SECTION 4: OPERATIONAL AMPLIFIERS

ENGR 201 – Electrical Fundamentals I



Amplification

Amplification

Multiplication of electrical signals

- Voltage or current
- Scaling factor: gain
- Performed by *amplifiers*



Amplification – Why?

Often want to increase the amplitude of small electrical signals

- Sensor outputs, e.g.:
 - Strain gauges
 - Pressure sensors
 - Flow meters
 - Temperature sensors
 - Photo-detectors, etc.
- Wireless communication signals
- Audio signals
- Larger signals are easier to measure
 - Utilize the full *dynamic range* of the measurement system
 - Higher accuracy

Amplification – Why?

- Amplifiers are also useful for *impedance conversion*
 - Make a high-resistance source look like a low resistance
 - Make a low-resistance load look like a high resistance
 - Buffering a high- resistance source from a lowresistance load
- A single amplifier circuit can provide amplification and impedance conversion



Amplifier Characteristics

Key amplifier characteristics:

🗖 Gain

May be designed for voltage, current, or power gain

- Input resistance
- Output resistance



Amplifier Characteristics - Gain



□ Voltage gain

Ratio of output to input voltage

$$A_V = \frac{v_o}{v_i}$$

Current gain

Ratio of output to input current

$$A_i = \frac{i_o}{i_i}$$

Dever gain

Ratio of output to input power

$$G = \frac{P_o}{P_i}$$

Amplifier Characteristics – Input Resistance



□ Input resistance, R_i

Equivalent resistance seen looking into the amplifier

- Amplifier *loads* the source
 - **\square** Source appears to be connected to a resistance of R_i
 - Possible voltage division between source resistance and input resistance

Amplifier Characteristics – Output Resistance



Output resistance, R_o

Thévenin equivalent resistance of the amplifier output

- From the perspective of the load, amplifier output is the source
 - Modeled as Thévenin equivalent circuit, with resistance, R_o
 - Possible voltage division between R_o and the load

Amplifier Characteristics – Cascaded Amplifiers

Consider two amplifiers connected in *cascade*



Input to the second amplifier is the output from the first

$$v_{o1}(t) = A_{v1} \cdot v_i(t)$$

Output of the second amplifier is the output of the cascade

$$v_o(t) = A_{v2} \cdot v_{o1}(t) = A_{v1} \cdot A_{v2} \cdot v_i(t)$$

Overall gain is the *product* of the individual gains



Amplifier – Equivalent Circuit

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Amplifier equivalent circuit:



- $\square R_i$: input resistance
- □ R_o: output resistance

\Box A_{voc} : open-circuit voltage gain

• Note that, in general, due to loading:



¹³ Opamp Fundamentals

Operational Amplifiers - Opamps

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- How do we get the amplification we need?
- Different requirements for different applications
 - High gain/low gain Inverting/non-inverting gain
 - Accuracy
 - Adjustability
- Building amplifiers out of transistors is difficult, inconvenient
- Chip makers could make unique integrated circuits (ICs) for all possible applications
 - Impractical, not economical
- Instead: operational amplifiers (opamps)
 - **General-purpose** amplifier ICs
 - Gain set by a few external components

Operational Amplifiers - Opamps

- High-gain amplifiers built from many transistors, resistors, and capacitors
- Integrated circuits (ICs)
 - All components fabricated on a single semiconductor (e.g. Si) chip
- Gain set by a few external resistors





Ideal Opamps

Schematic symbol:



Equivalent circuit:



- \Box Open-loop voltage gain, A_{vol}
 - Gain of the opamp without feedback

Differential input, v_{id}

Difference between the two input voltages

$$v_{id} = v^+ - v^-$$

Common-mode input, v_{icm}

$$v_{icm} = \frac{(v^+ + v^-)}{2}$$

Ideal Opamp Characteristics

Ideal opamp characteristics:

- **1)** Infinite input resistances at v^+ and v^-
 - No current flows into either input terminal

2) Infinite open-loop gain

Any differential input will result in an infinite output

- 3) Zero output resistance
 - Immune to the effect of loading

Infinite Gain

Ideal opamps have infinite gain
How do we get a gain of 2 or 9 or -3?

Negative feedback

- By enclosing the opamp in a *feedback loop* we can create an amplifier with useable gain
- Before applying feedback to opamp circuits, we'll first introduce the concept of feedback

¹⁹ Introduction to Feedback

Feedback

Feedback

A process in which a portion of the output of some system is fed back to the input of that system

Positive feedback

The *addition* of a portion of the output to the input
Generally has a *destabilizing* effect

Negative feedback

- **•** The *subtraction* of portion of the output from the input
- Generally has a *stabilizing* effect
- All opamp *amplifiers* we will encounter in this course employ negative feedback

Feedback

Feedback is everywhere:

HVAC	Toilets
Cruise control	Ovens
Robotics	Autonomous vehicles
Amplifiers	Etc.

 A very important concept in many engineering fields, particularly *controls* and *electronics*

Signal Flow Diagrams

Signal flow diagrams

- Block diagrams
- Show the flow of signals energy, information, etc. through a system
- Used for all types of engineering systems: electrical, mechanical, etc.
- □ Amplifier, with open-loop gain A_{vol} , enclosed in a feedback loop:



- Amplifier is in a *closed-loop* configuration
- □ Feedback gain β , determines how much output is fed back
- Difference between input and feedback is amplifier input
- $\square \beta$ determines overall gain
 - Closed-loop gain

Closed-Loop Gain

Open-loop gain

Gain of amplifier without feedback

$$A_{vol} = \frac{v_o}{v_{id}}$$

$$V_i$$
 V_{id} A_{vol} V_o

Closed-loop gain

Gain of the closed-loop amplifier

$$A_{vcl} = \frac{v_o}{v_i}$$

Calculating the closed-loop gain:

$$v_{o} = v_{id} \cdot A_{vol} = (v_{i} - \beta v_{o})A_{vol}$$
$$v_{o} + \beta \cdot v_{o} \cdot A_{vol} = v_{i} \cdot A_{vol}$$
$$v_{o}(1 + \beta \cdot A_{vol}) = v_{i} \cdot A_{vol}$$

$$A_{vcl} = \frac{v_o}{v_i} = \frac{A_{vol}}{1 + \beta \cdot A_{vol}}$$

Closed-Loop Gain

$$A_{vcl} = \frac{v_o}{v_i} = \frac{A_{vol}}{1 + \beta \cdot A_{vol}}$$

E.g., an ideal opamp

$$\lim_{A_{vol} \to \infty} A_{vcl} = \lim_{A_{vol} \to \infty} \frac{A_{vol}}{1 + \beta \cdot A_{vol}}$$
$$\lim_{A_{vol} \to \infty} A_{vcl} = \lim_{A_{vol} \to \infty} \frac{A_{vol}}{\beta \cdot A_{vol}} = \frac{1}{\beta}$$

Feedback gain, alone, determines the closed-loop gain
For example, to get a gain of four, feed back one quarter of the output signal



Summing Point Constraint

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For $A_{vol} = \infty$, the differential input is

$$v_{id} = v_i - \beta v_o = v_i - \beta \frac{1}{\beta} v_i$$
$$v_{id} = v_i - v_i = 0$$

□ A very important result, the *summing-point constraint*:

The input to an infinite-gain amplifier, enclosed in a negative feedback loop, is zero!

Any non-zero input would yield infinite output

 Along with the properties of ideal opamps, the summingpoint constraint will be essential for analyzing opamp circuits

Opamps as Feedback Systems

- Any amplifier enclosed in a negative feedback loop:
 - Output scaled by β and fed back
 - Feedback signal subtracted from the input at a summing junction

• If
$$A_{vol} = \infty$$
, then $v_{id} = 0$



Ideal opamp enclosed in a negative feedback loop:

- Output scaled by β and fed back
- Differential input is a built-in summing junction
- $v_{id} = 0$, so $v^+ = v^-$
- A virtual short between input terminals



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An amplifier with a gain of 4 is used to amplify the output of a sensor. The amplifier has an input resistance of $1 \ k\Omega$ and an output resistance of 100 Ω . The sensor has an open-circuit voltage of 1 V, and an output resistance of 50 Ω . The amplifier drives a 5 k Ω load. What is the amplifier's output voltage? An amplifier is enclosed in a negative feedback loop with a feedback gain of 0.5. Determine the closed-loop gain for an amplifier with an open-loop gain of:

a) 10

b) 100E3

A 1 V input is applied to an opamp enclosed in a negative feedback loop with a feedback gain of 0.2. Determine the differential input voltage for an opamp with an open-loop gain of:

a) 10

b) 100E3

³⁴ Opamp Amplifiers

Analysis of Ideal Opamp Circuits

Key properties for analysis of opamp circuits:

- 1) Infinite input resistance no input current
- 2) Virtual short between inverting and non-inverting input terminals (as long as there is negative feedback)
- 3) Infinite open-loop gain

If there is negative feedback, 1 and 2 are sufficient

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Opamp Amplifiers

Two basic opamp amplifier configurations:





Inverting amplifier:


³⁷ Non-Inverting Amplifier

Non-Inverting Amplifier – Gain

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- Non-inverting amplifier gain:
- Negative feedback, so the summing-point constraint applies

$$v^- = v^+ = v_i$$



Zero opamp input current, so KCL at the inverting terminal gives:

$$i_1 = i_2$$
$$\frac{v_o - v^-}{R_2} = \frac{v^-}{R_1}$$

Applying the summing-point constraint

$$\frac{v_o - v_i}{R_2} = \frac{v_i}{R_1}$$

Non-Inverting Amplifier – Gain

$$\frac{v_o - v_i}{R_2} = \frac{v_i}{R_1}$$

Solving for the amplifier output

$$v_o = v_i \left(\frac{1}{R_1} + \frac{1}{R_2}\right) R_2$$



 \Box Dividing both sides by v_i gives the non-inverting amplifier gain:

$$A_{v} = \frac{v_{o}}{v_{i}} = \frac{R_{1} + R_{2}}{R_{1}}$$

Note that this is the inverse of the feedback path gain

$$A_{v} = \frac{1}{\beta} = \frac{1}{\frac{R_{1}}{R_{1} + R_{2}}}$$

Non-Inverting Amplifier – Gain



$$A_{\nu} = \frac{\nu_o}{\nu_i} = \frac{R_1 + R_2}{R_1}$$

- Gain determined entirely by the relative value of two external resistors
 - Any gain value is possible
 - Resistor tolerance sets amplifier gain tolerance
- Gain is *positive* (non-inverting)
 - As input goes up, output goes up
- Gain can never be less than one
 Unity gain for R₂ = 0 Ω

Non-Inverting Amplifier – R_i and R_o



- Input resistance, R_i :
 - Input connected directly to input terminal of ideal opamp

$$R_i = \infty$$

- \Box Output resistance, R_o :
 - Output is the output from an ideal opamp

$$R_o = 0 \ \Omega$$

⁴² Inverting Amplifier

Inverting Amplifier – Gain

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- Inverting amplifier gain:
- Negative feedback, so the summing-point constraint applies

$$v^- = v^+ = 0$$



□ *Zero opamp input current*, so KCL at the inverting terminal gives:

$$i_1 = i_2$$
$$\frac{v_i}{R_1} = \frac{-v_o}{R_2}$$

□ Gain of the inverting amplifier:

$$A_{v} = \frac{v_o}{v_i} = -\frac{R_2}{R_1}$$

Inverting Amplifier – Gain





Again, gain determined entirely by external resistor values

Gain is *negative* (inverting)

- As input goes up, output goes down
- □ Gain can be any value
 - $\square A_{v} < 1 \text{ for } R_{1} > R_{2}$

Inverting Amplifier – R_i

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- Input resistance, R_i :
 - Inverting terminal is a *virtual* ground, so

$$i_i = \frac{v_i}{R_1}$$



By definition, input resistance is

$$R_{i} = \frac{v_{i}}{i_{i}} = v_{i} \frac{R_{1}}{V_{i}}$$
$$R_{i} = R_{1}$$

■ Input is a resistance, R₁, to (virtual) ground

Inverting Amplifier – R_o

- Output resistance, R_o :
 - Output is the output of an ideal opamp, so

$$R_o = 0 \ \Omega$$

- Consider a non-ideal opamp with non-zero output resistance:
 - *R*₃ is driven by a *dependent source*
 - Determining R_o is a bit trickier





Inverting Amplifier – R_o

- To determine a terminal resistance in the presence of *dependent* sources:
 - Set all *independent* sources to zero
 - Here, ground the input, v_i
 - Apply a test current, *i_t*, to the terminal of interest
 - Analyze the circuit to determine the voltage at that terminal
 - Resistance at that terminal is given by

$$R_o = \frac{v_o}{i_t}$$



Inverting Amplifier – R_o

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- Still a virtual ground at the inverting terminal, so

$$i_{1} = i_{2} = \frac{v_{i}}{R_{1}} = \frac{0 V}{R_{1}} = 0$$
$$i_{2} = -\frac{v_{o}}{R_{1}} = 0$$
$$v_{o} = 0$$



\square The output is zero, independent of i_t (and independent of R_3), so

$$R_o = 0 \Omega$$

Feedback around an infinite gain amplifier forces the closed-loop output resistance to zero, even if the output resistance of the amplifier itself is non-zero

















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Unity-Gain Buffer

 Start with a non-inverting amplifier, and let the feedback path gain go to unity



Inverting terminal is connected directly to the output

 $v^- = v_o$

Virtual short at the input terminals

$$v^- = v^+ = v_i = v_o$$

□ Closed-loop gain is *unity*

$$A_v = \frac{v_0}{v_i} = 1$$

Unity-Gain Buffer – Buffering

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- What good is an amplifier with unity gain?
 - Impedance conversion
 - A *buffer* between a high-resistance source and a lower-resistance load
 - Eliminates loading effects
- Consider the following scenario:
 - Sensor with $R_{th} = 10 \ k\Omega$ drives a load of $R_L = 1 \ k\Omega$



Unity-Gain Buffer – Buffering

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- To prevent signal attenuation due to loading:
 Add a unity-gain buffer between the source and the load
- Buffer input is the load for the source

$$\square R_i = \infty$$

Buffer output is the source for the load

$$\square R_o = 0 \Omega$$



Now, full sensor signal appears across the load

$$v_L = v^+ = v_s$$

⁶¹ Summing & Difference Amplifiers

Summing Amplifier

KCL at the inverting node:

$$i_A + i_B + i_C = i_f$$
$$\frac{v_A}{R_A} + \frac{v_B}{R_B} + \frac{v_C}{R_C} = -\frac{v_o}{R_E}$$



Output is the (inverted) weighted sum of all of the inputs

$$v_o = -R_F \left(\frac{v_A}{R_A} + \frac{v_B}{R_B} + \frac{v_C}{R_C} \right)$$

□ For the special case of $R_A = R_B = R_C = R_i$:

$$v_o = -\frac{R_F}{R_i}(v_A + v_B + v_C)$$

Difference Amplifier

- This looks a bit like a combination of a non-inverting and an inverting amplifier
- □ Analyze by applying *superposition*
- \Box First, set $v_2 = 0$
 - A non-inverting amplifier with a voltage divider at the input

$$v_o \Big|_{v_1} = v^+ \frac{R_1 + R_2}{R_1}$$

$$v^{+} = v_1 \frac{R_4}{R_3 + R_4}$$

$$v_o \Big|_{v_1} = v_1 \frac{R_4}{R_3 + R_4} \frac{R_1 + R_2}{R_1}$$





Difference Amplifier

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■ Next, set $v_1 = 0$ ■ No current through R_3 and R_4

 $v^+ = 0 V$

An inverting amplifier

$$v_o \Big|_{v_2} = -v_2 \frac{R_2}{R_1}$$



Summing the contributions from each output:

$$v_o = v_o \Big|_{v_1} + v_o \Big|_{v_2}$$

$$v_o = v_1 \frac{R_4}{R_3 + R_4} \frac{R_1 + R_2}{R_1} - v_2 \frac{R_2}{R_1}$$

Difference Amplifier

- Restricting resistor values: $\mathbf{R}_{3} = R_{1} \text{ and } R_{4} = R_{2}$ $v_{0} = v_{1} \frac{R_{2}}{R_{1} + R_{2}} \frac{R_{1} + R_{2}}{R_{1}} - v_{2} \frac{R_{2}}{R_{1}}$ $v_{0} = v_{1} \frac{R_{2}}{R_{1}} - v_{2} \frac{R_{2}}{R_{1}}$ $v_{0} = \frac{R_{2}}{R_{1}} (v_{1} - v_{2})$
- Inverting and non-inverting amplifiers are special cases of the difference amplifier
 - Inverting amplifier: v_1 is grounded
 - Non-inverting amplifier: v_2 is grounded









Design an opamp circuit to perform the following mathematical operation:

$$V_o = 3V_1 - 5V_2$$

Design an opamp circuit to perform the following mathematical operation:

$$V_o = 5V_1 - 3V_2$$


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Open-Loop Opamp Behavior

- So far, we've looked at opamp *amplifier* circuits
 Closed-loop configuration
 - Negative feedback
 - Output remains within the opamp's *linear* output range
- We'll now consider using an opamp *open-loop* without feedback – or with positive feedback
 - **\square** For ideal opamp, $A_{vol} = \infty$
 - For any non-zero input, $v_o \rightarrow \pm \infty$
 - But, v_o limits, or saturates, somewhere near the supply voltages, V_{o,max}
 - In practice, the output of an open-loop opamp is always saturated at $\pm V_{o,max}$

Comparators

 Open-loop opamp's output determined by relative values of the two inputs, differential input



An open-loop opamp acts as a *comparator Compares* the two input voltages
 Sets the output based on which input is higher

Comparators





Comparators – Applications

- Simple comparator application: thermostat
 - Inverting input connected to a temperature sensor
 - Non-inverting input connected to a variable reference voltage determined by the temperature setpoint



- Another example application: *motion-sensing light*
 - One input from a motion sensor variable analog voltage
 - Other input is a threshold voltage set by sensitivity setting
 - Want light to turn on for people and cars, not birds or insects

Comparators and Noise

- Consider the following comparator circuit:
 - Inverting input driven by sinusoidal voltage
 - Non-inverting input is threshold voltage – connected to ground (0 V)



Output switches cleanly at each input threshold crossing



Comparators and Noise

- Now, the sinusoidal input is corrupted by noise
 - Multiple input threshold crossings each time v_s(t) goes through 0 V
 - Multiple, unwanted, output transitions





Would like to be able to reject this noise at the input:
 Schmitt trigger

K. Webb

⁸² Hysteresis

Hysteresis – Schmitt Trigger

- Schmitt trigger employs *hysteresis*
 - Characteristics of the circuit are dependent on its previous state
- Looks like a non-inverting amplifier, but it is not

Positive feedback

 A comparator with a threshold voltage that depends on the output

$$v^+ = v_o \frac{R_2}{R_1 + R_2} = \beta v_o$$

□ Threshold voltage switches between two values as the output switches between $\pm V_{o,max}$

$$v^+ = \pm \beta V_{o,max}$$



Schmitt Trigger – Hysteresis voltage

Consider the case where the input is low

 $v_i < v^+$

Output will be high

$$v_o = +V_{o,max}$$

 The input then increases and exceeds the threshold voltage

$$v_i > v^+$$

The output will then switch low

$$v_o \rightarrow -V_{o,max}$$

The threshold voltage will switch low

 $v^+ \to -\beta V_{o,max}$

Threshold voltage switches away from the rising input

Similar thing is true for falling input



Schmitt Trigger – Hysteresis voltage

- Output, and threshold voltage, always *switches away from the input signal* at the first threshold crossing
- Hysteresis voltage:
 - Magnitude of the threshold voltage change

 $V_{hyst} = 2\beta V_{o,max}$



- Hysteresis voltage set for the amount of noise that is present
 - Threshold must switch far enough away from the noisy input that it will not be crossed multiple times by noise alone
 - V_{hyst} set greater than peak-to-peak noise on the input signal

Comparators and Noise – Hysteresis

- Feedback gain, β, set to provide adequate hysteresis to reject the input noise
 - Noisy input crosses threshold
 - Threshold voltage switches away from the input by the hysteresis voltage

$$V_{hyst} > V_{n,pp}$$





Hysteresis Voltage



Hysteresis Voltage

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- Can also measure V_{hyst} by performing a bidirectional DC sweep of the input

Low-to-high, then high-to-low



- Output traces a different path depending on direction of v_{in}
 - Vertical lines indicate threshold voltages
 - *V_{hyst}* given by difference between upper and lower thresholds

Adjustable Hysteresis

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- If noise level is not known ahead of time
 Use a potentiometer to get adjustable hysteresis
- \square R_2 adjustable from 0 Ω to $R_{2,max}$
- \Box Hysteresis varies as R_2 varies

$$V_{hyst} = 2\frac{R_2}{R_1 + R_2}V_{o,max}$$

□ Max hysteresis at
$$R_2 = R_{2,max}$$

$$V_{hyst,max} = 2 \frac{R_{2,max}}{R_1 + R_{2,max}} V_{o,max}$$



 \square Minimum hysteresis for $R_2 = 0 \Omega$

$$V_{hyst,min} = 2 \frac{0 \Omega}{R_1 + 0 \Omega} V_{o,max} = 0 V$$

- Consider the following scenario:
 - Test engine instrumented and running on a dynamometer
 - Want an engine temperature warning light (LED)
 - RTD (resistive temperature detector or resistive thermal device) installed to measure coolant temperature.
 - **\square** RTD is biased such that a 0 V output corresponds to T_{max}
 - Noise: $V_{n,pp} \approx 100 \ mV$
 - Opamp available for use as a comparator:

$$V_{supply} = \pm 5 V$$
 $V_{o,max} = \pm |V_{supply} - 500 \ mV$
 $I_{o,max} = \pm 10 \ mA$

 $\bullet I_{LED} = 9 mA$

- \Box When $T > T_{max}$
 - **RTD** output exceeds 0 V
 - Schmitt trigger output goes low: $v_o \rightarrow -4.5 V$
 - LED turns on: $I_{LED} = 9 mA$
 - Opamp output sinks LED current and feedback network current
 - **D** Feedback network current must not exceed 1 mA



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Required hysteresis is *at least* the peak-to-peak noise voltage, $V_{n,pp} \approx 100 \text{ mV}$

• Design for
$$V_{hyst} = 150 \ mV$$

• The opamp saturates at $\pm 4.5 V$, so

$$V_{hyst} = 2 \cdot 4.5 V \frac{R_2}{R_1 + R_2} = 150 mV$$

$$R_1 = 150 mV$$

$$\frac{R_2}{R_1 + R_2} = \frac{130 \text{ mV}}{9 \text{ V}} = 16.67 \times 10^{-3}$$

This specifies the ratio between the feedback resistors

$$R_2 = 16.95 \times 10^{-3} \cdot R_1$$

Absolute values will be selected to limit feedback path current

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- When the LED is on, the feedback path current flowing into the opamp output is

$$I_f = \frac{4.5 \, V}{R_1 + R_2} < 1 \, mA$$

□ So, the total feedback path resistance is

$$R_1 + R_2 > 4.5 \ k\Omega$$

Select a standard value for the larger resistor

 $R_1 = 5.1 \ k\Omega$

 \Box Calculate R_2 using the previously derived ratio

$$R_2 = 16.95 \times 10^{-3} \cdot 5.1 \ k\Omega = 86.4 \ \Omega$$

Select the next *larger* standard value: R₂ = 91 Ω
 Larger, so as not to decrease V_{hyst}



□ The resulting circuit:



RTD
$$V^{+5V}$$
 V_{0} V^{+} V_{0} V_{0} V^{+} V_{0} V_{0}

· EV

+5V

$$V_{hyst} = 9 V \frac{91 \Omega}{5.1 k\Omega + 91 \Omega}$$

$$V_{hyst} = 157 \ mV$$

Feedback path current:

$$I_f = \frac{4.5 \, V}{5191 \, \Omega} = 867 \, \mu A$$