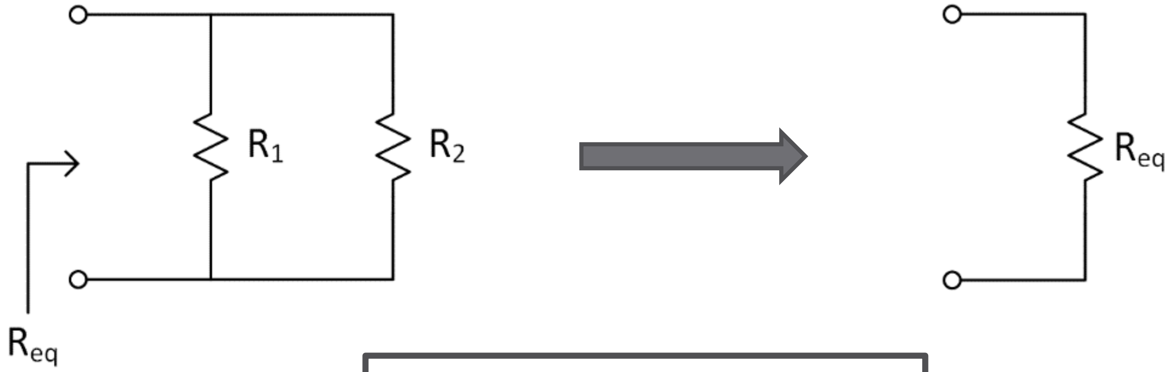
$$R_{eq} = R_1 + R_2$$

- In general,

$$R_{eq} = \sum R_i$$

Parallel Resistance

- ***Conductances in parallel add***



$$R_{eq} = \left(\sum \frac{1}{R_i} \right)^{-1}$$

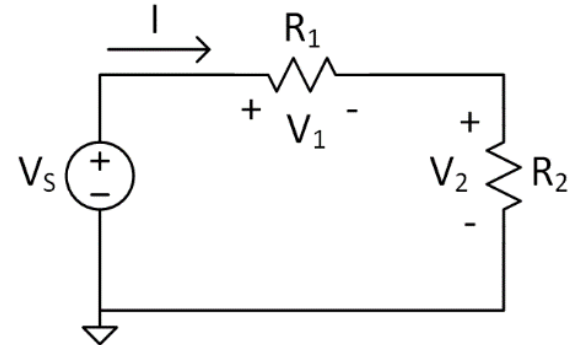
- For ***two*** parallel resistors (only):

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$

Voltage & Current Dividers

Voltage Dividers

- Voltage across series resistors divides proportional to resistance
- Consider two series resistors:
  - Current through the resistors



$$I = \frac{V_s}{R_1 + R_2}$$

  - Ohm's law gives the voltage across either resistor

$$V_n = I R_n$$

$$V_1 = \frac{V_s}{R_1 + R_2} R_1 = V_s \frac{R_1}{R_1 + R_2}$$

$$V_2 = \frac{V_s}{R_1 + R_2} R_2 = V_s \frac{R_2}{R_1 + R_2}$$

Voltage Dividers

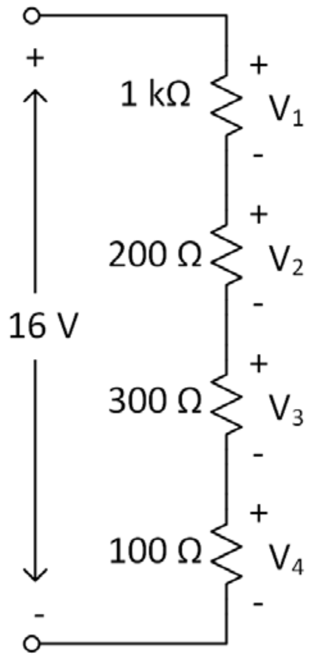
- In general, the voltage across one in a series of resistors is given by

$$V_n = V_{total} \cdot \frac{R_n}{\Sigma R_i}$$

- For example:

$$V_3 = 16\ V \frac{300\ \Omega}{1\ k\Omega + 200\ \Omega + 300\ \Omega + 100\ \Omega}$$

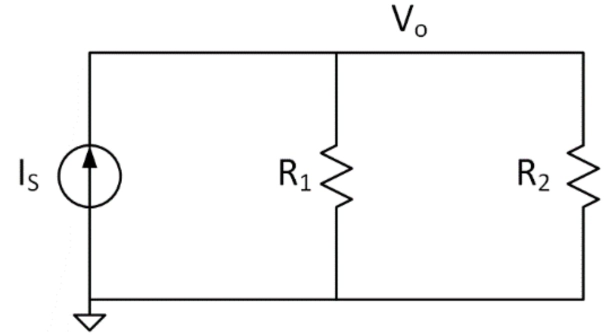
$$V_3 = 16\ V \frac{300\ \Omega}{1.6\ k\Omega} = 3\ V$$



Current Dividers

- Current through parallel-connected resistances divides proportional to conductance
- Consider two parallel resistors:
  - Voltage across the resistors

$$V_o = \frac{I_S}{G_1 + G_2} = \frac{I_S}{\frac{1}{R_1} + \frac{1}{R_2}} = I_S \frac{R_1 R_2}{R_1 + R_2}$$



  - Ohm's law gives the current through either resistor

$$I_n = \frac{V_o}{R_n}$$

$$I_1 = \frac{I_S}{R_1} \frac{R_1 R_2}{R_1 + R_2} = I_s \frac{R_2}{R_1 + R_2}$$

$$I_2 = \frac{I_S}{R_2} \frac{R_1 R_2}{R_1 + R_2} = I_s \frac{R_1}{R_1 + R_2}$$

Current Dividers

- Current through one of ***two*** parallel resistors is given by

$$I_1 = I_{total} \cdot \frac{R_2}{R_1 + R_2}$$

$$I_2 = I_{total} \cdot \frac{R_1}{R_1 + R_2}$$

  - One of the two resistors may be a parallel combination of multiple resistors

- More generally, expressed in terms of ***conductance***
  - Applies to any number of parallel resistances

$$I_n = I_{total} \cdot \frac{G_n}{\Sigma G_i}$$

Current Dividers

- For example, determine I_1
- First, combine the 300 Ω and 100 Ω resistors in parallel

$$R_{eq} = \left(\frac{1}{300\ \Omega} + \frac{1}{100\ \Omega}\right)^{-1} = 75\ \Omega$$

- Next, apply the current divider equation:

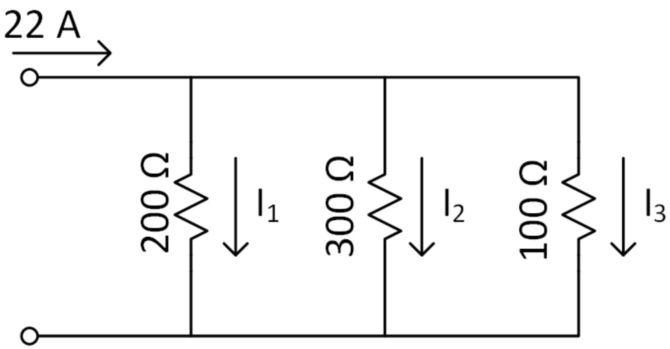
$$I_1 = I_{total}\frac{R_2}{R_1 + R_2}$$

$$I_1 = 22\ A\frac{75\ \Omega}{200\ \Omega + 75\ \Omega}$$

$$I_1 = 6\ A$$

Current Dividers

- Or, using conductances:



$$I_1 = 22\ A \frac{\frac{1}{200\ \Omega}}{\frac{1}{200\ \Omega} + \frac{1}{300\ \Omega} + \frac{1}{100\ \Omega}}$$

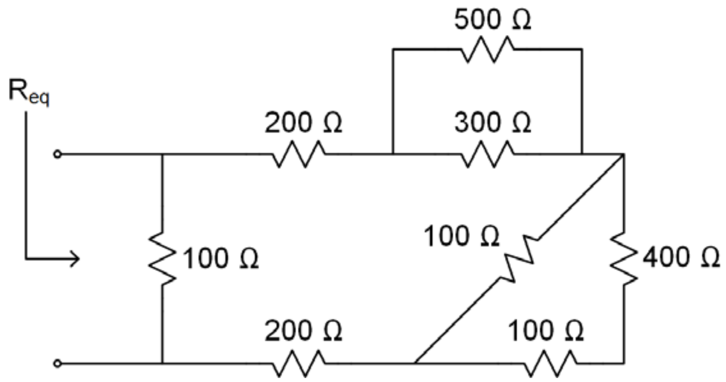
$$I_1 = 22\ A \cdot \frac{5\ mS}{5\ mS + 3.33\ mS + 10\ mS}$$

$$I_1 = 22\ A \cdot 0.2727$$

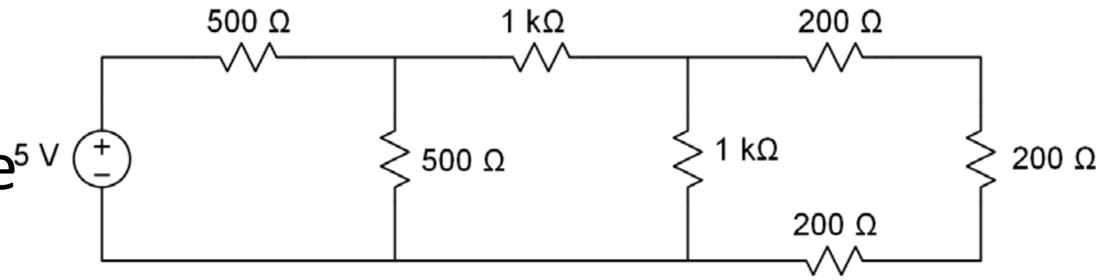
$$I_1 = 6\ A$$

Example Problems

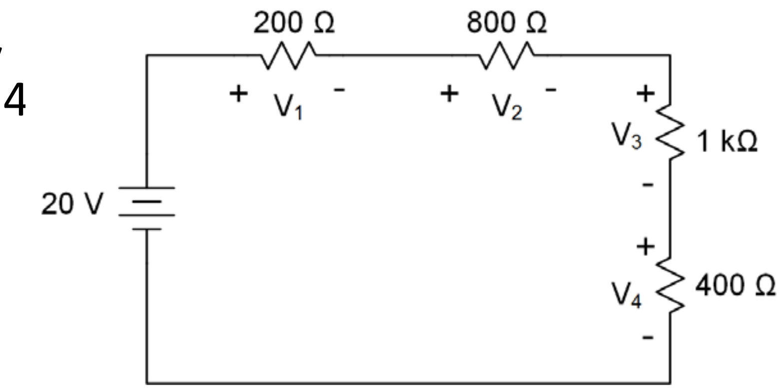
Determine the equivalent input resistance, R_{eq}, for the following network.



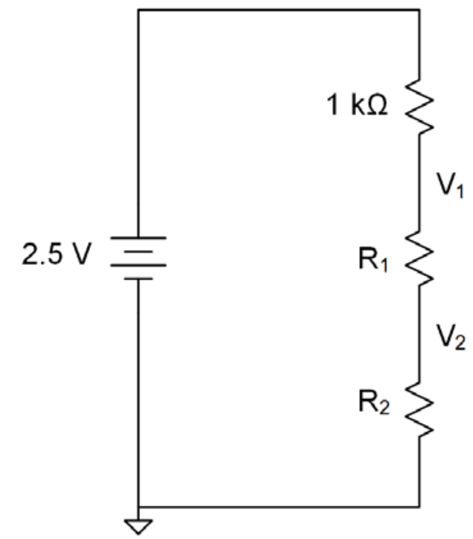
How much power is delivered by the voltage source in the following network?



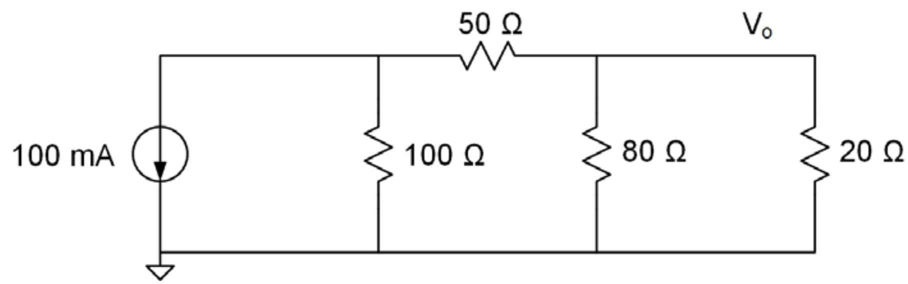
Determine V_1, V_2, V_3, and V_4



Determine R_1 and R_2, such that $V_1 = 2$ V, and $V_2 = 1.25$ V.

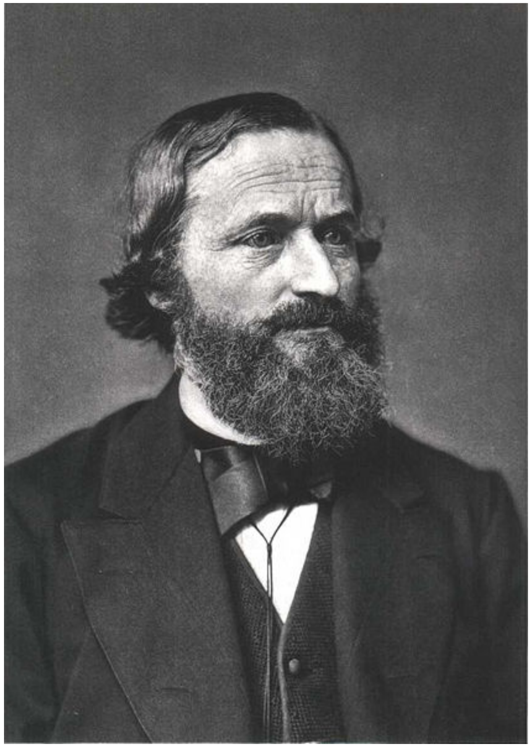


Determine V_o in the following circuit.



Kirchhoff’s Laws

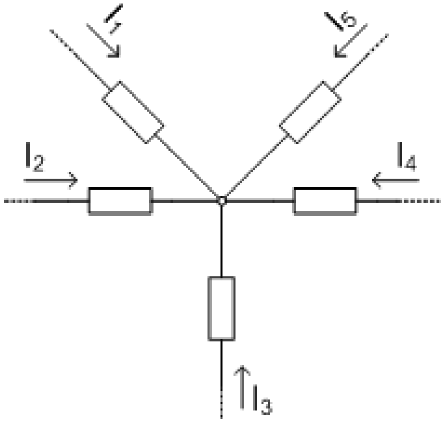
Kirchhoff's Current Law - KCL



Gustav Kirchhoff, 1824 – 1887

"The algebraic sum of currents entering any node must be zero."

- Charge cannot accumulate in a node
- What flows in, must flow out



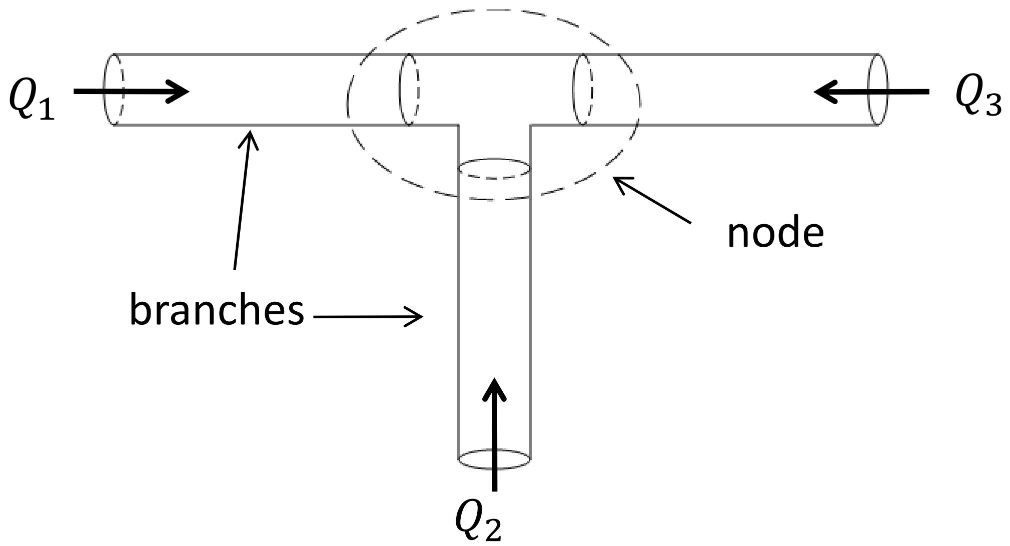
$$I_1 + I_2 + I_3 + I_4 + I_5 = 0$$

$$\sum_i I_i = 0$$

- Analogous to the conservation of mass

KCL & the Conservation of Mass

- Consider fluid-carrying pipes connected in a Tee
  - Tee connector is analogous to electrical ***node***
  - Pipes analogous to ***branches***
- According to the conservation of mass, what flows in must flow out
  - ***Sum of the flow rates must be zero***



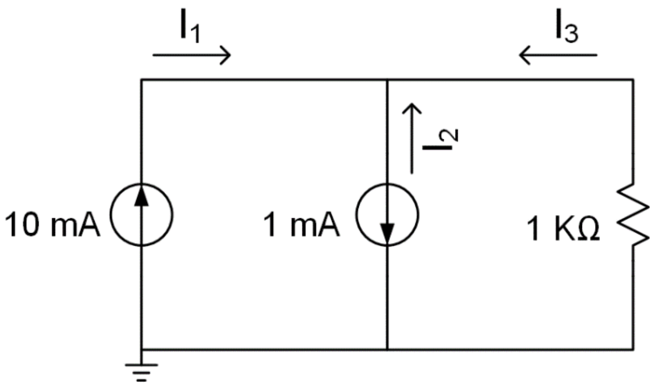
$$\sum_i Q_i = 0$$

KCL - Example

- Determine the current through the $1\ k\Omega$ resistor, I_3
- Applying KCL

$$I_1 + I_2 + I_3 = 0$$

$$I_3 = -I_1 - I_2$$



- I_1 and I_2 are known:

$$I_1 = 10\ mA, \quad I_2 = -1mA$$

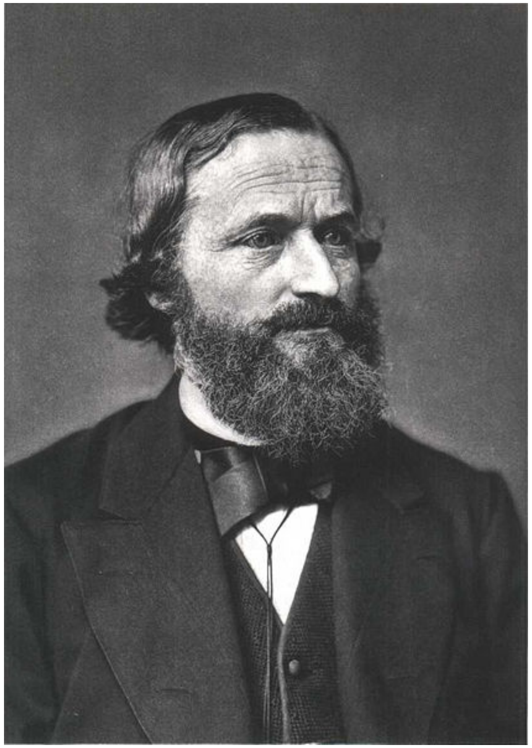
- Solving for I_3:

$$I_3 = -10\ mA + 1\ mA$$

$$I_3 = -9\ mA$$

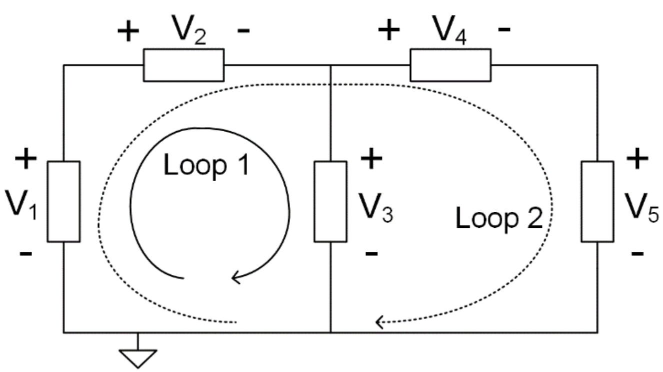
- The negative sign indicates that I_3 flows in the opposite direction of what was assumed

Kirchhoff's Voltage Law - KVL



Gustav Kirchhoff, 1824 – 1887

"The algebraic sum of voltage changes taken around any loop in a network is equal to zero."



KVL around Loop 1

$$V_1 - V_2 - V_3 = 0$$

KVL around Loop 2

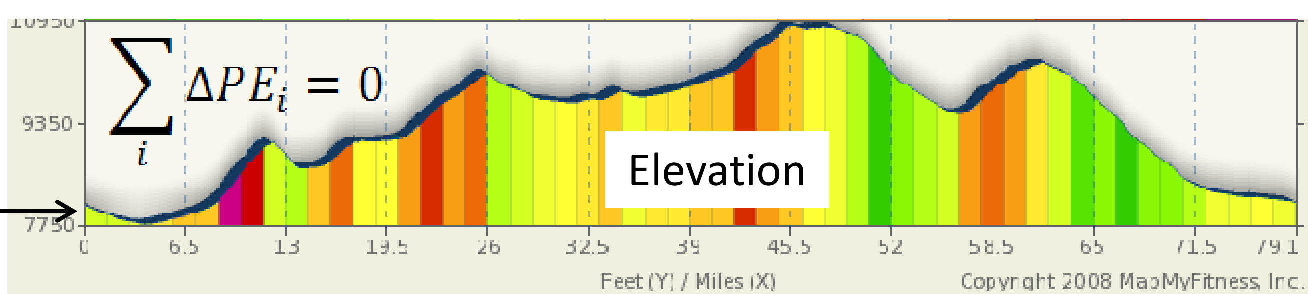
$$V_1 - V_2 - V_4 - V_5 = 0$$

- Conservation of energy applied to electric circuits

KVL & the Conservation of Energy

- Voltage drops around a circuit are analogous to changes in potential energy while traversing a loop
- PE varies with elevation
  - Increases with each climb
  - Decreases with each descent
- Initial/final elevation & PE are the same
  - Sum of PE rises/drops around the loop is zero
  - Just like voltage drops around a circuit

PE_i



$PE_f = PE_i$

KVL – Example

- Determine the voltage, V_2, across the $3.3\ k\Omega$ resistor
- Applying KVL

$$V_1 - V_2 - V_3 = 0$$

$$V_2 = V_1 - V_3$$

- V_1 and V_3 are known:

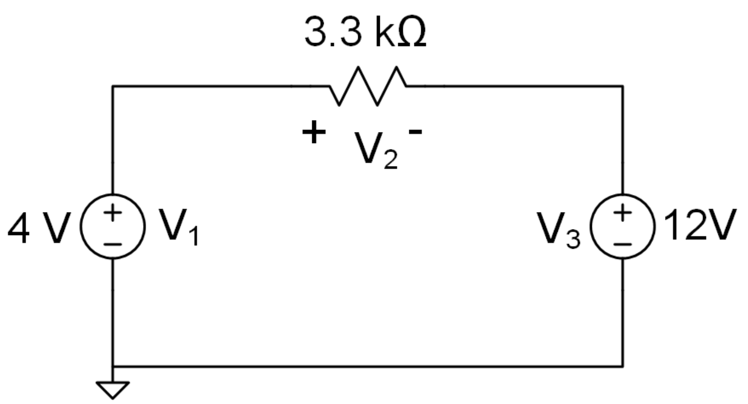
$$V_1 = 4\ V,\ \ V_3 = 12\ V$$

- Solving for V_2:

$$V_2 = 4\ V - 12\ V$$

$$V_2 = -8\ V$$

- The negative sign indicates that the polarity of V_2 is the opposite of what was assumed

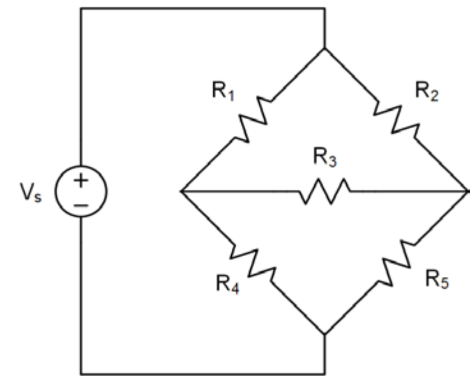


Delta and Wye Networks

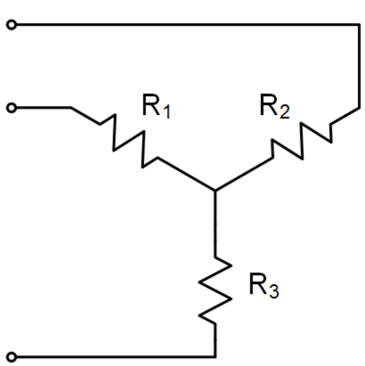
Delta and Wye Networks

- Circuits may comprise components that are connected ***neither in series nor in parallel***, e.g.:
  - Wheatstone bridge circuit
  - Three-phase AC power systems
    - Motors
    - Generators
    - Transformers
- Often, these include ***wye*** and/or ***delta*** networks

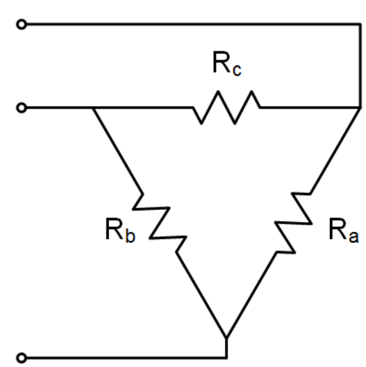
Bridge network



Wye network

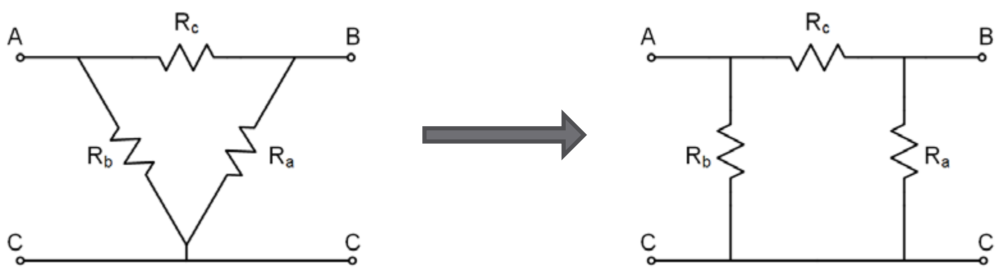


Delta network

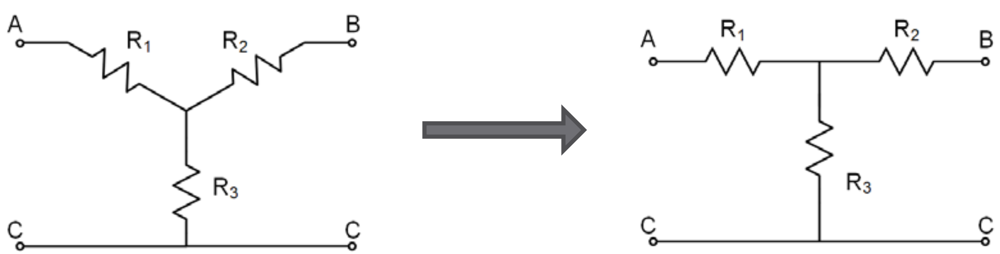


Y-Δ Transformations

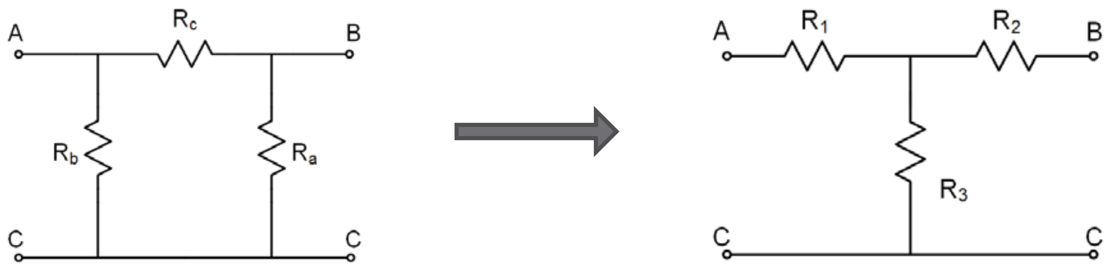
- Circuit analysis is often simplified if we are able to convert between Y and Δ networks
- To aid in developing the Y-Δ conversion relationships, we can redraw the Δ network as a Π network:



- And the Y network as a T network:



Δ-to-Y Conversion



- For a Y network and Δ network to be equivalent, they must have equal resistance between corresponding terminals
  - Between node A and node C:

$$R_{AC_Y} = R_1 + R_3 = R_{AC_\Delta} = R_b || (R_a + R_C)$$

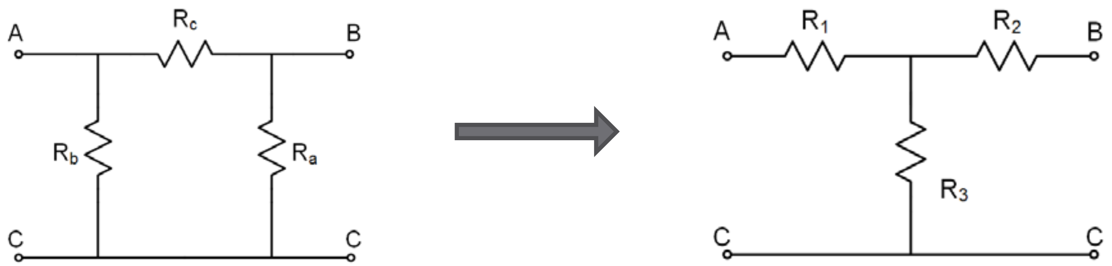
$$R_1 + R_3 = \frac{R_b(R_a+R_c)}{R_a+R_b+R_c} \qquad (1)$$

  - Between B and C:

$$R_{BC_Y} = R_2 + R_3 = R_{BC_\Delta} = R_a || (R_b + R_C)$$

$$R_2 + R_3 = \frac{R_a(R_b+R_c)}{R_a+R_b+R_c} \qquad (2)$$

Δ-to-Y Conversion



- Similarly, between nodes A and B:

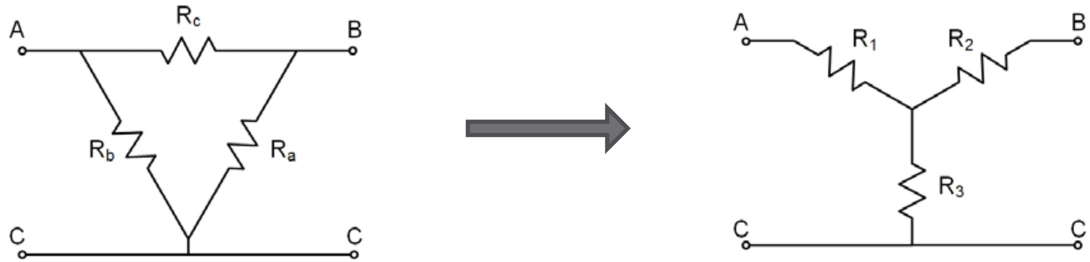
$$R_{AB_Y} = R_1 + R_2 = R_{AB_\Delta} = R_c || (R_a + R_b)$$

$$R_1 + R_2 = \frac{R_c(R_a+R_b)}{R_a+R_b+R_c} \qquad (3)$$

- Subtracting (2) from (1) yields:

$$R_1 - R_2 = \frac{R_c(R_b-R_a)}{R_a+R_b+R_c} \qquad (4)$$

Δ-to-Y Conversion



- Adding (4) to (3) gives an expression for R_1:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \tag{5}$$

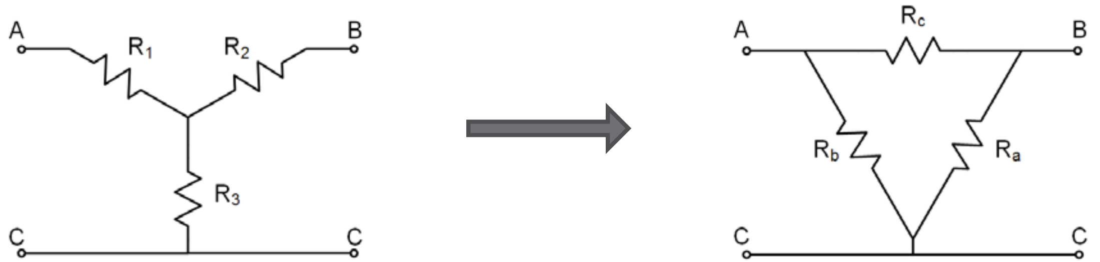
- Subtracting (4) from (3) gives

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} \tag{6}$$

- And, finally, subtracting (5) from (1) gives

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \tag{7}$$

Y-to-Δ Conversion



- We can derive a similar set of relationships for converting from a Y network to a Δ network

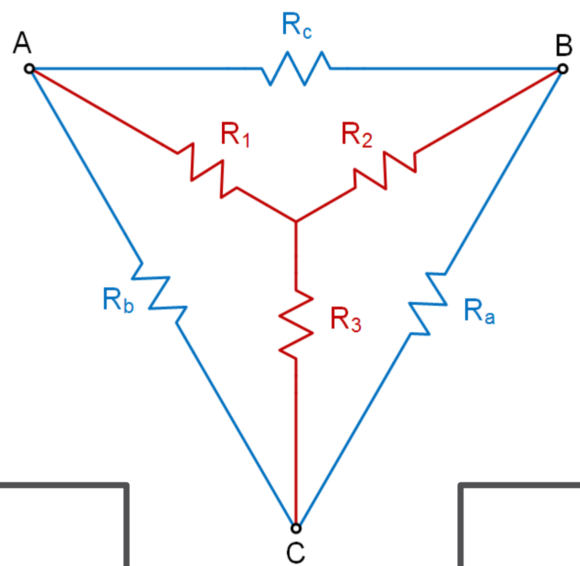
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Y-Δ Transformations

Each resistor in the equivalent Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors



Each resistor in the equivalent Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

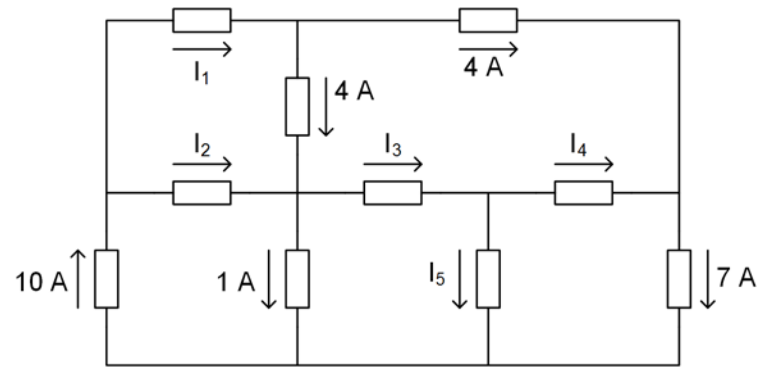
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

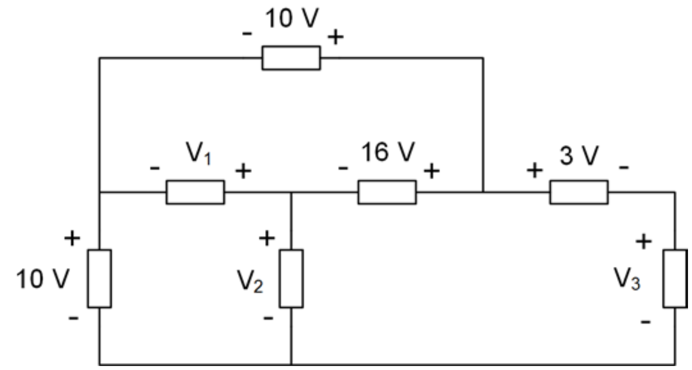
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Example Problems

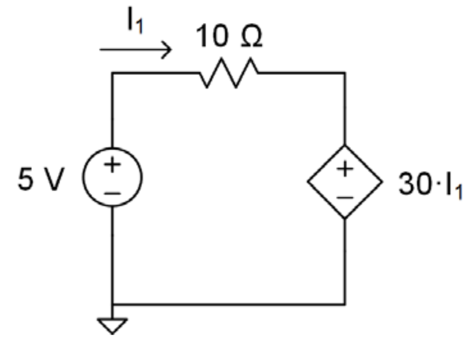
Determine I_1, I_2, I_3, I_4, and I_5.



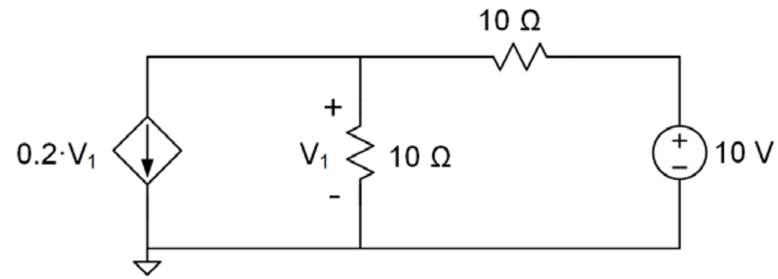
Find V_1, V_2, and V_3.



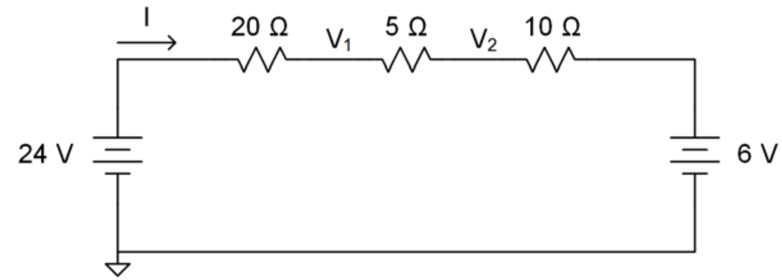
Determine the current through the circuit, I_1.



Determine the voltage, V_1.



Determine V_1 and V_2 in the following circuit.



Determine I and V_1 in the following circuit.