SECTION 6: AC POWER FUNDAMENTALS

ENGR 202 – Electrical Fundamentals II

Instantaneous Power

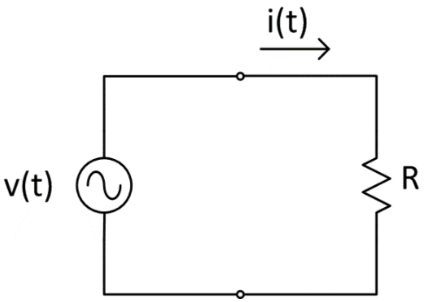
Instantaneous Power

- **Instantaneous power**:
  - Power supplied by a source or absorbed by a load or network element as a function of time

$$p(t) = v(t) \cdot i(t)$$

- The nature of this instantaneous power flow is determined by the impedance of the load
- Next, we'll look at the instantaneous power delivered to loads of different impedances
- Instantaneous power is a useful place to start our discussion of power, but is not how we typically characterize power

Instantaneous Power – Resistive Load



- The voltage across the resistive load is

$$v(t) = V_p \cos(\omega t + \delta)$$

- Current through the resistor is

$$i(t) = \frac{V_p}{R} \cos(\omega t + \delta)$$

- The instantaneous power absorbed by the resistor is

$$p_R(t) = v(t) \cdot i(t) = V_p \cos(\omega t + \delta) \cdot \frac{V_p}{R} \cos(\omega t + \delta)$$

$$p_R(t) = \frac{V_p^2}{R} \cos^2(\omega t + \delta) = \frac{V_p^2}{R} \frac{1}{2} [1 + \cos(2\omega t + 2\delta)]$$

Instantaneous Power – Resistive Load

$$p_R(t) = \frac{V_p^2}{2R}[1 + \cos(2\omega t + 2\delta)]$$

- Making use of the rms voltage

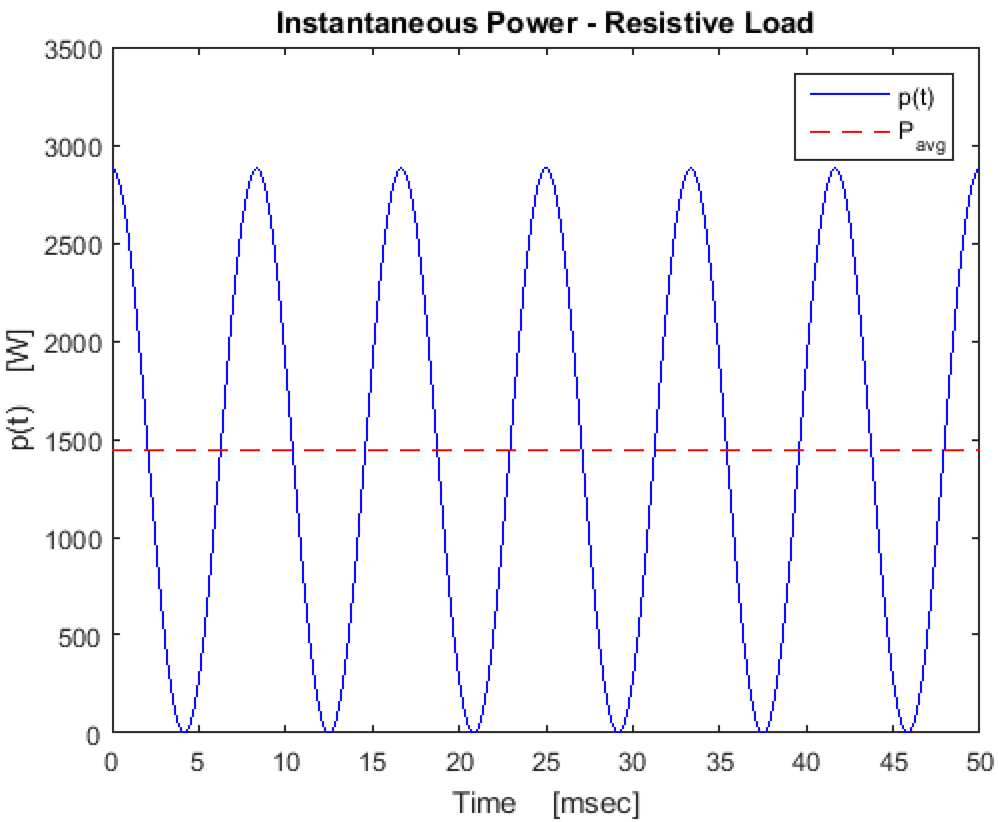
$$p_R(t) = \frac{\left(\sqrt{2}\,V_{rms}\right)^2}{2R}[1 + \cos(2\omega t + 2\delta)]$$

$$p_R(t) = \frac{V_{rms}^2}{R}[1 + \cos(2\omega t + 2\delta)]$$

- The instantaneous power absorbed by the resistor has a non-zero average value and a double-frequency component

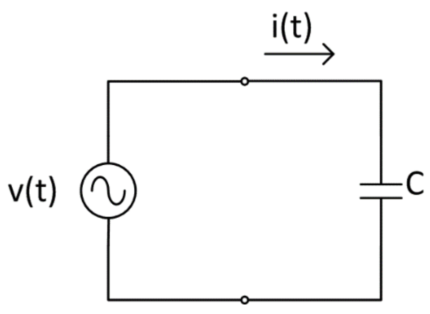
Instantaneous Power – Resistive Load

- Power delivered to the resistive load has a non-zero average value and a double-frequency component



Instantaneous Power – Capacitive Load

- Now consider the power absorbed by a purely capacitive load
  - Again, $v(t) = V_p \cos(\omega t + \delta)$



- The current flowing to the load is

$$i(t) = I_p \cos(\omega t + \delta + 90°)$$

where

$$I_p = \frac{V_p}{X_C} = \frac{V_p}{1/\omega C} = \omega C V_p$$

- The instantaneous power delivered to the capacitive load is

$$p_C(t) = v(t) \cdot i(t)$$

$$p_C(t) = V_p \cos(\omega t + \delta) \cdot \omega C V_p \cos(\omega t + \delta + 90°)$$

Instantaneous Power – Capacitive Load

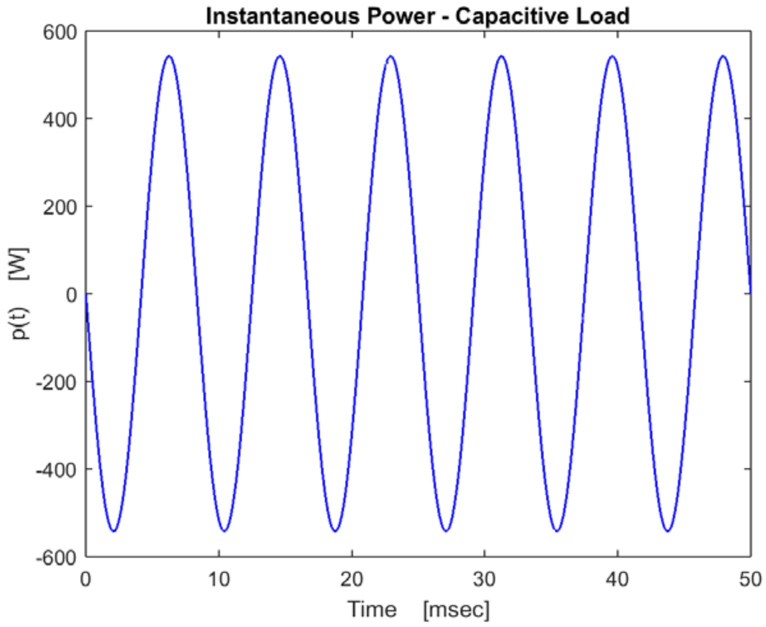
$$p_C(t) = \omega C V_p^2 \frac{1}{2}[\cos(-90°) + \cos(2\omega t + 2\delta + 90°)]$$

$$p_C(t) = \omega C \frac{V_p^2}{2} \cdot \cos(2\omega t + 2\delta + 90°)$$

- In terms of rms voltage

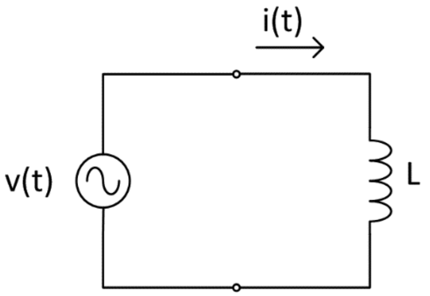
$$p_C(t) = \omega C V_{rms}^2 \cdot \cos(2\omega t + 2\delta + 90°)$$

- This is a double frequency sinusoid, but, unlike for the resistive load, the average value is zero



Instantaneous Power – Inductive Load

- Now consider the power absorbed by a purely inductive load
- Now the load current *lags* by 90°



$$i(t) = I_p \cos(\omega t + \delta - 90°)$$

where

$$I_p = \frac{V_p}{X_L} = \frac{V_p}{\omega L}$$

- The instantaneous power delivered to the inductive load is

$$p_L(t) = v(t) \cdot i(t)$$

$$p_L(t) = V_p \cos(\omega t + \delta) \cdot \frac{V_p}{\omega L} \cos(\omega t + \delta - 90°)$$

Instantaneous Power – Inductive Load

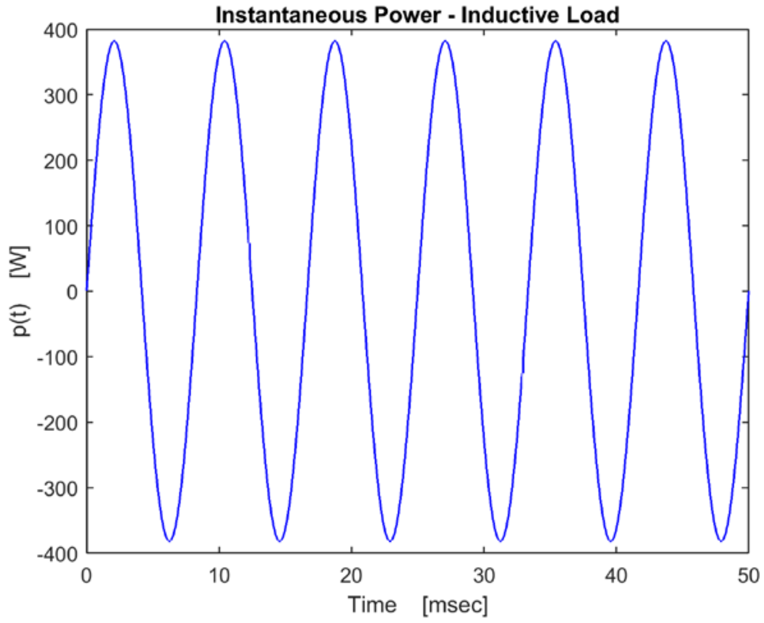
$$p_L(t) = \frac{V_p^2}{\omega L}\frac{1}{2}[\cos(90°) + \cos(2\omega t + 2\delta - 90°)]$$

$$p_L(t) = \frac{V_p^2}{2\omega L} \cdot \cos(2\omega t + 2\delta - 90°)$$

- In terms of rms voltage

$$p_L(t) = \frac{V_{rms}^2}{\omega L} \cdot \cos(2\omega t + 2\delta - 90°)$$

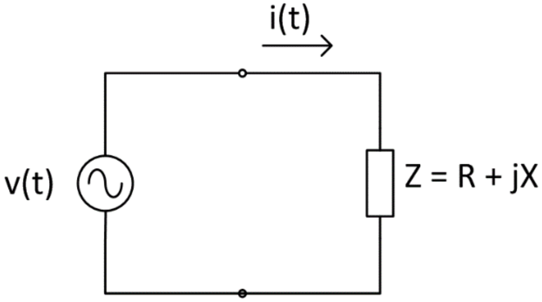
- As for the capacitive load, this is a double frequency sinusoid with an average value of zero



Instantaneous Power – General Impedance

- Finally, consider the instantaneous power absorbed by a general RLC load
- Phase angle of the current is determined by the angle of the impedance

$$i(t) = I_p \cos(\omega t + \beta)$$



- The instantaneous power is

$$p(t) = V_p \cos(\omega t + \delta) \cdot I_p \cos(\omega t + \beta)$$

$$p(t) = \frac{V_p I_p}{2}[\cos(\delta - \beta) + \cos(2\omega t + \delta + \beta)]$$

$$p(t) = V_{rms} I_{rms}\left[\cos(\delta - \beta) + \cos\left(2\omega t + 2\delta - (\delta - \beta)\right)\right]$$

Instantaneous Power – General Impedance

- Using the following trig identity

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

we get

$$p(t) = V_{rms}I_{rms}[\cos(\delta-\beta) + \cos(\delta-\beta)\cos(2\omega t+2\delta) + \sin(\delta-\beta)\sin(2\omega t+2\delta)]$$

and

$$p(t) = V_{rms}I_{rms}\cos(\delta-\beta)\,[1+\cos(2\omega t+2\delta)] + V_{rms}I_{rms}\sin(\delta-\beta)\sin(2\omega t+2\delta)$$

Instantaneous Power – General Impedance

- Letting

$$I_R = I_{rms}\cos(\delta - \beta) \quad \text{and} \quad I_X = I_{rms}\sin(\delta - \beta)$$

we have

$$p(t) = V_{rms} I_R [1 + \cos(2\omega t + 2\delta)]$$

$$+ V_{rms} I_X \sin(2\omega t + 2\delta)$$

- There are two components to the power:

$$p_R(t) = V_{rms} I_R [1 + \cos(2\omega t + 2\delta)]$$

is the power absorbed by the resistive component of the load, and

$$p_X(t) = V_{rms} I_X \sin(2\omega t + 2\delta)$$

is the power absorbed by the reactive component of the load

Real & Reactive Power

Real Power

- According to previous expressions, power delivered to a resistance has a non-zero average value
  - Purely resistive load or a load with a resistive component
- This is ***real power***, ***average power***, or ***active power***

$$P = V_{rms} I_R$$

$$\boxed{P = V_{rms} I_{rms} \cos(\delta - \beta)}$$

- Real power has units of ***watts*** (W)
- Real power is power that results in work (or heat dissipation)

Power Factor

- The phase angle $(\delta - \beta)$ represents the phase difference between the voltage and the current
  - This is the ***power factor angle***
  - The angle of the load impedance
- Note that the *real power* is a function of the *cosine of the power factor angle*

$$P = V_{rms} I_{rms} \cos(\delta - \beta)$$

- This is the ***power factor***

$$\boxed{p.f. = \cos(\delta - \beta)}$$

- For a purely resistive load, voltage and current are in phase

$$p.f. = \cos(\delta - \beta) = \cos(0°) = 1$$

$$P = V_{rms} I_{rms}$$

Power Factor

- For a purely capacitive load, current leads the voltage by $90°$

$$p.f. = \cos(\delta - \beta) = \cos(-90°) = 0$$

$$P = 0$$

  - This is referred to as a ***leading power factor***
  - Power factor is *leading* for loads with *capacitive* reactance
- For a purely inductive load, current lags the voltage by $90°$

$$p.f. = \cos(\delta - \beta) = \cos(90°) = 0$$

$$P = 0$$

  - Loads with inductive reactance have *lagging* power factors
- Note that power factor is defined to always be ***positive***

$$0 \leq p.f. \leq 1$$

Reactive Power

- The other part of instantaneous power is the power delivered to the reactive component of the load

$$p_X(t) = V_{rms} I_{rms} \sin(\delta - \beta) \sin(2\omega t + 2\delta)$$

- Unlike real power, this component of power has zero average value
- The *amplitude* is the ***reactive power***

$$Q = V_{rms} I_{rms} \sin(\delta - \beta) \;\; var$$

- Units are ***volts-amperes reactive***, or ***var***
- Power that flows to and from the load reactance
  - Does not result in work or heat dissipation

RMS Phasors

- Up to this point:
  - Phasor magnitude has been ***peak amplitude***

$$v(t) = V_p \sin(\omega t + \phi) \quad \leftrightarrow \quad \mathbf{V} = V_p e^{j\phi} = V_p \angle \phi$$

- For ***AC power systems***:
  - Phasor magnitude is the ***RMS value***

$$v(t) = \sqrt{2} \cdot V_{rms} \sin(\omega t + \phi) \quad \leftrightarrow \quad \mathbf{V} = V_{rms} e^{j\phi} = V_{rms} \angle \phi$$

  - Simplifies calculation of AC power quantities

Complex Power

Complex Power

- ***Complex power*** is defined as the product of the rms voltage phasor and *conjugate* rms current phasor

$$\boxed{\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^*}$$

where the voltage has phase angle δ

$$\boldsymbol{V} = V_{rms}\angle\delta$$

and the current has phase angle β

$$\boldsymbol{I} = I_{rms}\angle\beta \quad \rightarrow \quad \boldsymbol{I}^* = I_{rms}\angle-\beta$$

- The complex power is

$$\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^* = (V_{rms}\angle\delta)(I_{rms}\angle-\beta)$$

$$\boldsymbol{S} = V_{rms}I_{rms}\angle(\delta-\beta)$$

Complex Power

- Complex power has units of ***volts-amperes*** (VA)
- The *magnitude* of complex power is ***apparent power***

$$S = V_{rms} I_{rms} \;\; VA$$

- Apparent power also has units of volts-amperes
- Complex power is the vector sum of real power (in phase with $\boldsymbol{V}$) and reactive power ($\pm 90°$ out of phase with $\boldsymbol{V}$)

$$\boldsymbol{S} = P + jQ$$

Complex Power

- ***Real power*** can be expressed in terms of complex power

$$P = Re\{\boldsymbol{S}\}$$

or in terms of ***apparent power***

$$P = S \cdot \cos(\delta - \beta) = S \cdot p.f.$$

- Similarly, ***reactive power***, is the imaginary part of complex power

$$Q = Im\{\boldsymbol{S}\}$$

and can also be related to ***apparent power***

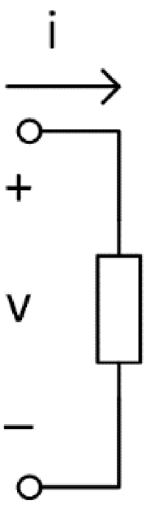
$$Q = S \cdot \sin(\delta - \beta)$$

- And, ***power factor*** is the ***ratio*** between ***real power*** and ***apparent power***

$$p.f. = \cos(\delta - \beta) = \frac{P}{S}$$

Power Convention – Load Convention

- Applying a consistent sign convention allows us to easily determine whether network elements supply or absorb real and reactive power
- ***Passive sign convention*** or ***load convention***
  - Positive current defined to enter the positive voltage terminal of an element
- If $P > 0$ or $Q > 0$, then real or reactive power is ***absorbed*** by the element
- If $P < 0$ or $Q < 0$, then real or reactive power is ***supplied*** by the element



Power Absorbed by Passive Elements

- Complex power absorbed by a ***resistor***

$$\boldsymbol{S_R} = \boldsymbol{V I_R^*} = (V\angle\delta)\left(\frac{V}{R}\angle-\delta\right)$$

$$\boldsymbol{S_R} = \frac{V^2}{R}$$

  - Positive and purely real
    - Resistors ***absorb real*** power
    - ***Reactive*** power is ***zero***

- Complex power absorbed by a ***capacitor***

$$\boldsymbol{S_C} = \boldsymbol{V I_C^*} = (V\angle\delta)(-j\omega C V\angle-\delta)$$

$$\boldsymbol{S_C} = -j\omega C V^2$$

  - Negative and purely imaginary
    - Capacitors ***supply reactive*** power
    - ***Real*** power is ***zero***

Power Absorbed by Passive Elements

- Complex power absorbed by an ***inductor***

$$\boldsymbol{S_L} = \boldsymbol{V}\boldsymbol{I_L^*} = (V\angle\delta)\left(\frac{V}{-j\omega L}\angle-\delta\right)$$

$$\boldsymbol{S_L} = j\frac{V^2}{\omega L}$$

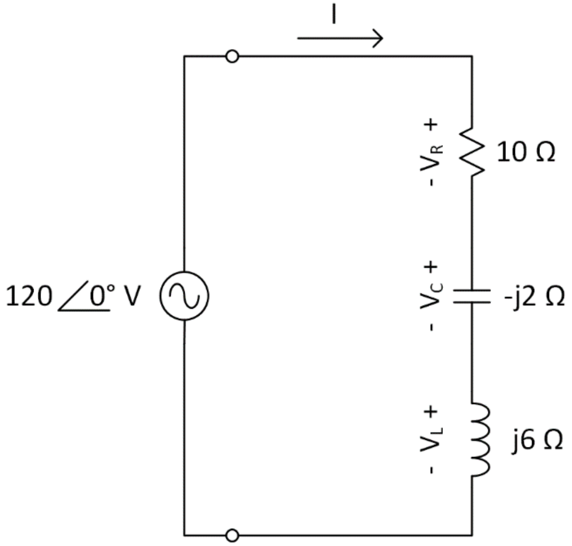
  - Positive and purely imaginary
    - Inductors ***absorb reactive*** power
    - ***Real*** power is ***zero***

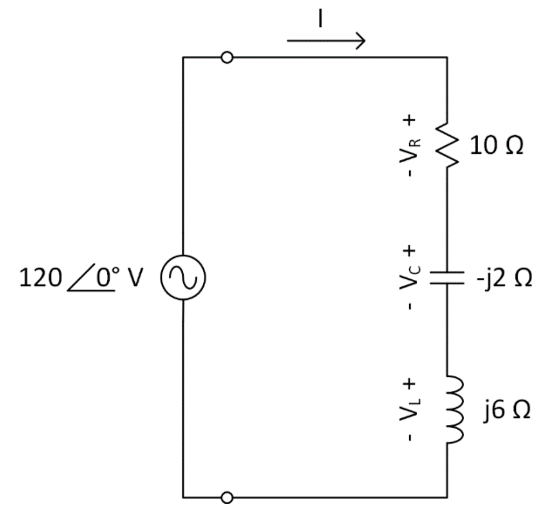
- In summary:
  - Resistors absorb real power, zero reactive power
  - Capacitors supply reactive power, zero real power
  - Inductors absorb reactive power, zero real power

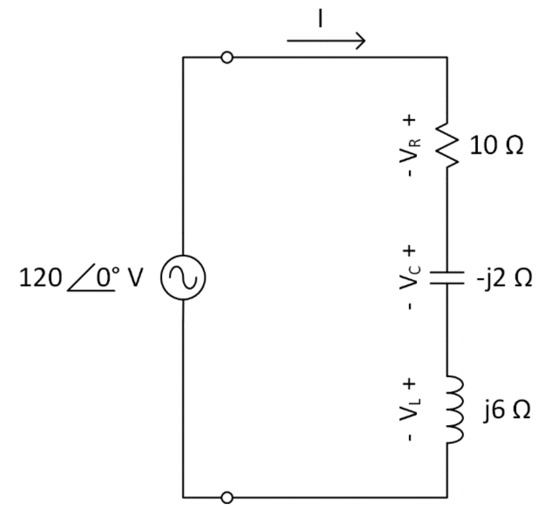
Example Problems

Determine:

- Complex, real, and reactive power delivered to the load
- Voltage across each element
- Power associated with each element
- Power factor




I
10 Ω
-j2 Ω
j6 Ω
120 ∠0° V
- V_R +
- V_C +
- V_L +



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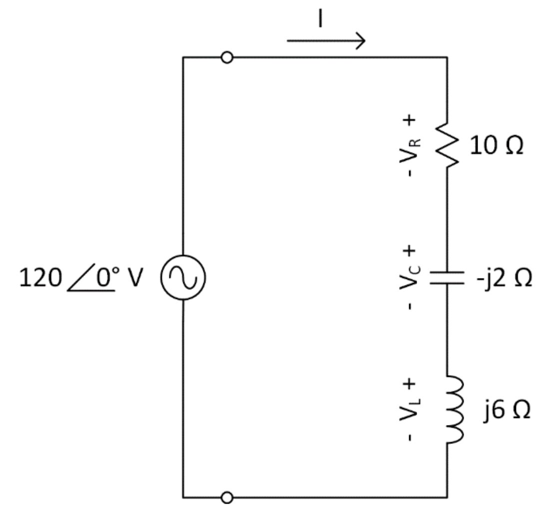
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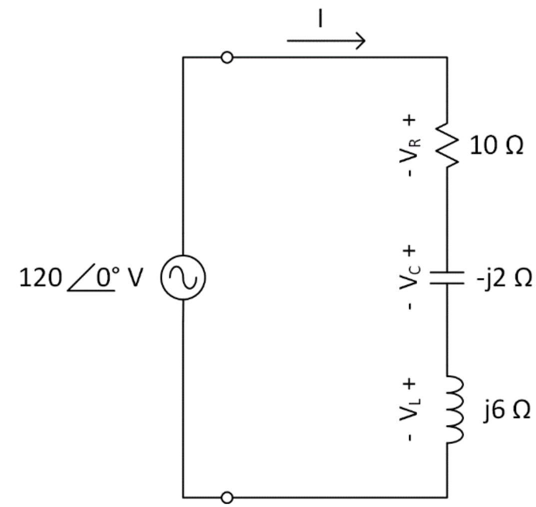

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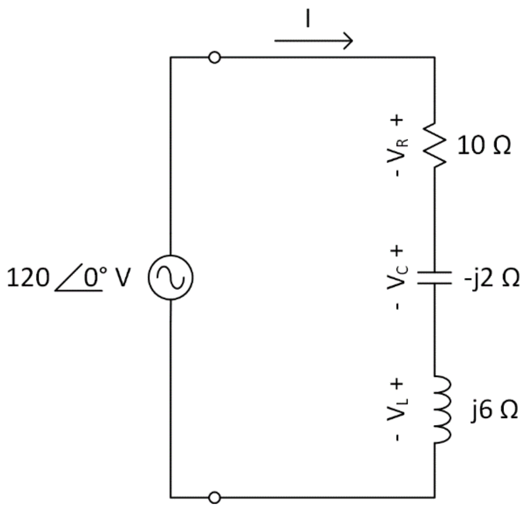

I
120∠0° V
10 Ω
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- V_R +
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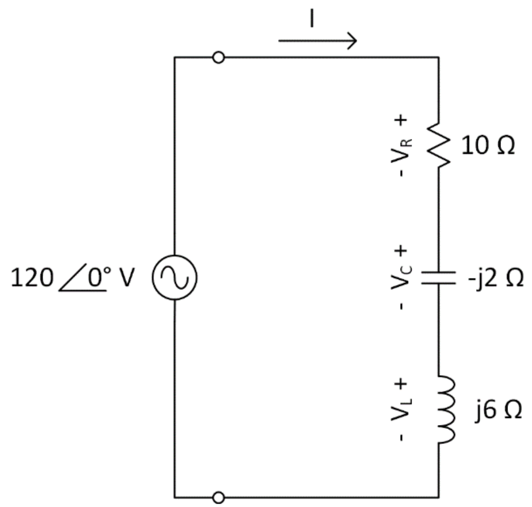
- V_L +


I
120 ∠0° V
10 Ω
-j2 Ω
j6 Ω
- V_R +
- V_C +
- V_L +

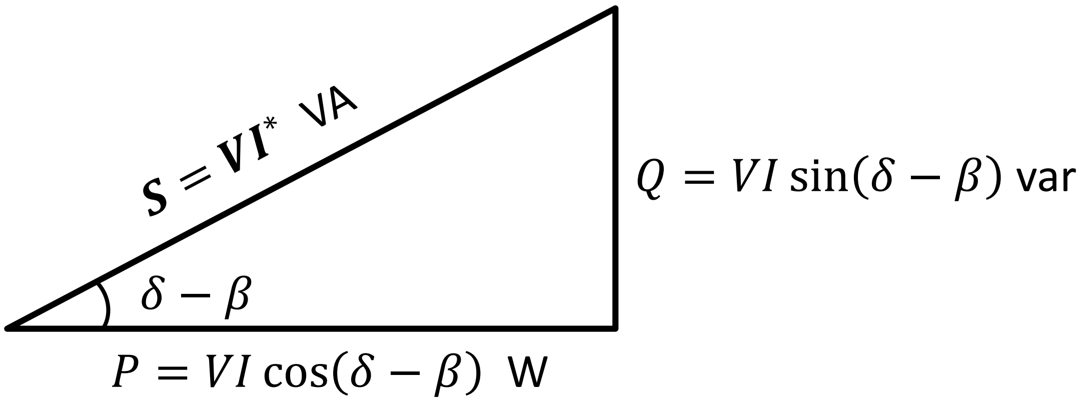
Power Triangle

Power Triangle

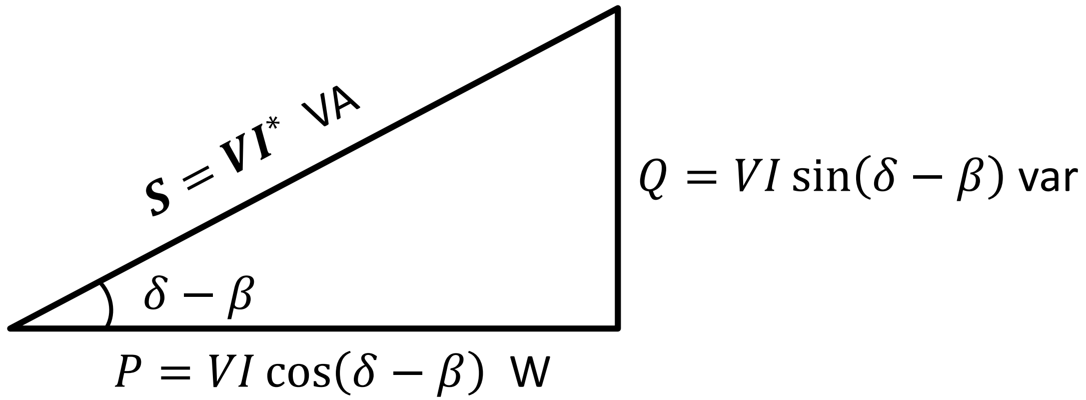
- Complex power is the vector sum of real power (in phase with $\boldsymbol{V}$) and reactive power ($\pm 90°$ out of phase with $\boldsymbol{V}$)

$$\boldsymbol{S} = P + jQ$$

- Complex, real, and reactive powers can be represented graphically, as a ***power triangle***



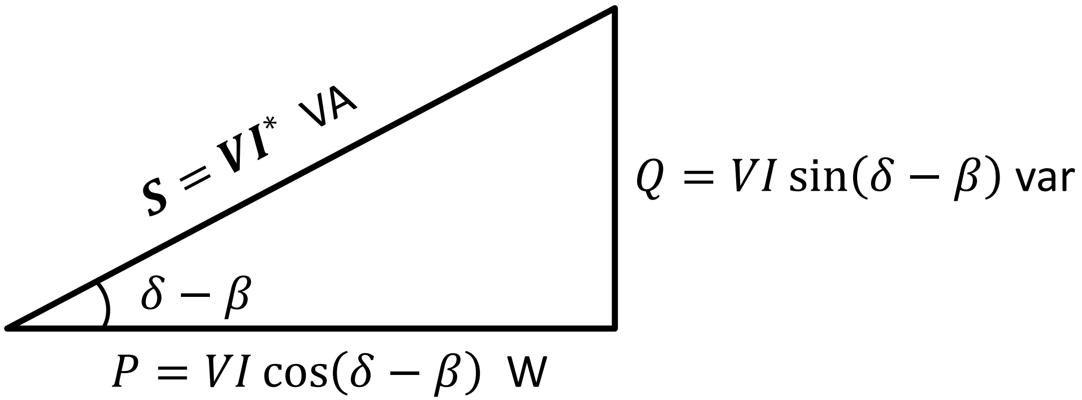
Power Triangle



- Quickly and graphically provides power information
  - ***Power factor*** and power factor angle
  - ***Leading*** or ***lagging*** power factor
  - Reactive nature of the load – ***capacitive*** or ***inductive***

Lagging Power Factor

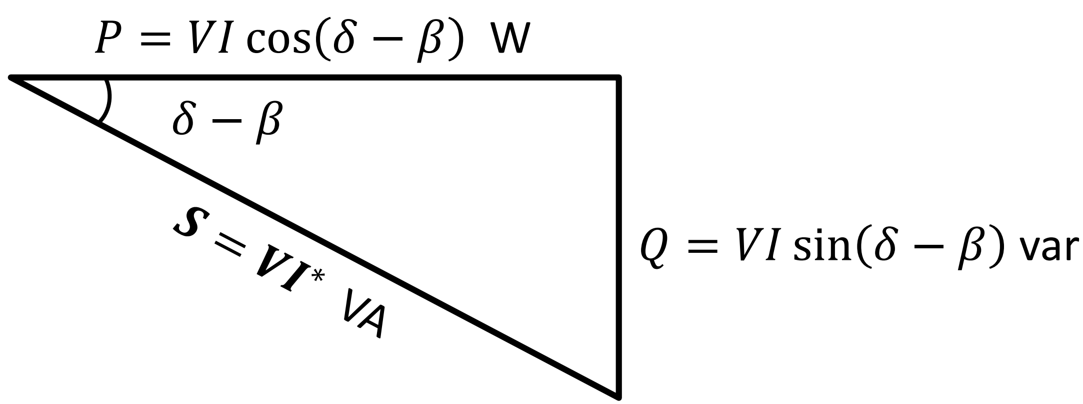
- For loads with ***inductive*** reactance
  - Impedance angle is positive
  - Power factor angle is positive
  - Power factor is ***lagging***



- Q is positive
  - The load ***absorbs*** reactive power

Leading Power Factor

- For loads with ***capacitive*** reactance
  - Impedance angle is negative
  - Power factor angle is negative
  - Power factor is ***leading***



- Q is negative
  - The load ***supplies*** reactive power

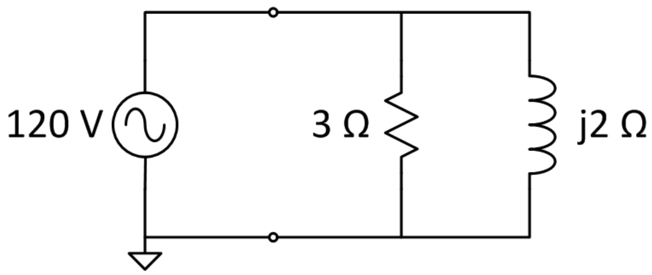
Power Factor Correction

Power Factor Correction

- The overall goal of power distribution is to supply power to do work
  - *Real* power
- Reactive power does not perform work, but
  - Must be supplied by the source
  - Still flows over the lines
- For a given amount of real power consumed by a load, we'd like to
  - Reduce reactive power, Q
  - Reduce S relative to P, that is
  - Reduce the p.f. angle, and
  - Increase the p.f.
- ***Power factor correction***

Power Factor Correction – Example

- Consider a source driving an inductive load



- Determine:
  - Real power absorbed by the load
  - Reactive power absorbed by the load
  - p.f. angle and p.f.
- Draw the power triangle

- Current through the resistance is

$$\boldsymbol{I_R} = \frac{120\ V}{3\ \Omega} = 40\ A$$

- Current through the inductance is

$$\boldsymbol{I_L} = \frac{120\ V}{j2\ \Omega} = 60\angle -90°\ A$$

- The total load current is

$$\boldsymbol{I} = \boldsymbol{I_R} + \boldsymbol{I_L} = (40 - j60)A = 72.1\angle -56.3°\ A$$

Power Factor Correction – Example

- The power factor angle is

$$\theta = (\delta - \beta) = 0° - (-56.3°)$$

$$\boxed{\theta = 56.3°}$$

- The power factor is

$$p.f. = \cos(\theta) = \cos(56.3°)$$

$$\boxed{p.f. = 0.55 \ \text{lagging}}$$

- Real power absorbed by the load is

$$P = VI\cos(\theta) = 120\ V \cdot 72.1\ A \cdot 0.55$$

$$\boxed{P = 4.8\ kW}$$

- Alternatively, recognizing that real power is power absorbed by the resistance

$$P = VI_R = 120\ V \cdot 40\ A = 4.8\ kW$$

Power Factor Correction – Example

- Reactive power absorbed by the load is

$$Q = VI\sin(\theta) = 120\,V \cdot 72.1\,A \cdot 0.832$$

$$\boxed{Q = 7.2\,kvar}$$

- This is also the power absorbed by the load inductance

$$Q = VI_L = 120\,V \cdot 60\,A = 7.2\,kvar$$

- Apparent power is

$$S = VI = 120\,V \cdot 72.1\,A = 8.65\,kVA$$

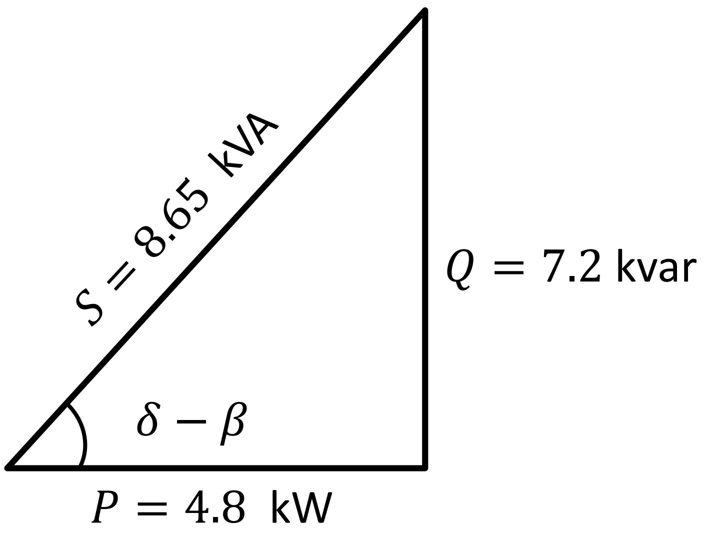
- Or, alternatively

$$S = \sqrt{P^2 + Q^2}$$

$$S = \sqrt{(4.8\,kW)^2 + (7.2\,kvar)^2} = 8.65\,kVA$$

Power Factor Correction – Example

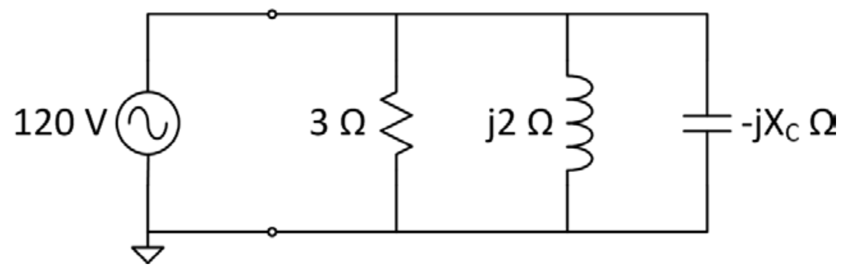
- The ***power triangle***:
- Here, the source is supplying 4.8 kW at a power factor of 0.55 lagging
- Let's say we want to reduce the apparent power supplied by the source
  - Deliver 4.8 kW at a p.f. of 0.9 lagging
- Add ***power factor correction***
- Add capacitors to *supply* reactive power



Power Factor Correction – Example

- For $p.f. = 0.9$, we need a power factor angle of

$$\theta' = \cos^{-1}(0.9) = 25.8°$$



- This corresponds to a reactive power of

$$Q' = P\tan(\theta') = 4.8\ kW \cdot \tan(25.8°)$$

$$Q' = 2.32\ kvar$$

- This means that the required reactive power absorbed (negative, so it is supplied) by the capacitors is

$$Q_C = 2.32\ kvar - 7.2\ kvar = -4.88\ kvar$$

Power Factor Correction – Example

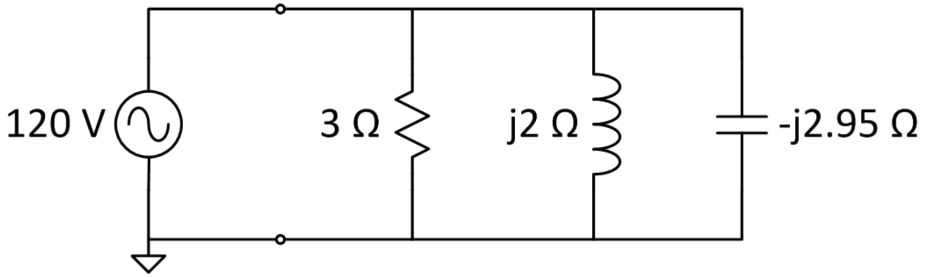
- Reactive power absorbed by the capacitor is

$$Q_C = \frac{V^2}{X_C}$$

- So the required capacitive reactance is

$$X_C = \frac{V^2}{Q_C} = \frac{(120\ V)^2}{-4.88\ kvar} = -2.95\ \Omega$$

- The addition of $-j2.95\ \Omega$ provides the desired power factor correction



Power Factor Correction – Example

- Before power factor correction, current supplied to the load was

$$\mathbf{I} = 72.1\angle-56.3°\ A$$

- Current through the added capacitor:

$$\mathbf{I_C} = \frac{\mathbf{V}}{Z_C} = \frac{120\angle 0°}{-j2.95\ \Omega} = 40.7\angle 90°\ A$$

- After power factor correction, current to the load is

$$\mathbf{I} = \mathbf{I_R} + \mathbf{I_L} + \mathbf{I_C} = 40\angle 0° + 60\angle-90° + 40.7\angle 90°$$

$$\mathbf{I} = 44.4\angle-25.8°\ A$$

- Power factor correction has reduced current to the load:

$$\Delta I = \frac{72.1 - 44.4}{72.1} \cdot 100\% = 38\%$$

- In practice, losses would be reduced
  - I^2R losses in resistances between source and load