SECTION 1: INTRODUCTION

ESE 330 – Modeling & Analysis of Dynamic Systems

Modeling and Analysis

Modeling and Analysis

- As engineers, we are interested in analyzing and designing physical ***systems***
- What is a ***system?***
  - Any entity comprised of ***interacting components***
  - Systems have ***inputs*** and ***outputs***
    - Not necessarily explicit
    - System characteristics determine how inputs translate to outputs
  - ***Separable from its surroundings or environment***
    - Physically or conceptually
    - May interact – via inputs and outputs – with its environment
  - May be composed of multiple integrated ***subsystems***
- Examples of systems:
  - Refrigeration unit
  - Mobile phone
  - Industrial robot
  - Computer software
  - Satellite
  - Engine
  - Stock market
  - Etc…

System Models

- Want to be able to describe these systems in a tractable, mathematical way
- We represent these systems with ***models:***
  - Abstracted representation of the real system
  - Captures *some* aspects of the real system's behavior – the behavior we care about – while ignoring others
  - ***Simplified*** in some way
    - Smaller
    - Less complex
    - Linear
    - Lossless, etc. …

System Models

- Model of a physical system may be:
  - A physical system itself, simplified in some way
    - e.g., scale model for wind-tunnel testing
  - A ***mathematical model***
    - An ***equation*** or ***system of equations*** that describe the aspects of system behavior that interest us (while ignoring others)
  - A physical model as an intermediate step in generating a mathematical model
    - An abstraction of the real system, whose behavior we can describe with mathematical expressions

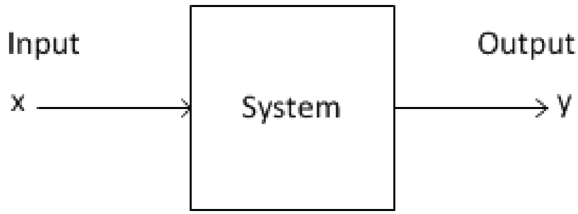
Analysis & Simulation

- Model used for ***analysis*** and ***simulation*** of the system
  - Analysis of system behavior
  - Could be ***physical simulation***, e.g. aerodynamic testing in a wind tunnel
  - Here, we're interested in ***mathematical simulation***
    - Could be either ***analytical*** or ***numerical***

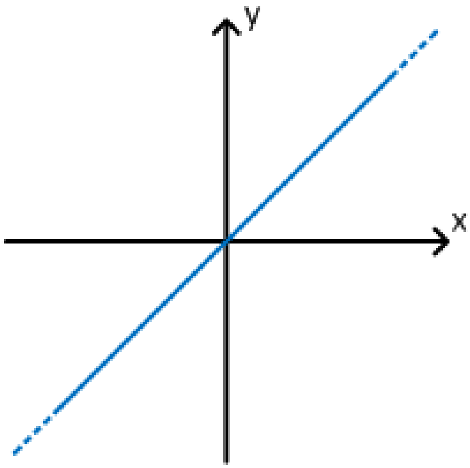
- Why simulate?
  - ***Analysis***
    - How does a system respond to different types of inputs?
    - How does the response depend on component parameters?...
  - ***Design***
    - Modifying the system parameters to achieve desired behavior
    - Control system design – adding feedback and a controller to the system to improve system performance

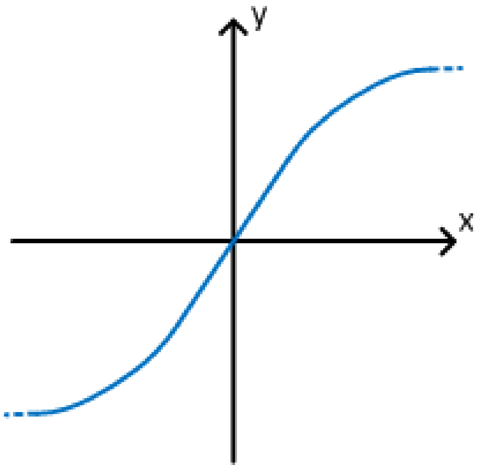
Linear vs. Nonlinear Systems

Linear vs. Nonlinear Systems



- Systems take inputs and yield outputs
  - Could be force, velocity, voltage, current, etc. …
- ***Transfer characteristics*** relate outputs to inputs These may be ***linear*** or ***nonlinear***





Linear vs. Nonlinear Systems

- ***Linear systems*** are comprised of ***linear components***
  - I.e., those with linear transfer characteristics
- Linear systems are described by ***linear differential equations***, e.g.

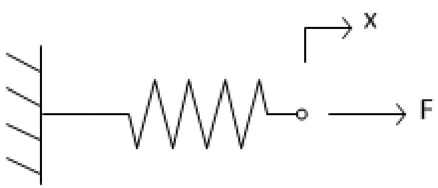
$$m\ddot{x} + b\dot{x} + kx = F_{in}(t)$$

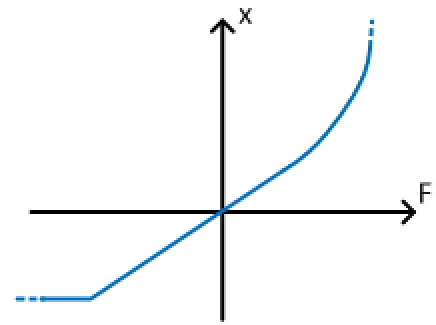
- Non-linear systems are described by ***nonlinear differential equations***, e.g.

$$m\ddot{x} + b \cdot ln(\dot{x}) + kx^2 = F_{in}(t)$$

Linear vs. Nonlinear Systems

- Consider, for example, a simple spring
  - Transfer characteristic relating displacement to force:





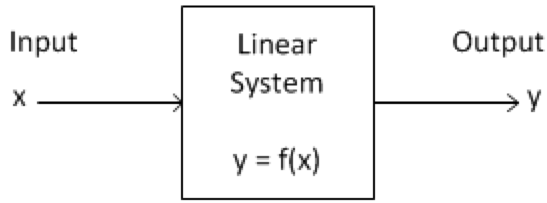
- Is the spring a linear component?
  - ***No*** – over a full range of force and displacement, it is clearly nonlinear
  - ***Yes*** – for small values of force and displacement the spring is accurately approximated as linear
    - Obeys Hooke's law: $x = \frac{1}{k} \cdot F$

No Such Thing as a Linear System

- ***Truly linear systems <u>do not exist</u> in reality***
  - All systems are inherently nonlinear
    - Some very nonlinear, others negligibly so
  - If stressed far enough, all systems will exhibit significant nonlinearity
- ***We will focus nearly exclusively on linear systems***
  - Simplifies modeling and analysis
  - Many systems can be accurately modeled as linear over a small enough range
  - Linear system theory serves as the basis for dealing with nonlinear systems as well

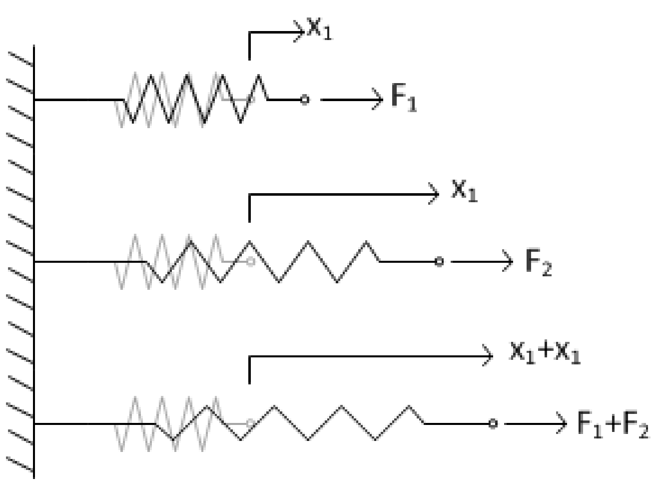
Superposition

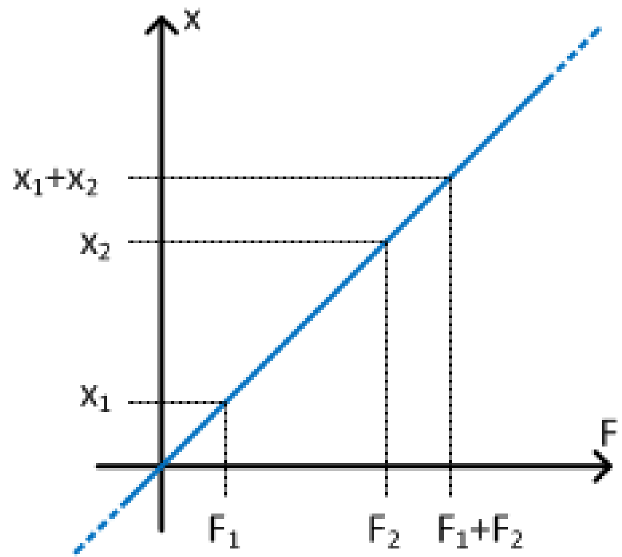
- The principle of ***superposition*** applies to linear systems



$$f(x_1) = y_1$$
$$f(x_2) = y_2$$
$$f(\alpha x_1 + \beta x_2) = \alpha y_1 + \beta y_2$$

- For example, a ***linear spring***:





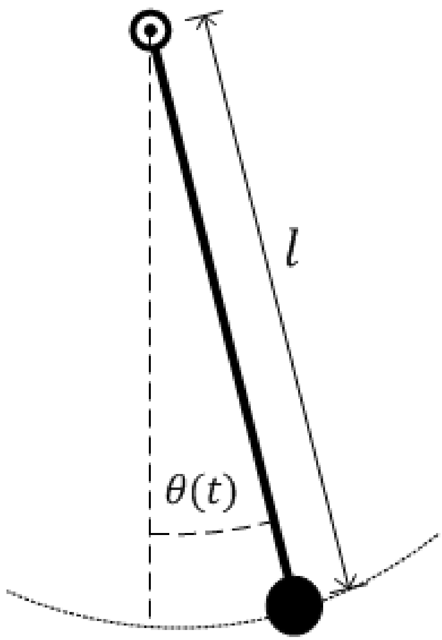
Linearization – Example

- A simple pendulum is a ***nonlinear system***

$$\ddot{\theta} = \frac{g}{l}\sin(\theta) - \frac{1}{ml}F_d(\dot{\theta}) - \frac{1}{ml^2}\tau_f(\dot{\theta})$$

  - ***Nonlinear air resistance*** term, $F_d(\dot{\theta})$
    - Neglect it altogether
  - ***Nonlinear friction*** term, $\tau_f(\dot{\theta})$
    - Treat it as linear viscous friction:

$$\tau_f = b_\tau\dot{\theta}$$



- Pendulum model becomes:

$$\ddot{\theta} = \frac{g}{l}\sin(\theta) - \frac{1}{ml^2}b_\tau\dot{\theta}$$

- Still have the ***nonlinear*** $sin(\theta)$ term
  - Restrict angular displacement to very small values, where $\sin(\theta) \approx \theta$
- The ***linearized pendulum model***

$$\ddot{\theta} = \frac{g}{l}\theta - \frac{1}{ml^2}b_\tau\dot{\theta}$$

Mechanical System – Example

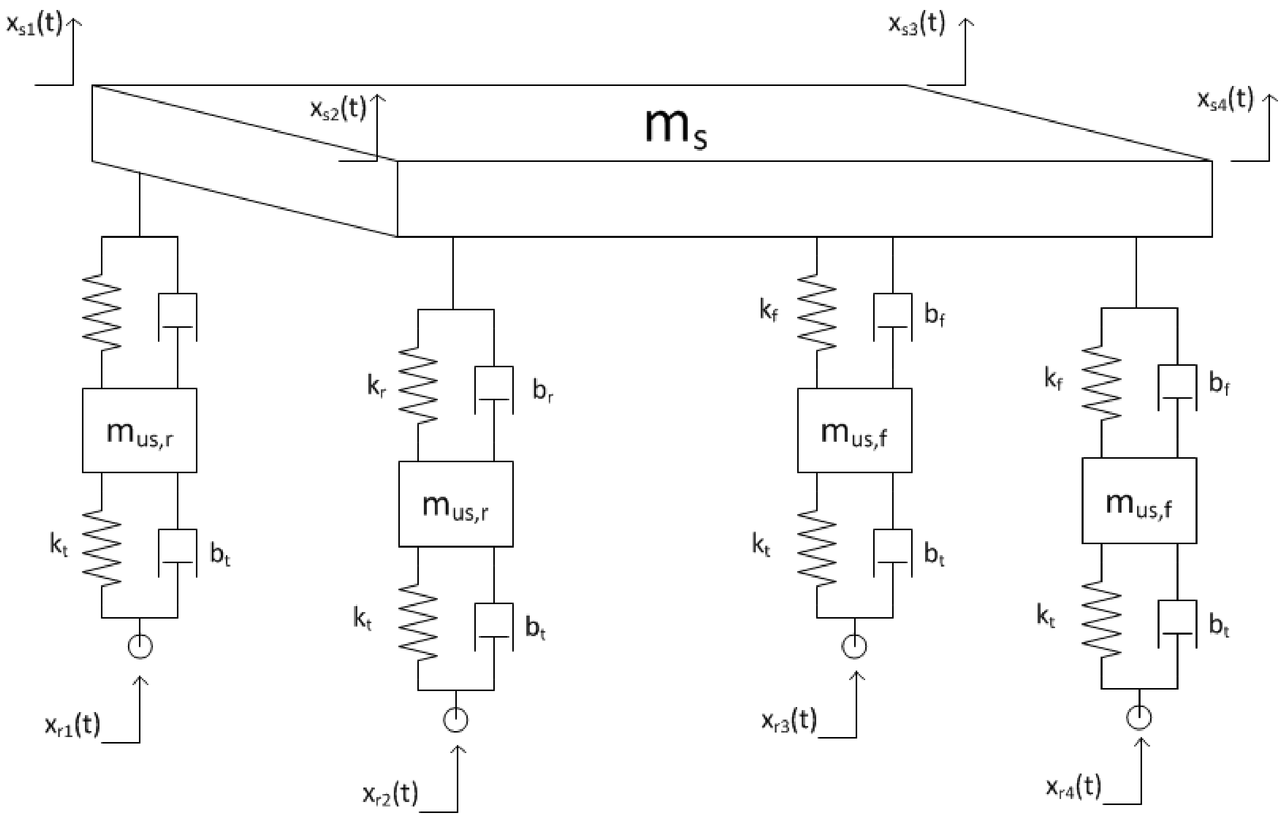
Without going into the details, we'll now walk through the process of modeling and simulating two different types of systems – the first mechanical, and the second electrical.

Vehicle Suspension System

- Suppose you want to analyze the performance of a vehicle suspension system
- Physical system:
  - Car body mass - the *sprung mass*
  - Four contact point to the road
    - Tires
      - Damped compliance
    - Wheels, etc. – the *unsprung mass*
    - Shock absorbers
      - A spring and a damper

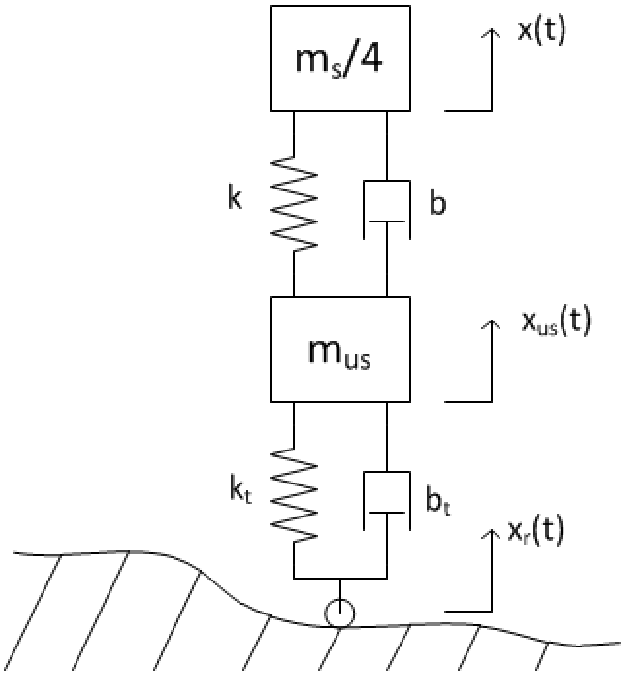
Initial Physical Model

- An initial model might look something like this:

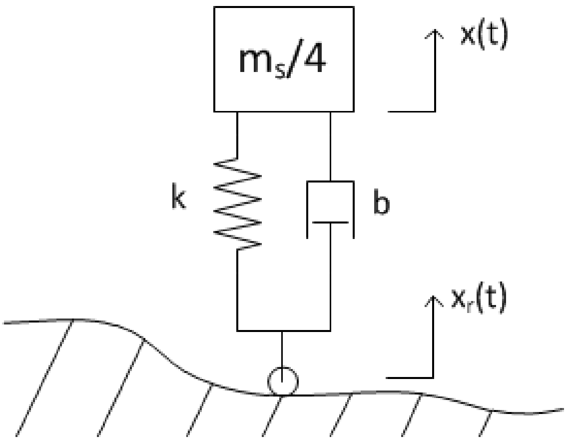


Simplified Physical Model

- Simplify the model by considering only one contact point at a time – the ***quarter-car model***
- Assume ***linear components*** – springs and dampers

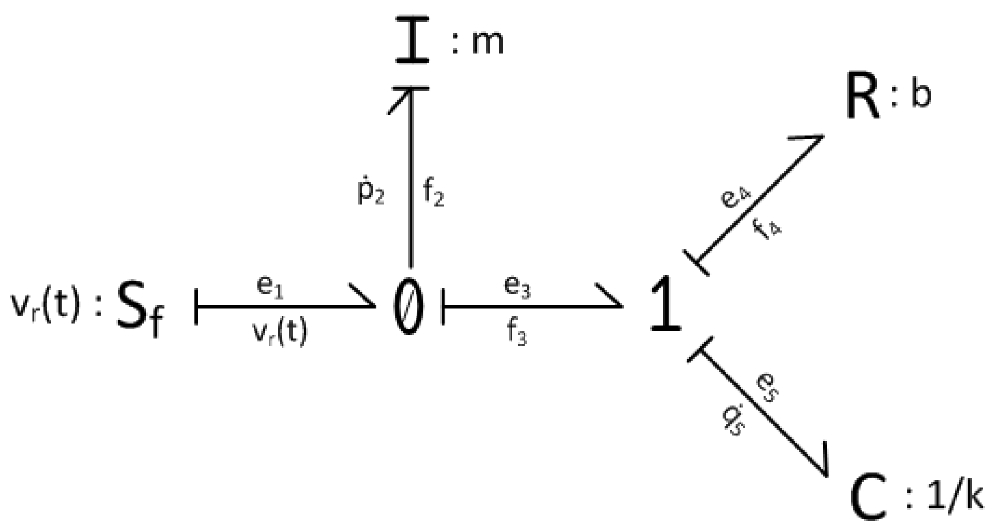


- Further simplify by neglecting the tire and unsprung mass
  - Less significant than suspension and sprung mass



Bond Graph Model

- The Physical model is specific to the type of system
  - Mechanical system – springs, masses, dampers
- A ***bond graph model*** is a universal model
  - Independent of domain
  - Based on the flows of energy within the system



Mathematical Model

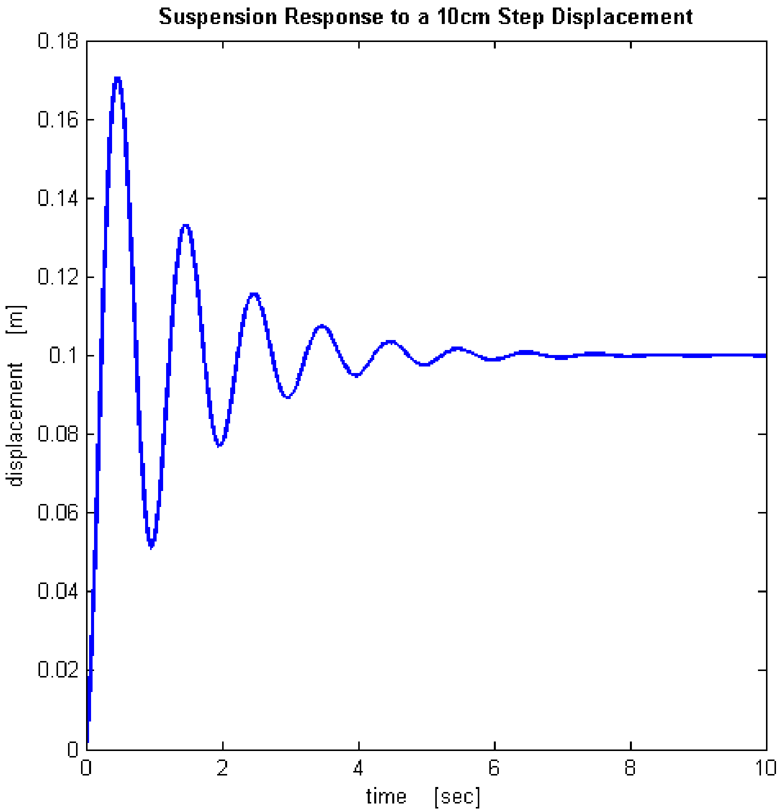
- Use the bond graph model to derive the ***mathematical model*** for the system
  - Governing differential equations in ***State-variable*** form

$$\begin{bmatrix} \dot{p}_2 \\ \dot{q}_5 \end{bmatrix} = \begin{bmatrix} -b/m & k \\ -1/m & 0 \end{bmatrix} \begin{bmatrix} p_2 \\ q_5 \end{bmatrix} + \begin{bmatrix} b \\ 1 \end{bmatrix} v_r(t)$$

- Note that we could have derived a ***similar***, though not necessarily identical, set of equations by skipping the bond graph model and simply applying Newton's 2nd law to the mass

Simulation

- Can now use the mathematical model to determine how the system will respond to various inputs, e.g.:
- How will the suspension respond to a 10 cm step displacement
  - Driving over a curb
- System parameters:
  - Sprung mass: $m = 500\,kg$
  - Spring constant: $k = 20\,\frac{kN}{m}$
  - Damping coeff.: $b = 750\,\frac{N\cdot s}{m}$
- Numerical simulation using MATLAB



Electrical System – Example

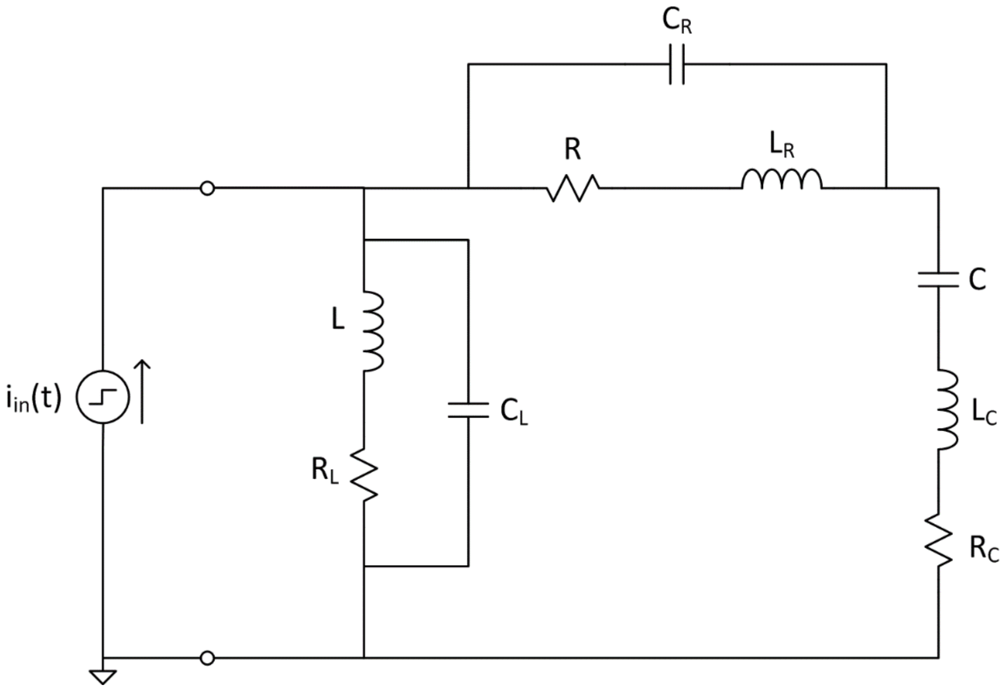
Just as we did for a mechanical system, we'll now step through the modeling and simulation procedure for an electrical system.

RLC Circuit

- Derive a model that can be used for the numerical simulation of an RLC electrical circuit
- Physical system is a circuit board, including the following:
  - ***Resistor***
    - Also includes some inductance and capacitance
  - ***Inductor***
    - Includes winding resistance and inter-turn capacitance
  - ***Capacitor***
    - Some equivalent series inductance and resistance
  - ***Traces***
    - Small amounts of series R and L, along with some shunt C – we'll neglect all trace parasitics immediately
  - ***Connectors***
    - Some small amount of R and/or L and/or C, depending on type of connector – we'll neglect this right away

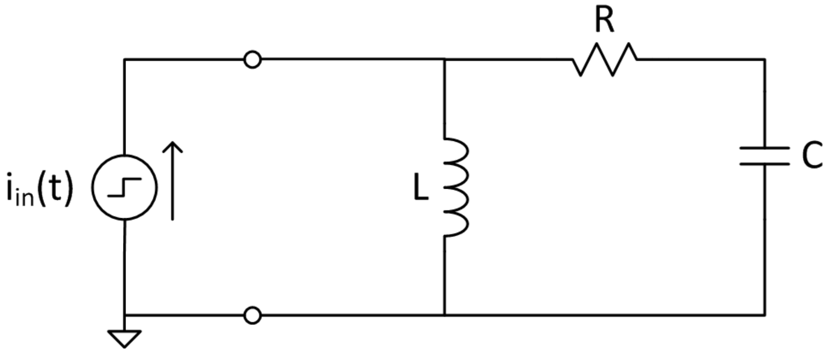
Initial System Model

- An all-inclusive model, accounting for component parasitics, may look like:



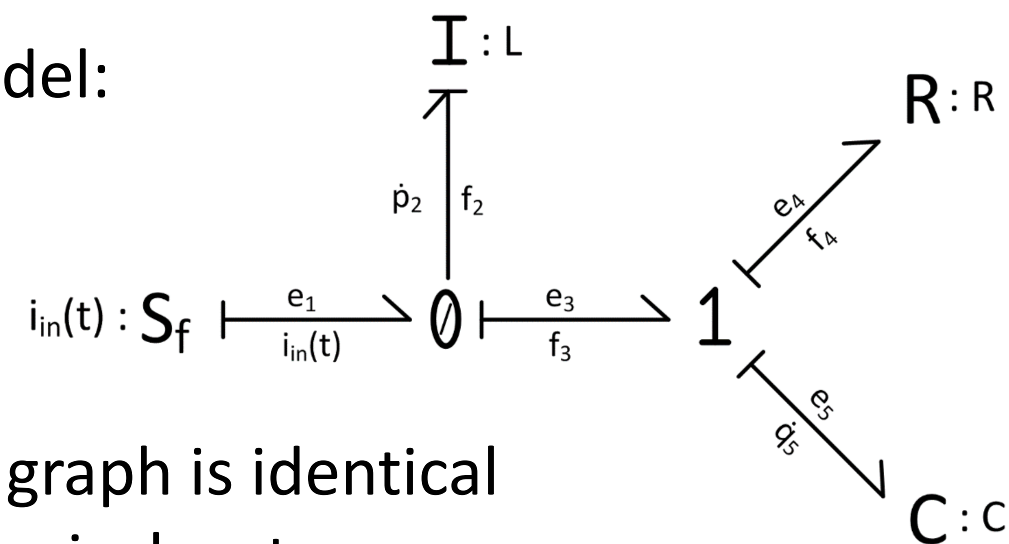
Simplified Model

- The model is already simplified in that we've neglected any parasitics associated with the connector and interconnect
- Further simplify by treating R, L, and C components as ideal – i.e. free of parasitics and linear



Bond Graph Model

- More natural to jump directly to the simplified RLC model for the electric system than for the mechanical system
  - In both cases tradeoffs must be made between accuracy and simplicity.
- The bond graph model:



- Note that the bond graph is identical to that of the mechanical system
  - A universal modeling approach

Mathematical Model

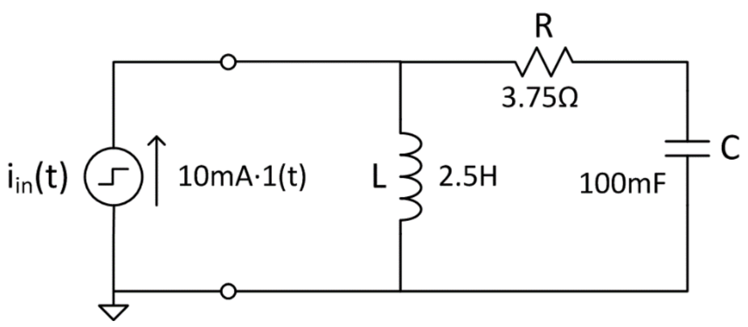
- Again, use the bond graph model to develop a ***state-variable*** mathematical model for the system

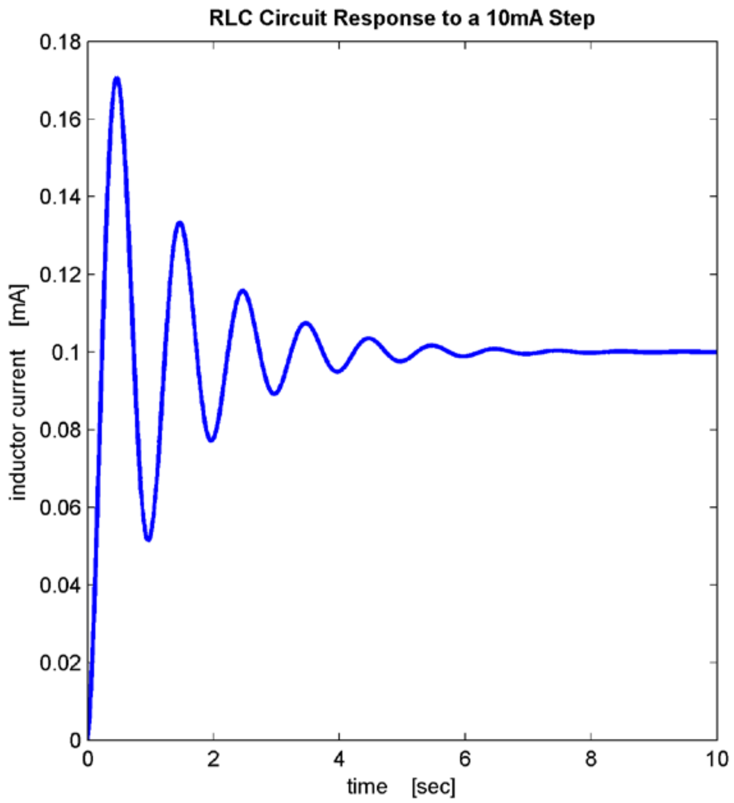
$$\begin{bmatrix} \dot{p}_2 \\ \dot{q}_5 \end{bmatrix} = \begin{bmatrix} -R/L & 1/C \\ -1/L & 0 \end{bmatrix} \begin{bmatrix} p_2 \\ q_5 \end{bmatrix} + \begin{bmatrix} R \\ 1 \end{bmatrix} i_{in}(t)$$

- Aside from variable names, state-space model is identical to that of the mechanical system
- Note that, again, we could have bypassed the bond graph model and derived a similar set of state-variable equations directly, though application of Kirchhoff's laws

Simulation

- Use the mathematical model to determine inductor current in response to a 10 mA input current step
- Numerical simulation in MATLAB
- Component values:



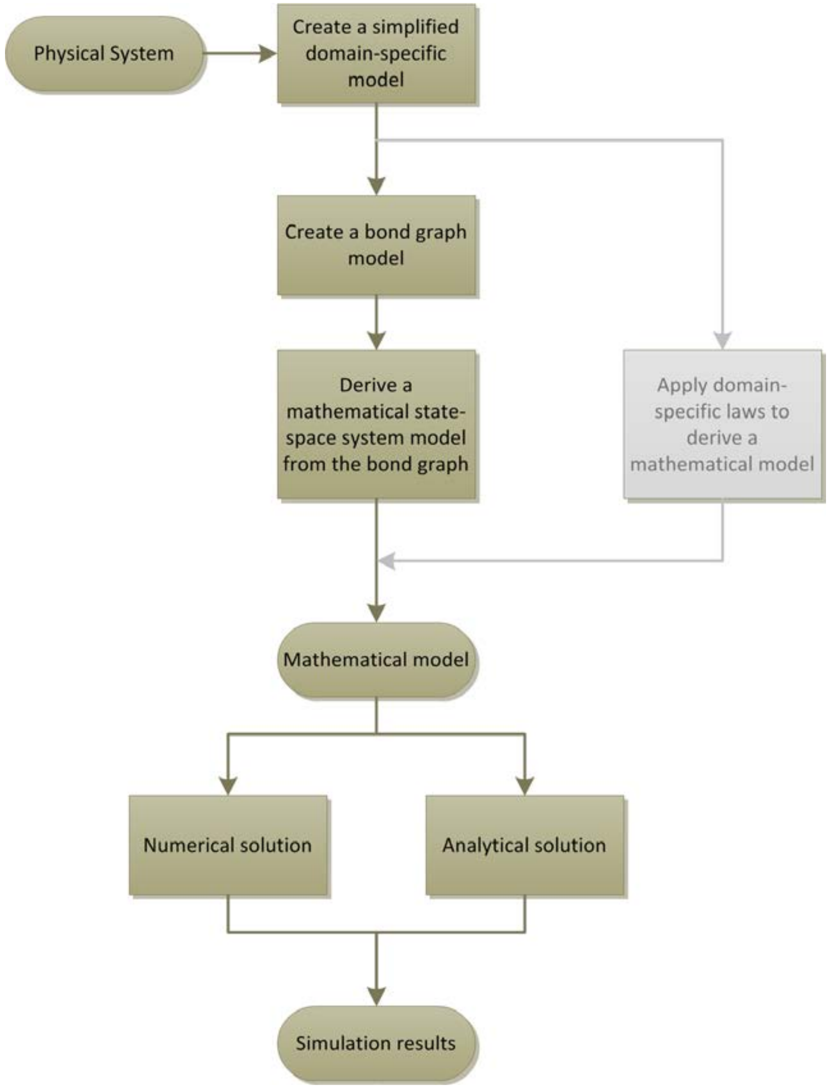


- Response is identical to that of the mechanical system

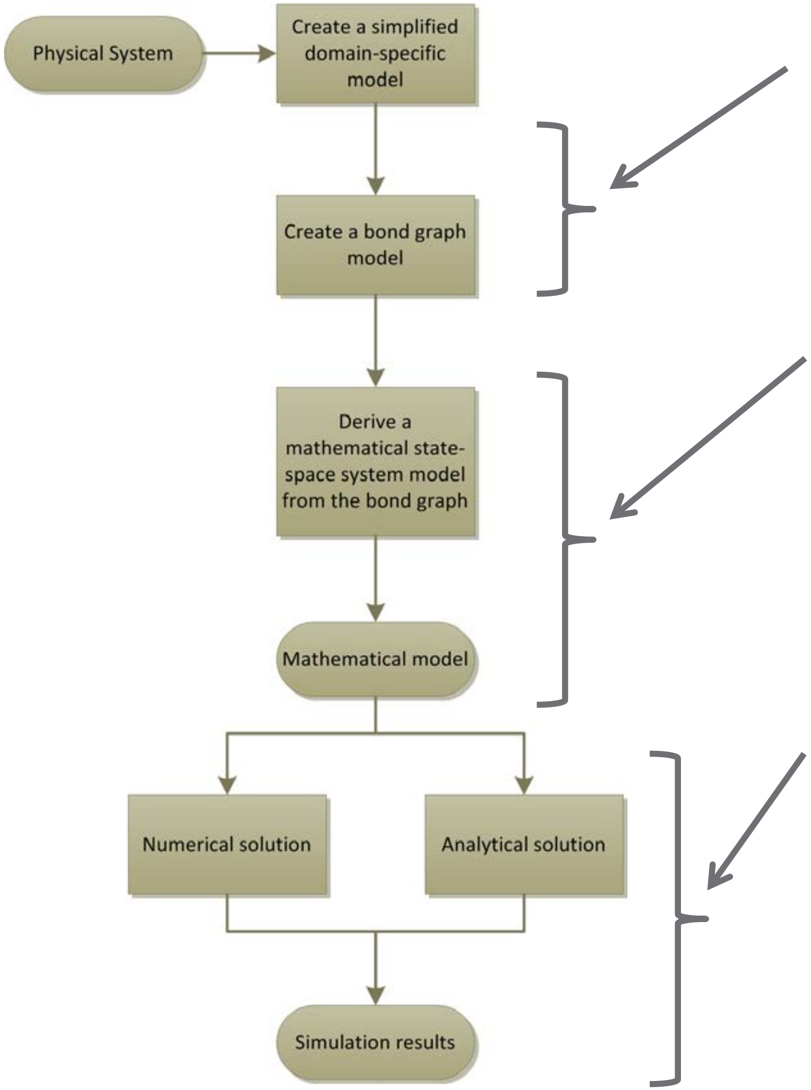
Course Overview

Basic Modeling & Analysis Procedure

- Our starting point will generally be a ***simplified domain-specific model***
- We'll focus on a ***bond graph modeling*** approach
  - A universal, energy-based approach
  - One, but not the only, method for deriving a mathematical model
- Both numerical and analytical solution will be addressed



Course Overview



- The first section of the course will cover bond graph fundamentals
- Next, we'll learn how to develop a state-variable mathematical models from bond graph models
- Finally, we'll cover how to use state-space models to determine system response

Motivation

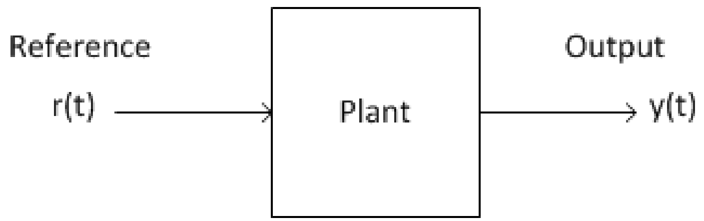
- Need to model systems in order to simulate them
- Want to simulate for two main reasons:
  - ***Analysis***
    - System response to various inputs
    - Dependence of response on parameters
  - ***Design***
    - Modifying a system to yield desired performance
    - ***Control system design*** – the addition of feedback and a controller to the system to improve performance
      - The subject of the following course in the series, ESE 430

Control of Dynamic Systems

- Example: ***automobile cruise control***
  - Maintain a constant desired speed
  - Modulate throttle position to vary speed
- Three modes of control:
  - ***Open-loop control*** – set the throttle to the angle that corresponds to the desired speed and leave it there
  - ***Human control*** – driver monitors vehicle speed and adjusts the throttle to maintain constant speed
  - ***Closed-loop control*** – a ***controller*** monitors vehicle speed, compares that to the desired speed, and modulates throttle position accordingly

Block Diagrams & Terminology

- We use ***block diagrams*** to represent control systems
  - For the cruise control system:



- The ***plant*** is the system we want to control – the car
- The ***reference input***, r(t), is the set point – the desired speed
- The ***output***, y(t), is the actual speed
- Arrows in the block diagram represent the flow of ***signals*** – information of some kind

Open-Loop Control

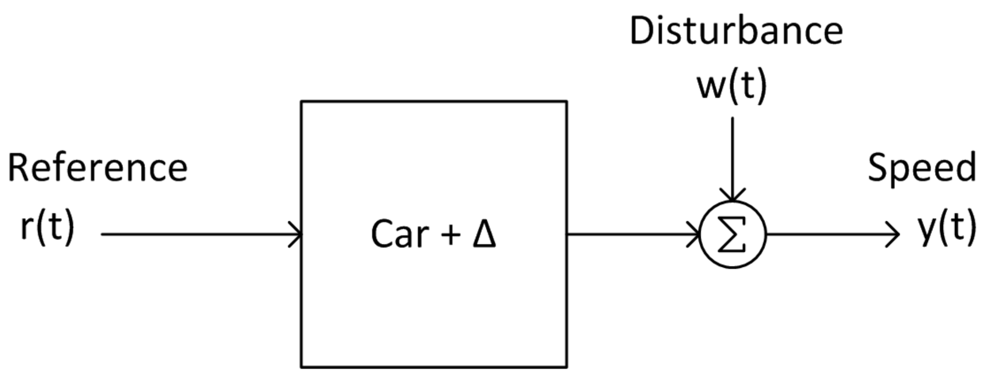
- Create a lookup table or formula relating throttle position to speed
  - Test a car or sampling of cars on a track at the factory to gather data
- Driver sets the cruise control to go 60 MPH – vehicle computer sets throttle to corresponding position

or

- Set throttle position to current value when cruise control is set – hold it there

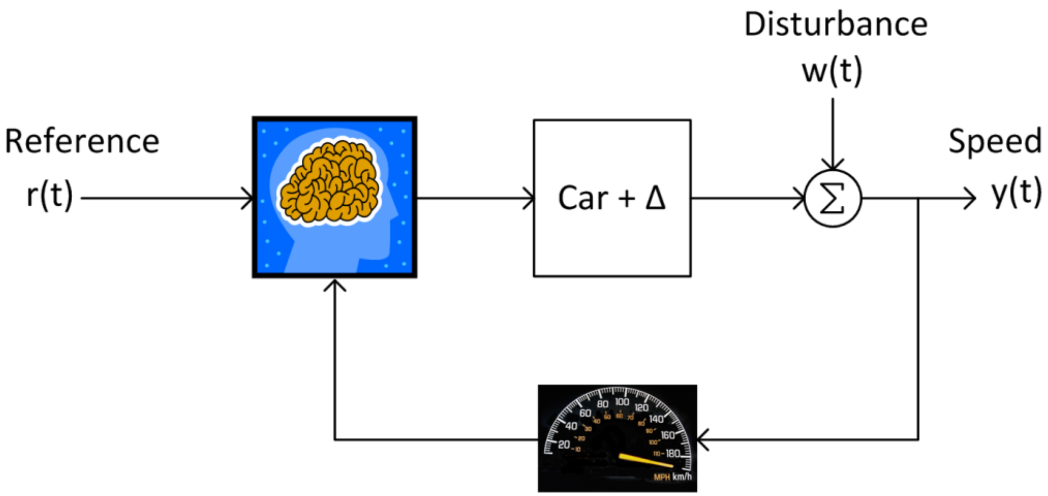
Open-Loop Control – Problems

- ***Plant variation***
  - Not all cars are the same
  - Throttle position/speed relationship affected by age, elevation, fuel, etc.
- ***Disturbances***
  - Hills, wind, road surface, etc.

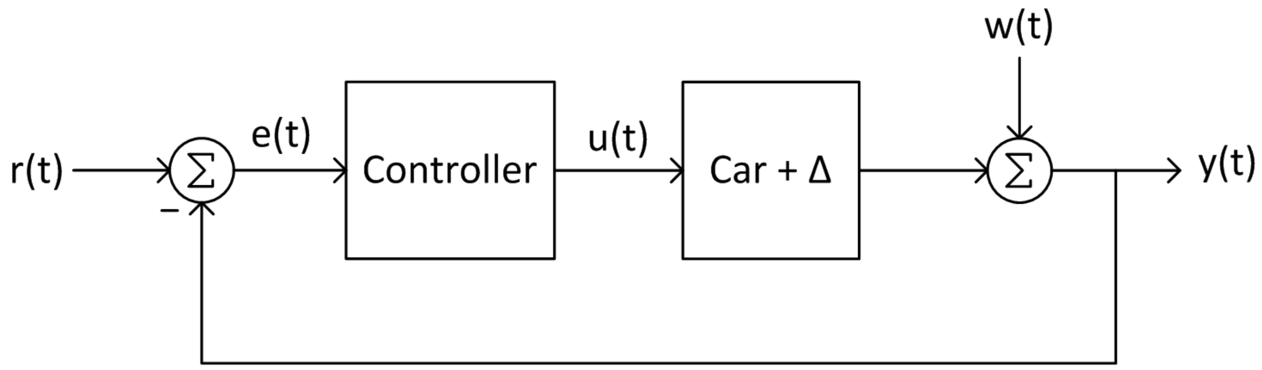


Human Control

- This *is* **feedback control**, but *not* **automatic control**
  - Driver chooses a desired speed, r(t)
  - Speedometer senses and displays current speed, y(t)
  - Driver visually monitors speedometer and adjusts the accelerator such that y(t) ≈ r(t)
- Output is fed back through the driver
  - Driver has some 'model' of the car in their head
  - Disturbances and plant variation are accounted for

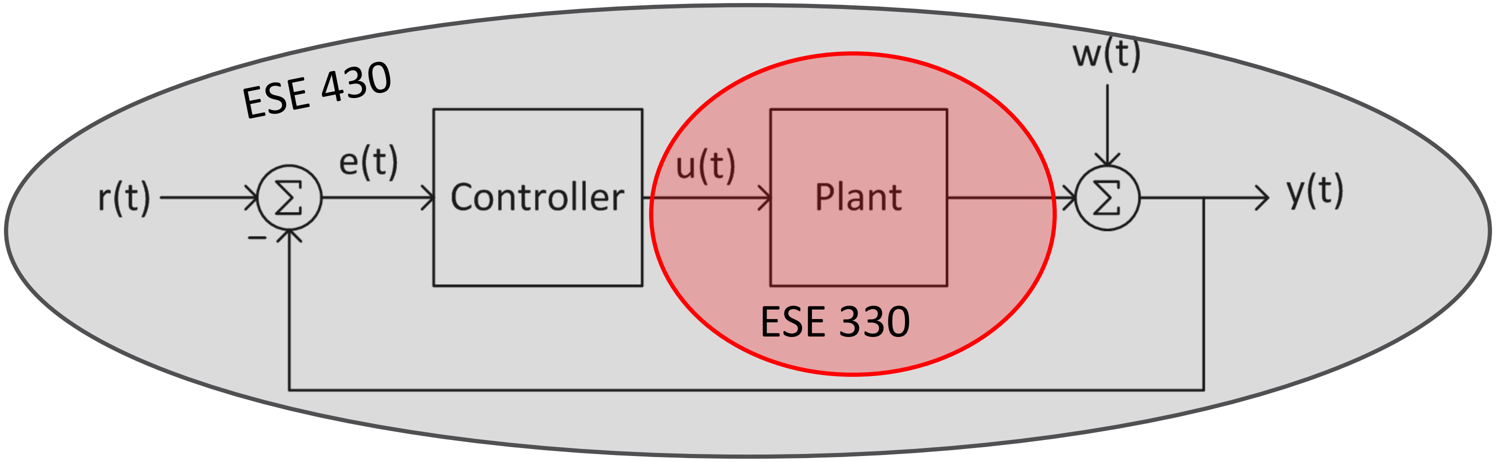


Closed-Loop Feedback Control



- ***Output fed back*** and subtracted from the reference
- ***Error signal***, e(t), is input to the ***controller***
  - Controller mathematically manipulates e(t) to generate the ***control signal***, u(t)
  - Here, u(t) would be a signal to change the throttle position
- Disturbances and plant variation are rejected

Closed-Loop Feedback Control



- Control system design involves ***designing the controller block*** to yield desired performance at y(t) – ESE 430
- Need to accurately ***model and simulate:***
  - The ***plant*** we want to control
  - The entire ***closed-loop control system***, including the plant and the controller
- The goal of this course, ESE 330, is to learn to ***model and simulate the plant block*** of the system above