SECTION 4: SECOND-ORDER TRANSIENT RESPONSE

ENGR 202 – Electrical Fundamentals II

Introduction

Second-Order Circuits

- In this and the previous section of notes, we consider second-order RLC circuits from two distinct perspectives:
  - ***Frequency-domain***
    - Second-order, RLC filters
  - ***Time-domain***
    - Second-order, RLC step response

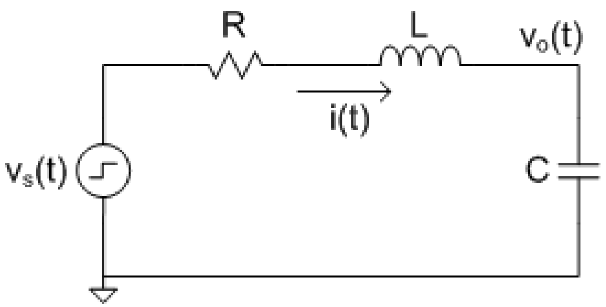
Transient Response of Second-Order Circuits

Second-Order Transient Response

- In ENGR 201 we looked at the transient response of first-order RC and RL circuits
  - Applied KVL
    - Governing differential equation
  - Solved the ODE
    - Expression for the step response
- For ***second-order circuits***, process is the same:
  - ***Apply KVL***
    - Second-order ODE
  - ***Solve the ODE***
    - Second-order step response

Step Response of RLC Circuit

- Determine the response of the following RLC circuit
  - Source is a voltage step: $v_s(t) = 1V \cdot u(t)$
  - Output is the voltage across the capacitor



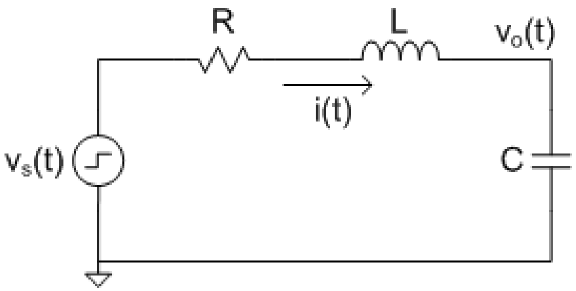
- Apply KVL around the loop

$$v_s(t) - i(t)R - L\frac{di}{dt} - v_o(t) = 0$$

- Want an ODE in terms of $v_o(t)$
  - Need to eliminate $i(t)$
  - Can express $i(t)$ in terms of the output voltage:

$$i(t) = C\frac{dv_o}{dt} \quad \text{so,} \quad \frac{di}{dt} = C\frac{d^2v_0}{dt^2}$$

Step Response of RLC Circuit



- Substituting in the expression for current, the KVL equation becomes

$$v_s(t) - C\frac{dv_o}{dt}R - LC\frac{d^2v_o}{dt^2} - v_o(t) = 0$$

- Rearranging gives the governing second-order ODE:

$$\frac{d^2v_o}{dt^2} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC}v_o(t) = \frac{1}{LC}v_s(t)$$

- A second-order, linear, non-homogeneous, ordinary differential equation
- Non-homogeneous, so solve in two parts
  1) Find the complementary solution to the homogeneous equation
  2) Find the particular solution for the step input
- General solution will be the sum of the two individual solutions:

$$v_o(t) = v_{oc}(t) + v_{op}(t)$$

Complementary Solution

Complementary Solution – $v_{oc}(t)$

- The ***homogeneous equation*** is obtained by setting the forcing function (input) to zero

$$\frac{d^2 v_o}{dt^2} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC} v_o(t) = 0$$

- For an ODE of this form, we assume a solution of the form

$$v_{oc}(t) = e^{st}$$

  - Where s is an unknown *complex* value. Then

$$\frac{dv_{oc}}{dt} = se^{st} \quad \text{and} \quad \frac{d^2 v_{oc}}{dt^2} = s^2 e^{st}$$

- Substituting back into the homogeneous ODE yields the ***characteristic equation***

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Complementary Solution – $v_{oc}(t)$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

- The **characteristic equation** can be rewritten as

$$s^2 + 2\alpha s + \omega_o^2 = 0$$

or

$$s^2 + 2\zeta\omega_o s + \omega_o^2 = 0$$

- The ***roots*** of the characteristic equation (also called ***poles***) tell us about the:
  - Form of the complementary solution
  - Nature of the response
- These roots (or ***poles***) are

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

where

$$\alpha = \frac{R}{2L} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Complementary Solution – $v_{oc}(t)$

- We've said we can write the ***characteristic equation*** as

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

or

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

- The ***damping ratio***, ζ, can be defined as

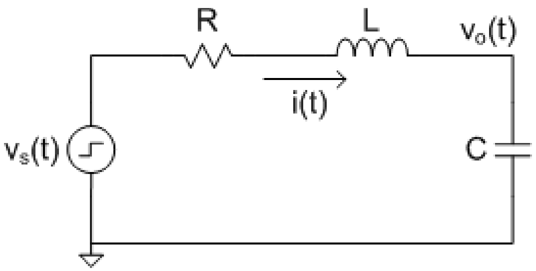
$$\zeta = \frac{\alpha}{\omega_0}$$

- A few key points:
  - ω_0 is the resonant frequency
  - ζ characterizes the nature (sharpness) of the resonance
  - Both are related to the roots of the characteristic equation

Complementary Solution – $v_{oc}(t)$

- Complementary solution has the same form as that of a first-order circuit:

$$v_{oc}(t) = e^{st}$$



  - s is the roots of the characteristic equation
    - Now two values – identical or distinct
    - May be complex
- Form of the solution depends on the values of s
  - Can be characterized in terms of the value of ζ:
    - $\boldsymbol{\zeta > 1}$ – ***over-damped*** case:

$$v_{oc}(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

    - $\boldsymbol{\zeta = 1}$ – ***critically-damped*** case:

$$v_{oc}(t) = K_1 e^{s_1 t} + K_2 t e^{s_1 t}$$

    - $\boldsymbol{\zeta < 1}$ – ***under-damped*** case:

$$v_{oc}(t) = K_1 e^{-\alpha t} \cos(\omega_d t) + K_2 e^{-\alpha t} \sin(\omega_d t)$$

Over-Damped RLC Circuit – $\zeta > 1$

- Roots of the characteristic equation are

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}\ , \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

  - These are related to the damping ratio as

$$\zeta = \frac{\alpha}{\omega_0}$$

- If $\zeta > 1$, then
  - $\alpha > \omega_0$
  - $\alpha^2 - \omega_0^2 > 0$ – i.e., the discriminant is ***positive***
  - s_1 and s_2 are ***real and distinct***
- Complimentary solution has the following form

$$v_{oc}(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

- Recall that $\zeta > 1$ (actually, $\zeta \geq 0.707$) corresponded to no peaking in the frequency domain

Critically-Damped RLC Circuit – $\zeta = 1$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}\ , \qquad s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

- If $\zeta = 1$, then
  - $\alpha = \omega_0$
  - $\alpha^2 - \omega_0^2 = 0$ – i.e., the discriminant is ***zero***
  - s_1 and s_2 are ***real and identical***
- Complimentary solution has the following form

$$v_{oc}(t) = K_1 e^{s_1 t} + K_2 t e^{s_1 t}$$

- This is the lowest value of ζ for which the step response is ***monotonic***
  - Constantly increasing
  - ***No overshoot***

Under-Damped RLC Circuit – $\zeta < 1$

- If $\zeta < 1$, then
  - $\alpha < \omega_0$
  - $\alpha^2 - \omega_0^2 < 0$ – i.e., the discriminant is ***negative***
  - s_1 and s_2 are a ***complex conjugate pair***
- Complimentary solution has the following form

$$v_{oc}(t) = K_1 e^{-\alpha t}\cos(\omega_d t) + K_2 e^{-\alpha t}\sin(\omega_d t)$$

  - ω_d is the ***damped natural frequency***

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \omega_0\sqrt{1-\zeta^2}$$

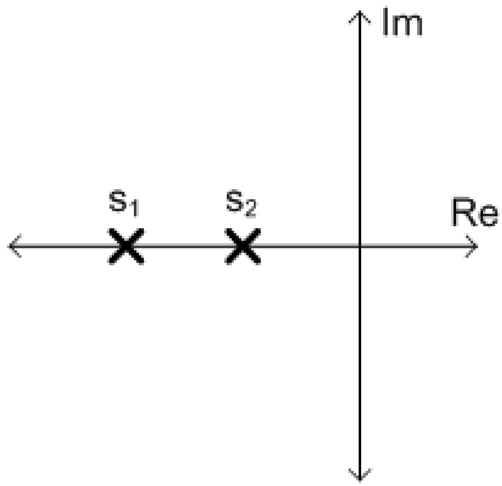
- Response now contains ***damped sinusoidal*** components
  - Will exhibit ***overshoot***
  - Possible ***ringing***

Damping Cases – Geometric Interpretation

- Roots of characteristic equation (system poles) are, in general, complex
  - Can plot them in the ***complex plane***
  - Pole locations tell us a lot about the nature of the response
    - Speed – risetime, settling time
    - Overshoot, ringing

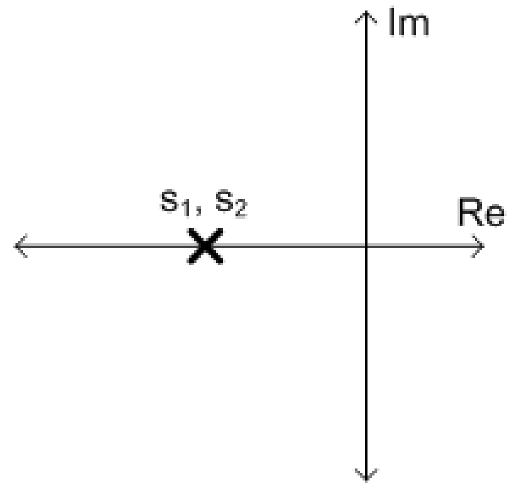
Case 1:

ζ > 1 – *overdamped*



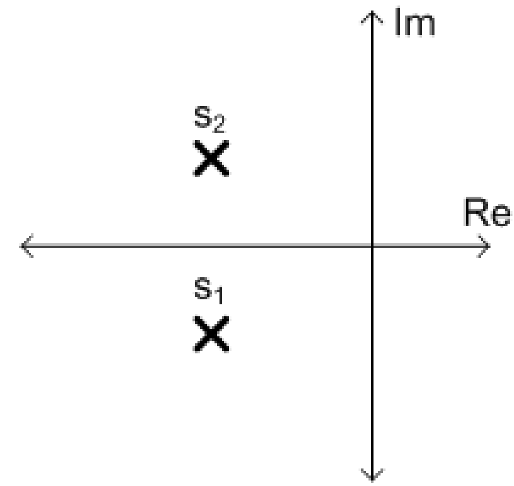
Case 2:

ζ = 1 – *critically-damped*

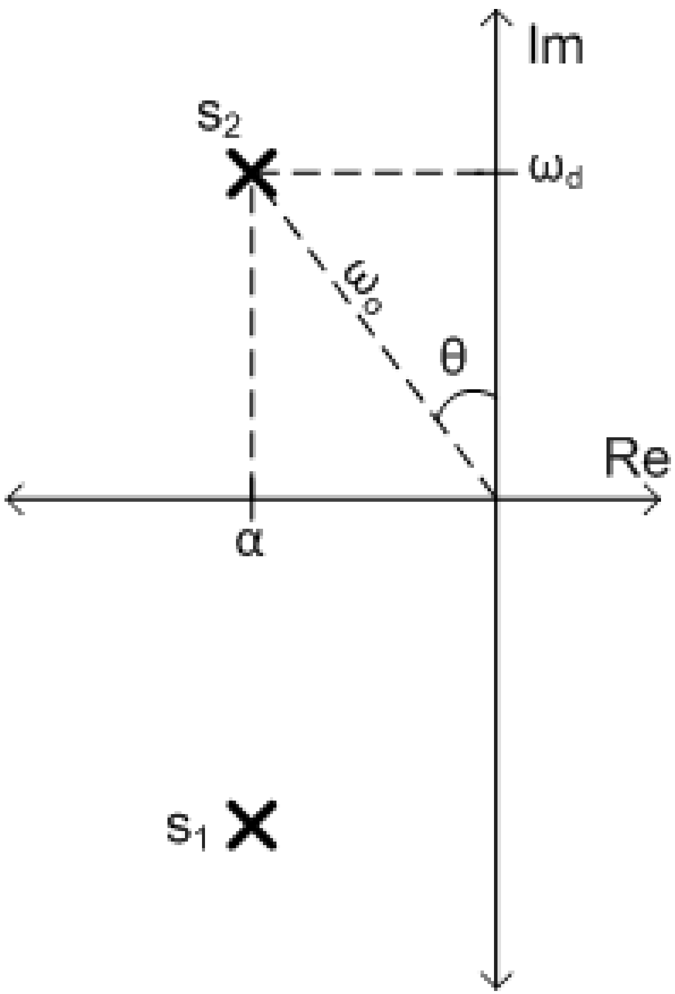


Case 3:

ζ < 1 – *underdamped*



Under-Damped Case – α, ζ, and ω_0



- Under-damped case – $\zeta < 1$
- Roots are a complex-conjugate pair

$$s_{1,2} = -\alpha \pm j\omega_d$$

  - α is the real part
  - ω_d is the imaginary part
  - The ***magnitude*** of the root is

$$\omega_0 = \sqrt{\alpha^2 + \omega_d^2}$$

  - Angle between imaginary axis and vector to the poles is related to damping

$$\zeta = \frac{\alpha}{\omega_0} = \sin(\theta)$$

Particular Solution

Particular solution – $v_{op}(t)$

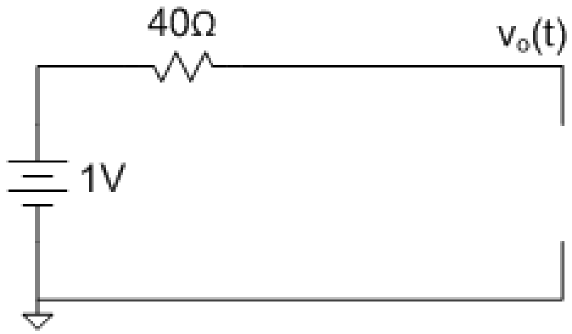
- General solution for the RLC step response is the sum of the complementary and particular solutions

$$v_o(t) = v_{oc}(t) + v_{op}(t)$$

- We now have the complementary solution with two unknown constants, K_1 and K_2
  - Constants to be determined later through application of ***initial conditions***
- Next, determine the ***particular solution***, $v_{op}(t)$
  - For a circuit driven by a step input, this is simply the circuit's ***steady-state response***

$$v_{op}(t) = v_o(t \to \infty)$$

Particular solution – $v_{op}(t)$



- Particular solution is the circuit's steady-state response
  - Inductor → short
  - Capacitor → open

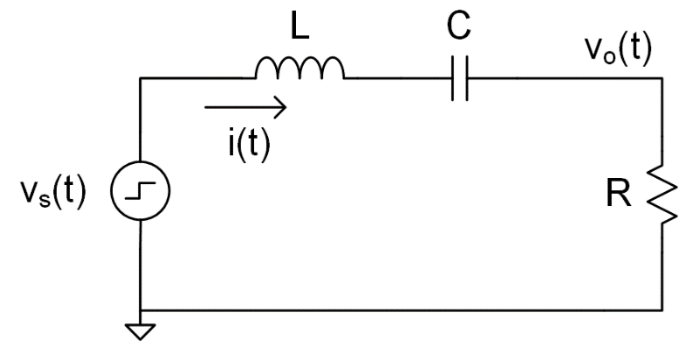
$$v_{op}(t) = v_o(t \to \infty) = v_s(t > 0)$$

- For a unit step input, the particular solution is

$$v_{op}(t) = 1\,V$$

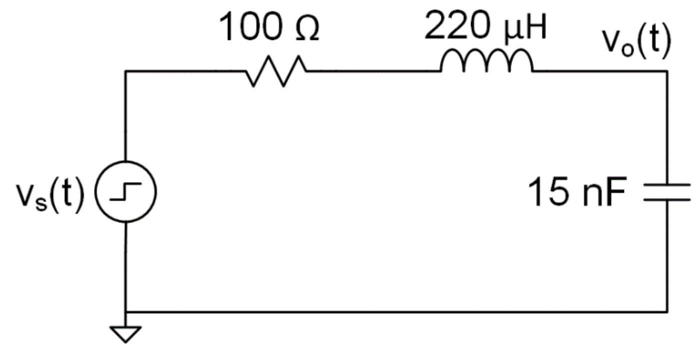
Example Problems

Derive the governing differential equation for the following circuit.



Determine:

- Damping ratio
- Damping case
- Characteristic equation
- Poles

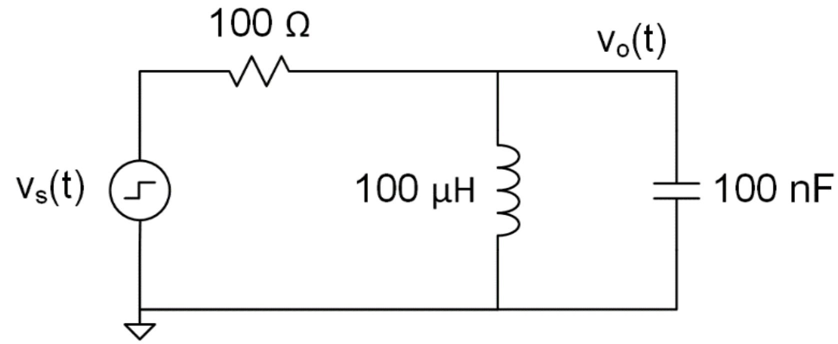


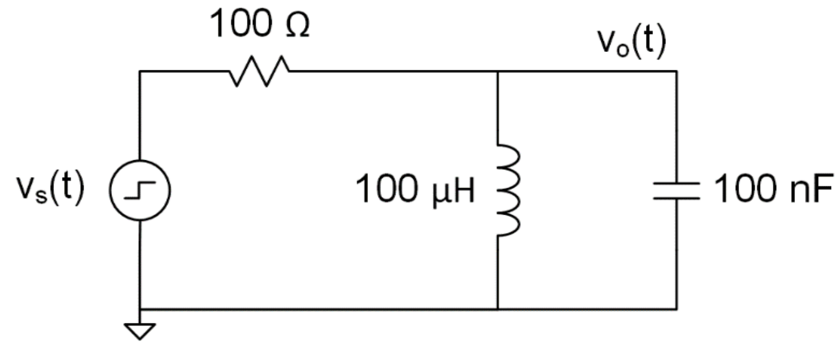
Determine:

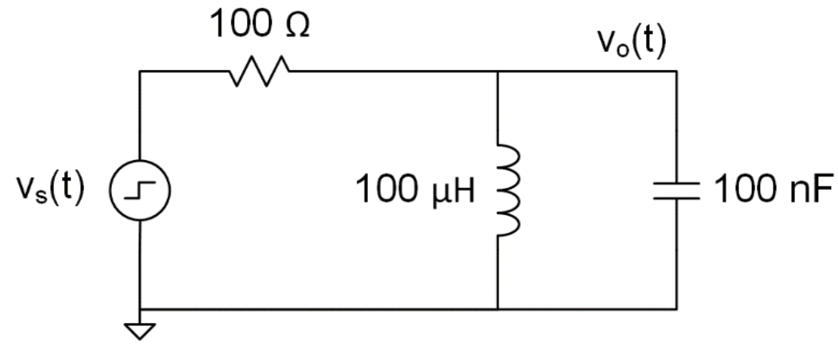
- Initial conditions
- Particular solution

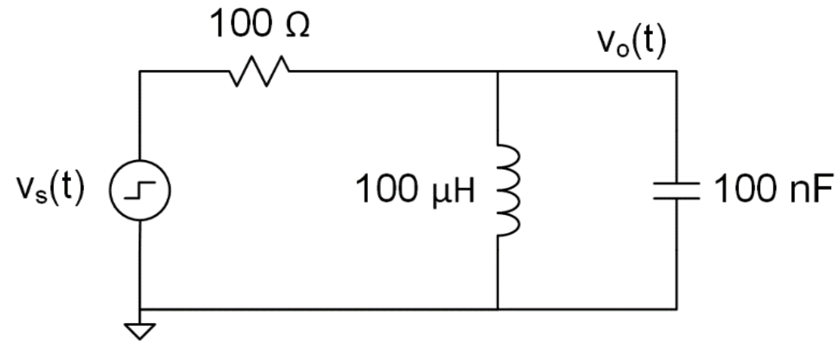
For

$$v_s(t) = -1V \cdot u(t) + 2V$$




100 Ω
vo(t)
vs(t)
100 µH
100 nF


100 Ω
$v_o(t)$
$v_s(t)$
100 μH
100 nF


100 Ω
vo(t)
vs(t)
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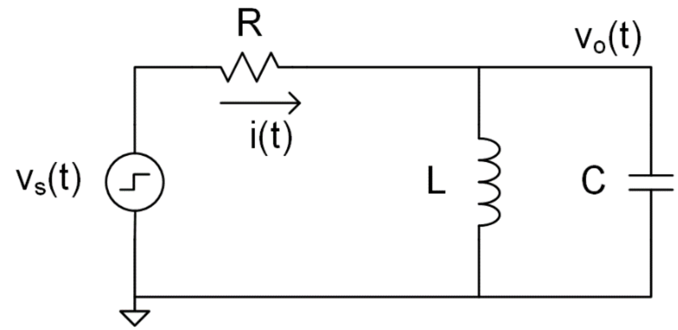
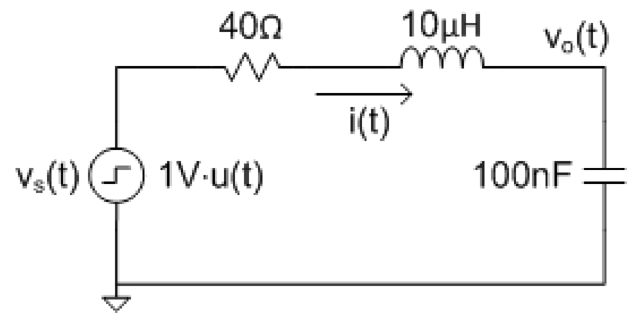


Over-Damped Circuit Response

RLC Step Response – Example 1

- Determine $v_o(t)$
- Input is a unit voltage step

$$v_s(t) = 1V \cdot u(t)$$



- First, apply KVL

$$v_s(t) - i(t)R - L\frac{di}{dt} - v_o(t) = 0$$

- Eliminate $i(t)$ using the i-v relationship for the capacitor

$$i(t) = C\frac{dv_o}{dt} \quad \text{and} \quad \frac{di}{dt} = C\frac{d^2v_o}{dt^2}$$

- This gives the second-order ODE in terms of $v_o(t)$, which can then be rearranged to standard form

$$v_s(t) - RC\frac{dv_o}{dt} - LC\frac{d^2v_o}{dt^2} - v_o(t) = 0 \quad \rightarrow \quad \frac{d^2v_o}{dt^2} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC}v_o(t) = \frac{1}{LC}v_s(t)$$

RLC Step Response – Example 1

- Find the complementary solution, $v_{oc}(t)$
- The homogeneous equation

$$\frac{d^2v_o}{dt^2}+\frac{R}{L}\frac{dv_o}{dt}+\frac{1}{LC}v_o(t)=0$$

- The characteristic equation

$$s^2+\frac{R}{L}s+\frac{1}{LC}=0$$

  - This can be rewritten as

$$s^2+2\alpha s+\omega_0^2=0$$

where

$$\alpha=\frac{R}{2L}=\frac{40\ \Omega}{2\cdot 10\ \mu H}=2\times 10^6\frac{rad}{sec}$$

and

$$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{10\ \mu H\cdot 100\ nF}}=1\times 10^6\frac{rad}{sec}$$

RLC Step Response – Example 1

- The damping ratio is

$$\zeta = \frac{\alpha}{\omega_0} = \frac{2 \times 10^6}{1 \times 10^6} = 2$$

  - $\zeta > 1$, so the circuit is over-damped
- Solution is of the form

$$v_{oc}(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

- s_1 and s_2 are the roots of the characteristic equation

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -268 \times 10^3 \frac{rad}{sec}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -3.73 \times 10^6 \frac{rad}{sec}$$

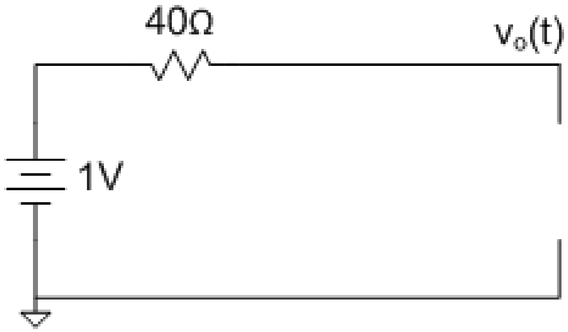
- The complementary solution is

$$v_{oc}(t) = K_1 e^{-268 \times 10^3 t} + K_2 e^{-3.73 \times 10^6 t}$$

RLC Step Response – Example 1

- The particular solution is the circuit's steady-state solution
- Steady-state equivalent circuit:
  - Capacitor → open
  - Inductor → short
- So, the ***particular solution*** is

$$v_{op}(t) = 1\ V$$



- The ***general solution***:

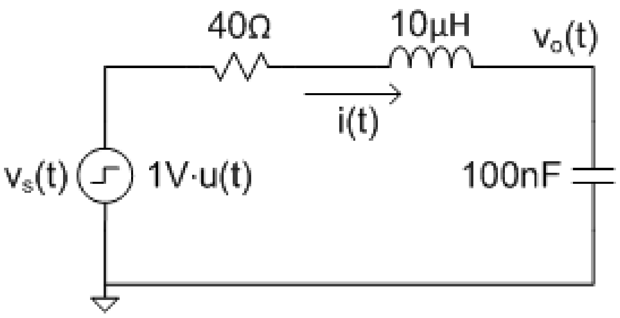
$$v_o(t) = v_{oc}(t) + v_{op}(t)$$

$$v_o(t) = K_1 e^{-268\times10^3 t} + K_2 e^{-3.73\times10^6 t} + 1\ V$$

- Next, we'll apply ***initial conditions*** to determine the unknown coefficients, K_1 and K_2

RLC Step Response – Example 1

- Initial Conditions:
  - For $t < 0$
    - $v_s(t < 0) = 0$
    - $i(t < 0) = 0$
    - $v_o(t < 0) = 0$
  - At $t = 0$
    - $v_s(0) = 1\ V$
    - Capacitor voltage cannot change instantaneously
      - $v_o(0) = v_o(t < 0) = 0\ V$
    - Inductor current cannot change instantaneously
      - $i(0) = i(t < 0) = 0\ A$
    - And, current is related to the output voltage, so
      - $\left.\frac{dv_o}{dt}\right|_{t=0} = \dot{v}_o(0) = 0$

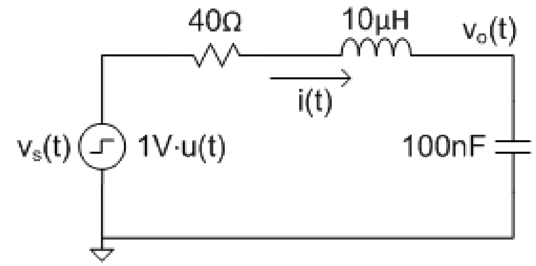


RLC Step Response – Example 1

- The two initial conditions are:

$$v_o(0) = 0 \qquad (1)$$

$$\dot{v}_o(0) = 0 \qquad (2)$$



- Use the initial conditions to determine K_1 and K_2
  - Applying the first initial condition

$$v_o(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} + 1\,V$$

$$v_o(0) = K_1 + K_2 + 1\,V = 0$$

$$K_2 = -K_1 - 1\,V \qquad (3)$$

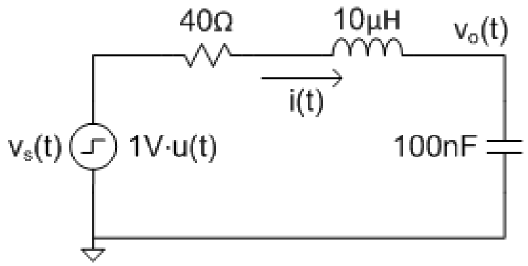
  - Applying the second initial condition

$$\dot{v}_o(t) = s_1 K_1 e^{s_1 t} + s_2 K_2 e^{s_2 t}$$

$$\dot{v}_o(0) = s_1 K_1 + s_2 K_2 = 0 \qquad (4)$$

RLC Step Response – Example 1

- Substituting (3) into (4)



$$s_1 K_1 - s_2(K_1 + 1\,V) = 0$$

$$K_1(s_1 - s_2) = s_2 \cdot 1\,V$$

$$K_1 = \frac{s_2}{s_1 - s_2} \cdot 1\,V = -1.08\,V$$

- Substituting the value of K_1 back into (3)

$$K_2 = -K_1 - 1\,V = 0.08\,V$$

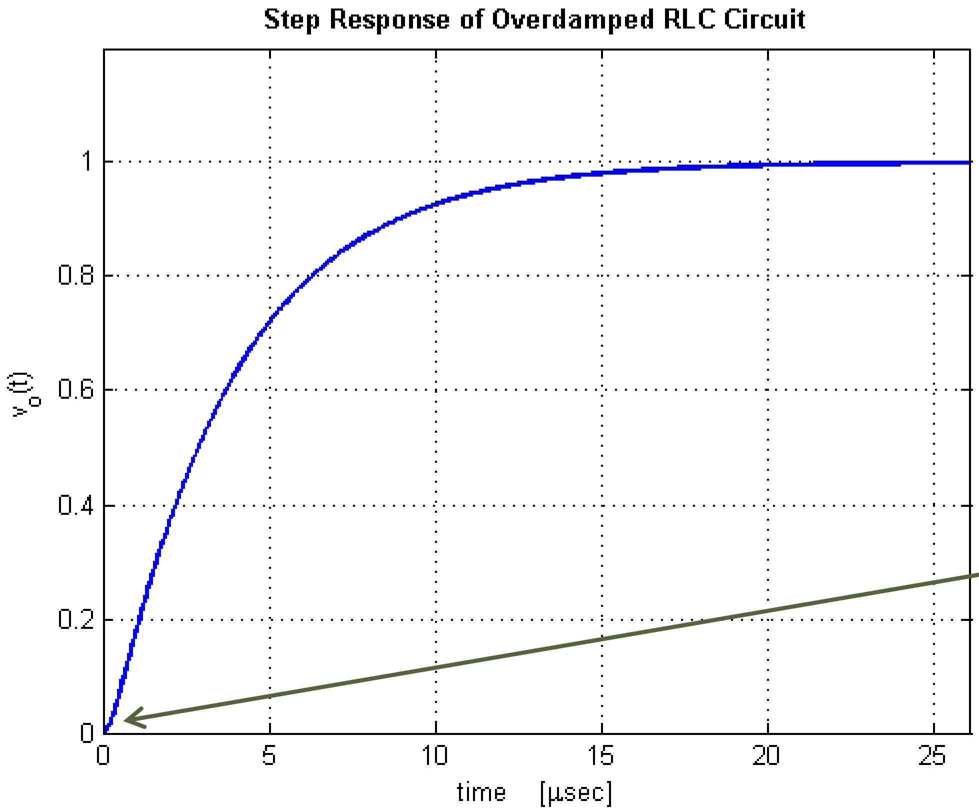
- The step response for this over-damped RLC circuit is

$$v_o(t) = -1.08\,V \cdot e^{s_1 t} + 0.08\,V \cdot e^{s_2 t} + 1\,V$$

$$\boxed{v_o(t) = -1.08\,V \cdot e^{-268\times 10^3 t} + 0.08\,V \cdot e^{-3.73\times 10^6 t} + 1\,V}$$

RLC Step Response – Example 1

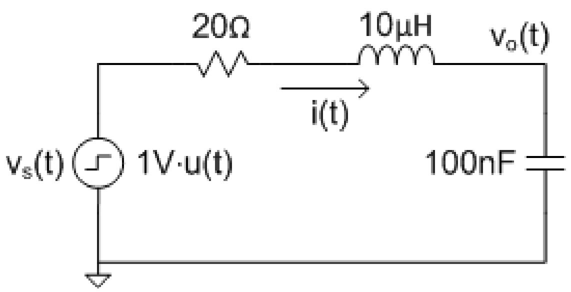
$$v_o(t) = -1.08\,V \cdot e^{-268\times 10^3 t} + 0.08\,V \cdot e^{-3.73\times 10^6 t} + 1\,V$$



- Similar to first-order response
  - Sum of *two* decaying exponentials
  - Monotonic increase to final value
- Initial slope differs from first-order response
  - Increases after $t = 0$

Critically-Damped Circuit Response

RLC Step Response – Example 2



- Now consider the same circuit with decreased resistance
- To determine the form of the response, first determine the ***damping ratio***, ζ

$$\zeta = \frac{\alpha}{\omega_0}$$

where

$$\alpha = \frac{R}{2L} = \frac{20\ \Omega}{2 \cdot 10\ \mu H} = 1 \times 10^6 \frac{rad}{sec}$$

and

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\ \mu H \cdot 100\ nF}} = 1 \times 10^6 \frac{rad}{sec}$$

- The damping ratio is $\zeta = 1$, and the circuit is ***critically-damped***
  - The ***complementary solution*** will be of the following form:

$$v_{oc}(t) = K_1 e^{s_{1,2}t} + K_2 t e^{s_{1,2}t}$$

RLC Step Response – Example 2

- The ***critically-damped*** circuit will have two real, identical poles

$$s_{1,2} = -\alpha = -1 \times 10^6 \frac{rad}{sec}$$

- The ***complementary solution*** is

$$v_{oc}(t) = K_1 e^{-1\times 10^6 t} + K_2 t e^{-1\times 10^6 t}$$

- The ***particular solution*** is still given by the steady-state response, and has not changed

$$v_{op}(t) = v_o(t \rightarrow \infty) = v_s(t > 0) = 1\ V$$

- The ***general solution*** is the sum of the complementary and particular solutions

$$v_o(t) = K_1 e^{-1\times 10^6 t} + K_2 t e^{-1\times 10^6 t} + 1\ V$$

RLC Step Response – Example 2

$$v_o(t) = K_1 e^{-1\times10^6 t} + K_2 t e^{-1\times10^6 t} + 1\ V$$

- Next, determine the unknown coefficients by applying initial conditions
  - Following the same reasoning as in the previous example, initial conditions are the same

$$v_o(0) = 0 \quad \text{and} \quad \dot{v}_o(0) = 0$$

- Applying the first initial condition

$$v_o(0) = K_1 + 1\ V = 0 \quad \rightarrow \quad K_1 = -1\ V$$

- Applying the second initial condition

$$\dot{v}_o(t) = K_1 s_{1,2} e^{s_{1,2} t} + K_2\left(t s_{1,2} e^{s_{1,2} t} + e^{s_{1,2} t}\right)$$

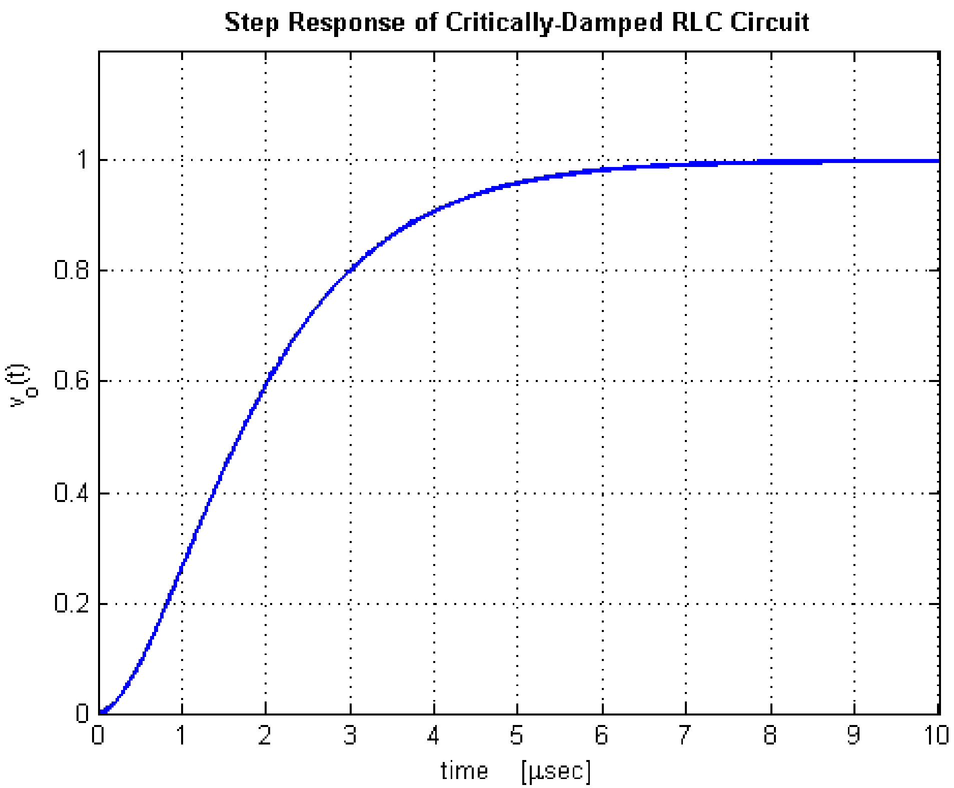
$$\dot{v}_o(0) = K_1 s_{1,2} + K_2 = 0 \quad \rightarrow \quad K_2 = -s_{1,2} K_1 = -1 \times 10^6\ V$$

- The step response of this critically-damped circuit:

$$\boxed{v_o(t) = -e^{-1\times10^6 t} - 1 \times 10^6 t e^{-1\times10^6 t} + 1\ V}$$

RLC Step Response – Example 2

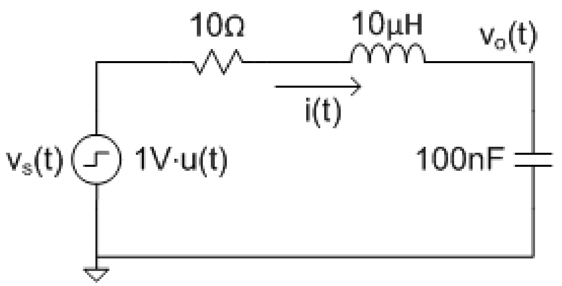
$$v_o(t) = -e^{-1\times10^6 t} - 1\times10^6 t e^{-1\times10^6 t} + 1\,V$$



- Similar to over-damped response
- Faster risetime
  - Dominant, slow pole replaced by higher-frequency double pole
- Again, increasing initial slope differs from first-order response
- Response never exceeds its final value

Under-Damped Circuit Response

RLC Step Response – Example 3



- Again decrease the resistance
- First, determine the ***damping ratio***, ζ

$$\zeta = \frac{\alpha}{\omega_0}$$

where, now

$$\alpha = \frac{R}{2L} = \frac{10\ \Omega}{2 \cdot 10\ \mu H} = 500 \times 10^3\, \frac{rad}{sec}$$

and

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\ \mu H \cdot 100\ nF}} = 1 \times 10^6\, \frac{rad}{sec}$$

- The damping ratio is $\zeta = 0.5$, and the circuit is ***under-damped***
  - The ***complementary solution*** will be of the following form:

$$v_{oc}(t) = K_1 e^{-\alpha t}\cos(\omega_d t) + K_2 e^{-\alpha t}\sin(\omega_d t)$$

RLC Step Response – Example 3

$$v_{oc}(t) = K_1 e^{-\alpha t} \cos(\omega_d t) + K_2 e^{-\alpha t} \sin(\omega_d t)$$

- The ***damped natural frequency*** is

$$\omega_d = \omega_0 \sqrt{1-\zeta^2} = \sqrt{\omega_0^2 - \alpha^2} = 866 \times 10^3 \frac{rad}{sec}$$

- The ***complementary solution*** is

$$v_{oc}(t) = K_1 e^{-500\times 10^3 t} \cos(866 \times 10^3 t) + K_2 e^{-500\times 10^3 t} \sin(866 \times 10^3 t)$$

- Once again, the ***particular solution*** is

$$v_{op}(t) = v_o(t \to \infty) = v_s(t > 0) = 1\,V$$

- The ***general solution*** is the sum of the complementary and particular solutions

$$v_o(t) = K_1 e^{-500\times 10^3 t} \cos(866 \times 10^3 t) + K_2 e^{-500\times 10^3 t} \sin(866 \times 10^3 t) + 1\,V$$

RLC Step Response – Example 3

$$v_o(t) = K_1 e^{-\alpha t}\cos(\omega_d t) + K_2 e^{-\alpha t}\sin(\omega_d t) + 1\,V$$

- Next, determine the unknown coefficients by applying initial conditions

$$v_o(0) = 0 \quad \text{and} \quad \dot{v}_o(0) = 0$$

- Applying the first initial condition

$$v_o(0) = K_1 + 1\,V = 0$$
$$K_1 = -1\,V$$

- Applying the second initial condition

$$\dot{v}_o(t) = K_1[-\omega_d e^{-\alpha t}\sin(\omega_d t) - \alpha e^{-\alpha t}\cos(\omega_d t)]$$
$$+K_2[\omega_d e^{-\alpha t}\cos(\omega_d t) - \alpha e^{-\alpha t}\sin(\omega_d t)]$$

$$\dot{v}_o(0) = -K_1\alpha + K_2\omega_d = 0$$

$$K_2 = K_1\frac{\alpha}{\omega_d} = -\frac{500\times 10^3}{866\times 10^3} = -0.58$$

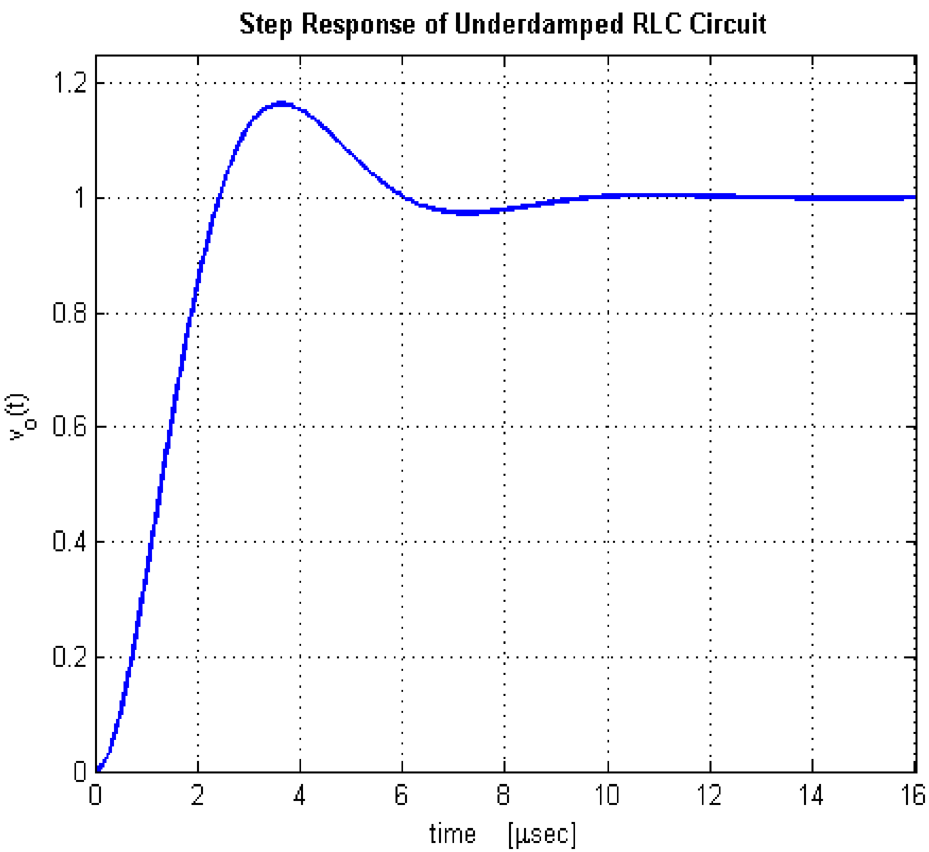
RLC Step Response – Example 3

- The step response for this under-damped RLC circuit is

$$v_o(t) = -e^{-500\times 10^3 t}\cos(866\times 10^3 t) - 0.58e^{-500\times 10^3 t}\sin(866\times 10^3 t) + 1\ V$$

  - Damped oscillatory components
    - Overshoot
    - Possible ringing
  - Exponential damping
    - Oscillatory components decay to zero
    - Rate of decay determined by α, real part of poles

RLC Step Response – Example 3



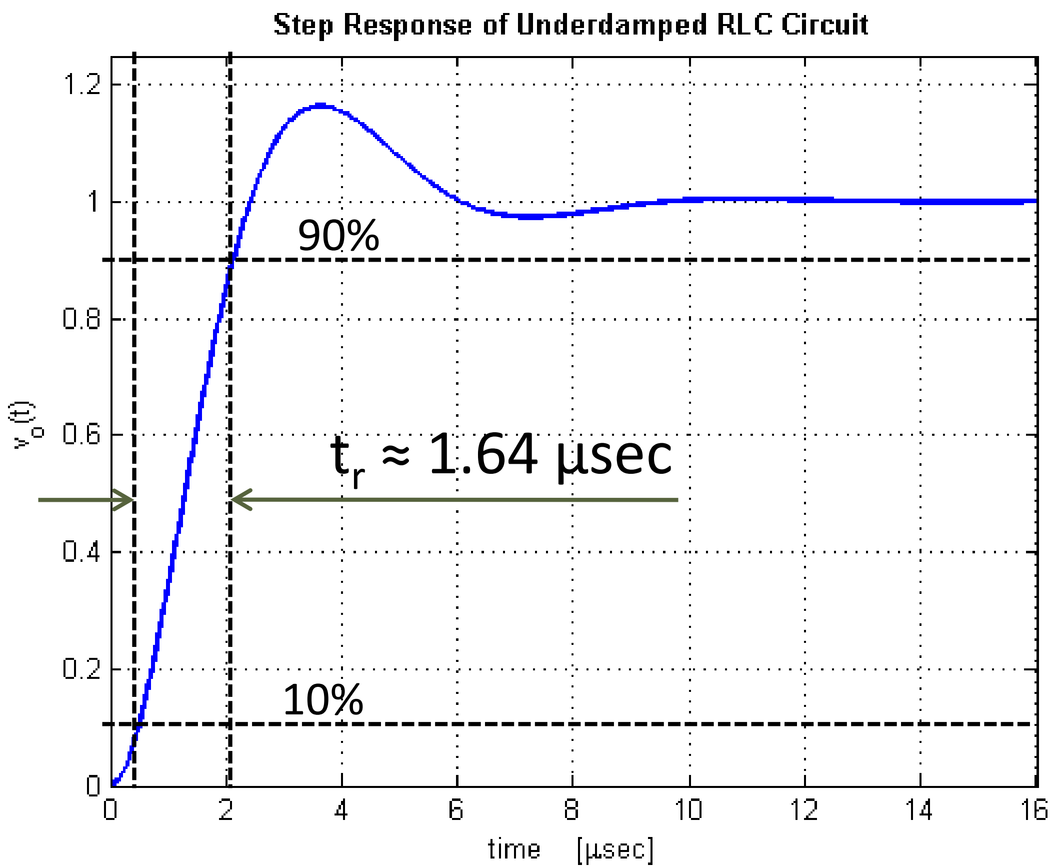
- ***Overshoot***
  - Response exceeds its final value
- ***Ringing***
  - Response oscillate about its final value
  - Not much ringing in this example
- ***Damping ratio***
  - Overshoot and ringing are inversely proportional to ζ

Step Response Characteristics

Step Response – Risetime

- ***Risetime***
  - The time it takes a response to transition between two set levels
  - Typically 10% and 90% of full swing
  - Occasionally 20% and 80%
  - Very rough approximation:

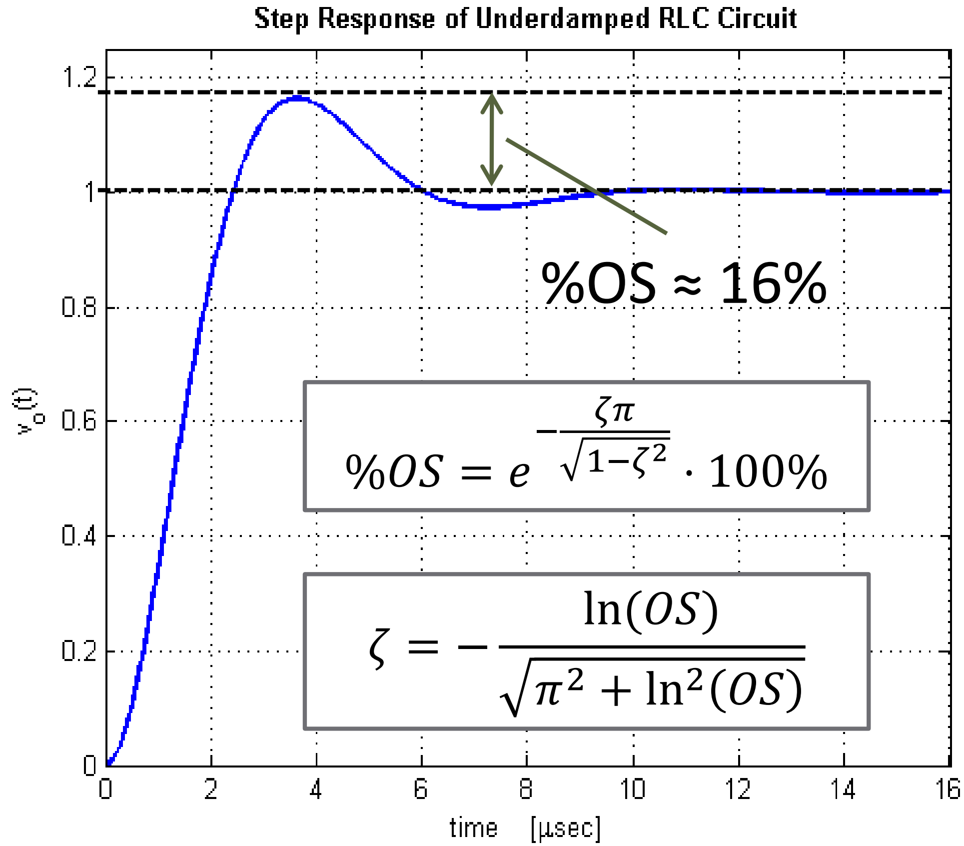
$$t_r \approx \frac{1.8}{\omega_0}$$



Step Response – Overshoot

- ***Overshoot***
  - Response's excursion beyond its final value
  - Expressed as a percentage of the full-scale swing
  - Inversely proportional to damping ratio

ζ	%OS
0.45	20
0.5	16
0.6	10
0.7	5



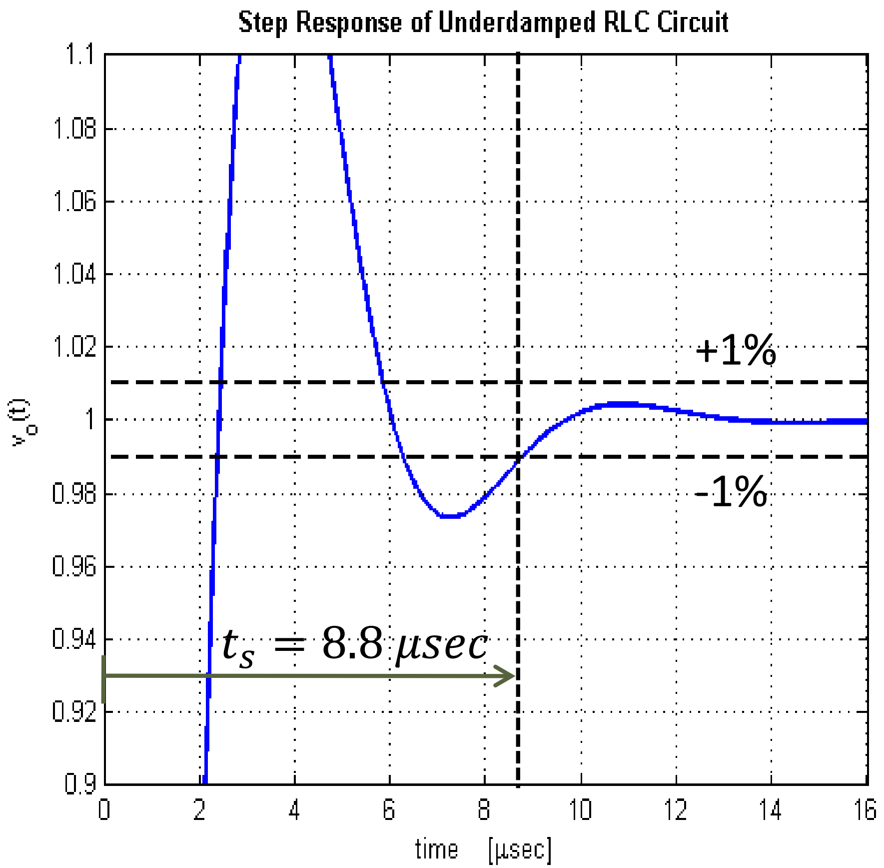
$$\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \cdot 100\%$$

$$\zeta = -\frac{\ln(OS)}{\sqrt{\pi^2 + \ln^2(OS)}}$$

Step Response – Settling Time

- ***Settling time***
  - The time it takes a response to settle (finally) to within some percentage of the final value
  - Typically $\pm 1\%$, $\pm 2\%$, or $\pm 5\%$
  - Inversely proportional to the real part of the circuit's poles (roots of the characteristic equation)
  - For $\pm 1\%$ settling time:

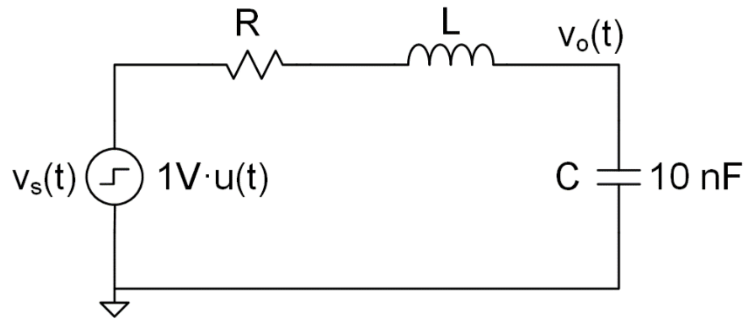
$$t_s \approx \frac{4.6}{\alpha} = \frac{4.6}{\zeta\omega_0}$$

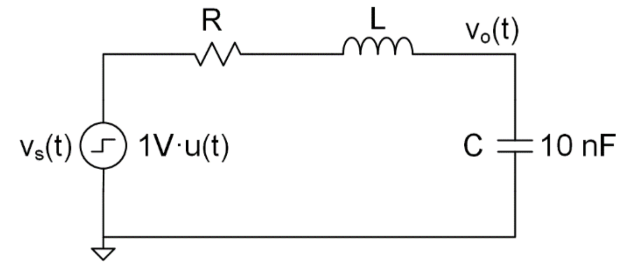


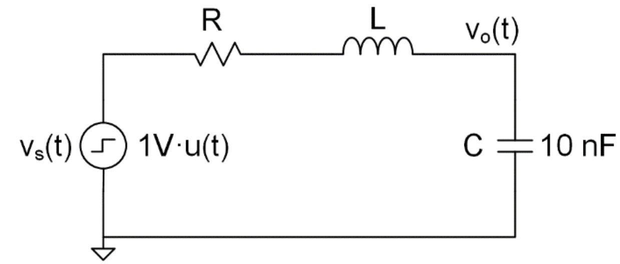
Example Problems

Determine:

- R and L, such that
  - $O.S. = 10\%$
  - $t_s \approx 2\mu sec$ ($\pm 1\%$)
- System poles
- $v_o(t)$ for $t \geq 0$




R
L
$v_o(t)$
$v_s(t)$
1V·u(t)
C
10 nF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$v_o(t)$
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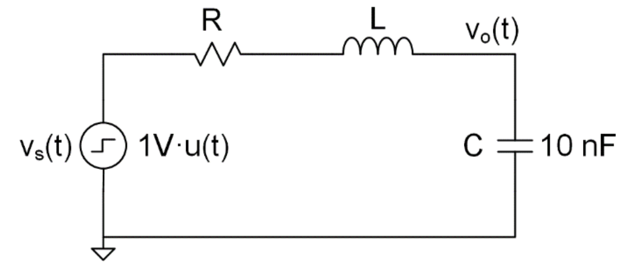
R
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$v_o(t)$
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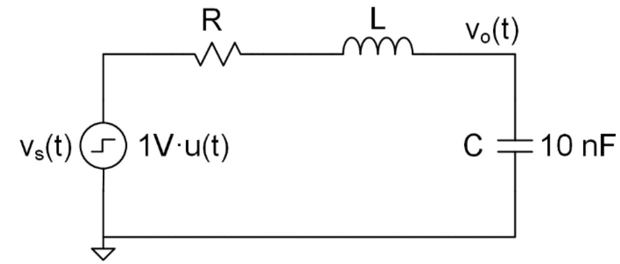
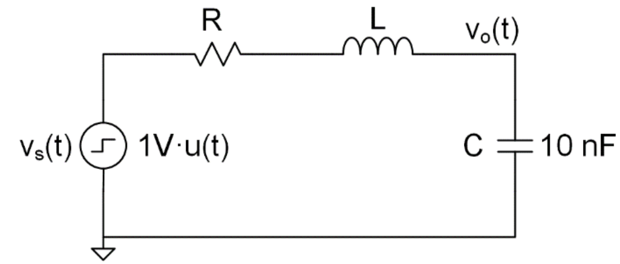
10 nF


R
L
vo(t)
vs(t)
1V·u(t)
C
10 nF


R
L
$v_o(t)$
$v_s(t)$
1V·u(t)
C
10 nF

Determine the minimum resistance for 0% overshoot.