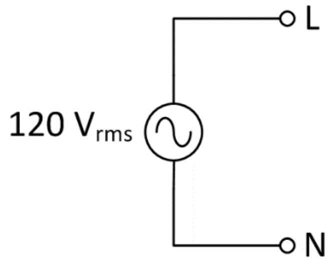
SECTION 7: THREE-PHASE CIRCUIT FUNDAMENTALS

ENGR 202 – Electrical Fundamentals II

Balanced Three-Phase Networks

Balanced Three-Phase Networks

- We are accustomed to ***single-phase*** power in our homes and offices
  - A single ***line*** voltage referenced to a ***neutral***

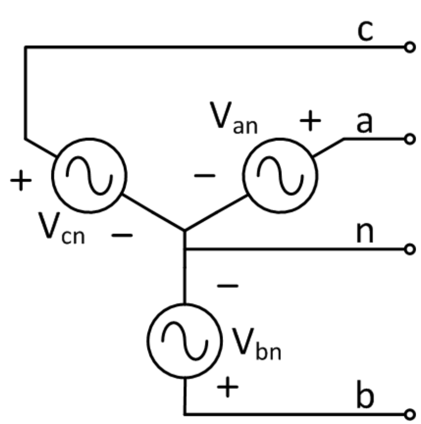


- Electrical power is generated, transmitted, and largely consumed (by industrial customers) as ***three-phase power***
  - Three individual line voltages and (possibly) a neutral
  - Line voltages all differ in phase by $\pm 120°$

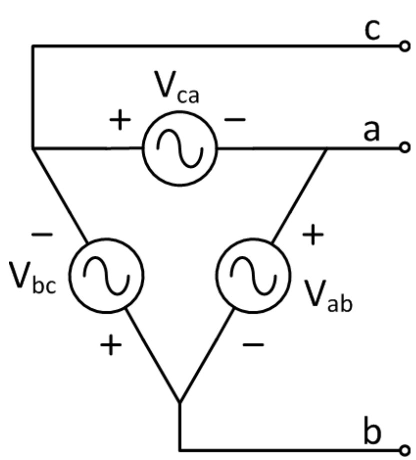
Δ- and Y-Connected Networks

- Two possible three-phase configurations
  - Applies to both ***sources*** and ***loads***

Y-Connected Source



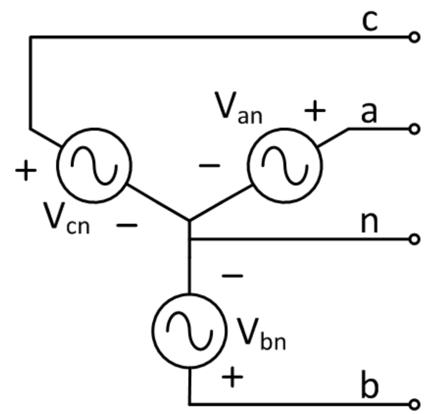
Δ-Connected Source



- Y-connected network has a neutral node
- Δ-connected network has no neutral

Line-to-Neutral Voltages

- In the Y network, voltages V_{an}, V_{bn}, and V_{cn} are ***line-to-neutral voltages***
- A three-phase source is ***balanced*** if
  - Line-to-neutral voltages have equal magnitudes
  - Line-to-neutral voltage are each $120°$ out of phase with one another



- A three-phase network is balanced if
  - Sources are balanced
  - The impedances connected to each phase are equal

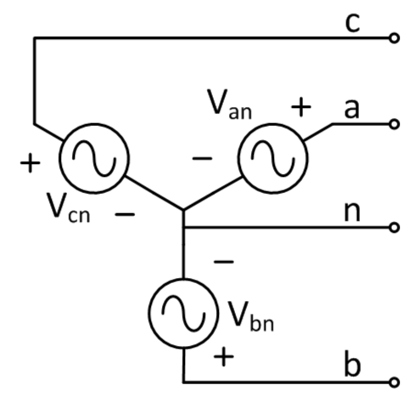
Line-to-Neutral Voltages

- The line-to-neutral voltages are

$$V_{an} = V_{LN}\angle 0°$$

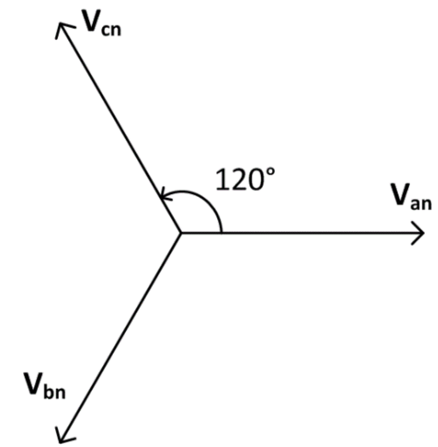
$$V_{bn} = V_{LN}\angle -120°$$

$$V_{cn} = V_{LN}\angle -240° = V_{LN}\angle +120°$$



- This is a ***positive-sequence*** or ***abc-sequence*** source
  - V_{an} leads V_{bn}, which leads V_{cn}
- Can also have a ***negative-*** or ***acb-sequence*** source
  - V_{an} leads V_{cn}, which leads V_{bn}
- We'll always assume *positive*-sequence sources

Positive-Sequence Phasor Diagram:

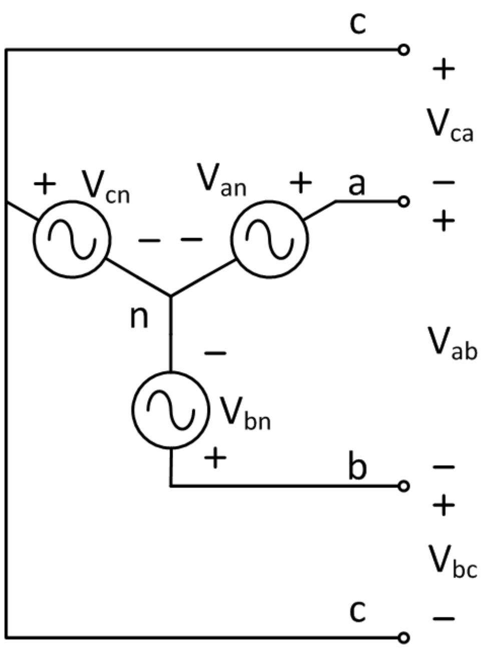


Line-to-Line Voltages

- The voltages between the three phases are ***line-to-line voltages***
- Apply KVL to relate line-to-line voltages to line-to-neutral voltages

$$\boldsymbol{V_{ab}} - \boldsymbol{V_{an}} + \boldsymbol{V_{bn}} = \boldsymbol{0}$$

$$\boldsymbol{V_{ab}} = \boldsymbol{V_{an}} - \boldsymbol{V_{bn}}$$



- We know that

$$\boldsymbol{V_{an}} = V_{LN}\angle 0°$$

and

$$\boldsymbol{V_{bn}} = V_{LN}\angle -120°$$

so

$$\boldsymbol{V_{ab}} = V_{LN}\angle 0° - V_{LN}\angle -120° = V_{LN}(1\angle 0° - 1\angle -120°)$$

$$\boldsymbol{V_{ab}} = V_{LN}\left[1 - \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right] = V_{LN}\left[\frac{3}{2} + j\frac{\sqrt{3}}{2}\right]$$

$$\boxed{\boldsymbol{V_{ab}} = \sqrt{3}V_{LN}\angle 30°}$$

Line-to-Line Voltages

- Again applying KVL, we can find $\boldsymbol{V_{bc}}$

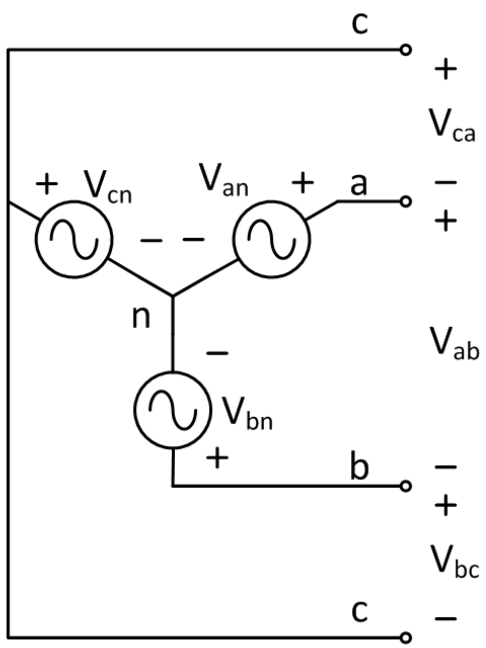
$$\boldsymbol{V_{bc}} = \boldsymbol{V_{bn}} - \boldsymbol{V_{cn}}$$

$$\boldsymbol{V_{bc}} = V_{LN}\angle -120° - V_{LN}\angle 120°$$

$$\boldsymbol{V_{bc}} = V_{LN}\left[\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right]$$

$$\boldsymbol{V_{bc}} = V_{LN}\left(-j\sqrt{3}\right)$$

$$\boxed{\boldsymbol{V_{bc}} = \sqrt{3}V_{LN}\angle -90°}$$



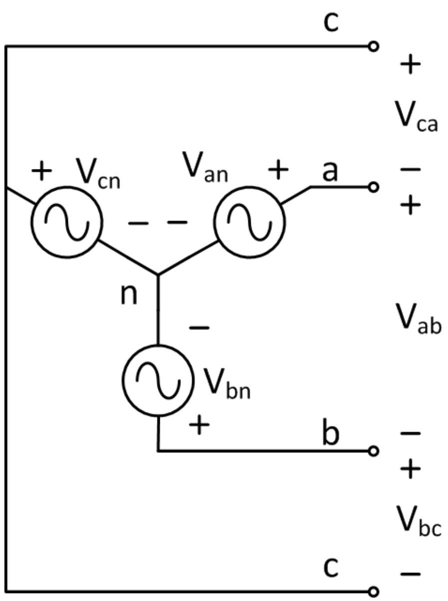
- Similarly,

$$\boxed{\boldsymbol{V_{ca}} = \sqrt{3}V_{LN}\angle 150°}$$

Line-to-Line Voltages

- The line-to-line voltages, with V_{an} as the reference:

$$\boldsymbol{V_{ab}} = \sqrt{3}V_{LN}\angle 30°$$
$$\boldsymbol{V_{bc}} = \sqrt{3}V_{LN}\angle -90°$$
$$\boldsymbol{V_{ca}} = \sqrt{3}V_{LN}\angle 150°$$

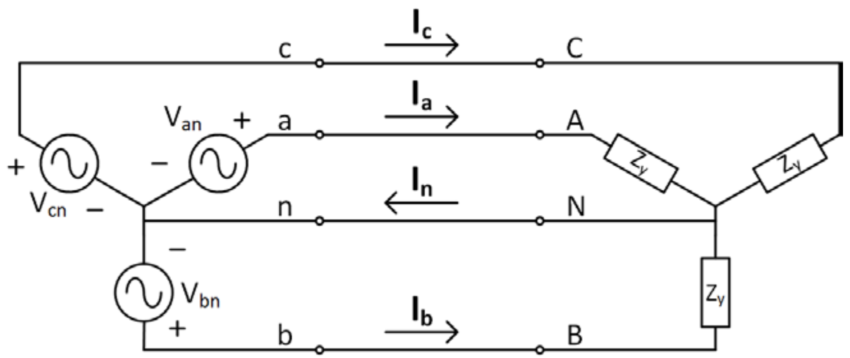


- Line-to-line voltages are $\sqrt{3}$ times the line-to-neutral voltage
- Can also express in terms of individual line-to-neutral voltages:

$$\boldsymbol{V_{ab}} = \sqrt{3}\boldsymbol{V_{an}}\angle 30°$$
$$\boldsymbol{V_{bc}} = \sqrt{3}\boldsymbol{V_{bn}}\angle 30°$$
$$\boldsymbol{V_{ca}} = \sqrt{3}\boldsymbol{V_{cn}}\angle 30°$$

Line Currents in Balanced 3ϕ Networks

- We can use the line-to-neutral voltages to determine the line currents
  - Y-connected source and load
  - Balanced load – all impedances are equal: Z_Y



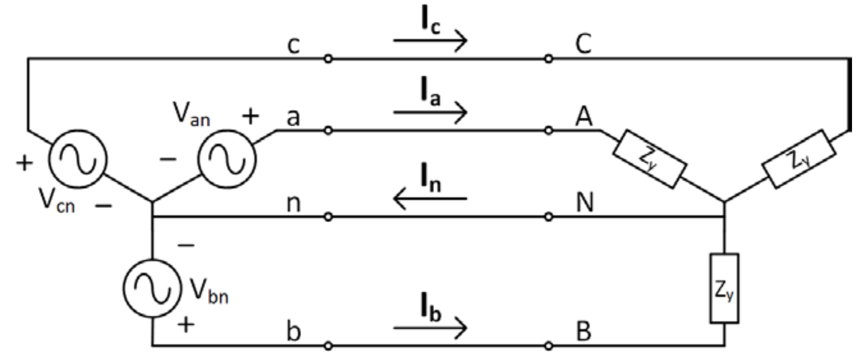
$$\boldsymbol{I_a} = \frac{\boldsymbol{V_{AN}}}{Z_Y} = \frac{V_{LN}\angle 0°}{Z_Y}$$

$$\boldsymbol{I_b} = \frac{\boldsymbol{V_{BN}}}{Z_Y} = \frac{V_{LN}\angle -120°}{Z_Y}$$

$$\boldsymbol{I_c} = \frac{\boldsymbol{V_{CN}}}{Z_Y} = \frac{V_{LN}\angle +120°}{Z_Y}$$

- Line currents are balanced as long as the source and load are balanced

Neutral Current in Balanced 3ϕ Networks



- Apply KCL to determine the neutral current

$$\boldsymbol{I_n} = \boldsymbol{I_a} + \boldsymbol{I_b} + \boldsymbol{I_c}$$

$$\boldsymbol{I_n} = \frac{V_{LN}}{Z_Y}[1\angle 0° + 1\angle -120° + 1\angle 120°]$$

$$\boldsymbol{I_n} = \frac{V_{LN}}{Z_Y}\left[1 + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right]$$

$$\boldsymbol{I_n} = 0$$

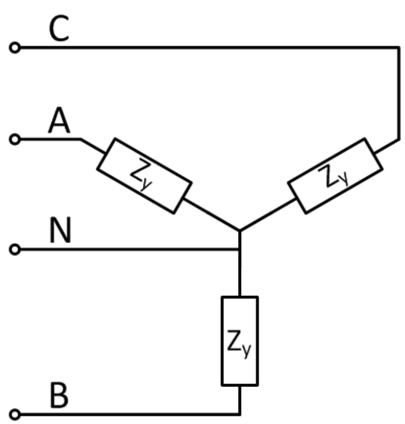
- The neutral conductor carries no current in a balanced three-phase network

Delta- & Wye-Connected Networks

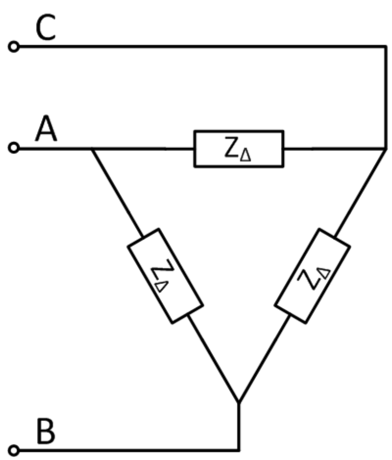
Three-Phase Network Configurations

- As for sources, three-phase loads can also be connected in two different configurations

Y-Connected Load



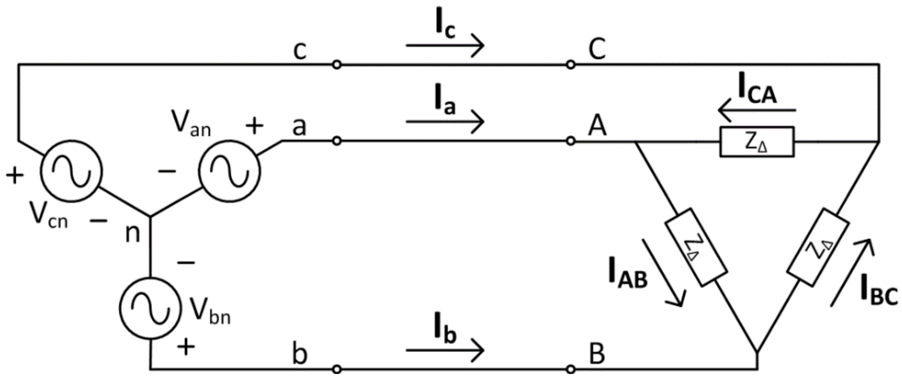
Δ-Connected Load



- The Y load has a neutral connection, but the Δ load does not
- Currents in a Y-connected load are the line currents we just determined
- Next, we'll look at currents in a Δ-connected load

Balanced Δ-Connected Loads

- We can use line-to-line voltages to determine the currents in Δ-connected loads



$$\boldsymbol{I}_{AB} = \frac{\boldsymbol{V}_{AB}}{Z_\Delta} = \frac{\sqrt{3}\boldsymbol{V}_{AN}\angle 30°}{Z_\Delta} = \frac{\sqrt{3}V_{LN}\angle 30°}{Z_\Delta}$$

$$\boldsymbol{I}_{BC} = \frac{\boldsymbol{V}_{BC}}{Z_\Delta} = \frac{\sqrt{3}\boldsymbol{V}_{BN}\angle 30°}{Z_\Delta} = \frac{\sqrt{3}V_{LN}\angle -90°}{Z_\Delta}$$

$$\boldsymbol{I}_{CA} = \frac{\boldsymbol{V}_{CA}}{Z_\Delta} = \frac{\sqrt{3}\boldsymbol{V}_{CN}\angle 30°}{Z_\Delta} = \frac{\sqrt{3}V_{LN}\angle 150°}{Z_\Delta}$$

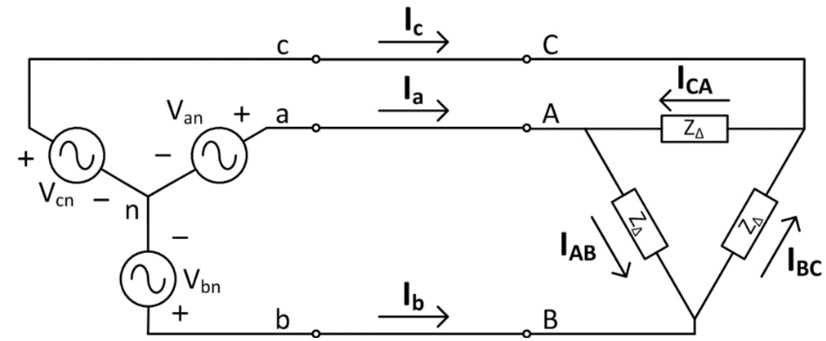
Balanced Δ-Connected Loads

- Applying KCL, we can determine the line currents

$$\boldsymbol{I_a} = \boldsymbol{I_{AB}} - \boldsymbol{I_{CA}}$$

$$\boldsymbol{I_a} = \frac{\sqrt{3}V_{LN}}{Z_\Delta}[1\angle 30° - 1\angle 150°]$$

$$\boldsymbol{I_a} = \frac{\sqrt{3}V_{LN}}{Z_\Delta}\left[\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) - \left(-\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)\right] = \frac{\sqrt{3}V_{LN}}{Z_\Delta}\left[\sqrt{3}\right] = \frac{3V_{LN}}{Z_\Delta}$$



- The other line currents can be found similarly:

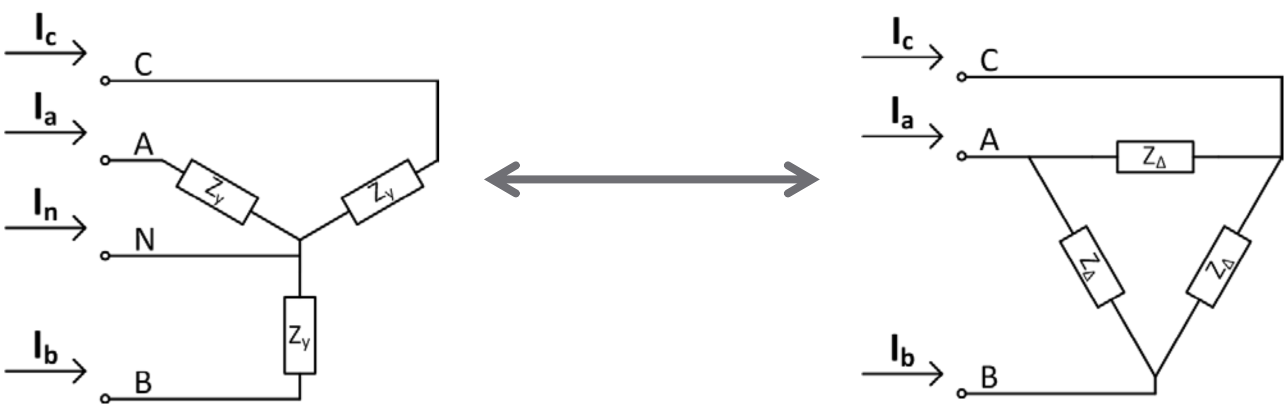
$$\boldsymbol{I_a} = \frac{3V_{LN}\angle 0°}{Z_\Delta} = \sqrt{3}\boldsymbol{I_{AB}}\angle -30°$$

$$\boldsymbol{I_b} = \frac{3V_{LN}\angle -120°}{Z_\Delta} = \sqrt{3}\boldsymbol{I_{BC}}\angle -30°$$

$$\boldsymbol{I_c} = \frac{3V_{LN}\angle 120°}{Z_\Delta} = \sqrt{3}\boldsymbol{I_{CA}}\angle -30°$$

$\Delta - Y$ Conversion

- Analysis is often simpler when dealing with Y-connected loads
  - Would like a way to convert Δ loads to Y loads (and vice versa)



- For a Y load and a Δ load to be equivalent, they must result in equal line currents

$\Delta - Y$ Conversion

- Line currents for a Y-connected load:

$$\boldsymbol{I_a} = \frac{V_{LN}\angle 0°}{Z_Y}$$

$$\boldsymbol{I_b} = \frac{V_{LN}\angle -120°}{Z_Y}$$

$$\boldsymbol{I_c} = \frac{V_{LN}\angle 120°}{Z_Y}$$

- For a Δ-connected load:

$$\boldsymbol{I_a} = \frac{3V_{LN}\angle 0°}{Z_\Delta}$$

$$\boldsymbol{I_b} = \frac{3V_{LN}\angle -120°}{Z_\Delta}$$

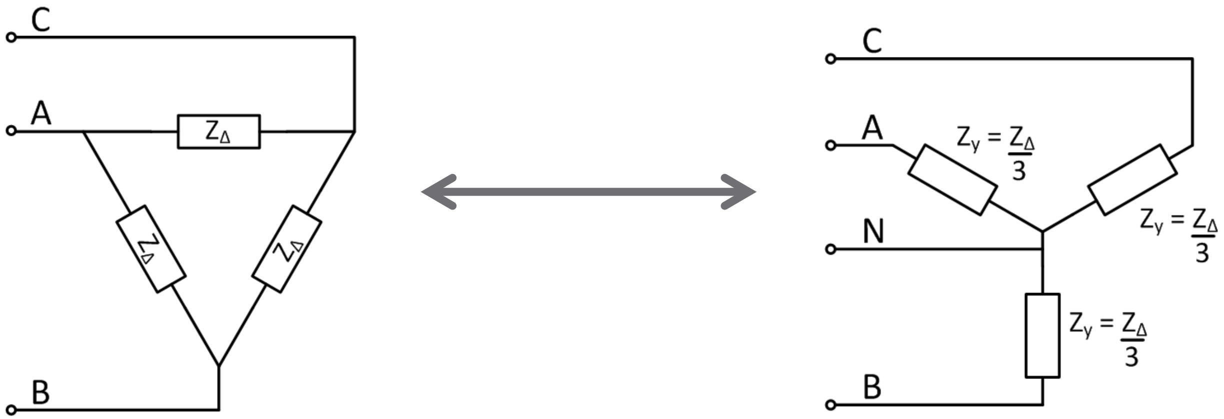
$$\boldsymbol{I_c} = \frac{3V_{LN}\angle 120°}{Z_\Delta}$$

$\Delta - Y$ Conversion

- Equating any of the three line currents, we can determine the impedance relationship

$$\frac{V_{LN}\angle 0°}{Z_Y} = \frac{3V_{LN}\angle 0°}{Z_\Delta}$$

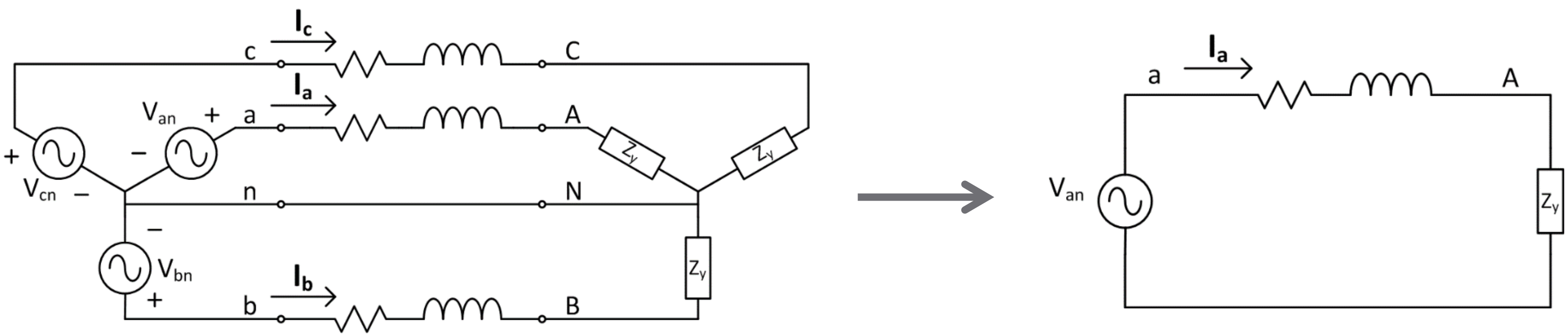
$$\boxed{Z_Y = \frac{Z_\Delta}{3}} \quad \text{and} \quad \boxed{Z\Delta = 3Z_Y}$$



Line-to-Neutral Schematics

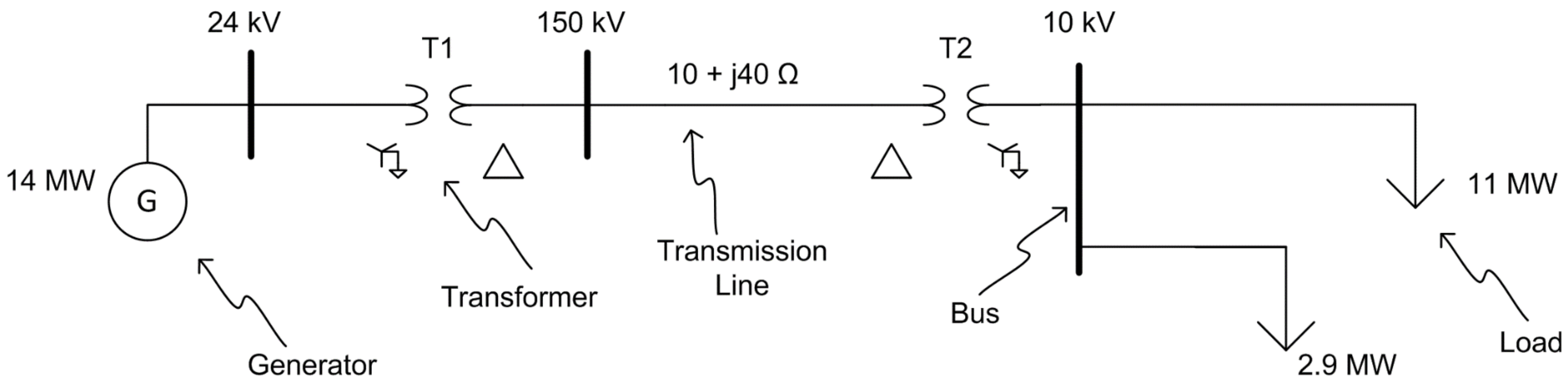
- For balanced networks, we can simplify our analysis by considering only a single phase
  - A ***per-phase analysis***
  - Other phases are simply shifted by $\pm 120°$

- For example, a balanced Y-Y circuit:



One-Line Diagrams

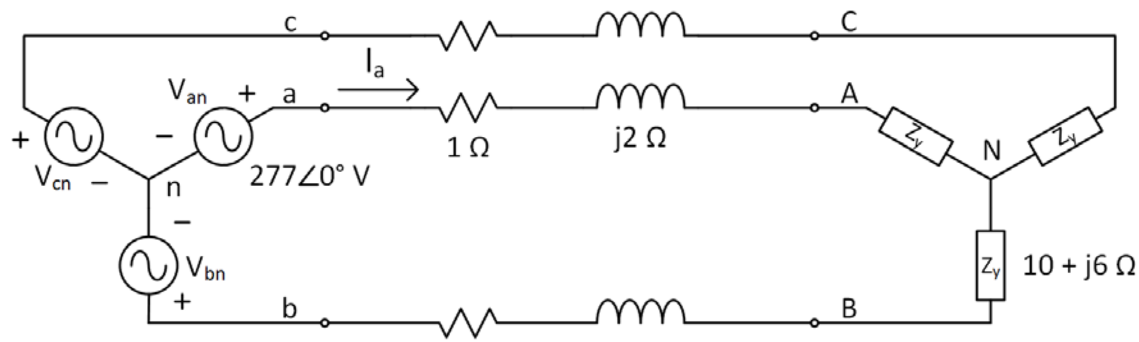
- Power systems are often depicted using ***one-line diagrams*** or ***single-line diagrams***
  - Not a schematic – not all wiring is shown
- For example:



Example Problems

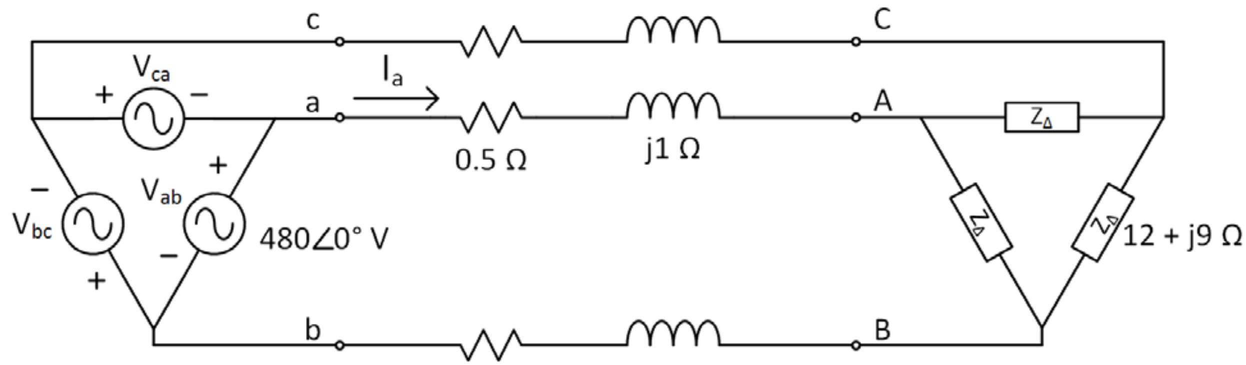
Determine:

- $\mathbf{I_a}$
- $\mathbf{V_{AN}}$
- $\mathbf{V_{AB}}$



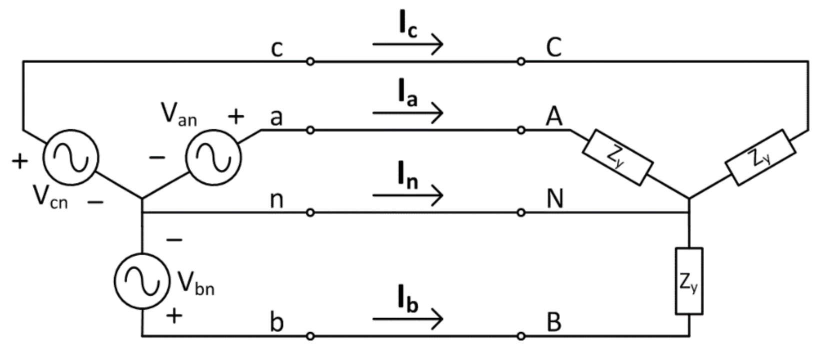
Determine:

- $\mathbf{I_a}$
- $\mathbf{V_{AB}}$
- $\mathbf{V_{AN}}$
- $\mathbf{V_{BC}}$



Power in Balanced 3ϕ Networks

Instantaneous Power



- We'll first determine the instantaneous power supplied by the source
  - Neglecting line impedance, this is also the power absorbed by the load
- The phase a line-to-neutral voltage is

$$v_{an}(t) = \sqrt{2}V_{LN}\cos(\omega t + \delta)$$

- The phase a current is

$$i_a(t) = \sqrt{2}I_L\cos(\omega t + \beta)$$

where β depends on the load impedance

Instantaneous Power

- The instantaneous power delivered out of phase a of the source is

$$p_a(t) = v_{an}(t) i_a(t)$$

$$p_a(t) = 2V_{LN} I_L \cos(\omega t + \delta) \cos(\omega t + \beta)$$

$$p_a(t) = V_{LN} I_L \cos(\delta - \beta) + V_{LN} I_L \cos(2\omega t + \delta + \beta)$$

- The b and c phases are shifted by $\pm 120°$
  - Power from each of these phases is

$$p_b(t) = V_{LN} I_L \cos(\delta - \beta) + V_{LN} I_L \cos(2\omega t + \delta + \beta - 240°)$$

$$p_c(t) = V_{LN} I_L \cos(\delta - \beta) + V_{LN} I_L \cos(2\omega t + \delta + \beta + 240°)$$

Instantaneous Power

- The *total* power delivered by the source is the sum of the power from each phase

$$p_{3\phi}(t) = p_a(t) + p_b(t) + p_c(t)$$

$$\begin{aligned} p_{3\phi}(t) = 3V_{LN}I_L\cos(\delta-\beta) \\ +V_{LN}I_L[\cos(2\omega t+\delta+\beta) \\ +\cos(2\omega t+\delta+\beta-240°) \\ +\cos(2\omega t+\delta+\beta+240°)] \end{aligned}$$

- Everything in the square brackets cancels, leaving

$$\boxed{p_{3\phi}(t) = 3V_{LN}I_L\cos(\delta-\beta) = P_{3\phi}}$$

- ***Power in a balanced*** $\boldsymbol{3\phi}$ ***network is constant***
- In terms of line-to-line voltages, the power is

$$\boxed{P_{3\phi} = \sqrt{3}V_{LL}I_L\cos(\delta-\beta)}$$

Complex Power

- The ***complex power*** delivered by phase a is

$$\boldsymbol{S_a} = \boldsymbol{V_{an}}\boldsymbol{I_a^*} = V_{LN}\angle\delta(I_L\angle\beta)^*$$

$$\boldsymbol{S_a} = V_{LN}I_L\angle(\delta - \beta)$$

$$\boldsymbol{S_a} = V_{LN}I_L\cos(\delta - \beta) + jV_{LN}I_L\sin(\delta - \beta)$$

- For phase b, complex power is

$$\boldsymbol{S_b} = \boldsymbol{V_{bn}}\boldsymbol{I_b^*} = V_{LN}\angle(\delta - 120°)\big(I_L\angle(\beta - 120°)\big)^*$$

$$\boldsymbol{S_b} = V_{LN}I_L\angle(\delta - \beta)$$

$$\boldsymbol{S_b} = V_{LN}I_L\cos(\delta - \beta) + jV_{LN}I_L\sin(\delta - \beta)$$

- This is equal to $\boldsymbol{S_a}$ and also to phase $\boldsymbol{S_c}$

Complex Power

- The ***total complex power*** is

$$\boldsymbol{S}_{\boldsymbol{3\phi}} = \boldsymbol{S}_{\boldsymbol{a}} + \boldsymbol{S}_{\boldsymbol{b}} + \boldsymbol{S}_{\boldsymbol{c}}$$

$$\boldsymbol{S}_{\boldsymbol{3\phi}} = 3V_{LN}I_L\angle(\delta - \beta)$$

$$\boldsymbol{S}_{\boldsymbol{3\phi}} = 3V_{LN}I_L\cos(\delta - \beta) + j3V_{LN}I_L\sin(\delta - \beta)$$

- The ***apparent power*** is the magnitude of the complex power

$$S_{3\phi} = 3V_{LN}I_L$$

Complex Power

- Complex power can be expressed in terms of the real and reactive power

$$\boldsymbol{S}_{\boldsymbol{3\phi}} = P_{3\phi} + jQ_{3\phi}$$

- The ***real power***, as we've already seen is

$$P_{3\phi} = 3V_{LN}I_L\cos(\delta - \beta)$$

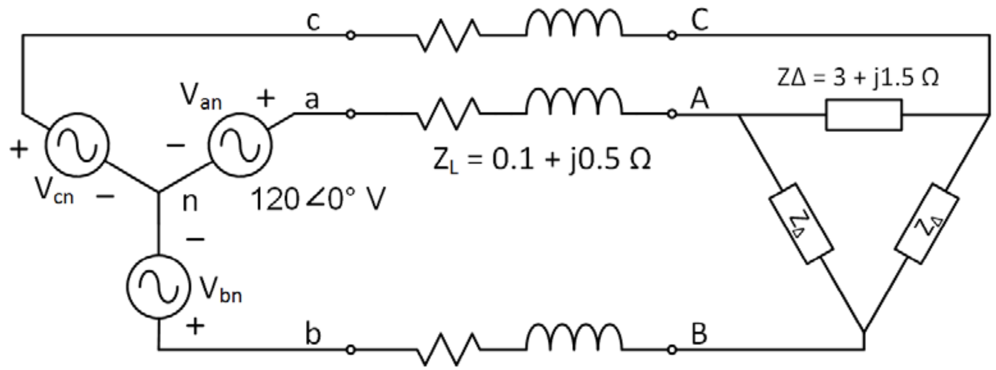
- The ***reactive power*** is

$$Q_{3\phi} = 3V_{LN}I_L\sin(\delta - \beta)$$

Advantages of Three-Phase Power

- Advantages of three-phase power:
  - For a given amount of power, ***half the amount of wire required*** compared to single-phase
    - No return current on neutral conductor
  - ***Constant real power***
    - Constant motor torque
    - Less noise and vibration of machinery

Three-Phase Power – Example



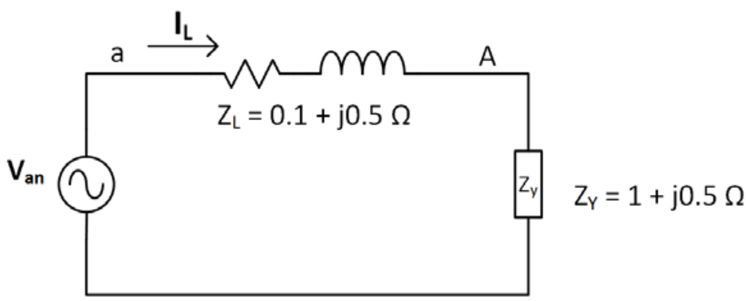
- Determine
  - Load voltage, $\boldsymbol{V_{AB}}$
  - Power triangle for the load
  - Power factor at the load

- We'll do a per-phase analysis, so first convert the Δ load to a Y load

$$Z_Y = \frac{Z_\Delta}{3} = 1 + j0.5\ \Omega$$

Three-Phase Power – Example

- The per-phase circuit:



- The line current is

$$\boldsymbol{I_L} = \frac{\boldsymbol{V_{an}}}{Z_L + Z_Y} = \frac{120\angle 0^\circ\ V}{1.1 + j1\ \Omega} = \frac{120\angle 0^\circ\ V}{1.45\angle 42.3^\circ\ \Omega}$$

$$\boldsymbol{I_L} = 80.7\angle -42.3^\circ\ A$$

- The line-to-neutral voltage at the load is

$$\boldsymbol{V_{AN}} = \boldsymbol{I_L} Z_Y = (80.7\angle -42.3^\circ\ A)(1 + j0.5\ \Omega)$$

$$\boldsymbol{V_{AN}} = (80.7\angle -42.3^\circ\ A)(1.12\angle 26.6^\circ\ \Omega)$$

$$\boldsymbol{V_{AN}} = 90.25\angle -15.71^\circ\ \mathrm{V}$$

Three-Phase Power – Example

- Calculate the line-to-line voltage from the line-to-neutral voltage

$$\boldsymbol{V_{AB}} = \sqrt{3}\boldsymbol{V_{AN}}\angle 30°$$

$$\boxed{\boldsymbol{V_{AB}} = 156\angle 14.3°\ V}$$

- Alternatively, we could calculate line-to-line voltage from the two line-to-neutral voltages.
  - The line-to-neutral voltage at phase B is

$$\boldsymbol{V_{BN}} = 90.25\angle - 135.71°\ V$$

  - So the line-to-line voltage is given by

$$\boldsymbol{V_{AB}} = \boldsymbol{V_{AN}} - \boldsymbol{V_{BN}} = 156\angle 14.3°\ V$$

Three-Phase Power – Example

- The complex power absorbed by the load is

$$\boldsymbol{S}_{3\phi} = 3\boldsymbol{S}_{A} = 3\boldsymbol{V}_{AN}\boldsymbol{I}_{L}^{*}$$

$$\boldsymbol{S}_{3\phi} = 3(90.25\angle -15.71°\,V)(80.7\angle -42.3°\,A)^{*}$$

$$\boldsymbol{S}_{3\phi} = 21.85\,\angle 26.6°\,kVA$$

$$\boldsymbol{S}_{3\phi} = 19.53 + j9.78\,kVA$$

- The apparent power:

$$S_{3\phi} = 21.85\,kVA$$

- Real power:

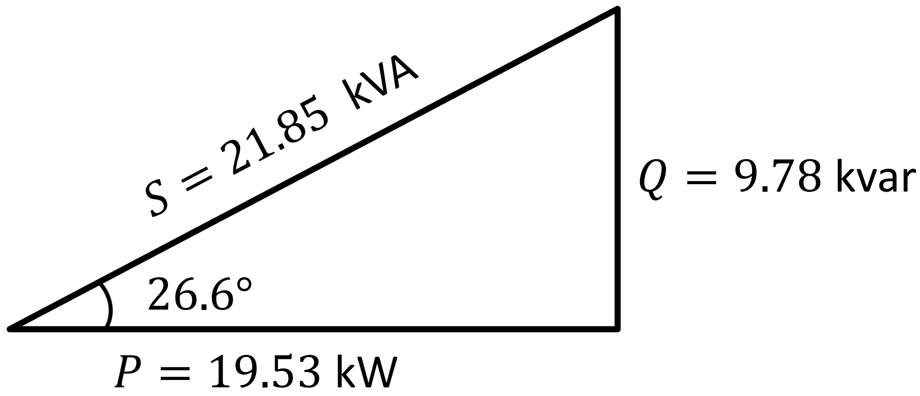
$$P = 19.53\,kW$$

- Reactive power:

$$Q = 9.78\,kvar$$

Three-Phase Power – Example

- The power triangle at the load:



- The power factor at the load is

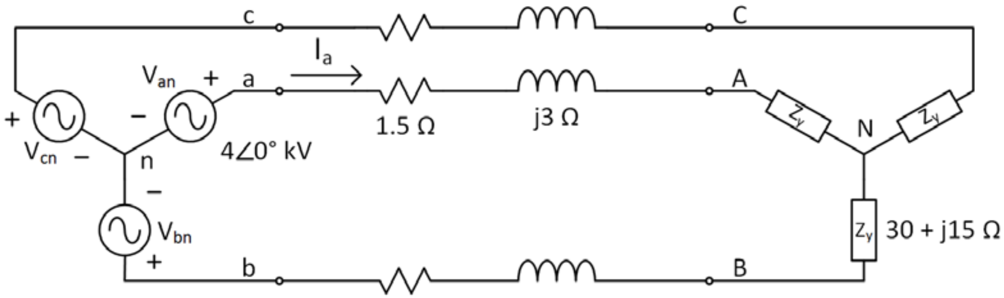
$$p.f. = \cos(26.6°) = \frac{P}{S} = \frac{19.53\ kW}{21.85\ kVA}$$

$$\boxed{p.f. = 0.89\ lagging}$$

Example Problems

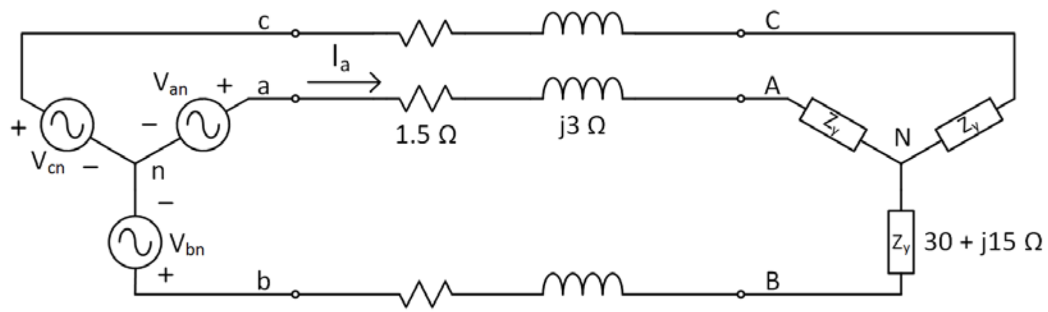
Determine complex power:

- From the source
- To the load
- To the line



Line-to-line voltage at the load is maintained at 4.16 kV.

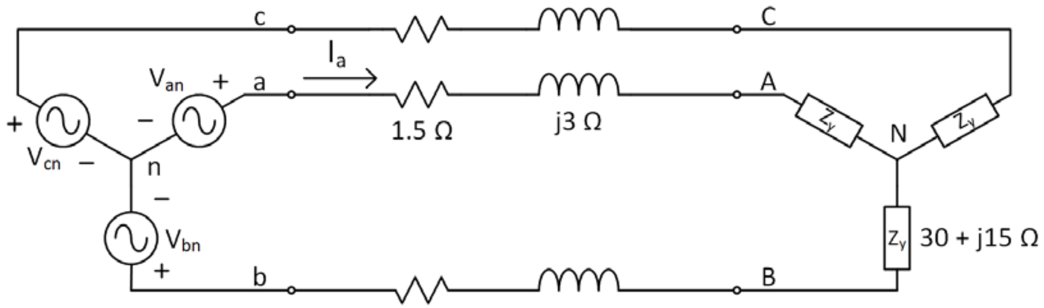
What is the voltage at the source? How much complex power is delivered by the source?



Line-to-line voltage at the load is maintained at 4.16 kV.

Determine:

- Power factor at load
- Power triangle at load
- Loss in Lines



Add power factor correction to improve p.f. to 0.98, lagging.

Determine:

- Power triangle at load
- Loss in Lines