SECTION 2: FIRST-ORDER FILTERS

ENGR 202 – Electrical Fundamentals II

Introduction

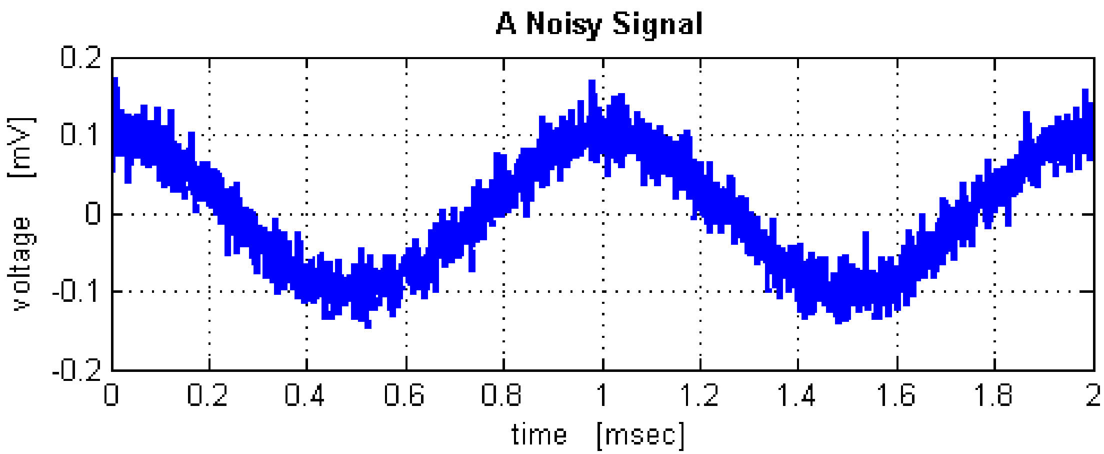
Filters

- We are all familiar with ***water*** and ***air filters***
  - Basis for operation is ***size selectivity***
    - Small particles (e.g. air or water molecules) are allowed to pass
    - Larger particles (e.g. dust, sediment) are not
  - Unwanted components are ***filtered out*** of the flow.

- ***Electrical filters*** are similar
  - Basis for operation is ***frequency selectivity***
    - Signal components in certain frequency ranges are ***filtered out***
    - Signal components at other frequencies are allowed to pass

Noise

- All real-world electrical signals are ***noisy***
  - You've seen this in the lab
  - Zoom in closely on a low-amplitude sinusoid with the scope (even one supplied directly from the function generator) – it won't look like a perfectly clean sinusoid

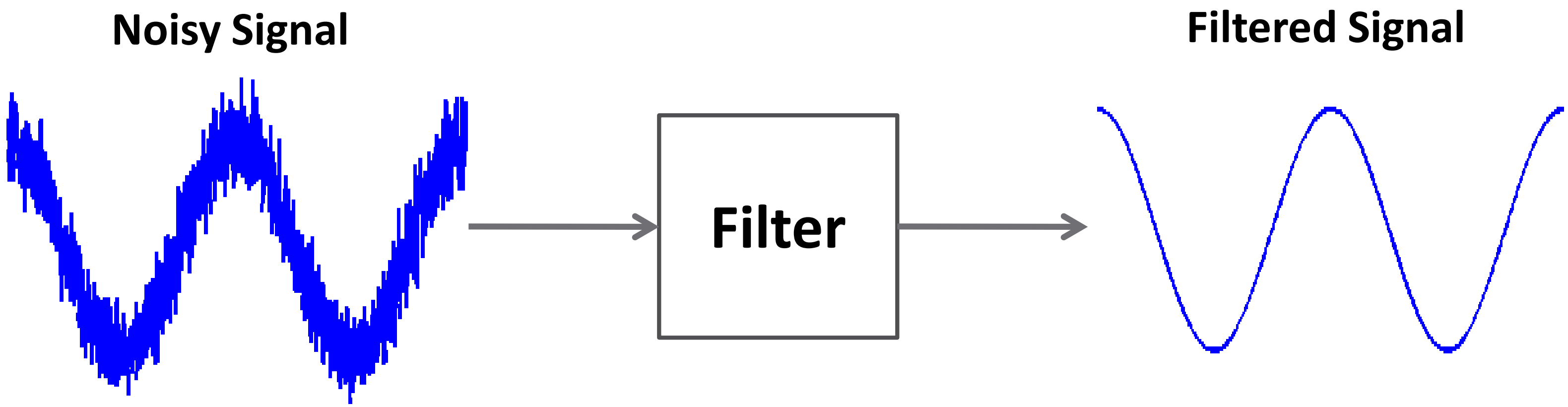


Noise

- We will use the term ***noise*** to mean any electrical signal that interferes with or corrupts a signal we are trying to measure.
- Noise has many sources:
  - Measurement instruments themselves
  - 60Hz power line interference
  - Electrical components – resistors, transistors, etc.
  - Wireless LAN, fluorescent lights, computers, etc.
- We'd like to be able to remove, or filter out, this noise
  - Improve the accuracy of measurements
  - Often possible, if we know the ***frequency characteristics*** of the signal and the noise

Filtering Noise

- We'll learn how to design filters to remove noise



- First, we must introduce two important concepts:
  - ***Frequency-domain representation of electrical signals***
    - What is meant by "frequency characteristics" of an electrical signal?
  - ***Frequency response of linear systems***
    - How does a linear system (e.g. a filter) behave as a function of frequency?

Frequency Spectrum

Frequency Domain

- We are accustomed to looking at electrical signals in the ***time domain***
  - Amplitude plotted as ***function of time***

- Can also be represented in the ***frequency domain***
  - Amplitude plotted as a ***function of frequency***
  - ***Frequency spectrum***
    - Describes the ***frequency content*** of a signal
  - Can think of signals as a sum of different frequency sinusoids
    - What frequencies (sinusoids) are present

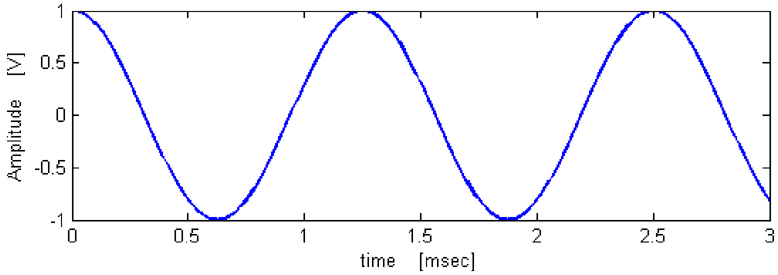
Frequency Spectrum

- ***Frequency spectrum***
  - An ***amplitude*** vs. ***frequency*** plot
  - X-axis is frequency – not time
  - Y-axis is amplitude
  - Amplitude units may be in ***decibels* (dB)**
- Shows the relative amount of energy at each frequency
- Time-domain plot and frequency spectrum are alternate representations of the same signal

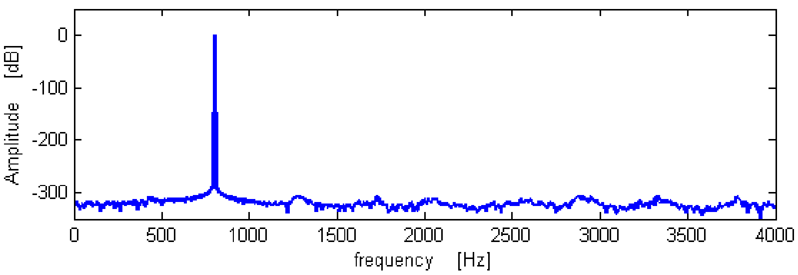
Frequency Spectra – Examples

- Single sinusoid: $v(t) = 1V\cos(2\pi \cdot 800Hz \cdot t)$

Time-domain



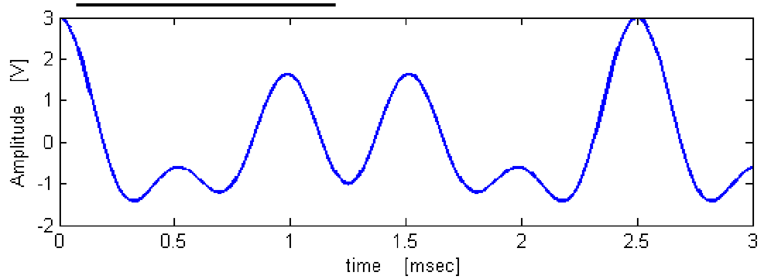
Frequency-domain



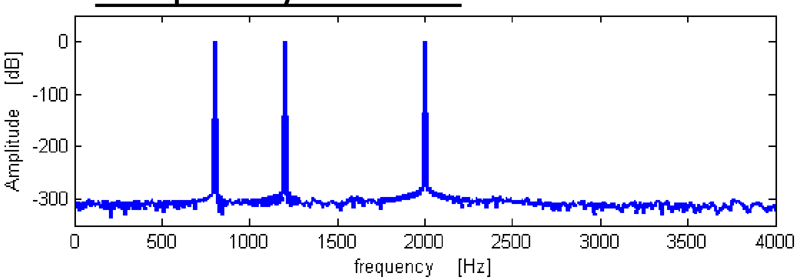
- Sum of three sinusoids:

$$v(t) = 1V[\cos(2\pi \cdot 800Hz \cdot t) + \cos(2\pi \cdot 1200Hz \cdot t) + \cos(2\pi \cdot 2000Hz \cdot t)$$

Time-domain



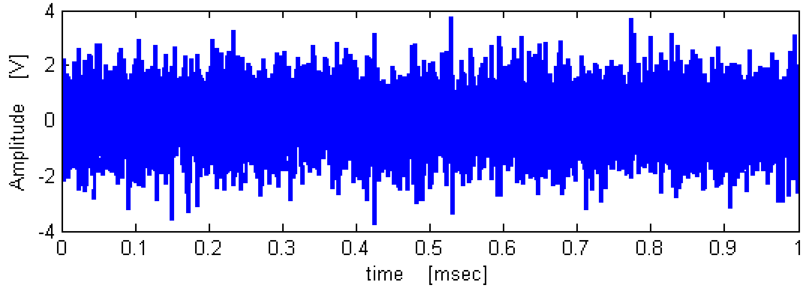
Frequency-domain



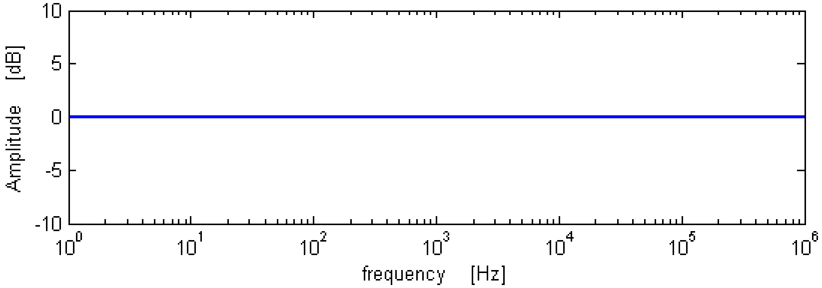
Frequency Spectra – Examples

- White noise:

Time-domain

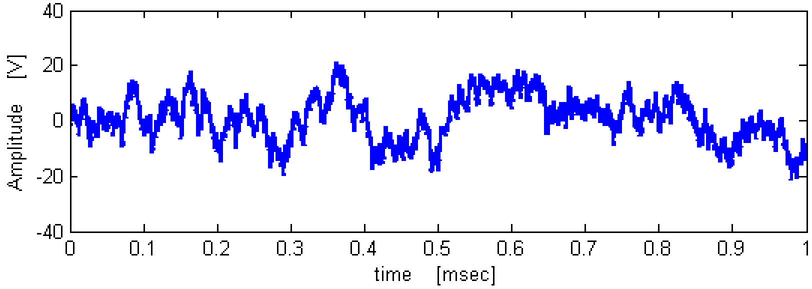


Frequency-domain

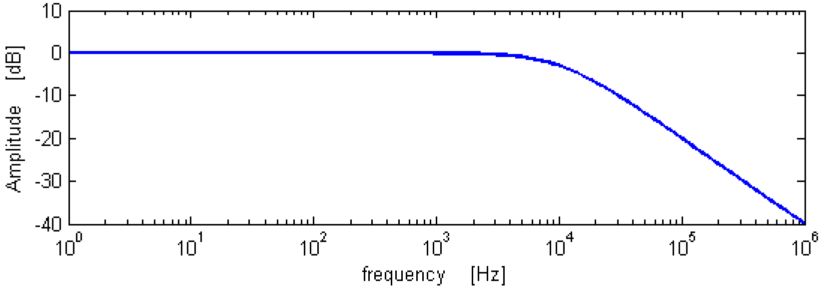


- Band-limited (colored) noise:

Time-domain

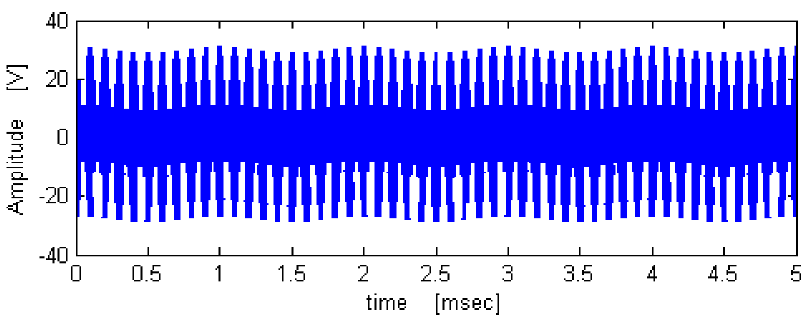


Frequency-domain



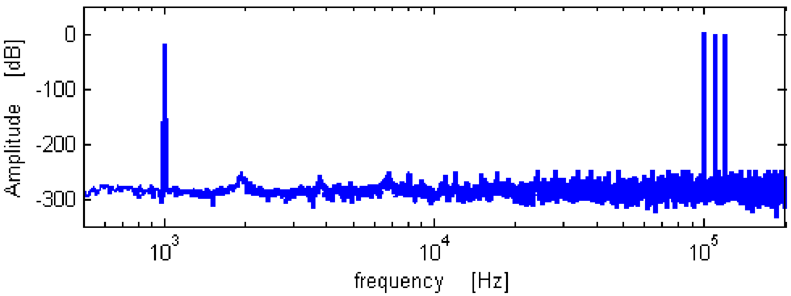
Frequency Spectra - Examples

- Consider the following scenario
  - Measuring a sensor output in the lab
  - Know the signal is roughly sinusoidal
  - Suspected frequency: ~1 kHz

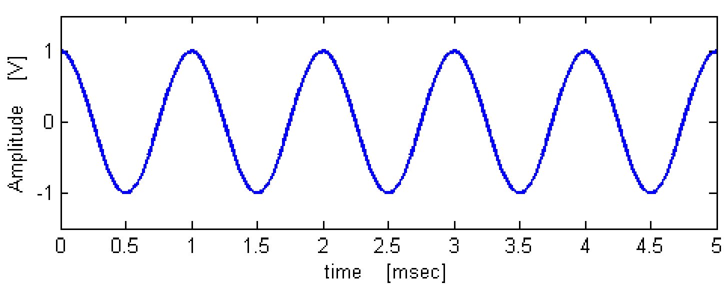


- Measured signal corrupted by noise/interference
  - Difficult to identify the interfering signal from the time-domain plot

- Same signal in the frequency domain:



- Three interfering tones
  - All near 100 kHz
- Can now design a filter to remove the noise:



Fourier Transform

- ***Fourier transform***
  - Transforms a time-domain representation to a frequency spectrum

$$V(\omega) = \int_{-\infty}^{\infty} v(t)\, e^{-j\omega t} dt$$

- ***Inverse Fourier transform***
  - Transforms from the frequency domain to the time domain

$$v(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V(\omega)\, e^{j\omega t} d\omega$$

- A mathematical transform
  - Two different ways of looking at the same signal
  - A ***change in perspective*** not a change of the signal itself

Frequency Response of Linear Systems

Frequency Response Function

- Linear systems (electrical, mechanical, etc.) can be described by their ***frequency responses***
- Frequency response
  - ***Ratio of the system output phasor to the system input phasor***
  - In general, a ***complex function of frequency***

$$H(\omega) = \frac{\mathbf{Y}}{\mathbf{X}} = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)}$$

- Complex-valued – has both magnitude and phase
  - Magnitude: ratio of output to input magnitudes
    - ***Gain*** of the system
  - Phase: difference in phase between output and input
    - ***Phase shift*** through the system

Frequency Response – Bode Plots

- ***Frequency response***
  - Description of system behavior *as a function of frequency*
  - ***Gain*** and ***phase***
- Represented graphically – formatted as a ***Bode plot***
  - Magnitude plot on top, phase plot below
  - Logarithmic frequency axes
  - Magnitude usually has units of decibels (dB)
  - Phase has units of degrees

Bode Plots

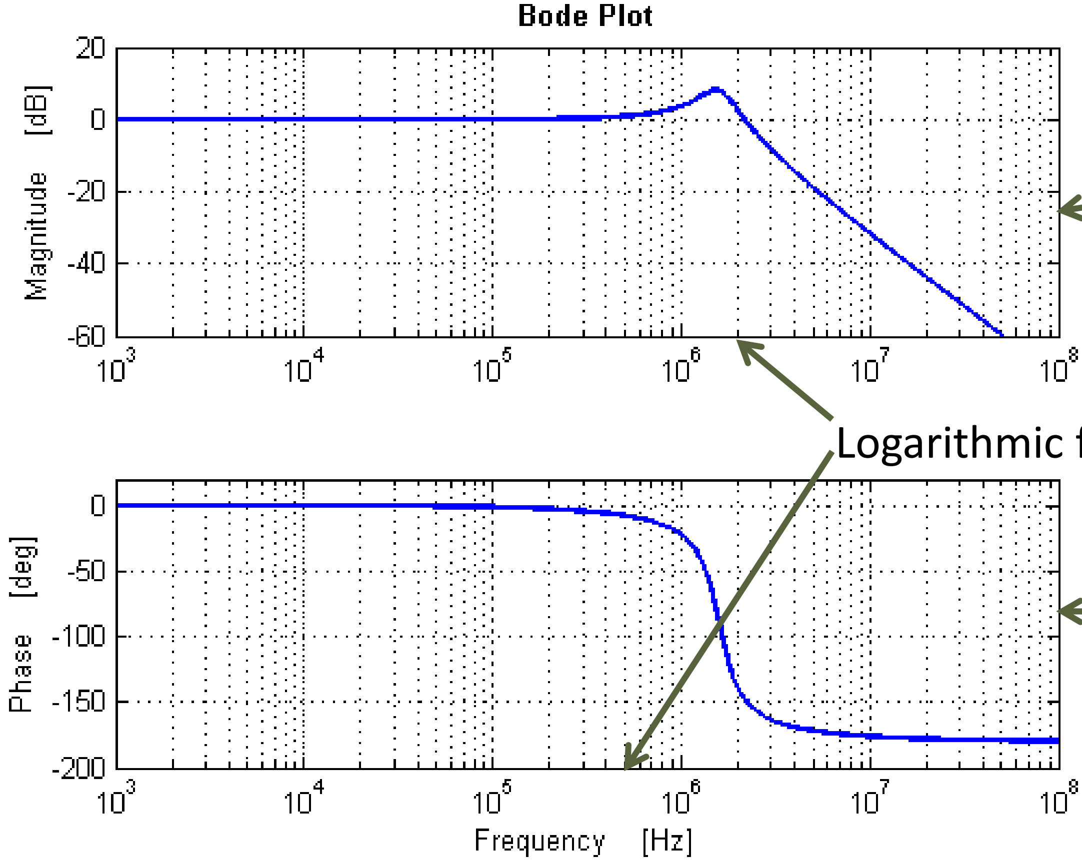
Units of magnitude are dB

Magnitude plot on top

Logarithmic frequency axes

Units of phase are degrees

Phase plot below



Decibels - dB

- Frequency response gain most often expressed and plotted with units of ***decibels*** (dB)
  - A ***logarithmic*** scale
  - Provides detail of very large and very small values on the same plot
  - Commonly used for ***ratios*** of powers or amplitudes

- Conversion from a linear scale to dB:

$$|H(\omega)|_{dB} = 20 \cdot \log_{10}(|H(\omega)|)$$

- Conversion from dB to a linear scale:

$$|H(\omega)| = 10^{\frac{|H(\omega)|_{dB}}{20}}$$

Decibels – dB

- Multiplying two gain values corresponds to adding their values in dB
  - E.g., the overall gain of cascaded systems

$$|H_1(\omega) \cdot H_2(\omega)|_{dB} = |H_1(\omega)|_{dB} + |H_2(\omega)|_{dB}$$

- Negative dB values corresponds to sub-unity gain
- Positive dB values are gains greater than one

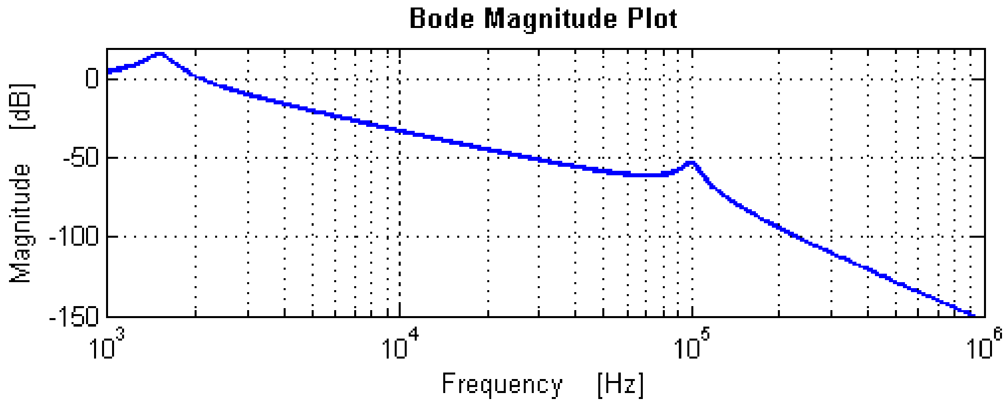
dB	Linear
60	1000
40	100
20	10
0	1

dB	Linear
6	2
-3	$1/\sqrt{2} = 0.707$
-6	0.5
-20	0.1

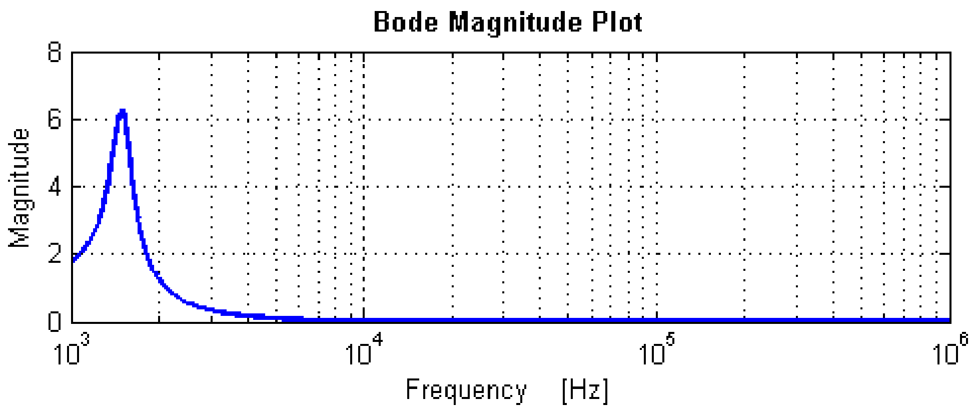
Value of Logarithmic Axes - dB

- Gain axis is linear in dB
  - A logarithmic scale
  - Allows for displaying detail at very large and very small levels on the same plot

- Gain plotted in dB
  - Two resonant peaks clearly visible



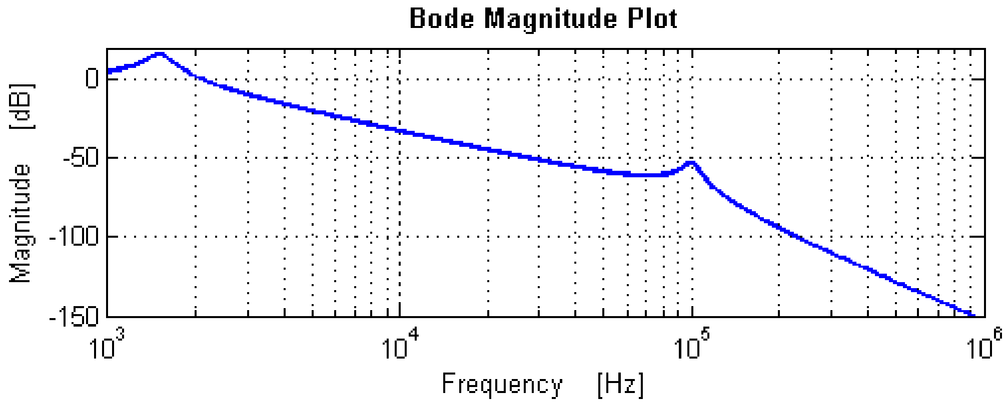
- Linear gain scale
  - Smaller peak has disappeared



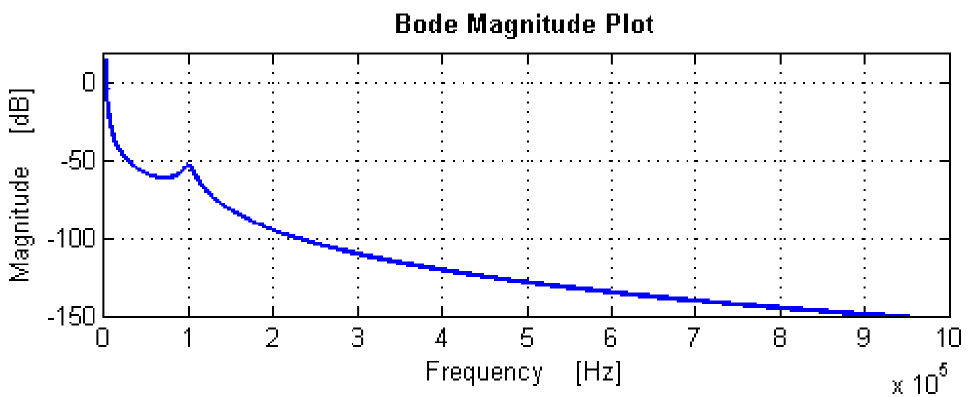
Value of Logarithmic Axes - Frequency

- Frequency axis is logarithmic
  - Allows for displaying detail at very low and very high frequencies on the same plot

- Log frequency axis
  - Can resolve frequency of both resonant peaks



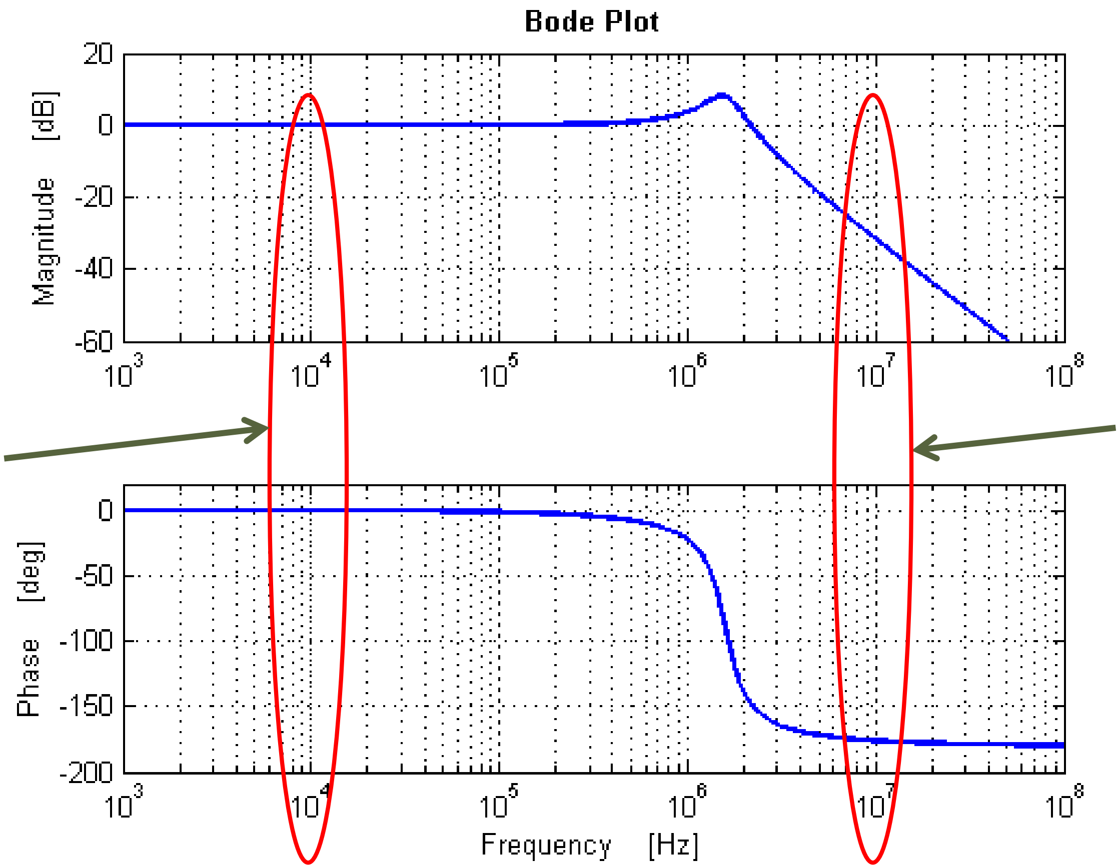
- Linear frequency axis
  - Lower resonant frequency is unclear



Interpreting Bode Plots

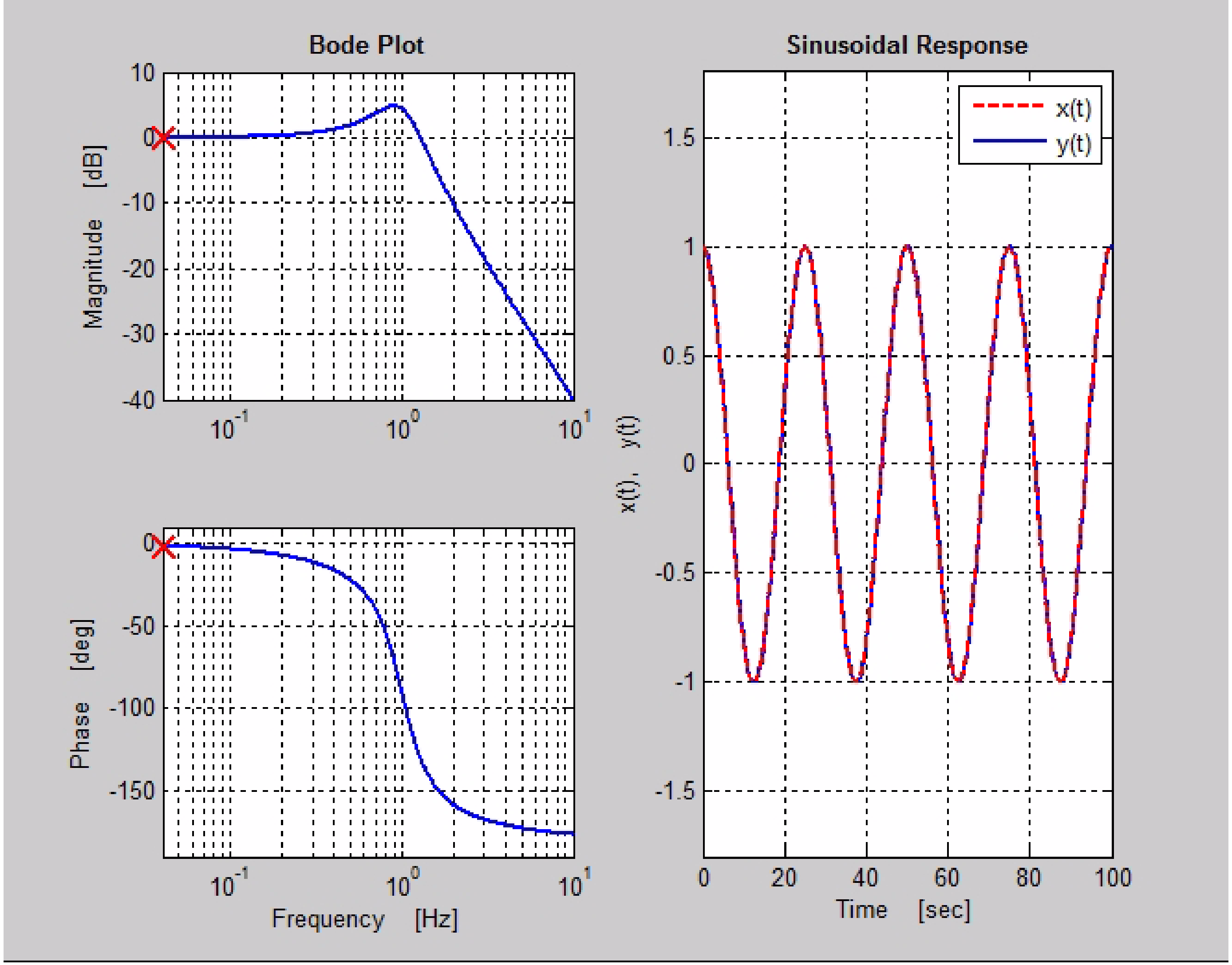
Bode plots tell you the gain and phase shift at all frequencies:
choose a frequency, read gain and phase values from the plot

For a 10KHz sinusoidal input, the gain is 0dB (1) and the phase shift is 0°.



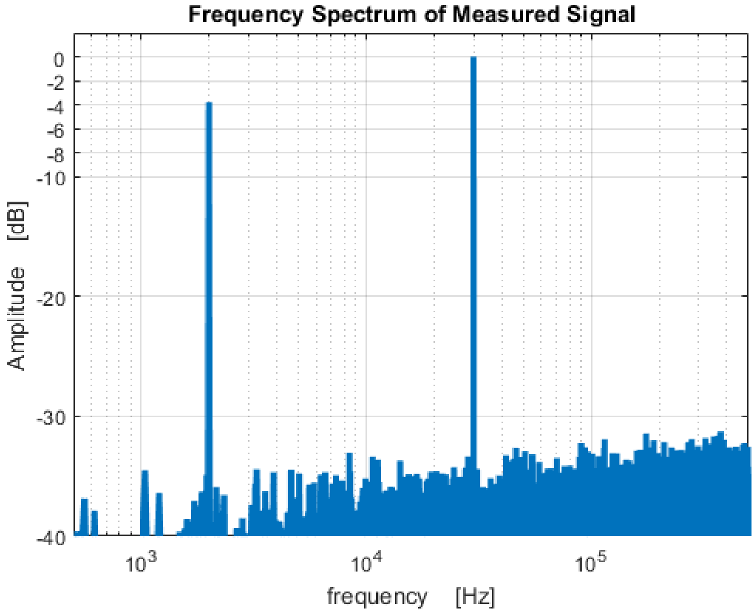
For a 10MHz sinusoidal input, the gain is -32dB (0.025), and the phase shift is -176°.

Interpreting Bode Plots



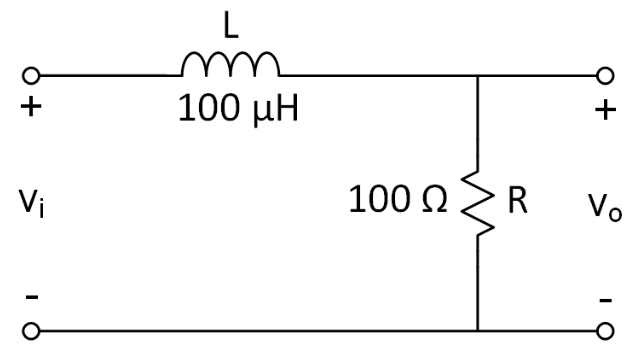
Example Problems

A measured signal has the frequency spectrum shown here. Assuming the larger signal component has an amplitude of 500 mV, and that both signal components are in phase, write a time-domain expression for the measured signal.



Determine the frequency response function, $H(\omega)$, for the following circuit.

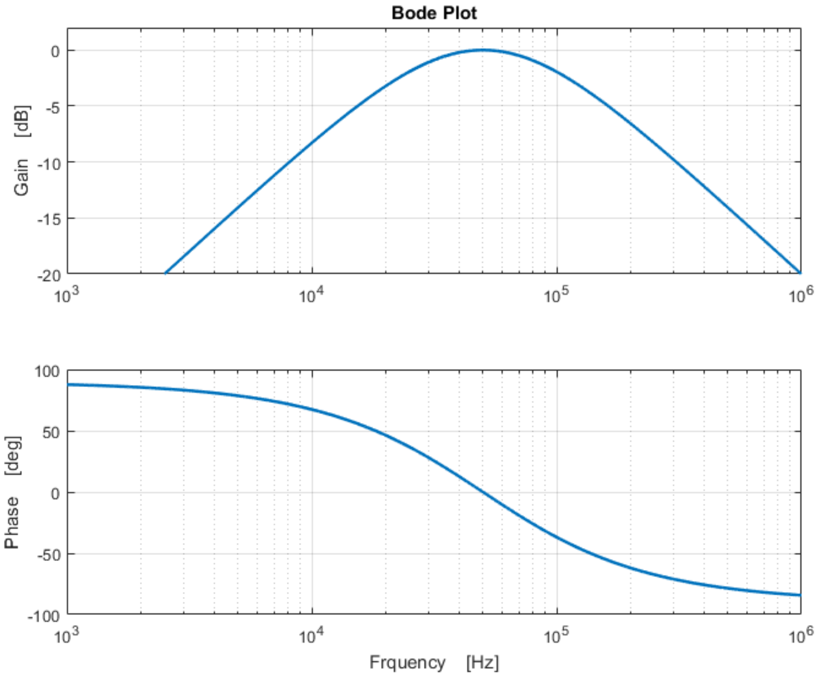
What are the circuit's gain and phase at 200 kHz?



The input to a circuit with the following Bode plot is

$$v_i(t) = 1.2V \cdot \cos(2\pi \cdot 10kHz \cdot t)$$

What is the output, $v_o(t)$?



Types of Filters

Filters are classified by the ranges of frequencies they pass and those they filter out

Filter Operation

- ***Frequency spectrum*** describes frequency content of electrical signals
- ***Frequency response*** describes system (circuit) gain and phase at different frequencies
- Can design circuits (i.e. ***filters***) to have high gain at desirable frequencies and low gain at undesirable frequencies
  - Want to filter out high frequencies?
    - Design a filter with low gain at high frequencies and high gain at low frequencies.
  - Want to filter out all signals between 1 MHz and 10 MHz?
    - Design a filter with low gain in this range and high gain everywhere else.

Types of Filters

- Filters are classified according to the ranges of frequencies they pass and those they filter out
  - **Low pass filters:** pass low frequencies, filter out high frequencies
  - **High pass filters:** pass high frequencies, filter out low frequencies
  - **Band pass filters:** pass only a range of frequencies, filter out everything else
  - **Band stop (notch) filters:** filter out only a certain range of frequencies, pass all others

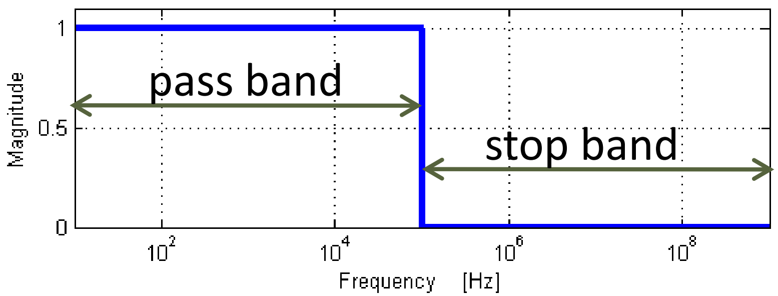
Ideal Filters

- ***Ideal filter*** gain characteristics:
  - ***Unity gain*** in the ***pass band***
    - Range of frequencies to be passed
  - ***Zero gain*** in the ***stop band***
    - Range of frequencies to be filtered out
  - ***Abrupt transition*** between pass band and stop band

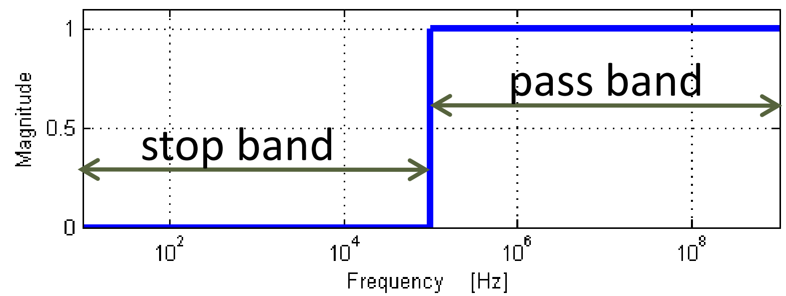
- Signals with frequency components in the pass band pass through the filter unaltered
- Signals with frequency components in the stop band are completely filtered out – removed from the signal

Ideal Filters – Magnitude Response

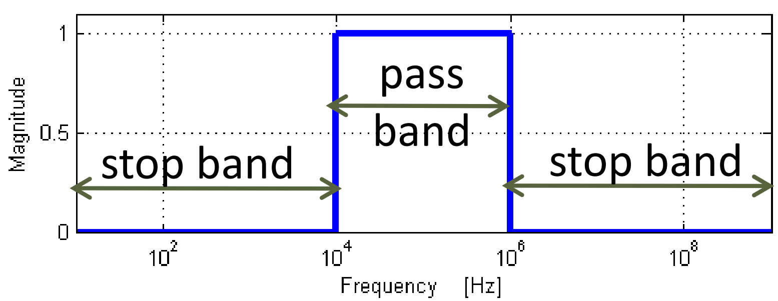
Ideal Low Pass Filter



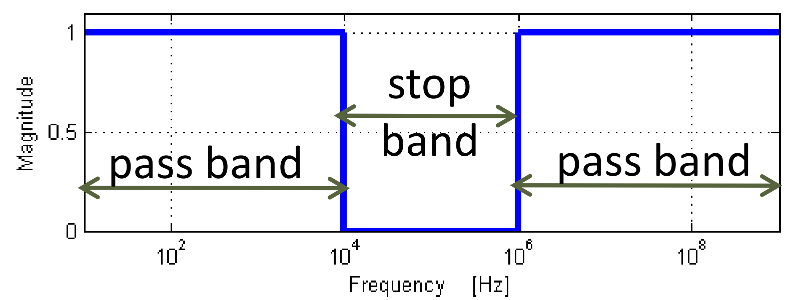
Ideal High Pass Filter



Ideal Band Pass Filter



Ideal Band Stop Filter

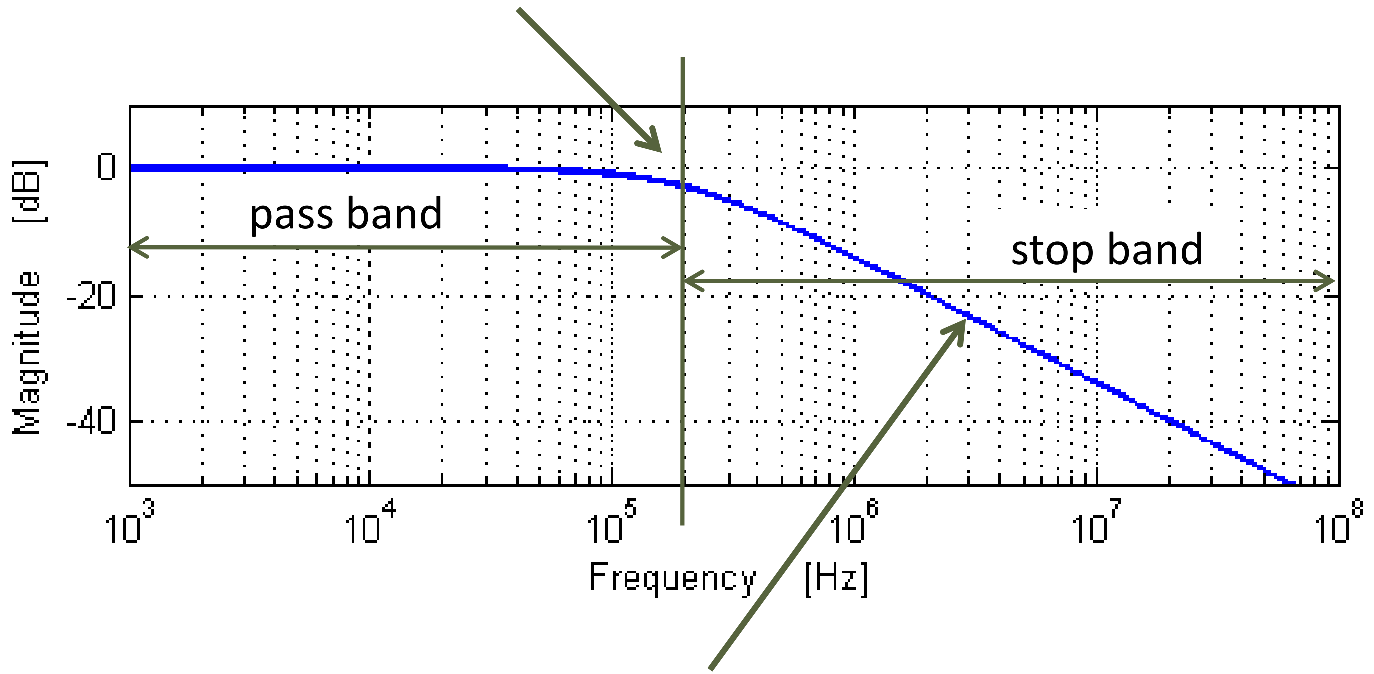


- Note the use of a linear gain scale here
  - Stop band gain of zero translates to $-\infty$ dB
- Ideal filters often referred to as ***brick wall filters***

Real Filters – Magnitude Response

Magnitude response for a real low pass filter:

Pass band edge is freq. at which gain is down by 3 dB – the ***-3 dB frequency***.
This is the filter's ***bandwidth***.



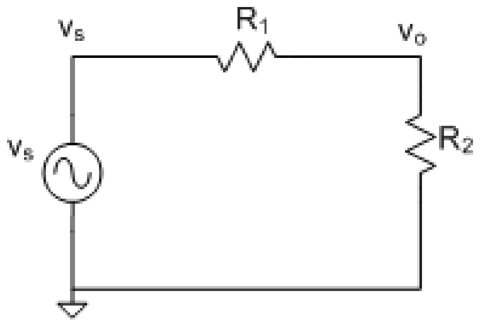
Roll-off rate between pass band and stop band depends on the type of filter – particularly, the ***order*** of the filter.

First-Order Passive Filters

First-order – only one energy-storage element

Passive – contain only resistors and capacitors or inductors – no opamps or transistors

Filters as Voltage Dividers



- Already familiar with resistive voltage dividers:

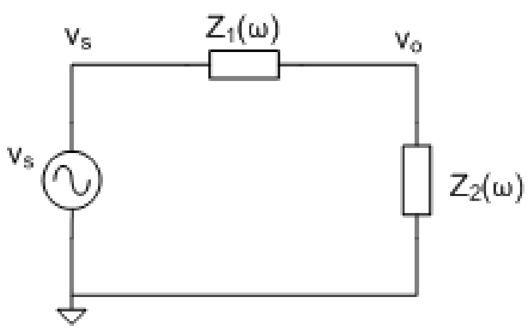
$$v_o = v_s \frac{R_2}{R_1 + R_2}$$

- Frequency response function:

$$H(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{R_2}{R_1 + R_2}$$

- A real constant – independent of frequency
  - Same gain at all frequencies
  - No phase shift at any frequency

- Now consider a circuit whose resistances have been replaced with impedances :



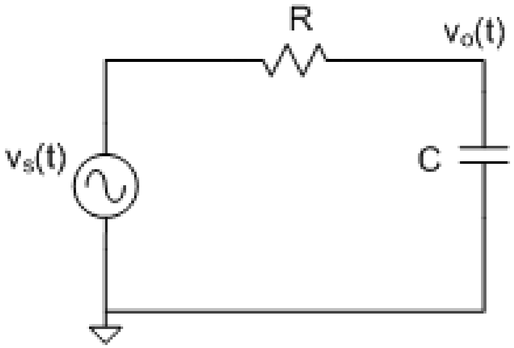
$$H(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{Z_2}{Z_1 + Z_2}$$

- Frequency response is now a complex function of frequency
  - Gain and phase vary as a function of frequency
  - Basis for the design of first-order filters

RC Low Pass Filter

RC Low Pass Filter

- Now, let Z_1 be resistive and Z_2 be capacitive



- Frequency response:

$$H(\omega) = \frac{V_o}{V_s} = \frac{Z_2}{Z_1 + Z_2} = \frac{{}^1/_{j\omega C}}{R + {}^1/_{j\omega C}}$$

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

- Recall from ENGR 201 that the transient response of this same circuit is characterized by its time constant, $\tau = RC$
- In the frequency domain, this is the ***corner frequency*** or ***break frequency***

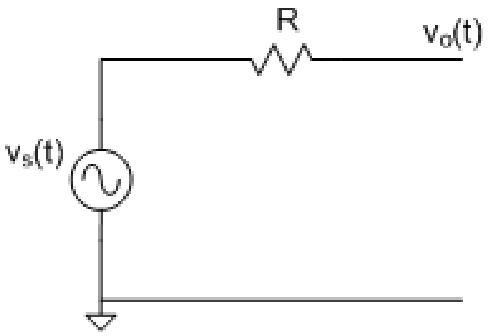
$$\omega_c = \frac{1}{\tau} = \frac{1}{RC}$$ and $$f_c = \frac{1}{2\pi RC}$$

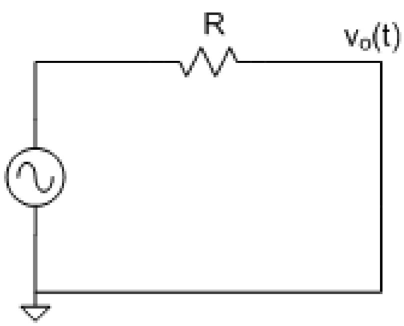
- The frequency at which gain is down by 3 dB
- The ***-3 dB frequency***
- Frequency at which the magnitude of R and C impedances are equal

RC Low Pass Filter

- To gain insight into the behavior of this filter circuit, consider two limiting cases

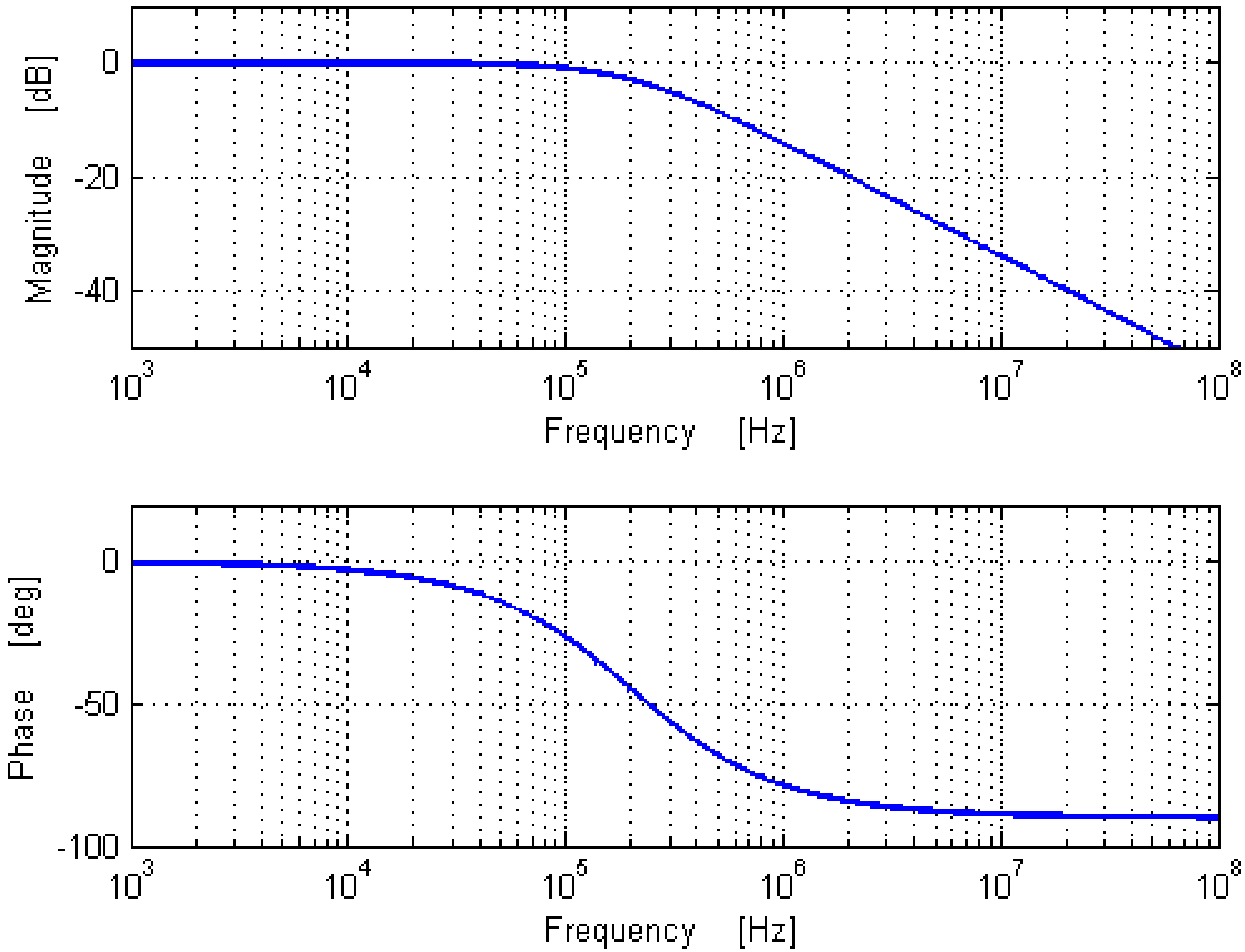
- As $f \rightarrow 0$,
  - Capacitor→ open circuit
  - $i(t) \rightarrow 0$
  - $v_o \rightarrow v_s$
  - ***Gain → unity***



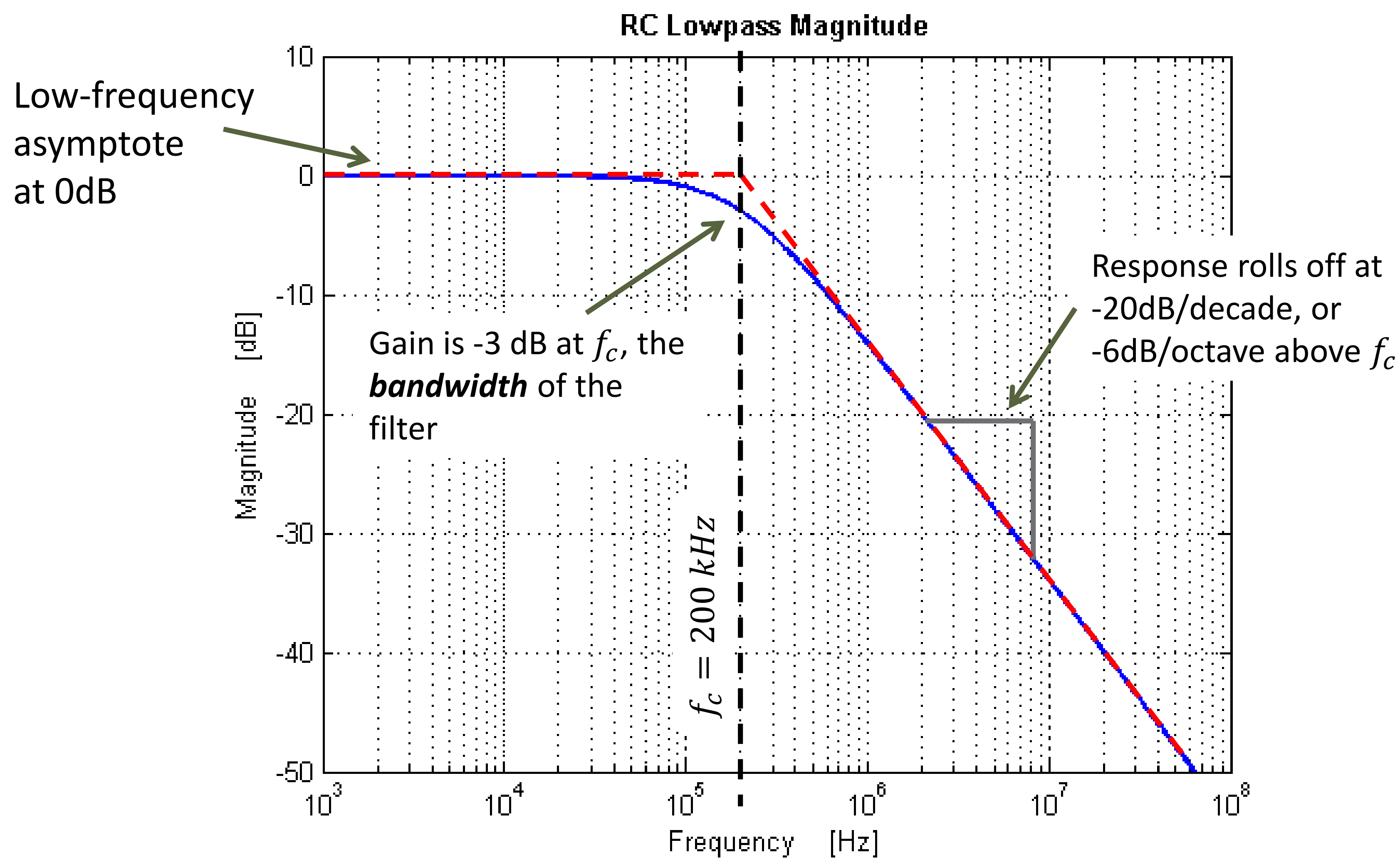


- As $f \rightarrow \infty$
  - Capacitor → short circuit
  - v_o shorted to ground
  - ***Gain → zero***

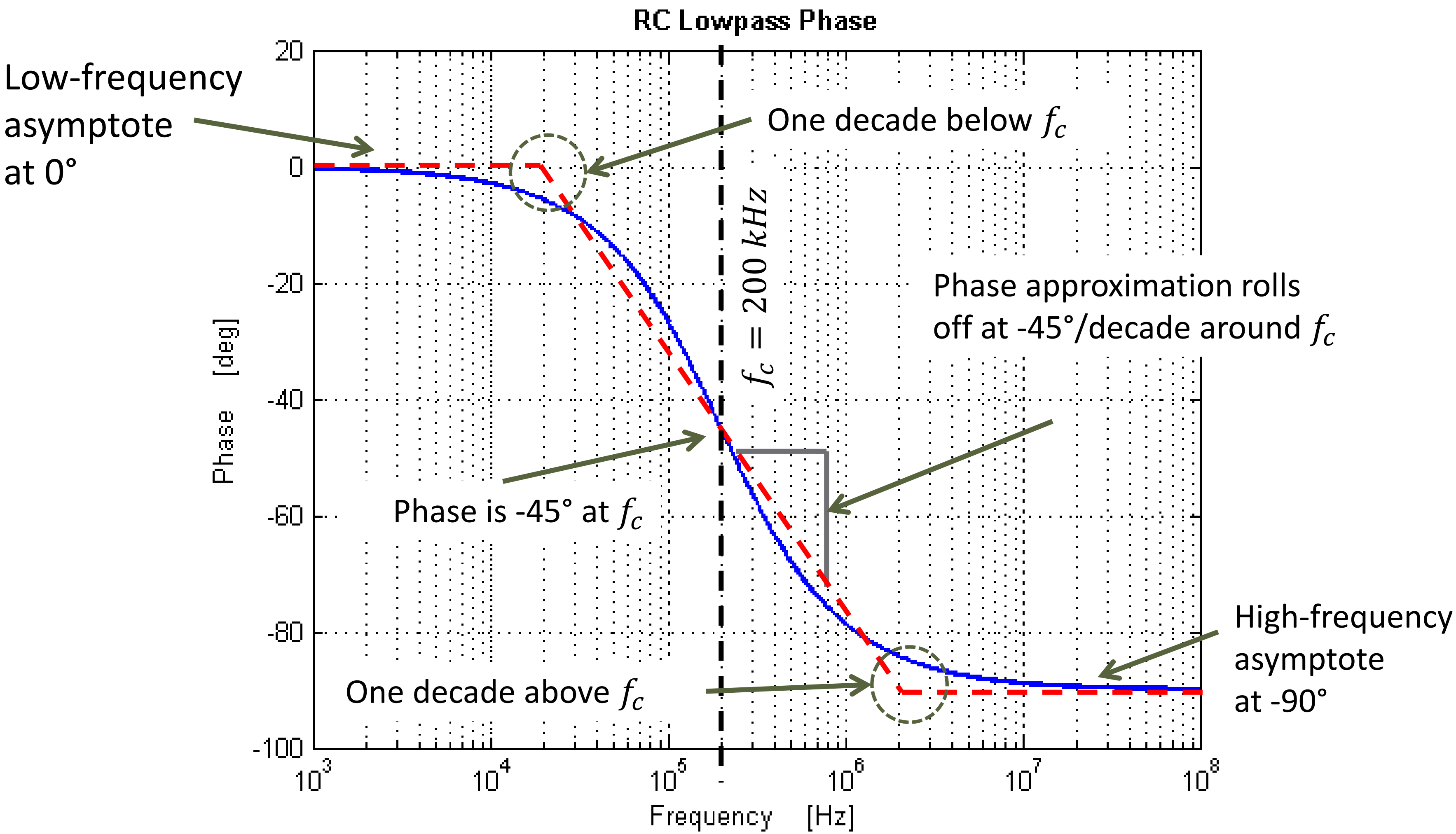
RC Low Pass Filter – Bode Plot



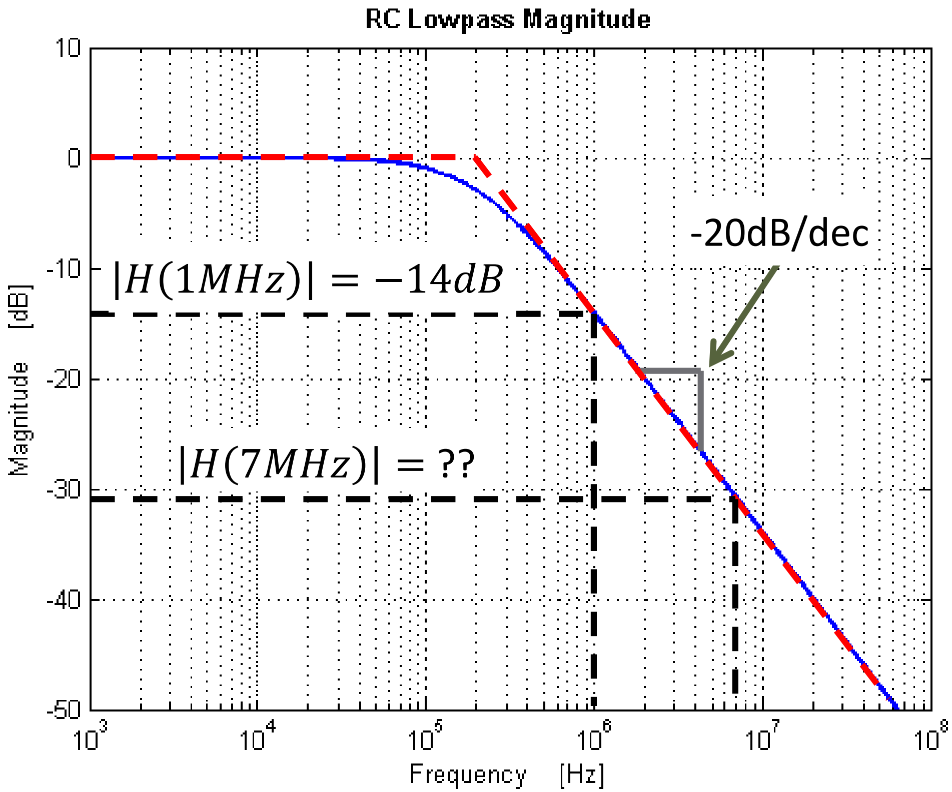
RC Low Pass Filter – Magnitude Response



RC Low Pass Filter – Phase Response



RC Low Pass Filter – Magnitude Response



- Known slope can be used to relate gains at different frequencies

$$Slope\left[\frac{dB}{dec}\right] = \frac{|H(f_2)|_{dB} - |H(f_1)|_{dB}}{\log_{10}(f_2) - \log_{10}(f_1)}$$

- For example:

$$-20\,\frac{dB}{dec} = \frac{|H(7MHz)| - |H(1MHz)|_{dB}}{\log_{10}(7MHz) - \log_{10}(1MHz)}$$

$$-20\,\frac{dB}{dec} = \frac{|H(7MHz)| - (-14dB)}{6.845 - 6}$$

$$|H(7MHz)| = -30.9dB$$

RC LP Filter – Application Example

- Simple first-order RC low pass filters provide a quick and easy way to remove noise from electrical signals

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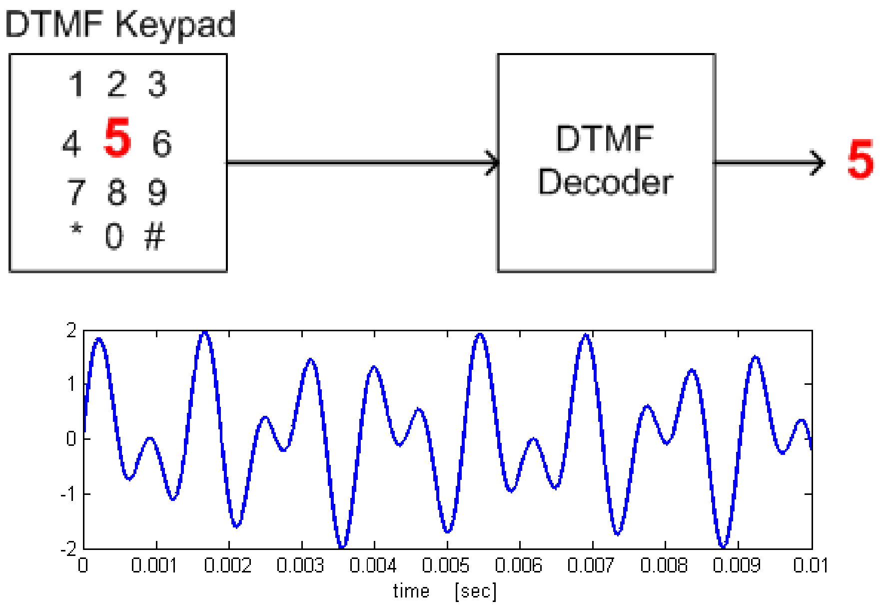
- Consider for example a *dual-tone multi-frequency* (***DTMF***) signal
  - Touch-tone telephone signal (key **5** in this example)
  - Tone is the sum of two sinusoids (key 5 = 1336Hz and 770Hz)
  - Pressing the "5" key generates the DTMF signal
  - Noise on the DTMF signal makes decoding impossible
  - Filter noise to enabling decoding

RC LP Filter – Application Example

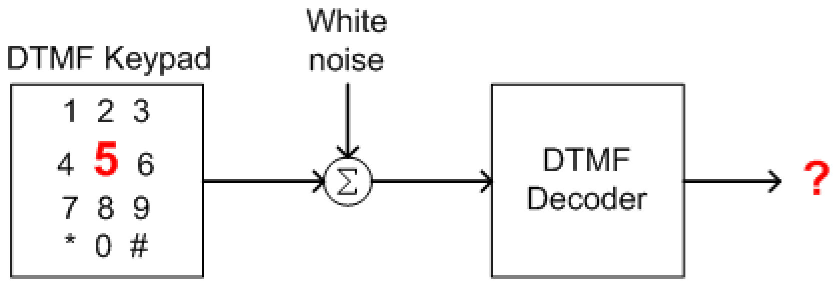
- Key number 5 is pressed
  - DTMF signal generated
    - Sum of 770 Hz and 1336 Hz sinusoids



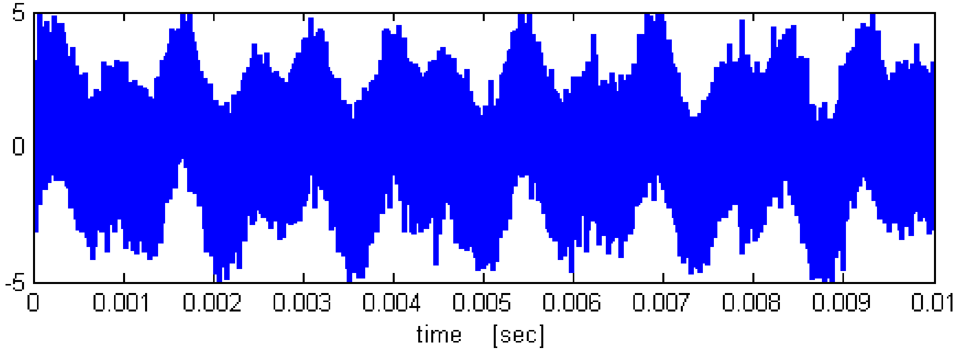
  - Decoder at the receiving end decodes the DTMF signal and determines that a 5 was pressed

RC LP Filter – Application Example

- Consider a more realistic scenario
  - DTMF signal corrupted by a significant amount of noise

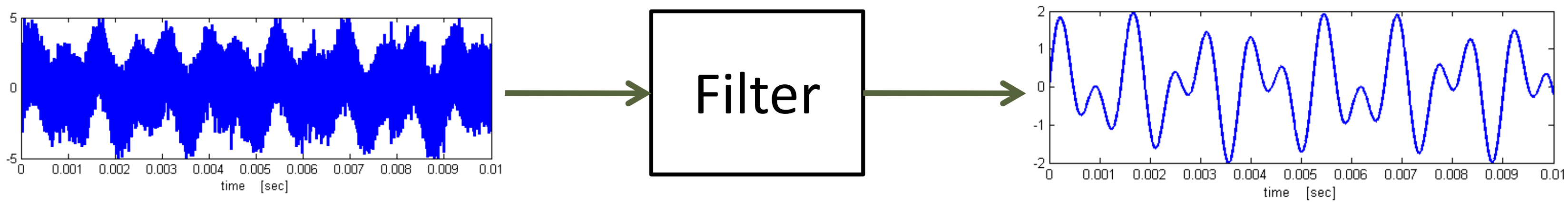


  - The decoder is no longer able to determine that a 5 was pressed



RC LP Filter – Application Example

- The goal is to filter the received signal so that the decoder can accurately interpret the DTMF signal

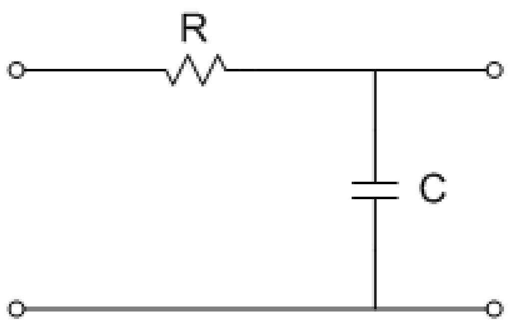


- Designing the low pass filter
  - White noise
    - Flat frequency spectrum
    - Equal power at all frequencies
  - DTMF frequency range: 697 Hz – 1633 Hz
  - Want to attenuate as much noise as possible
  - Want to attenuate DTMF signals as little as possible
  - RC LPF with corner frequency at 10 kHz will limit DTMF-band attenuation to < 0.2 dB

RC LP Filter – Application Example

- RC LPF design
  - Need to select R and C to set the corner frequency

$$f_c = \frac{1}{2\pi RC} = 10\ kHz$$



  - Say we have a 0.1 μF capacitor available
  - Solve for R

$$R = \frac{1}{2\pi f_C C}$$

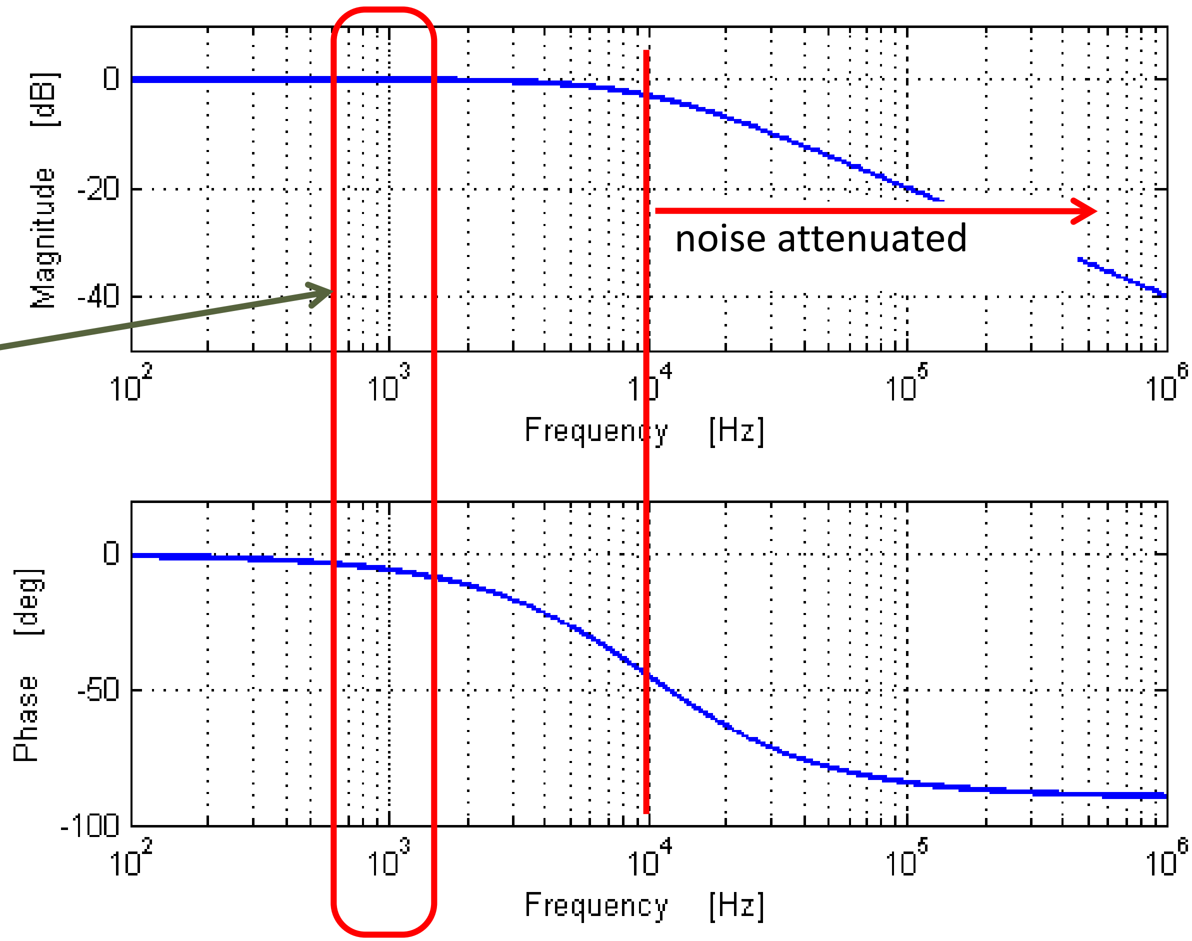
$$R = \frac{1}{2\pi \cdot 10\ kHz \cdot 0.1\mu F} = 159\ \Omega$$

$$R = 159\ \Omega, \quad C = 0.1\ \mu F$$

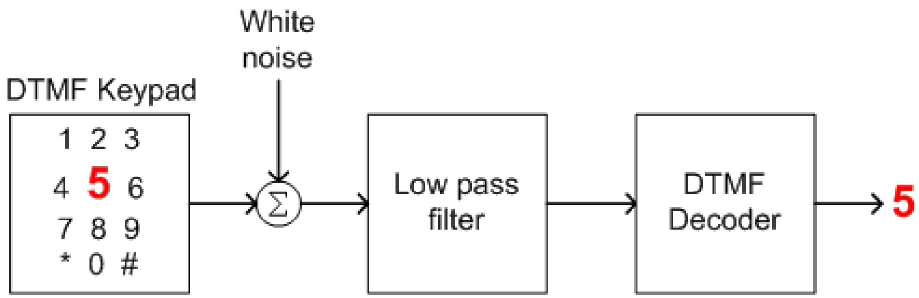
RC LP Filter – Application Example

Bode plot of the resulting filter:

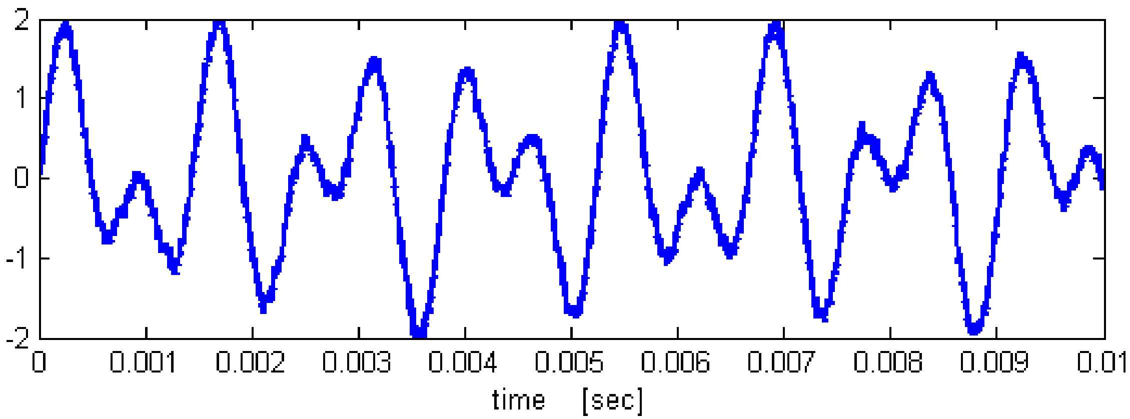
DTMF signals lie in this range – passed through the filter with little attenuation



RC LP Filter – Application Example



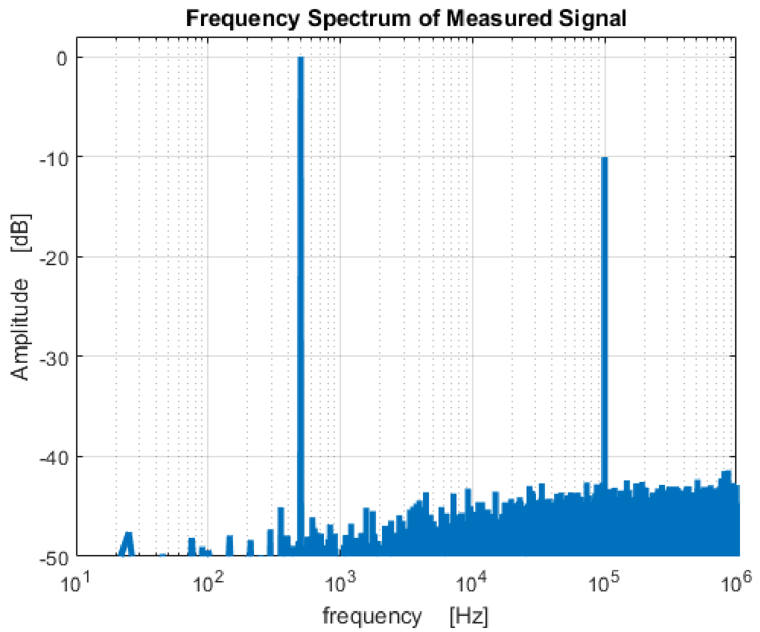
- Filter allows DTMF signal to pass mostly unaltered
- Noise below 10 kHz is mostly passed through
- Noise above 10 kHz is attenuated, but not removed
- Received signal is not noiseless, but clean enough to be decoded

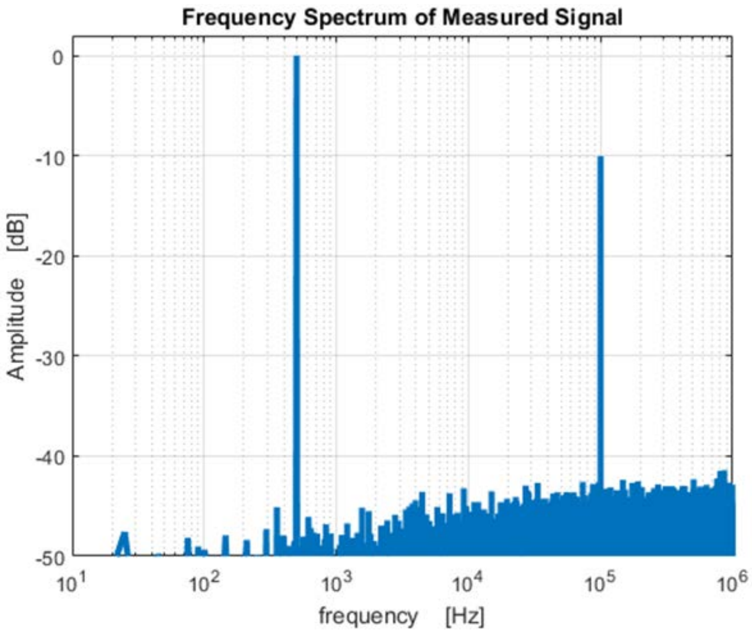


Example Problems

Design a filter to pass the desired 500 Hz signal and to attenuate the unwanted 100 kHz by 40 dB.

What is the signal-to-noise ratio (SNR) at the output of the filter?

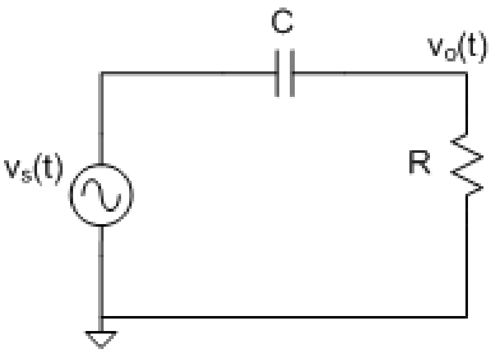



Frequency Spectrum of Measured Signal
0
-10
-20
-30
-40
-50
Amplitude [dB]
10^1
10^2
10^3
10^4
10^5
10^6
frequency [Hz]

RC High Pass Filter

RC High Pass Filter

- Now, swap the locations of the resistor and capacitor



- Frequency response:

$$H(\omega) = \frac{V_o}{V_s} = \frac{Z_2}{Z_1 + Z_2} = \frac{R}{R + {}^{1}/_{j\omega C}}$$

$$\boxed{H(\omega) = \frac{j\omega RC}{1 + j\omega RC}}$$

- Corner frequency is the same as for the low pass filter

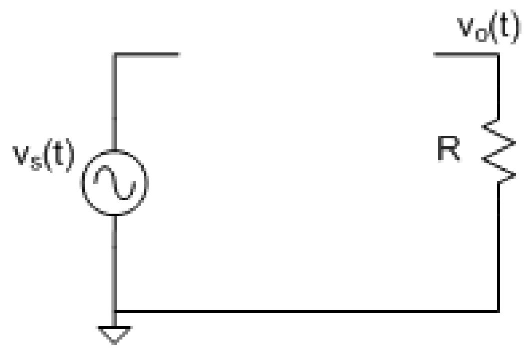
$$\boxed{\omega_c = \frac{1}{\tau} = \frac{1}{RC}} \quad \text{and} \quad \boxed{f_c = \frac{1}{2\pi RC}}$$

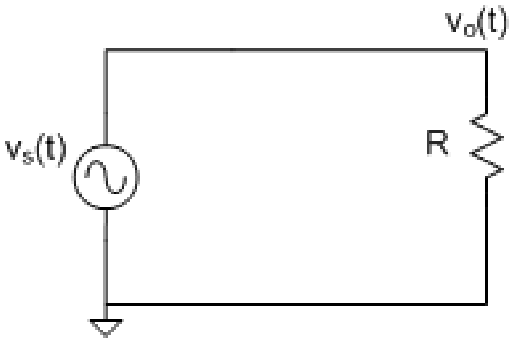
- The frequency at which gain is down by 3 dB
- Frequency at which the capacitor impedance magnitude is equal to the resistor impedance magnitude
- Now, gain is constant *above* f_c and rolls off *below* f_c

RC High Pass Filter

- To gain insight into the behavior of this filter circuit, consider two limiting cases

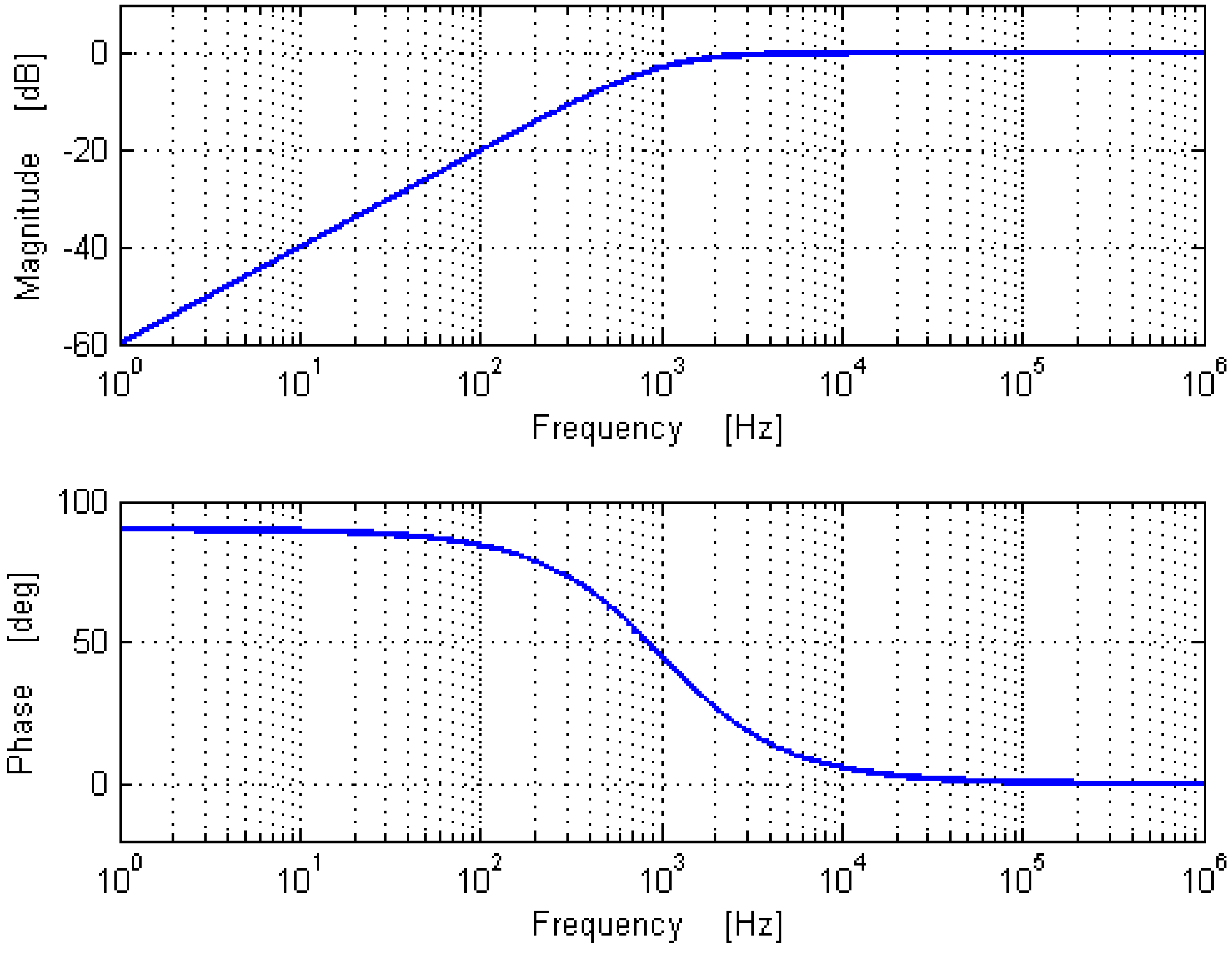
- As $f \rightarrow 0$,
  - Capacitor→ open circuit
  - $i(t) \rightarrow 0$
  - $v_o \rightarrow 0$
  - ***Gain → zero***



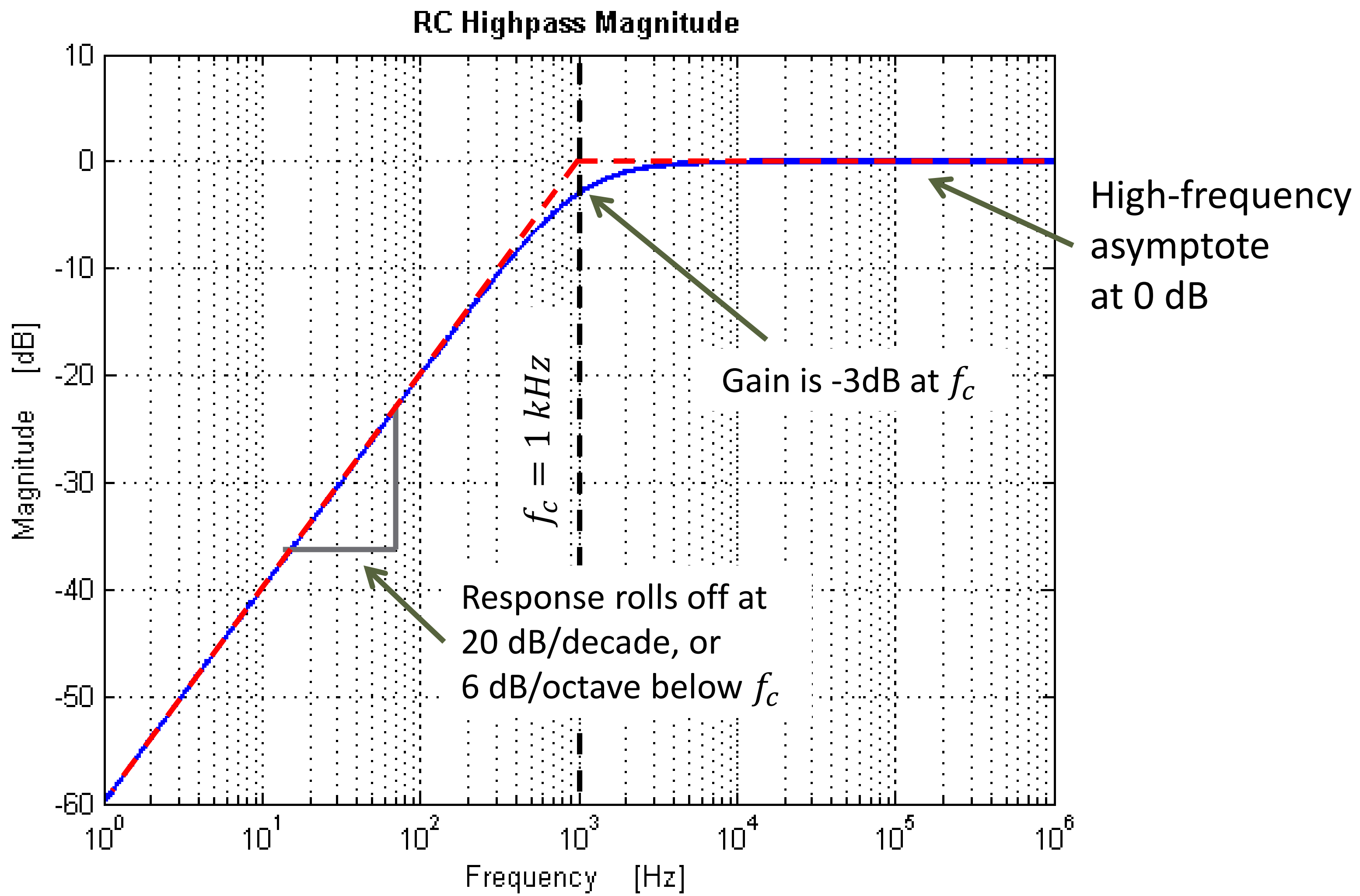


- As $f \rightarrow \infty$
  - Capacitor → short circuit
  - v_o shorted to v_s
  - ***Gain → unity***

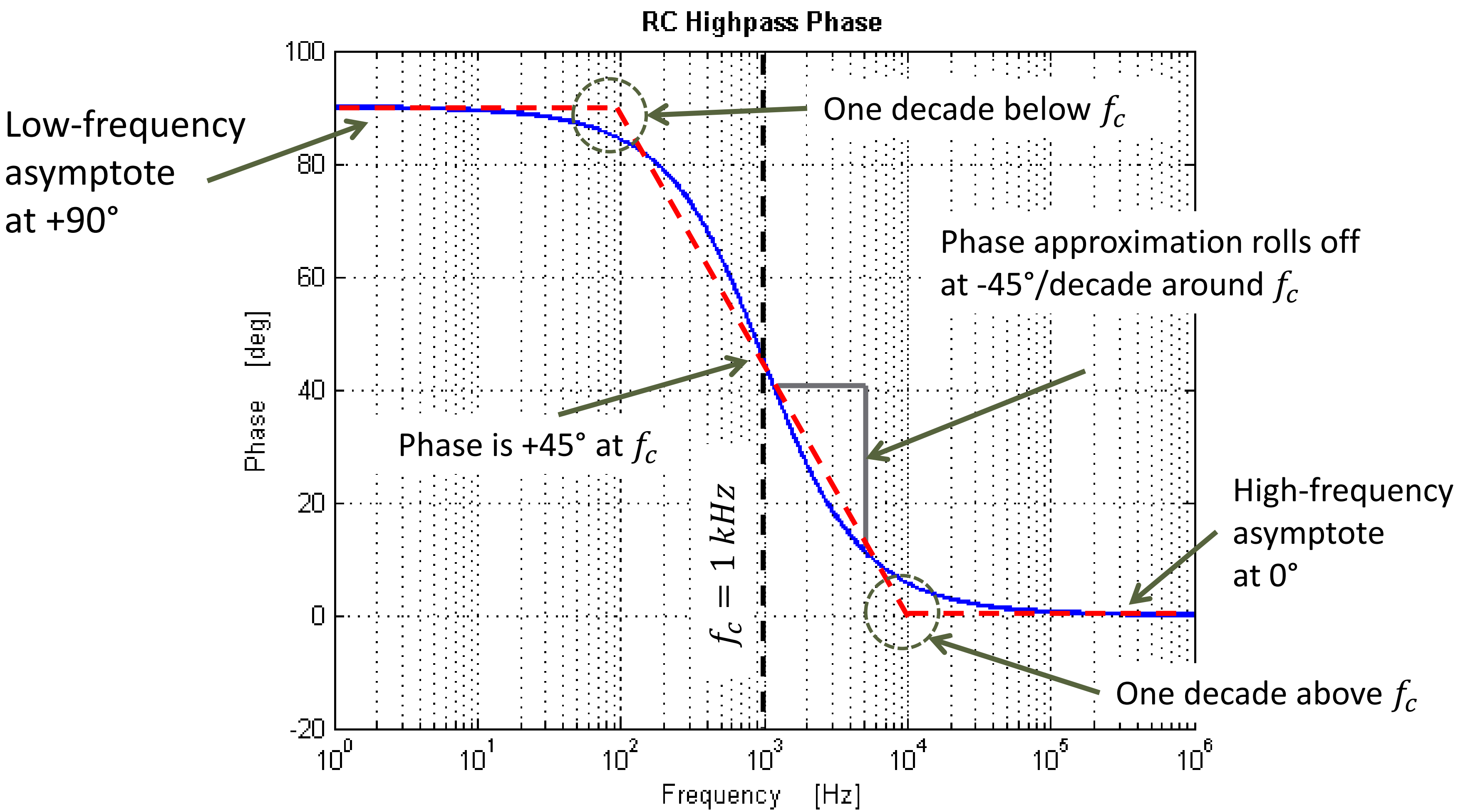
RC High Pass Filter – Bode Plot



RC High Pass Filter – Magnitude Response



RC High Pass Filter – Phase Response



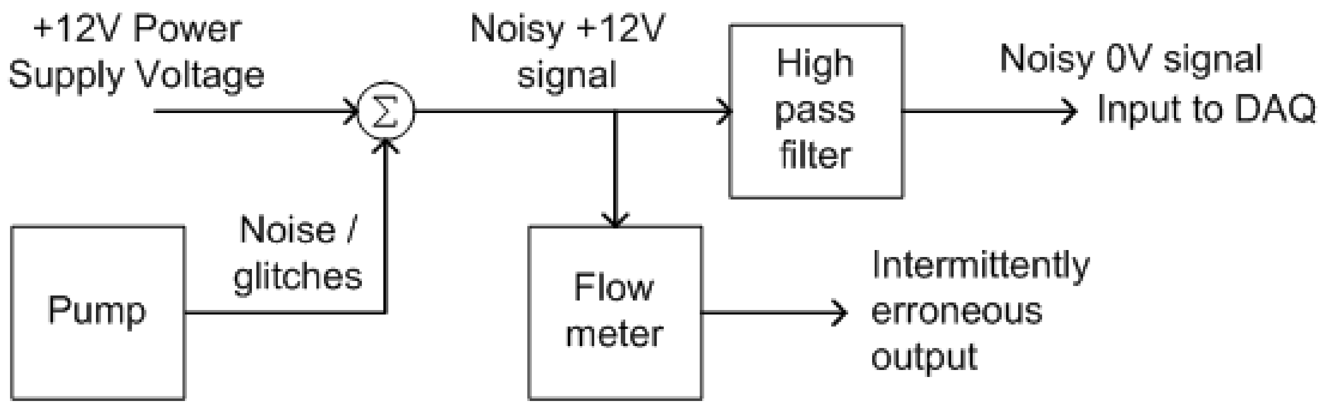
RC HP Filter – Application Example

- High pass filters are useful for removing low-frequency content, including DC, from electrical signals.

- For example, consider the following scenario:
  - Instrumented a flow loop in the lab
    - Pumps, temperature sensors, pressure sensors, and flow meters
  - Flow meter output seems to be erroneous every ~1 msec
  - Suspected cause: coupled through the +12V power supply from one of the pumps
  - Want to measure the flow meter's +12V power supply with a channel on our data acquisition system (DAQ)
    - Dynamic range of DAQ input: ±5 V
  - Use a high-pass filter to remove the +12V DC component from the power supply voltage

RC HP Filter – Application Example

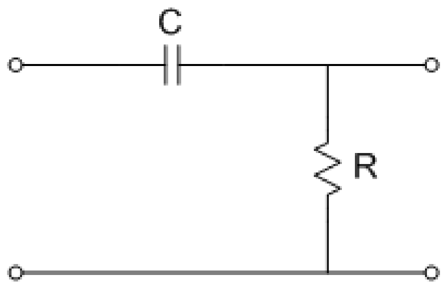
- Want to a +12 V supply with a $\pm$5 V DAQ input
- High pass filter will remove the DC component of the supply voltage



- High pass filter used to remove DC signal components
- Couples only AC signal components to the DAQ input
  - AC coupling
  - Similar to the AC coupling setting on the scopes in the lab

RC HP Filter – Application Example

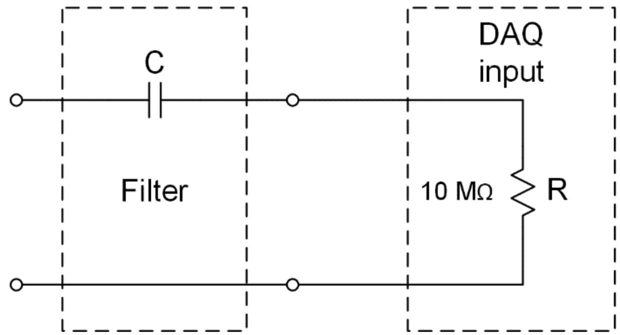
- High pass filter design
  - Want to remove DC
    - Low corner frequency
  - High RC time constant
    - Large R and C
  - Arbitrarily set $f_c = 10\ Hz$



- DAQ system
  - Datasheet says $R_{in} = 10\ M\Omega$
    - Let R_{in} be the filter resistance
- Calculate C to get desired f_c

$$C = \frac{1}{2\pi f_c R} = \frac{1}{2\pi \cdot 10Hz \cdot 10M\Omega}$$

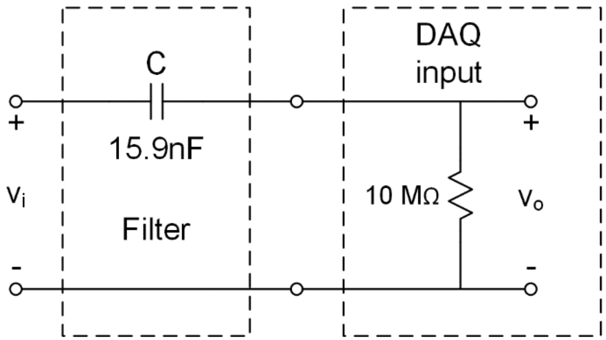
$$C = 15.9\ nF$$



  - Or anything in that neighborhood
  - Not critical – just want to block DC

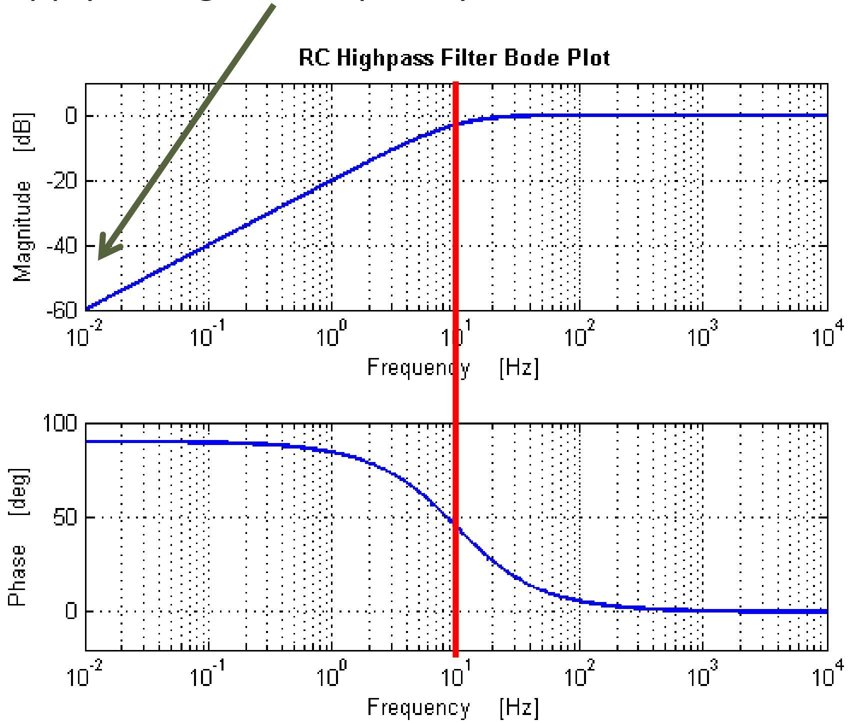
RC HP Filter – Application Example

RC high pass filter:



High pass filter Bode plot:

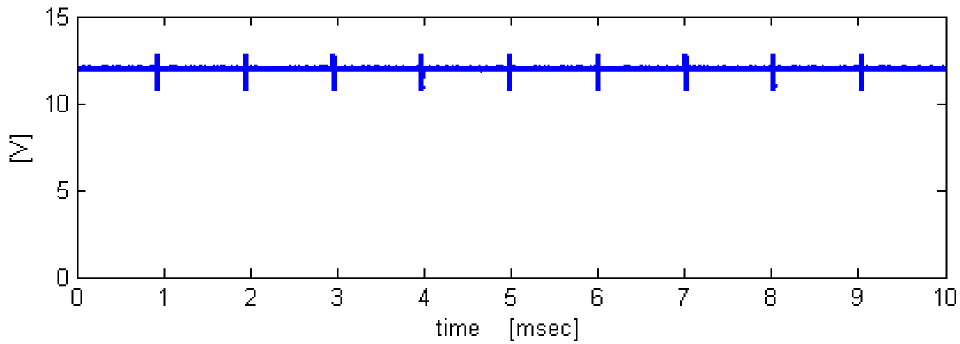
The +12V DC component of the power supply voltage is completely removed.



RC HP Filter – Application Example

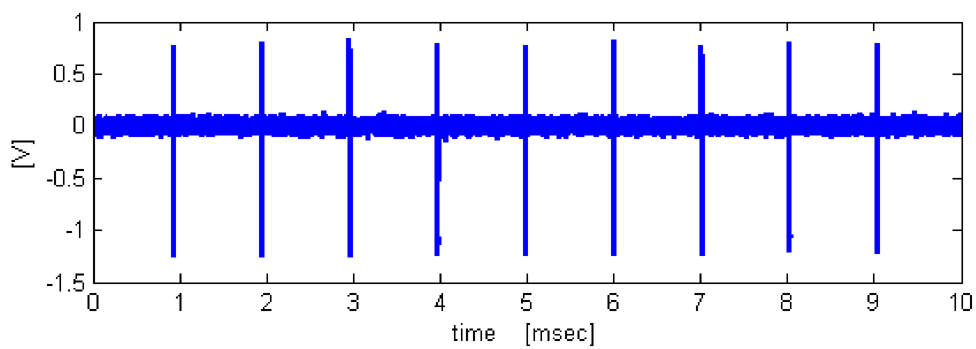
<u>The noisy +12V power supply at the malfunctioning flow meter</u>:

- DC value of signal is +12 V
- Outside $\pm$5 V DAQ input dynamic range



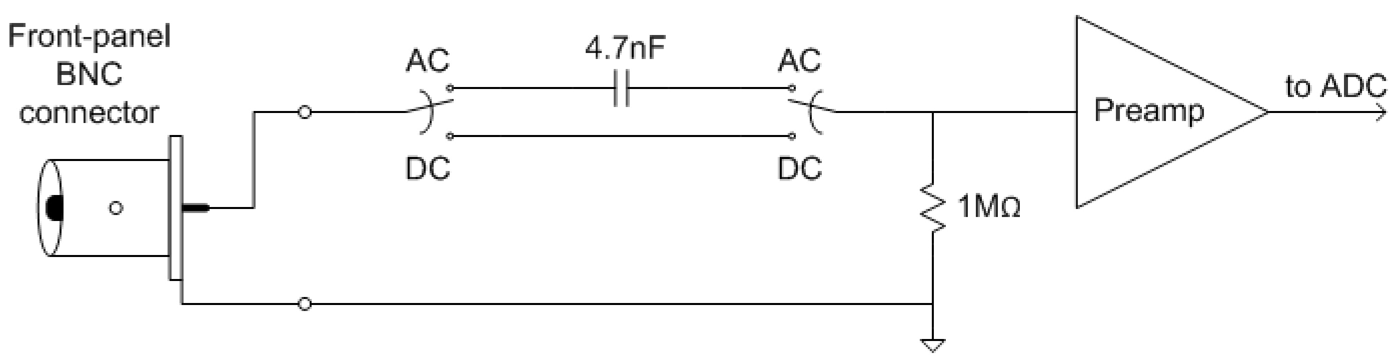
<u>High pass filter output – AC coupled power supply voltage</u>:

- DC value of signal is now 0 V
- Within $\pm$5 V DAQ input range
- Glitches clearly measured with the DAQ



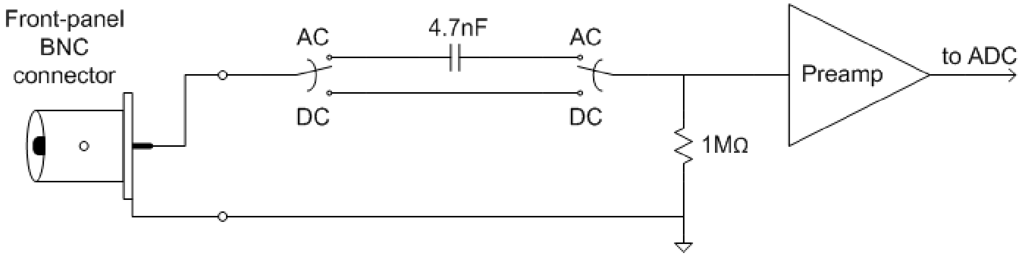
Oscilloscopes – AC Coupling

- Scope inputs allow you to select between DC and AC coupling
  - Usually under the *channel* menu
  - ***DC coupling***: input signal is terminated in 1MΩ and connected directly to the preamp and ADC in the scope
  - ***AC coupling***: input signal is switched through a capacitor that forms a high pass filter with the 1MΩ input resistor
    - $f_c \approx 3.5\ Hz$ – removes DC
    - Useful for looking at power supply ripple, etc.

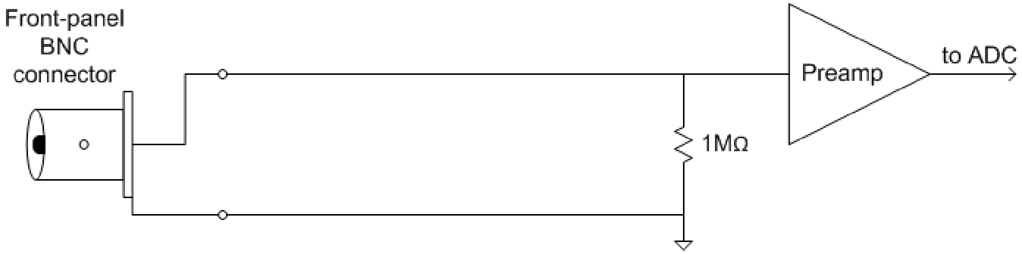


Oscilloscopes – AC Coupling

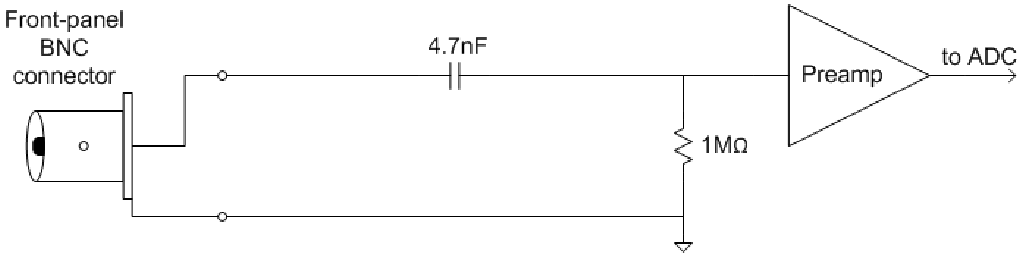
<u>High-impedance scope front-end</u>:



<u>Configured for DC coupling</u>:



<u>Configured for AC coupling</u>:

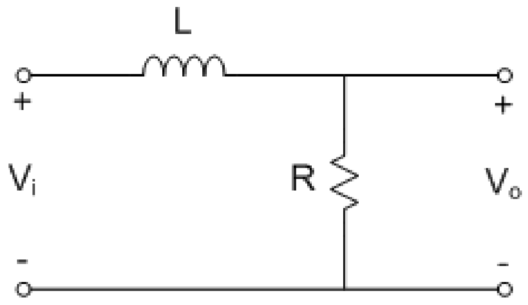


RL Filters

First-order RL filters

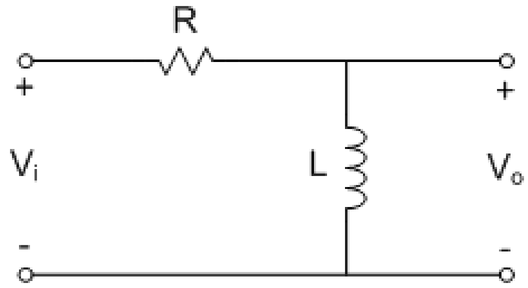
- Can also use ***inductors*** to make ***RL*** low pass and high pass filters
- Capacitors are usually preferable for simple first-order filters
  - Smaller
  - Cheaper
  - Draw no DC current

RL low pass filter:



Corner frequency: $f_c = \frac{R}{2\pi L}$

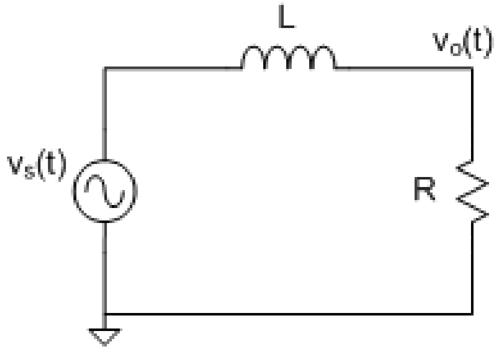
RL high pass filter:



Corner frequency: $f_c = \frac{R}{2\pi L}$

RL Low Pass Filter

- RL low pass filter



- Frequency response:

$$H(\omega) = \frac{V_o}{V_s} = \frac{Z_2}{Z_1 + Z_2} = \frac{R}{R + j\omega L}$$

$$H(\omega) = \frac{R}{R + j\omega L}$$

- Corner frequency is one over the time constant

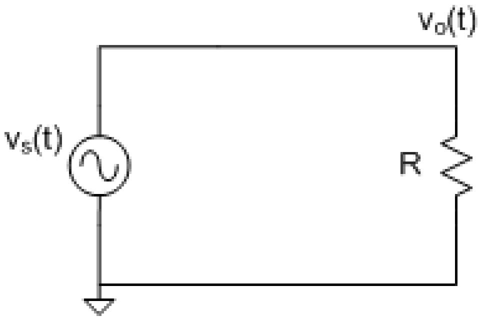
$$\omega_c = \frac{1}{\tau} = \frac{R}{L}$$ and $$f_c = \frac{R}{2\pi L}$$

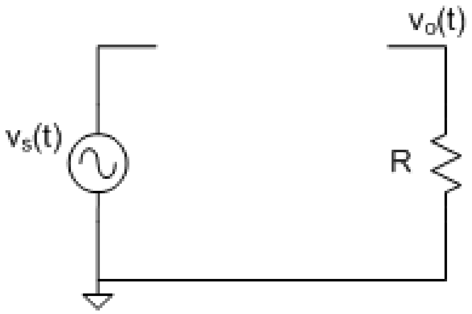
- The frequency at which gain is down by 3 dB
- Frequency at which the inductor impedance magnitude is equal to the resistor impedance magnitude
- Bode plot identical to that of the RC low pass filter
  - As it is for all first-order low pass systems

RL Low Pass Filter

- Again consider the filter's behavior for two limiting cases

- As $f \rightarrow 0$,
  - Inductor → short circuit
  - v_o shorted to v_s
  - ***Gain → unity***

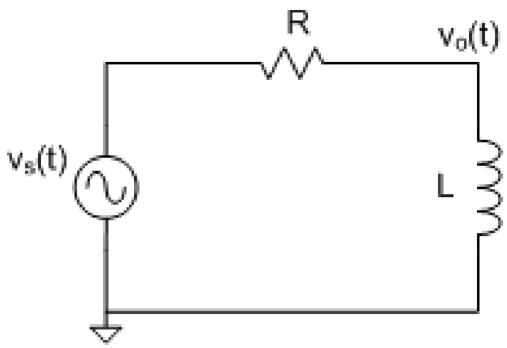




- As $f \rightarrow \infty$
  - Inductor → open circuit
  - $i(t) \rightarrow 0$
  - $v_o \rightarrow 0$
  - ***Gain → zero***

RL High Pass Filter

- Now, swap the locations of the resistor and inductor



  - Frequency response:

$$H(\omega) = \frac{V_o}{V_s} = \frac{Z_2}{Z_1 + Z_2} = \frac{j\omega L}{R + j\omega L}$$

$$\boxed{H(\omega) = \frac{j\omega L}{R + j\omega L}}$$

  - Corner frequency is the same as for the low pass filter

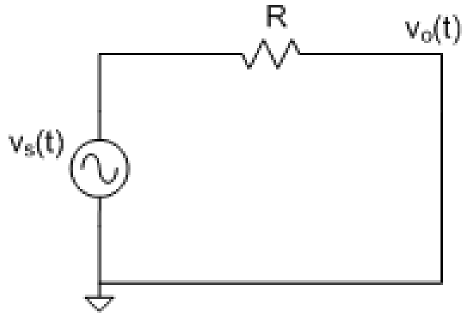
$$\boxed{\omega_c = \frac{1}{\tau} = \frac{R}{L}} \quad \text{and} \quad \boxed{f_c = \frac{R}{2\pi L}}$$

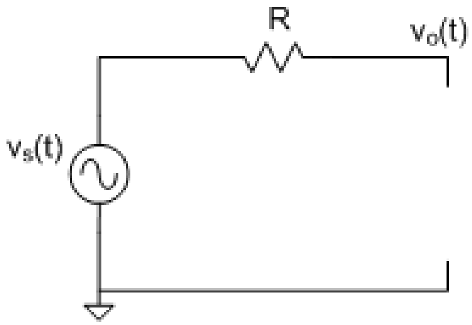
  - Bode plot is identical to that of the RC high pass filter
  - Gain is constant *above* f_c and rolls off *below* f_c

RL High Pass Filter

- Again, consider the two limiting frequency cases

  - As $f \rightarrow 0$,
    - Inductor → short circuit
    - v_o shorted to ground
    - ***Gain → zero***





  - As $f \rightarrow \infty$
    - Inductor → open circuit
    - $i(t) \rightarrow 0$
    - $v_o \rightarrow v_s$
    - ***Gain → unity***

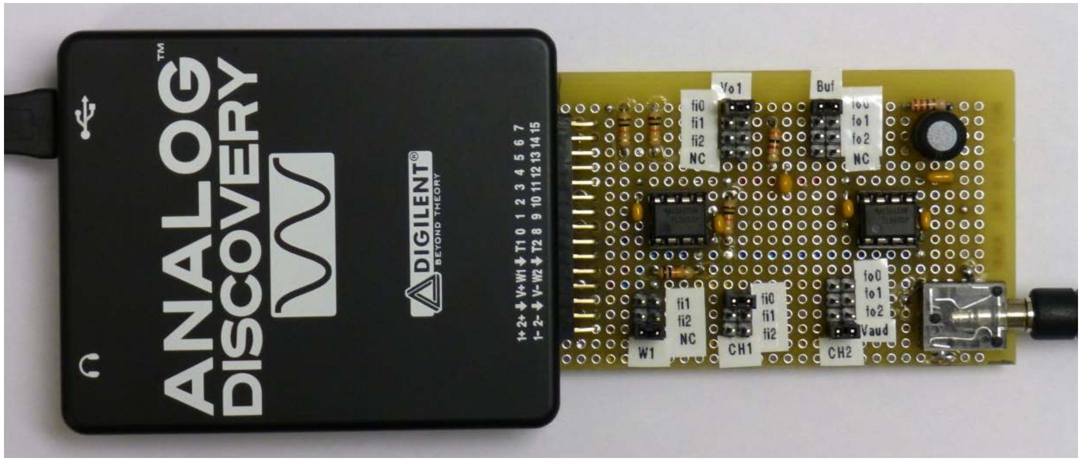
Audio Filter Demo

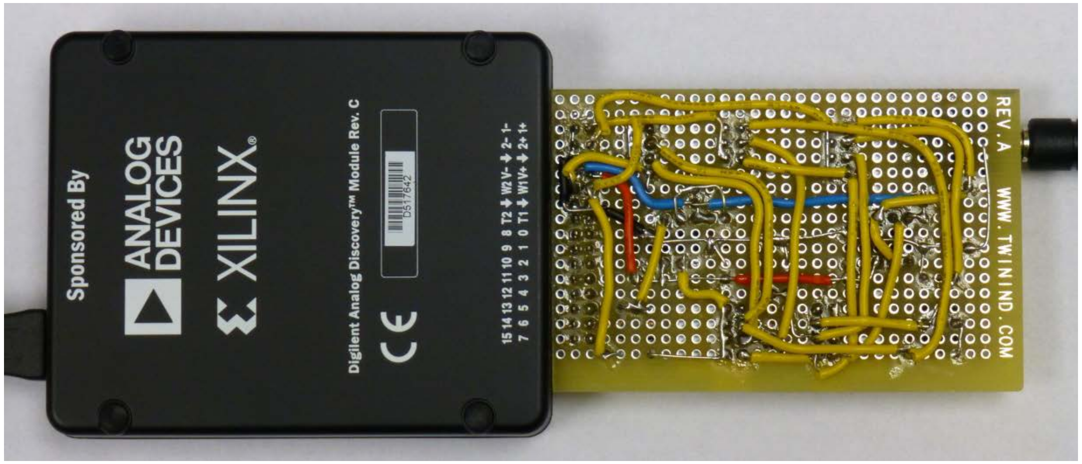
Analog Discovery Instrument

- 2-chan. Scope
  - 14-bit, 100MSa/s
  - 5MHz bandwidth
- 2-chan. function generator
  - 14-bit, 100MSa/s
  - 5MHz bandwidth
- 2-chan. spectrum analyzer
- Network analyzer
- Voltmeter
- ±5V power supplies
- 16-chan. logic analyzer
- 16-chan. digital pattern generator
- USB connectivity

Analog Discovery – Audio Demo

- Demo board plugs in to Analog Discovery module
- Summation of multiple tones
- Optional filtering of audio signal
- 3.5 mm audio output jack





Analog Discovery – Audio Demo