SECTION 3: BOND GRAPH SYNTHESIS

ESE 330 – Modeling & Analysis of Dynamic Systems

Introduction

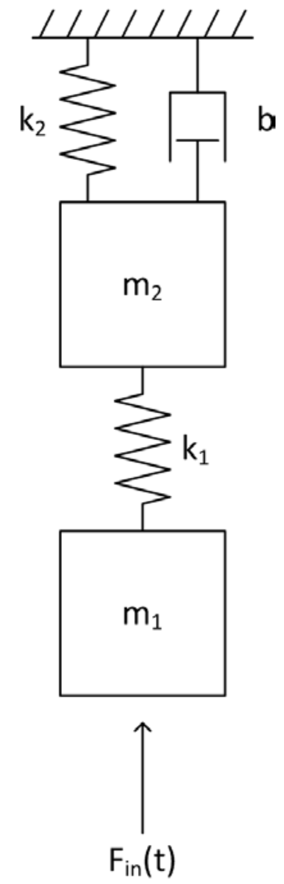
- Goal of this section of notes is learn how to ***generate a bond graph model*** for a physical system
  - Map system components to bond graph elements – I's, C's, R's, sources, etc.
- Starting point will be a ***physical system model***
  - A ***schematic***
  - Not the real system
  - All modeling decisions have already been made at this point
    - What to include in the model, what to neglect

- From here on out, we'll focus primarily on ***mechanical*** and ***electrical*** systems
  - Easily extended to other energy domains, e.g. fluid systems

Mechanical Systems

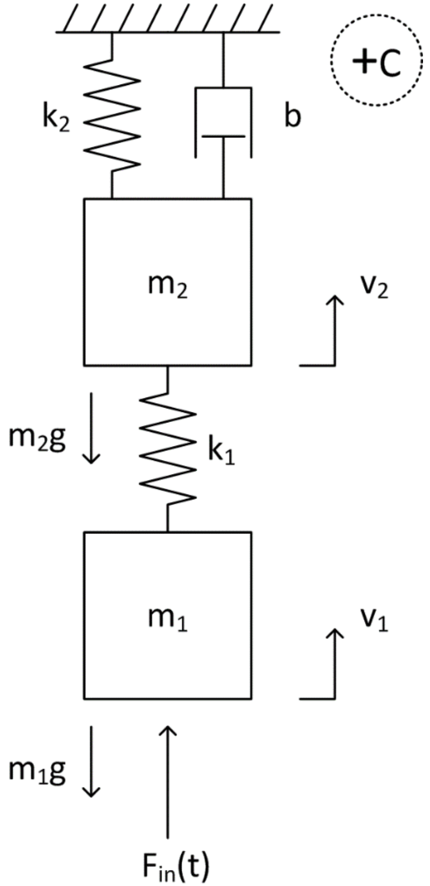
Bond Graphs of Mechanical Systems

- The following bond graph synthesis techniques apply equally to both ***translational*** and ***rotational mechanical systems***
- Illustrate the procedure with a simple example
  - Translational spring/mass/damper system
  - Starting from a schematic diagram
    - Already a system model
    - Real, physical system components have been reduced to springs, masses, and dampers
    - Some aspects have surely been neglected



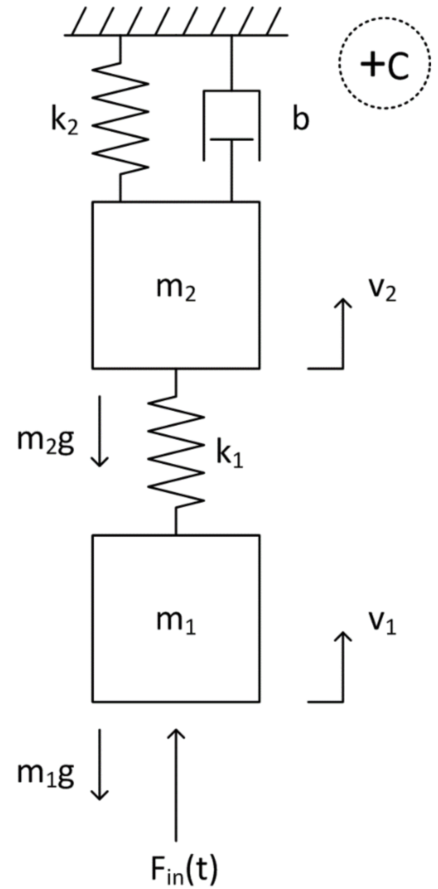
Mechanical Systems – Step 1

- ***Identify and label all distinct, non-zero, absolute velocities (flows) in the physical model (schematic)***
  - Velocities of masses
  - Velocities at the ends of springs and dampers
  - Indicate the arbitrarily-assumed positive velocity directions
    - Relative to an inertial reference
  - Choose the relative velocities of the springs and dampers to be positive either in compression or in tension
    - Indicate on the schematic as either +T or +C
  - Gravitational forces noted as well



Mechanical Systems – Step 2

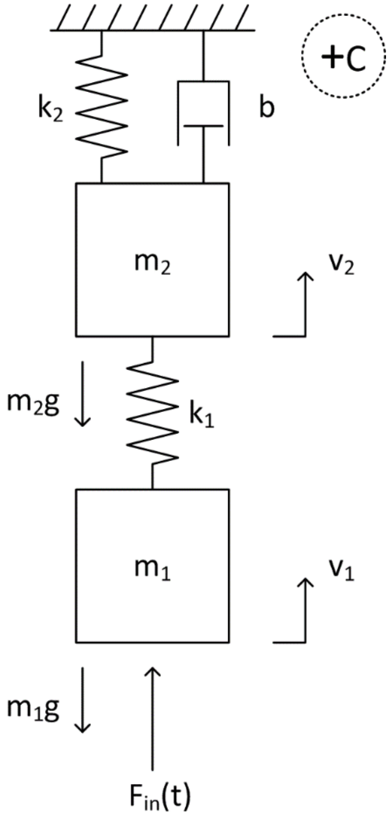
- ***List all one- and two-port elements along with their relevant velocities (flows)***
  - ***Include all TF and GY equations***
  - Map physical components to one-port bond graph components
    - Include component values
  - Define relative velocities as differences between absolute velocities
  - Sources move with the components to which they're connected
  - Include bonds connected to one-ports
    - Bonds point in toward all I's, R's, and C's
    - Direction of source bonds determined by power convention



Mechanical Systems – Step 2

- Input effort source acts in the same direction as v1, so its bond points out
- Gravitational effort sources oppose v1 and v2 – bonds point in

Element	Velocity
$m_1: I \leftharpoondown$	v_1
$1/k_1: C \leftharpoondown$	$v_{s1} = v_1 - v_2$
$m_2: I \leftharpoondown$	v_2
$1/k_2: C \leftharpoondown$	v_2
$b: R \leftharpoondown$	v_2
$m_1 g: S_e \leftharpoondown$	v_1
$m_2 g: S_e \leftharpoondown$	v_2
$F_{in}(t): S_e \rightharpoonup$	v_1



Mechanical Systems – Step 3

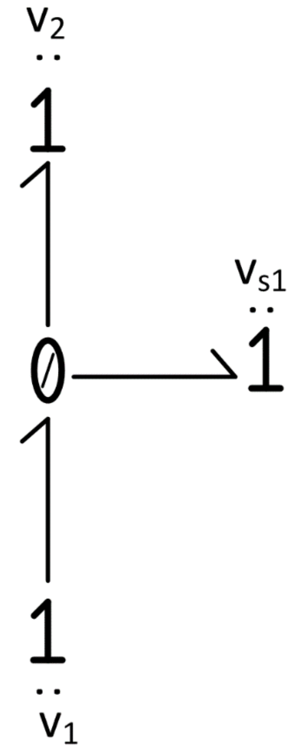
- ***Place a 1-junction for each distinct velocity (flow)***
  - Both absolute and relative velocities
  - Label the velocity of each 1-junction on the bond graph

Mechanical Systems – Step 4

- ***Relate velocities (flows, 1-jct.'s) together using 0-junctions, transformers, and gyrators***
  - Rewrite relative velocity equations from step 2, eliminating negative signs
    - Think of as 'ins' = 'outs'

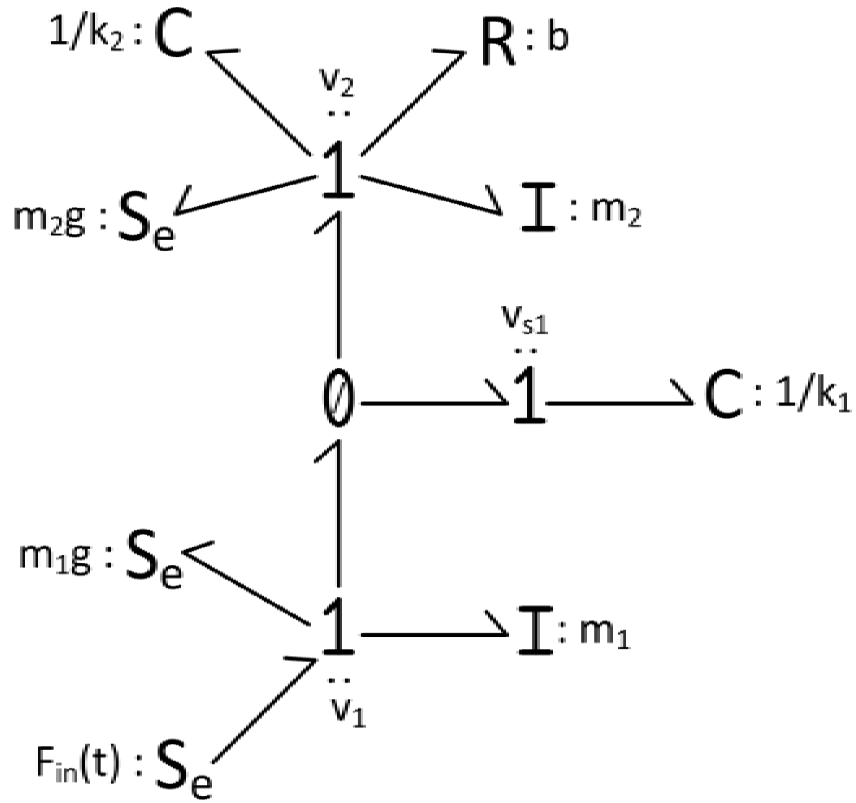
$$v_1 = v_{s1} + v_2$$

  - Use 0-junctions to sum absolute velocities, yielding relative velocities
  - Write TF and GY equations



Mechanical Systems – Step 5

- ***Attach one-port elements to appropriate 1-junctions***
  - Bonds point in toward I's, C's, and R's
  - Bond direction of sources dependent on power convention
    - As determined in step 2

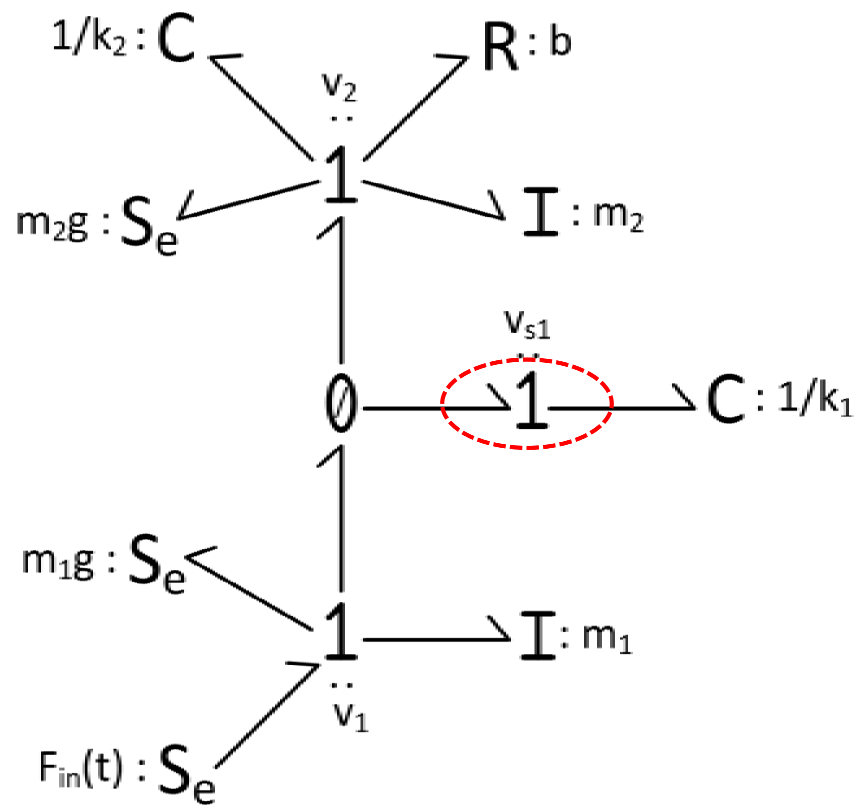


Mechanical Systems – Step 6

- ***Simplify the bond graph***
  - Eliminate any ***two-port 0- or 1-junctions*** with ***through power***, e.g.

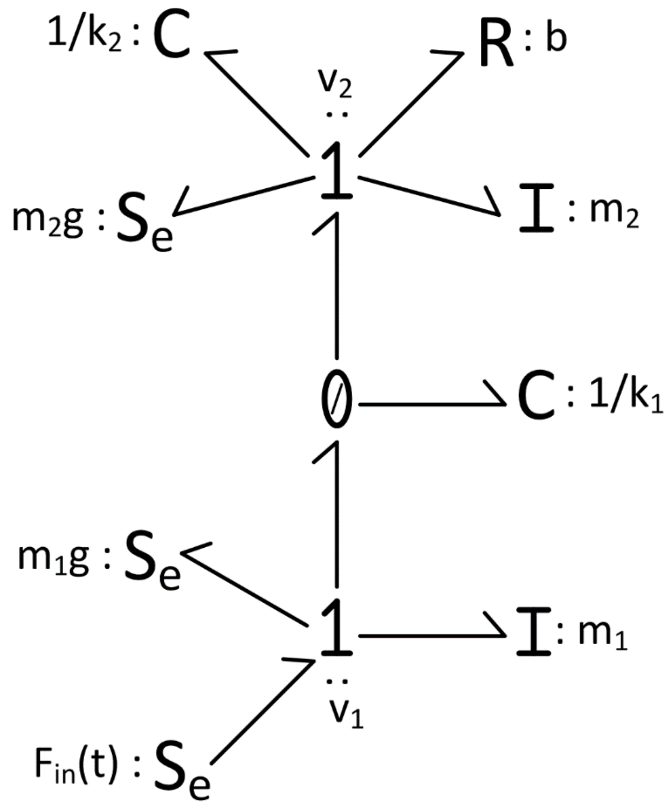
not

    - Replace with a single bond
  - Collapse cascaded 0- and 1-junctions



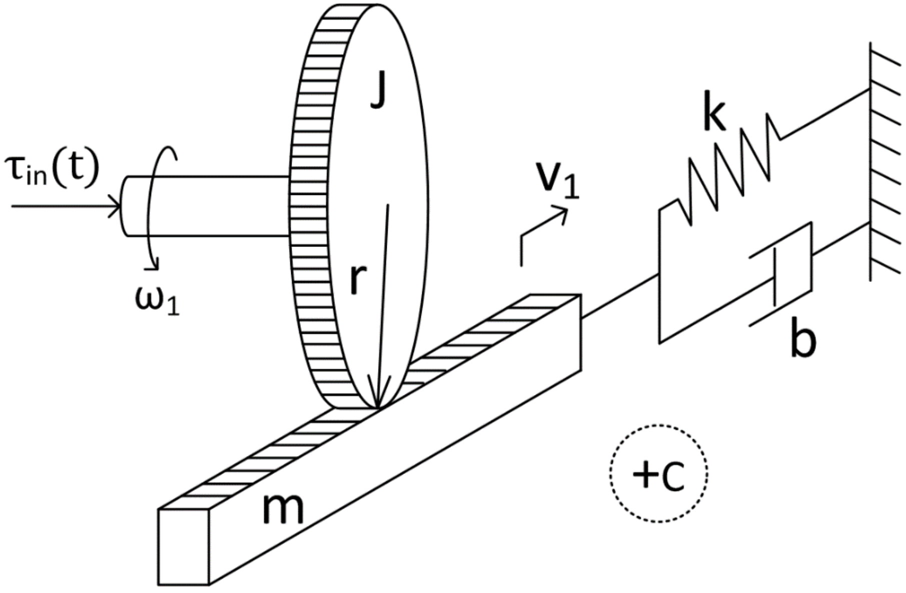
Mechanical Systems – Bond Graph

- ***The complete bond graph***
  - Understand how bond graph relates to physical system:
    - All components connected to a ***1-junction*** move at the ***same velocity***
      - C's and R's in ***parallel***
    - Spring's velocity is the difference between the velocities of its end points
      - Attached to a ***0-jct between its connection-point velocities***



Mechanical System - Example

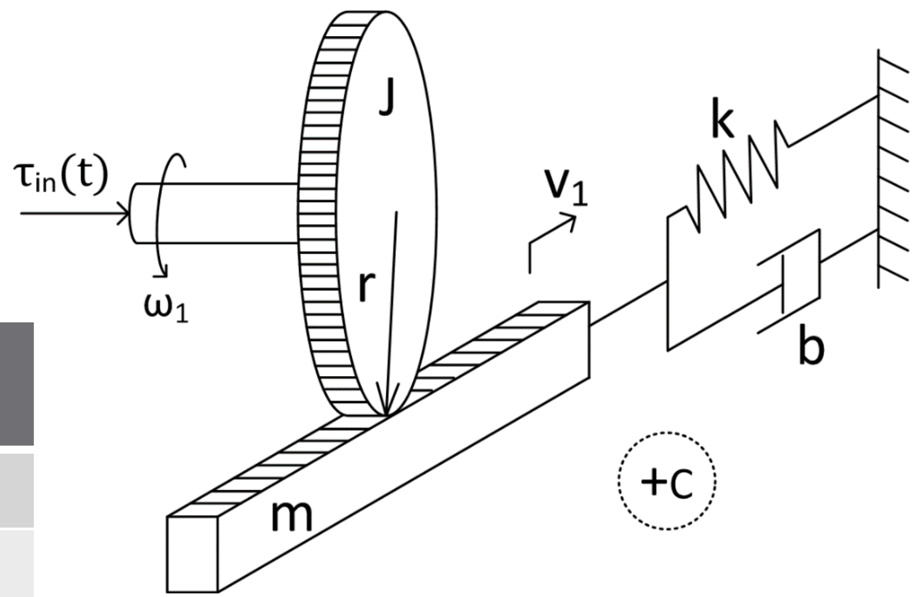
- Rack-and-pinion system
  - A hybrid rotational/ translational system
- **Step 1**: identify and label all distinct, non-zero, absolute velocities on the schematic diagram
  - Angular velocity of the pinion gear, ω_1
  - Linear velocity of the rack, v_1
  - Compression chosen to be positive



Mechanical System - Example

- **<u>Step 2</u>**: list all one- and two-port elements and their velocities
  - Include TF and GY equations

Element	Velocity
$J: I \leftharpoondown$	ω_1
$m: I \leftharpoondown$	v_1
$1/k: C \leftharpoondown$	v_1
$b: R \leftharpoondown$	v_1
$\tau_{in}(t): S_e \rightharpoonup$	ω_1
$\rightharpoonup TF \rightharpoonup$ $v_1 = r \cdot \omega_1$	ω_1 v_1



- Bond points out of effort source because it adds energy to the system

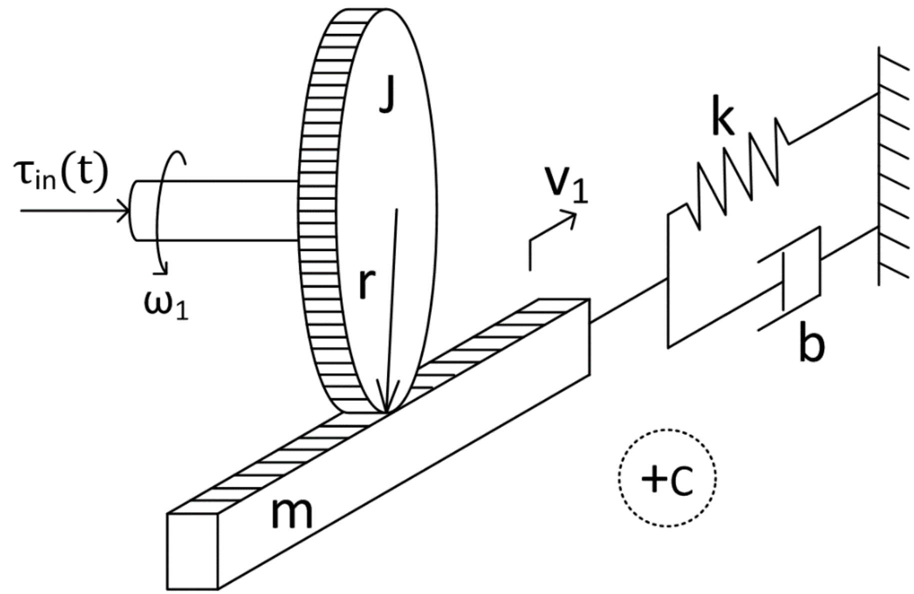
Mechanical System - Example

- **Step 3**: place a 1-jct for each distinct velocity
  - Only two distinct velocities: v_1 and ω_1

Element	Velocity
$J: I \leftharpoonup$	ω_1
$m: I \leftharpoonup$	v_1
$1/k: C \leftharpoonup$	v_1
$b: R \leftharpoonup$	v_1
$\tau_{in}(t): S_e \rightharpoonup$	ω_1
$\rightharpoonup TF \rightharpoonup$ $v_1 = r \cdot \omega_1$	ω_1 v_1

Mechanical System - Example

- **<u>Step 4</u>**: relate velocities to each other using 0-jct's, transformers, and gyrators

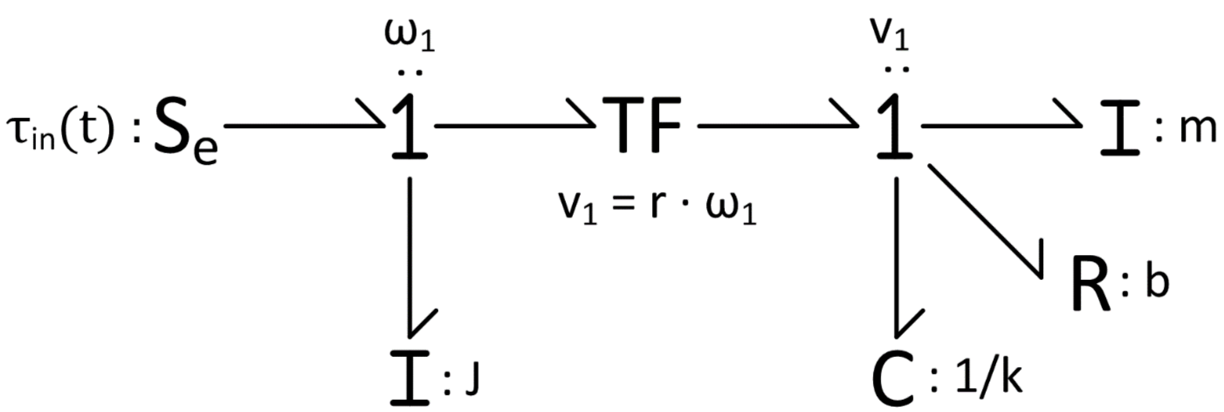


$$v_1 = r \cdot \omega_1$$

Mechanical System - Example

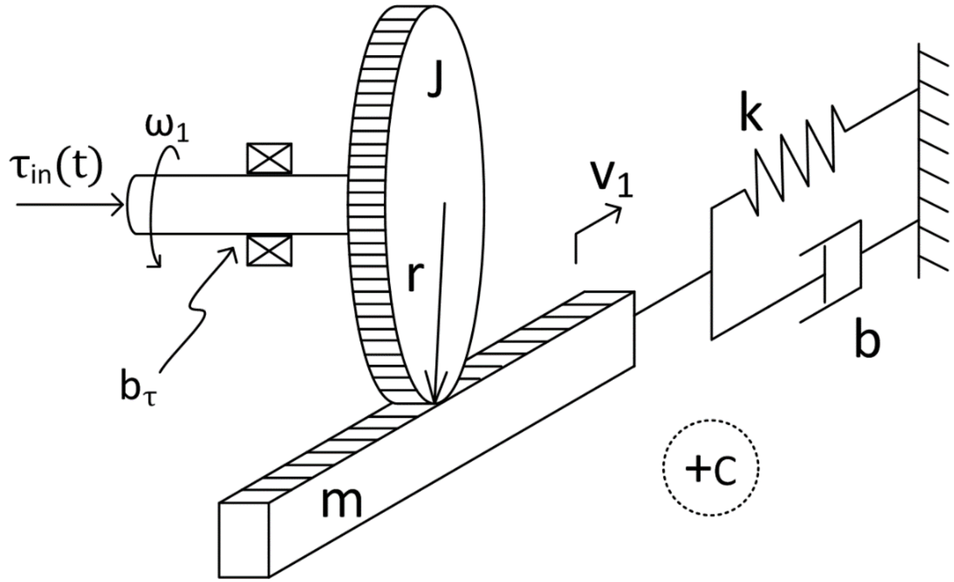
- **<u>Step 5</u>**: attach 1-port elements to the appropriate 1-jct's
- **<u>Step6</u>**: simplify
  - No simplifications

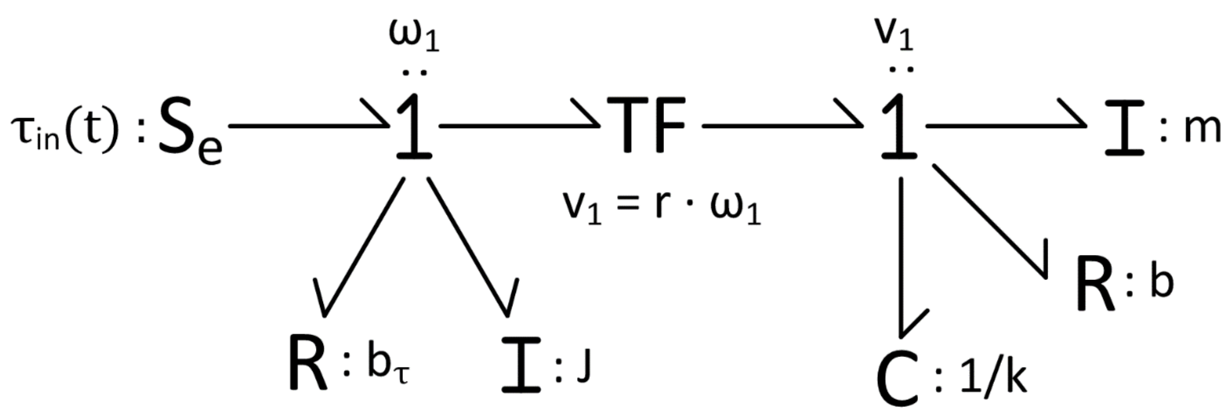
Element	Velocity
$J: I \leftharpoondown$	ω_1
$m: I \leftharpoondown$	v_1
$1/k: C \leftharpoondown$	v_1
$b: R \leftharpoondown$	v_1
$\tau_{in}(t): S_e \rightharpoonup$	ω_1
$\rightharpoonup TF \rightharpoonup$ $v_1 = r \cdot \omega_1$	ω_1 v_1



Mechanical System - Example

- Now, imagine that we want to modify the physical model to account for friction of a bearing
- No problem, simply add a resistor with flow ω_1

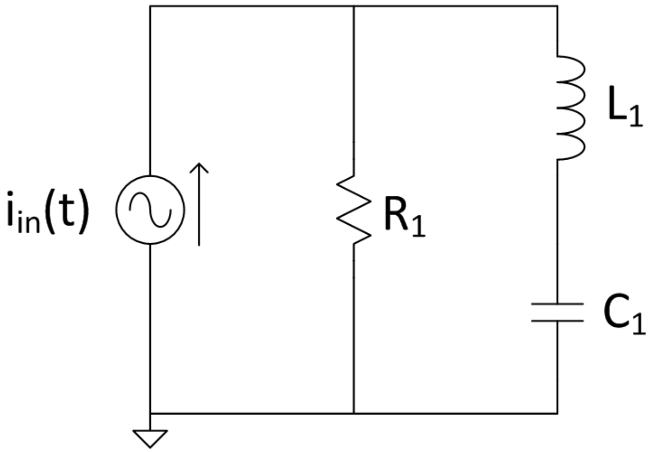




Electrical Systems

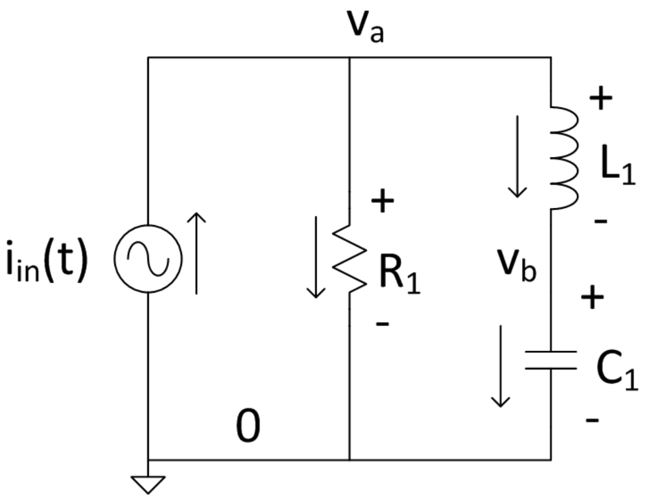
Bond Graphs of Electrical Systems

- Similar bond graph synthesis technique presented for electrical systems
  - A ***duality*** exists between mechanical and electrical systems
    - ***Series-connected components*** – constant effort in mechanical systems, constant flow in electrical
    - ***Parallel-connected components*** – constant flow in mechanical, constant effort in electrical
    - Relations of efforts and flows to the physical topology are swapped
  - Again, starting point is a schematic diagram



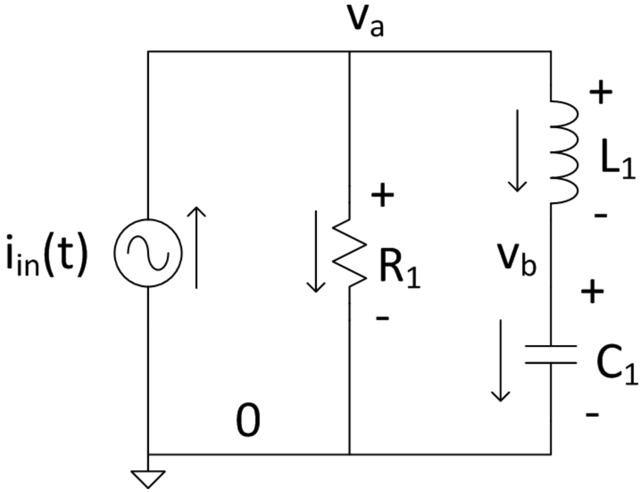
Electrical Systems – Step 1

- ***Identify all distinct node voltages (efforts) in the circuit and label them on the schematic***
  - All node voltages are relative to the ground node – 0V
    - Just as all mechanical velocities are relative to an inertial reference
  - Label an assumed voltage polarity across each component
    - Arbitrary – need not be correct
  - Label an assumed current direction through each component
    - Assume flow from high to low voltage



Electrical Systems – Step 2

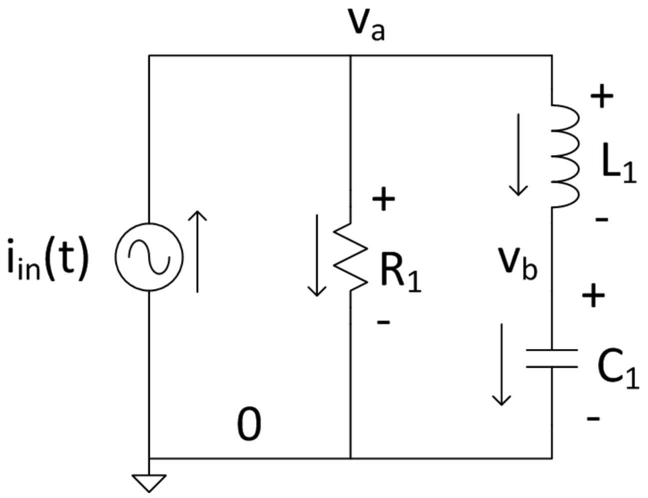
- ***List all one- and two-port elements along with their relevant voltages (efforts)***
  - Map physical components to one- and two-port bond graph components
    - Include component values and TF/GY equations
  - Define differential voltages as differences between node voltages
  - Include bonds connected to one-ports
    - Bonds point in toward all I's, R's, and C's
    - Direction of two-port and source bonds determined by power convention



Electrical Systems – Step 2

- Here, current flows out of the current source's assumed positive voltage terminal
  - Assumed to be supplying power
  - Bond points outward, away from the source

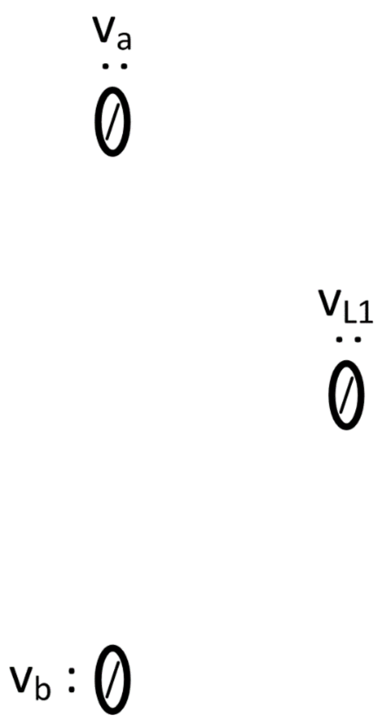
Element	Voltage
$i_{in}(t): S_f \rightharpoonup$	v_a
$R_1: R \leftharpoondown$	v_a
$L_1: I \leftharpoondown$	$v_{L1} = v_a - v_b$
$C_1: C \leftharpoondown$	v_b



Electrical Systems – Step 3

- ***Place a 0-junction for all distinct node voltages***

  - Voltages in the table from step 2
    - Node voltages
    - Differential voltages
  - Label the voltage of each 0-junction on the bond graph

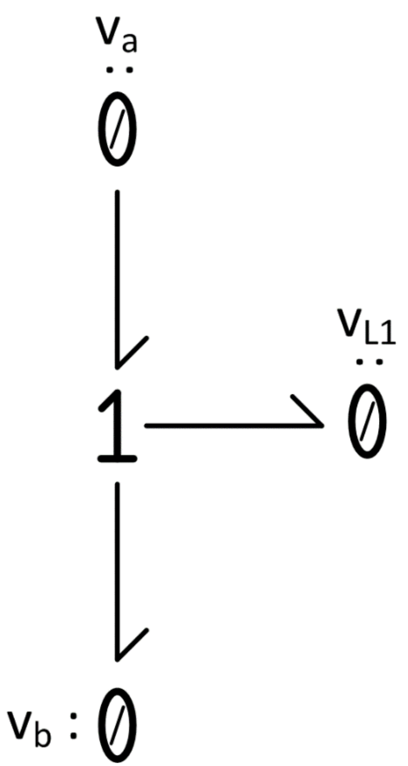


Electrical Systems – Step 4

- ***Relate voltages (efforts, 0-jct.'s) together using 1-junctions, transformers, and gyrators***
  - Rewrite relative voltage equations from step 2, eliminating negative signs
    - Think of as 'ins' = 'outs'

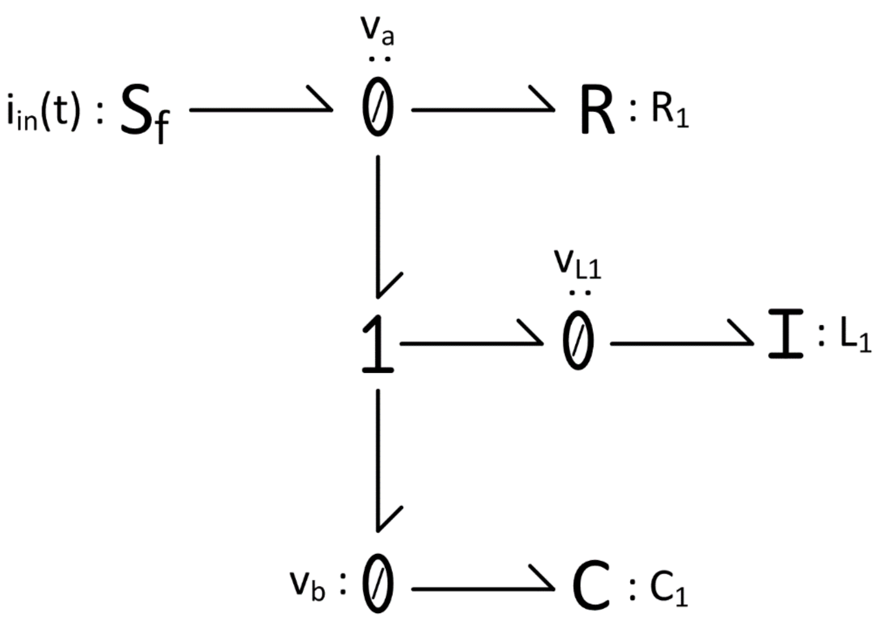
$$v_a = v_{L1} + v_b$$

  - Use 1-junctions to sum node voltages, yielding differential voltages
  - Annotate with TF and GY equations



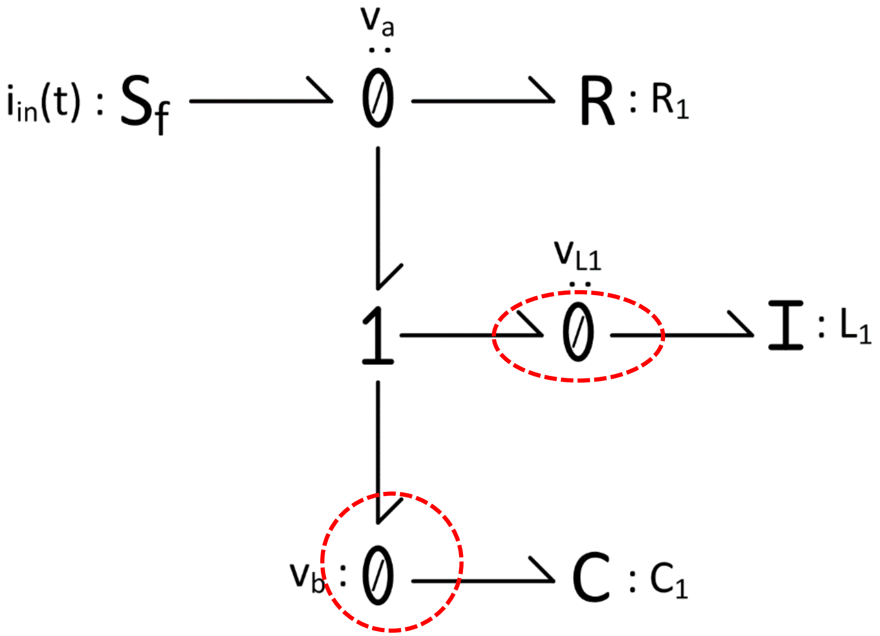
Electrical Systems – Step 5

- ***Attach one-port elements to appropriate 0-junctions***
  - Elements attach to the voltage that appears across them
  - Bonds point in toward I’s, C’s, and R’s
  - Bond direction of sources dependent on power convention
    - As determined in step 2



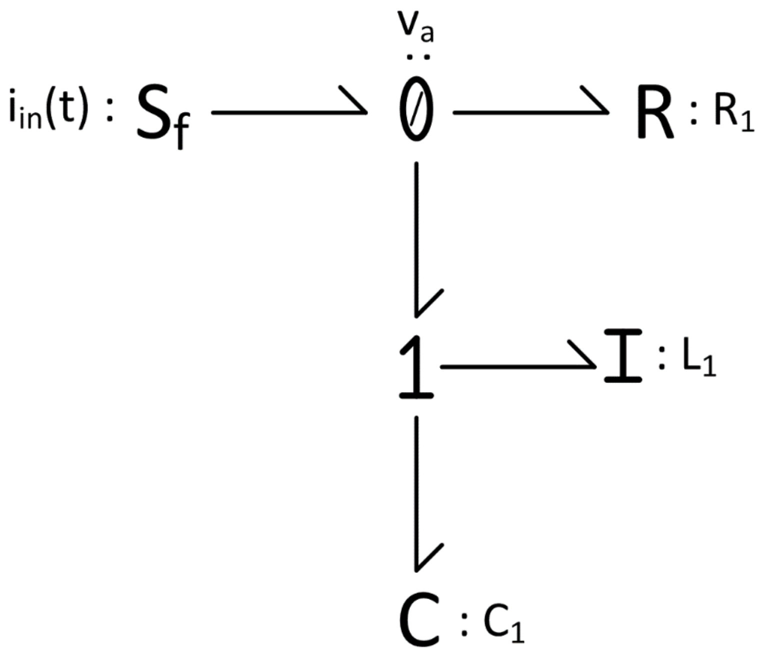
Electrical Systems – Step 6

- ***Simplify the bond graph***
  - Eliminate any ***two-port 0- or 1-junctions*** with ***through power***
    - Replace with a single bond
  - Collapse cascaded 0- and 1-junctions



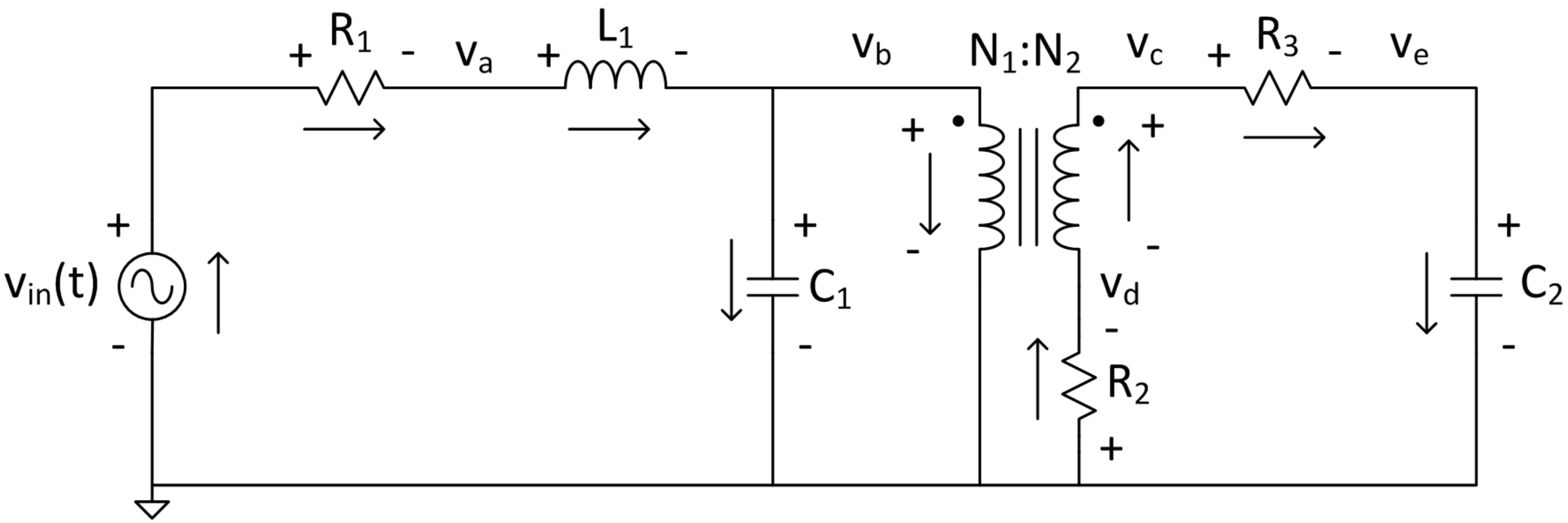
Electrical Systems – Bond Graph

- ***The complete bond graph***
  - Understand how bond graph relates to physical system:
    - Series-connected components connected to common 1-junctions
      - Equal current (flow) through components in series
    - Parallel connected components connected to common 0-junctions
      - Equal voltage across components connected in parallel



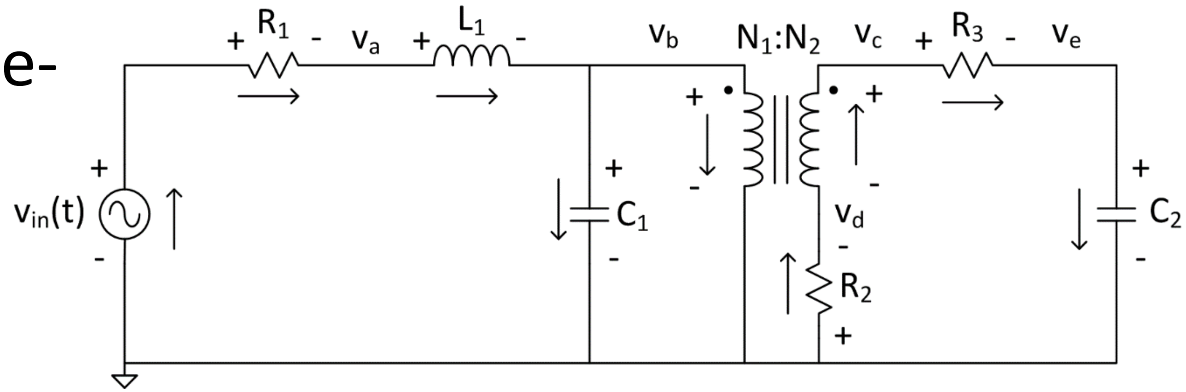
Electrical System – Example

- RLC circuit with a transformer
- **Step 1**: identify and label all distinct, non-zero, absolute voltages on the schematic diagram
  - Indicate assumed voltage polarities and directions of current flow



Electrical System - Example

- **Step 2**: list all one- and two-port elements and their voltages

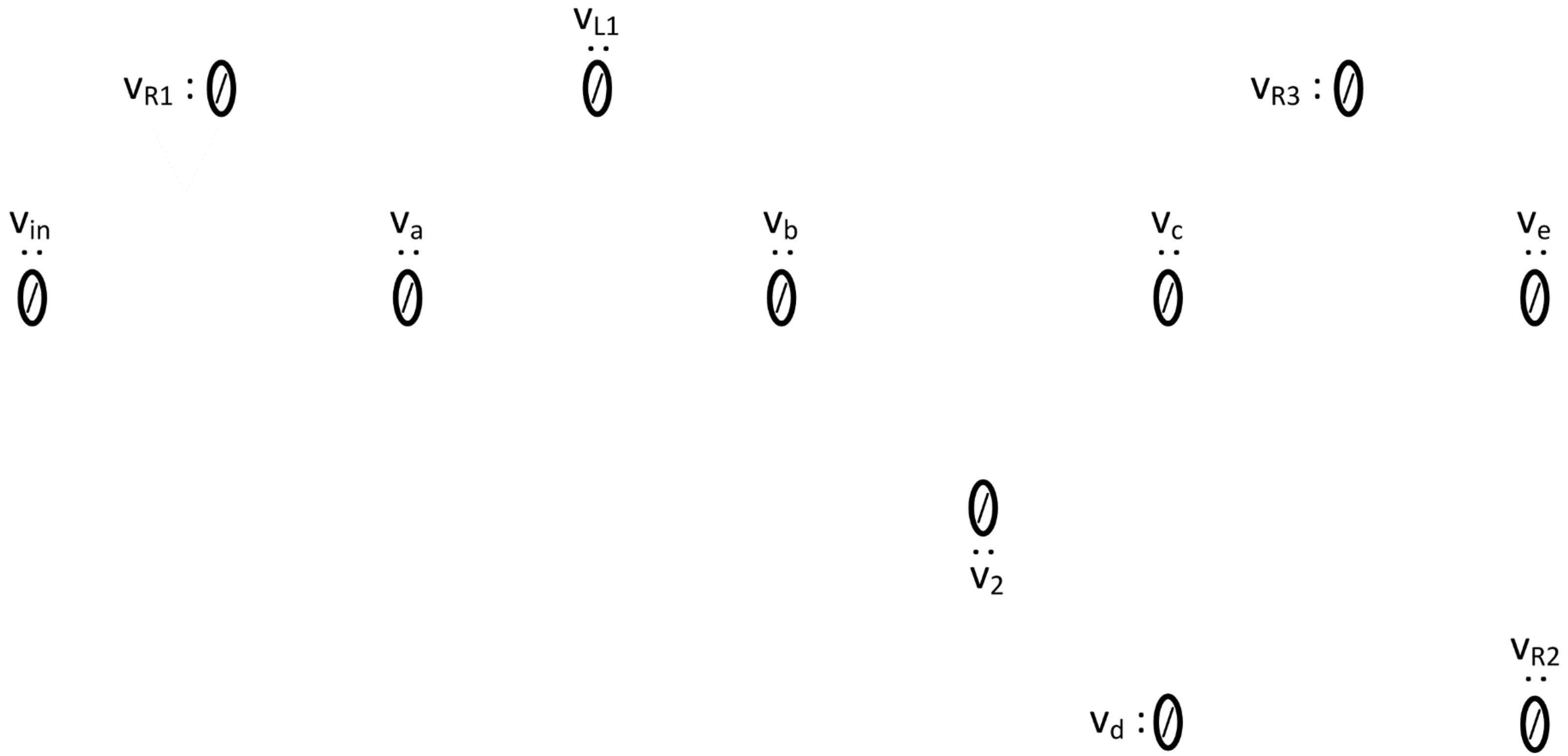


Element	Voltage
$v_{in}(t): S_e \rightharpoonup$	v_{in}
$R_1: R \leftharpoondown$	$v_{R1} = v_{in} - v_a$
$L_1: I \leftharpoondown$	$v_{L1} = v_a - v_b$
$C_1: C \leftharpoondown$	v_b

Element	Voltage
$R_2: R \leftharpoondown$	$v_{R2} = -v_d$
$R_3: R \leftharpoondown$	$v_{R3} = v_c - v_e$
$C_2: C \leftharpoondown$	v_e
$\rightharpoonup TF \rightharpoonup$ $v_2 = N_2/N_1 v_b$	v_b $v_2 = v_c - v_d$

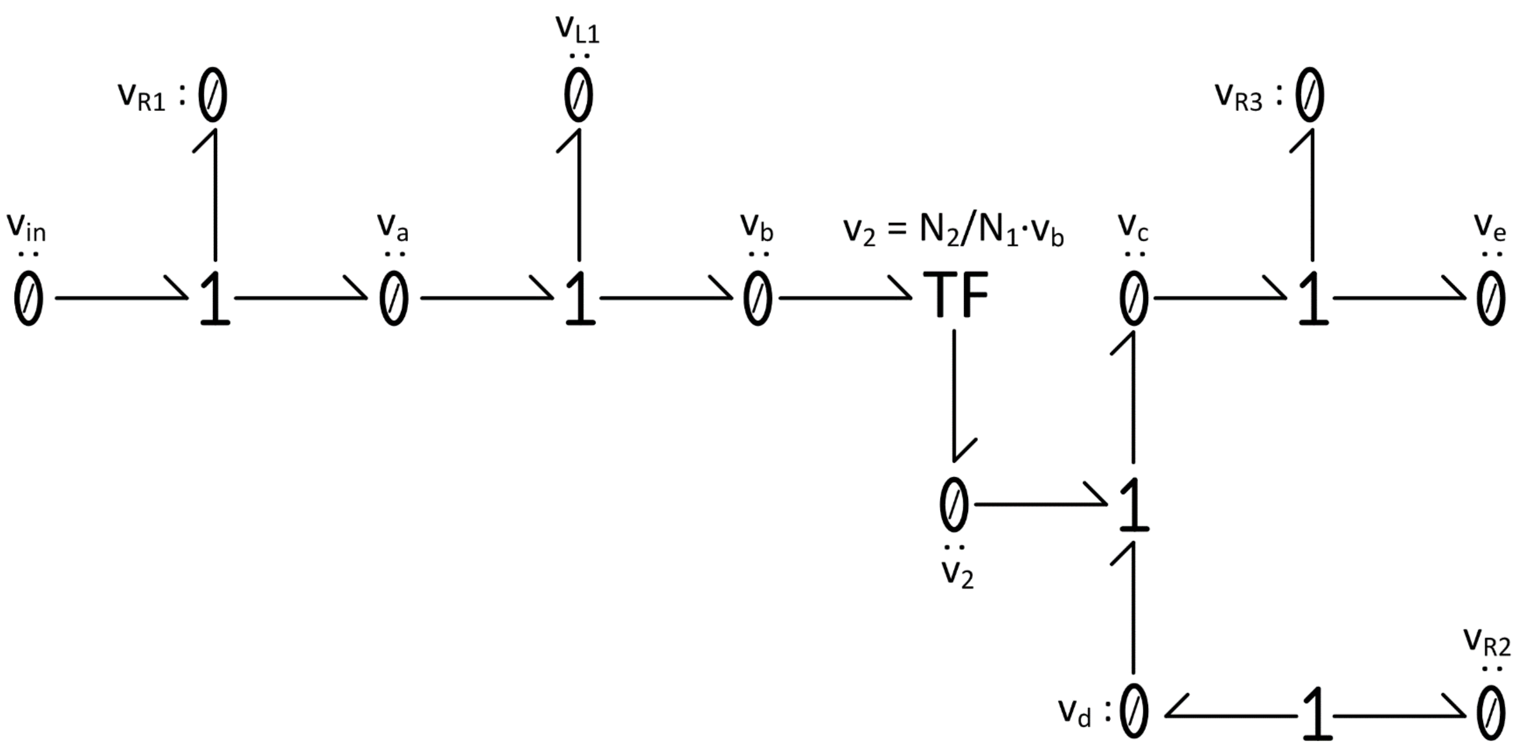
Electrical System – Example

- **Step 3**: place a 0-jct for each distinct voltage listed in the table from step 2



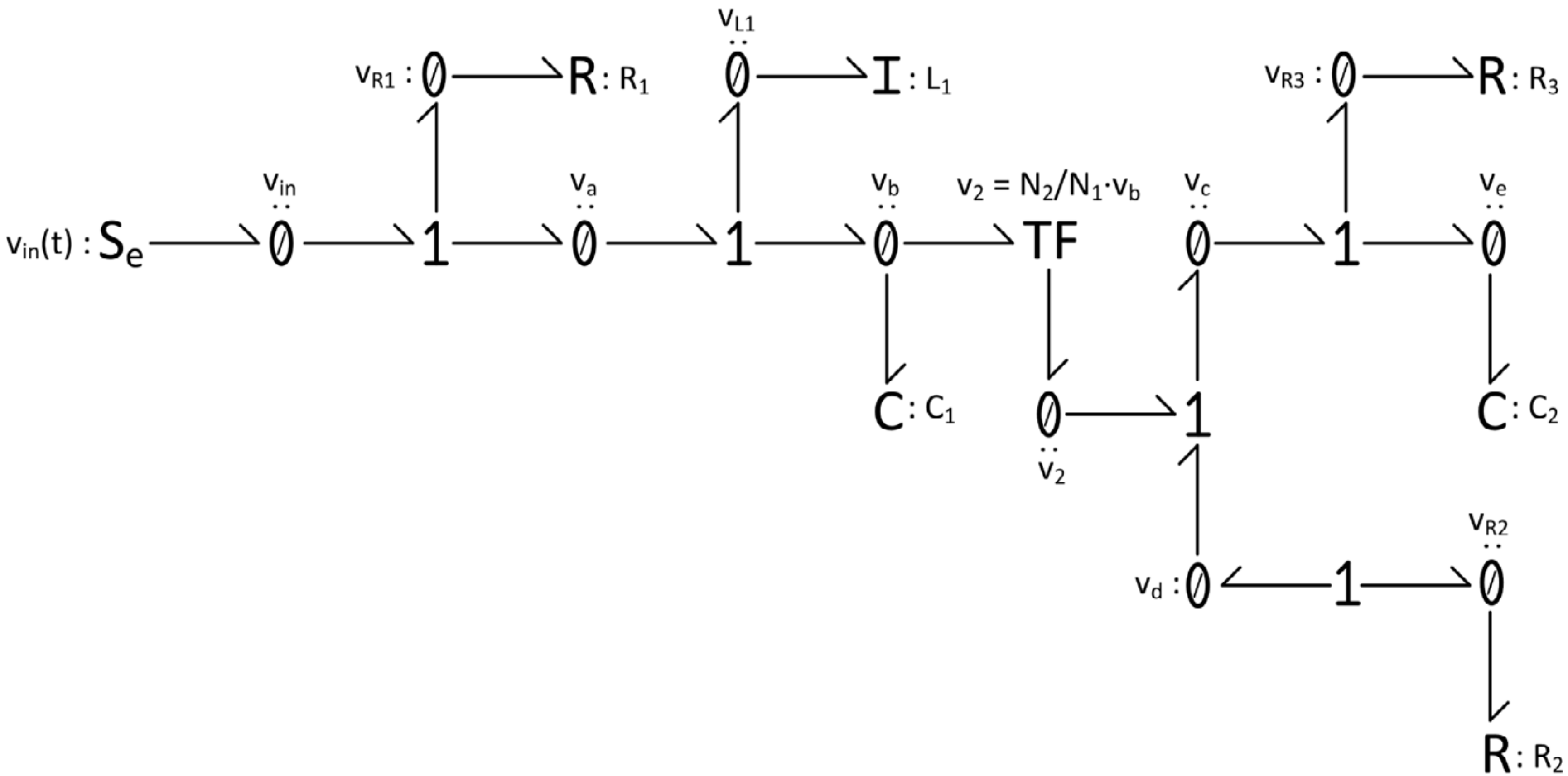
Electrical System – Example

- **Step 4**: relate voltages to one another using 1-jct's, transformers, and gyrators



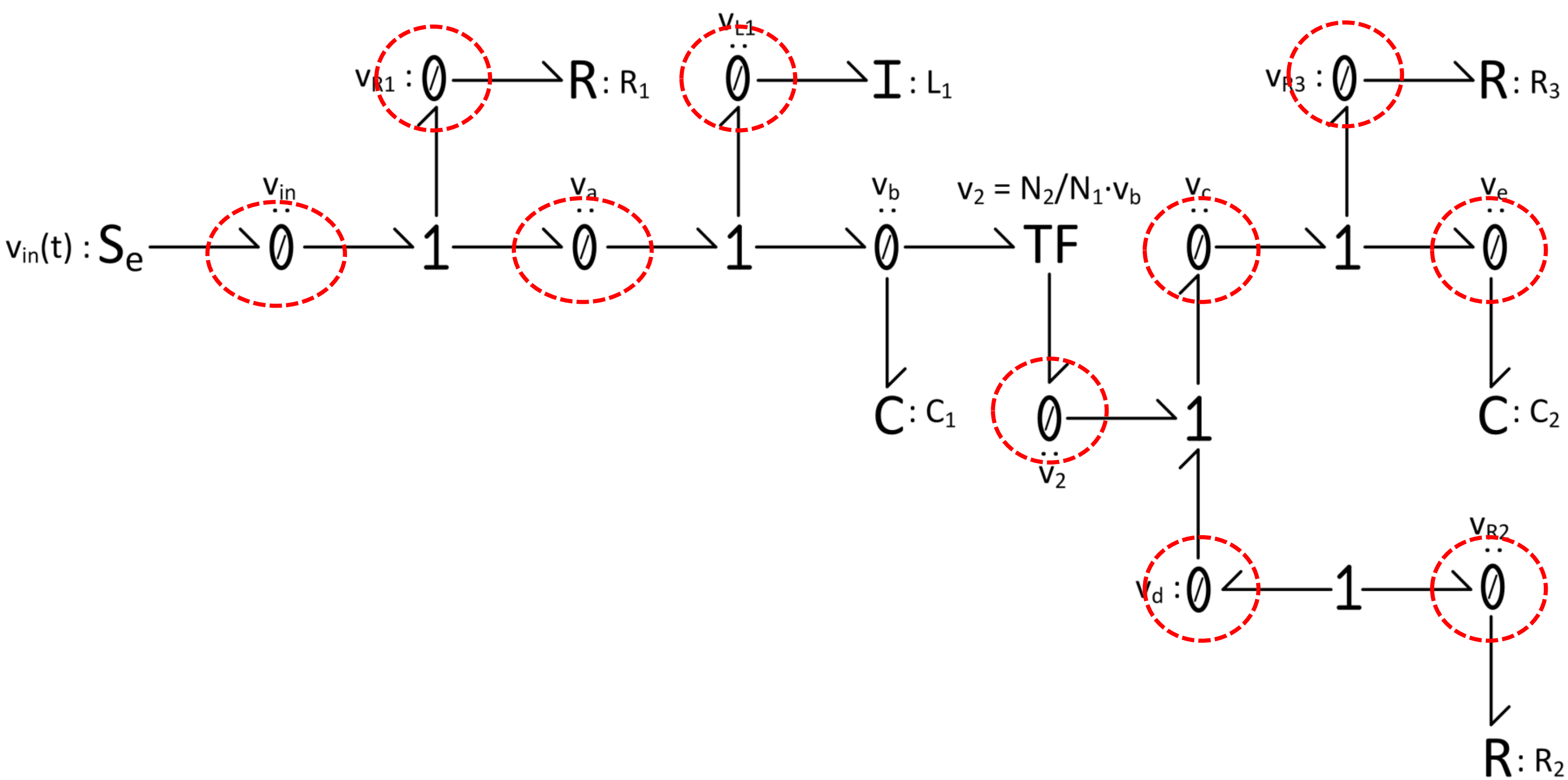
Electrical System – Example

- **Step 5**: attach 1-port elements to the appropriate 0-jct's



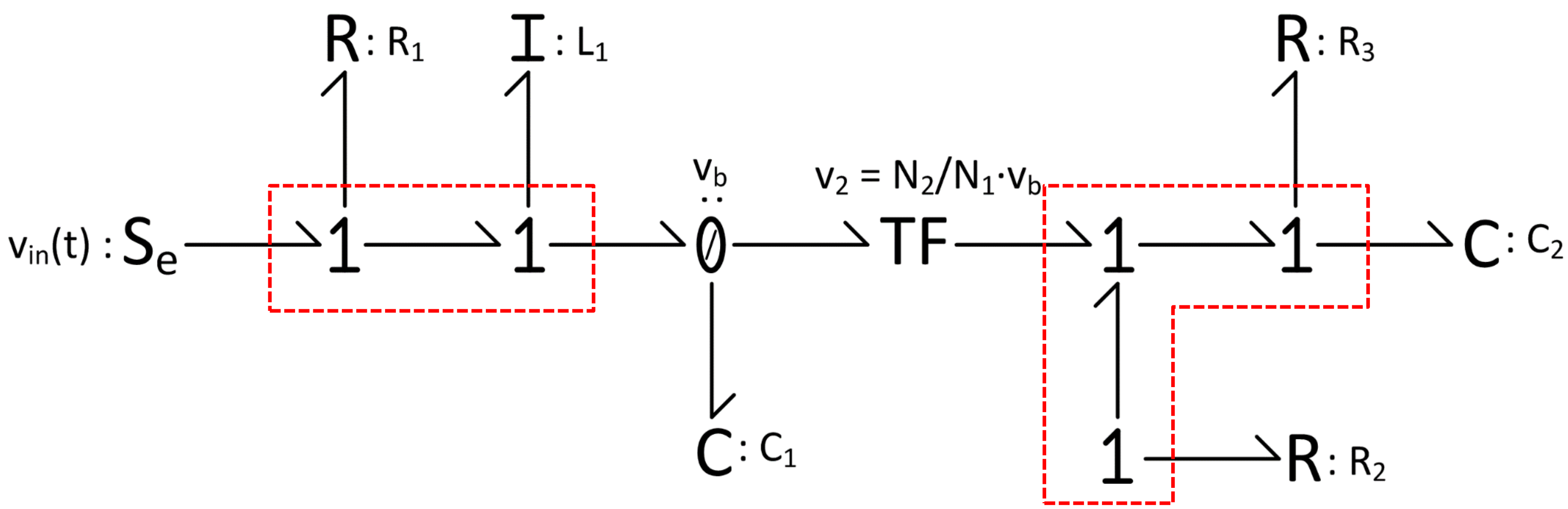
Electrical System – Example

- **Step 6**: simplify the bond graph
  - Eliminate any 0- or 1-junctions with through power
  - Note that the R_2 1-junction does not have through power



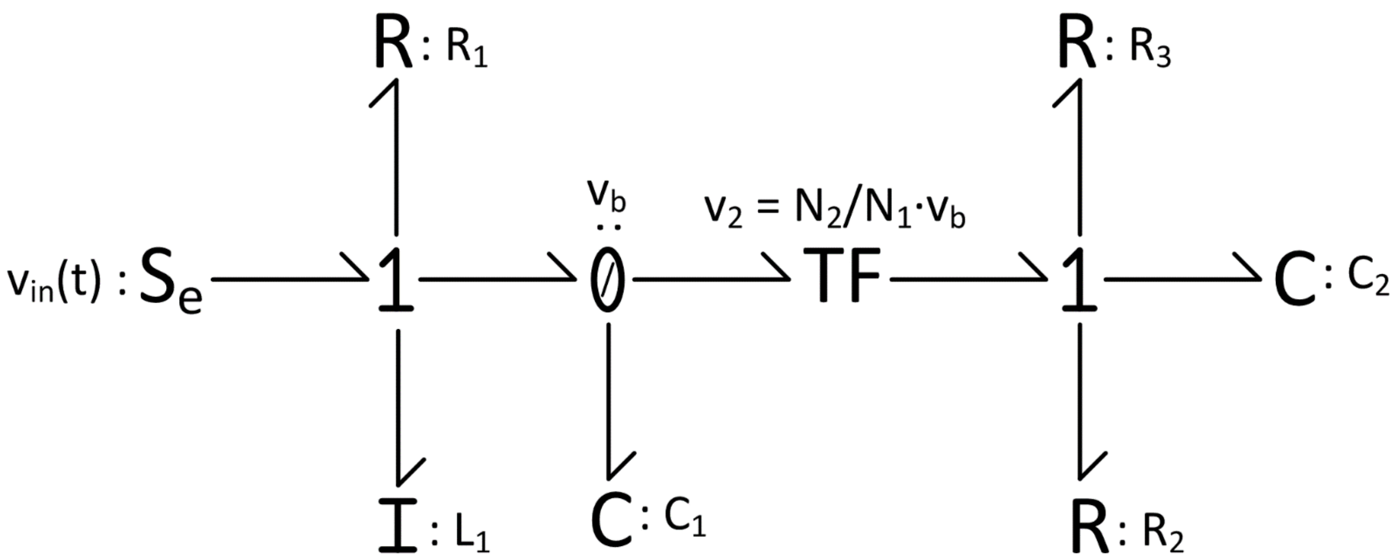
Electrical System – Example

- **Step 6**: simplify the bond graph
  - Collapse any cascaded 0- or 1-junctions



Electrical System – Example

- The final bond graph model:



- Think about how this model relates to the circuit
  - E.g., series combination of source, R_1, and L_1 is in parallel with C_1 and the primary side of the transformer, etc.

Augmenting the Bond Graph

Our goal in creating a bond graph system model is to use it to generate a mathematical system model. Next, we'll augment the bond graph to facilitate that task.

Augmenting the Bond Graph

1) ***Redraw a computational bond graph***

- Number the bonds sequentially
  - Assignment is arbitrary
- Drop the values associated with each element
  - Now, C_1 is the capacitor connected to bond 1, R_3 is the resistor connected to bond 3, etc.
  - Element names on the computational bond graph and physical schematic may not agree

2) ***Assign causality to each bond***

- Indicate causality by adding a ***causal stroke*** to each bond

Causality

Causality

- Bonds have associated ***effort*** and ***flow***
  - A component can set ***either*** the effort on a bond ***or*** the flow on a bond – ***not both***
- E.g., if you push a car, you can either determine how hard to push (effort), or you can determine how fast to push (flow)
  - You determine one quantity, and the car determines the other

The Causal Stroke

- Causality indicated by the addition of a causal stroke to the end of each bond

$$A \vdash\!\!\!\rightharpoonup B \qquad \text{or} \qquad A \rightharpoonup\!\!| \, B$$

- ***Flow*** is determined by the element ***near the causal stroke***
  - A determines flow, B determines effort:

$$A \vdash\!\!\!\rightharpoonup B$$

- ***Effort*** is determined by the element ***away from the causal stroke***
  - A determines effort, B determines flow:

$$A \rightharpoonup\!\!| \, B$$

Five Types of Causality

1) ***Required***
2) ***Restricted***
3) ***Integral***
4) ***Derivative***
5) ***Arbitrary***

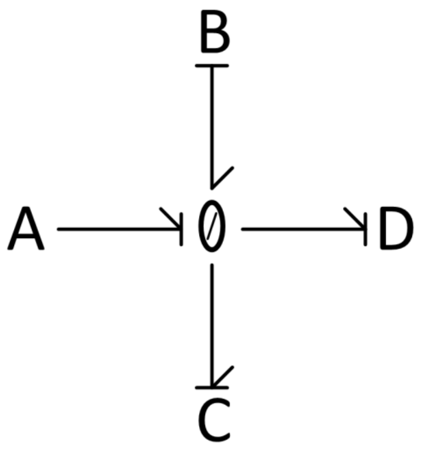
- ***Required causality*** – Sources
  - Effort sources can determine effort only
  - Flow sources can determine flow only

Restricted Causality

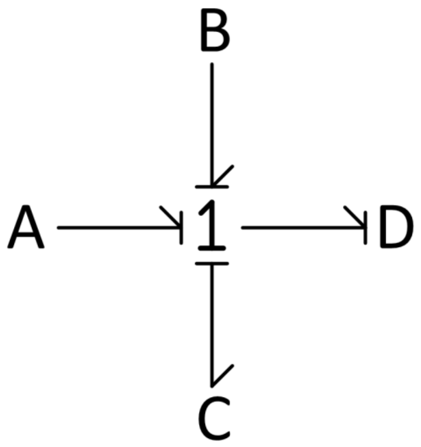
- ***<u>Restricted causality</u>*** – two-port elements and n-port junctions
  - 0-junctions
  - 1-junctions
  - Transformers
  - Gyrators
- Causality for all connected bonds and elements determined by the causality of one connected bond and element

Restricted Causality

- ***0-junction***
  - Constant effort, so only one element can set the effort
  - Only one causal stroke will be near the 0-junction

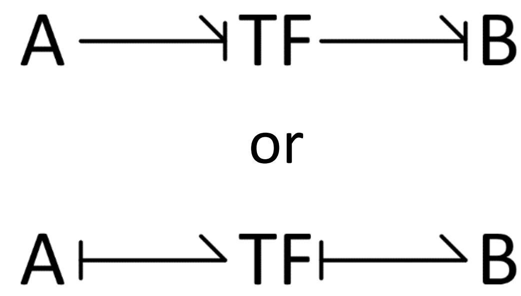


- ***1-junction***
  - Constant flow, so only one element can set the flow
  - All causal strokes, except for one, will be near the 1-junction

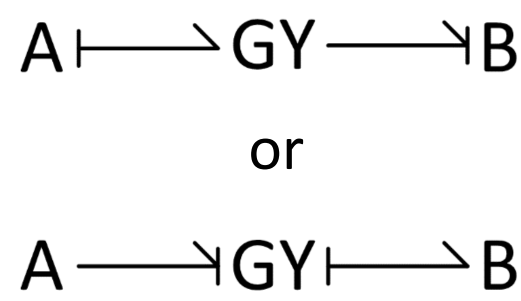


Restricted Causality

- ***Transformer***
  - Effort/flow at one port determines effort/flow at the other
  - If TF determines effort at one port, it will determine flow at the other
  - Only one causal stroke near the TF

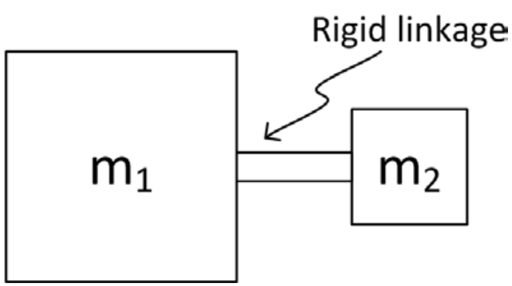


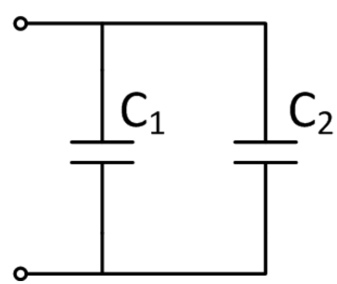
- ***Gyrator***
  - Effort at one port determines flow at the other and vice-versa
  - GY will determine both efforts or both flows
  - Both or neither causal strokes near the GY



Integral Causality

- ***Integral Causality*** – ***independent*** energy-storage elements (I's and C's)
  - A component in integral causality will either:
    - Integrate effort to determine flow, or
    - Integrate flow to determine effort
- ***Independent*** energy-storage elements:
  - Energy storage not directly tied to – not algebraically determined by – any other energy-storage element
  - Elements that are *not independent*:



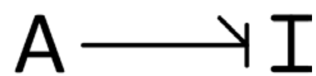


Integral Causality

- ***Inertia***

$$f = \frac{1}{I} p = \frac{1}{I} \int e \, dt$$

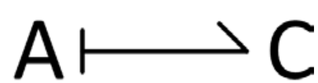
  - Inertias in integral causality integrate applied effort to determine flow



- ***Capacitor***

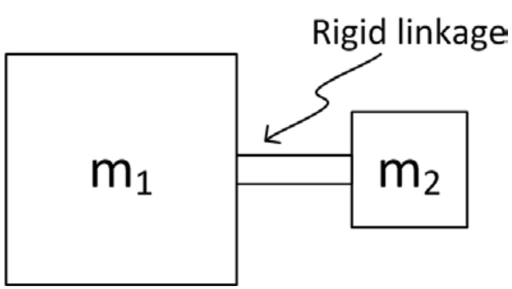
$$e = \frac{1}{C} q = \frac{1}{C} \int f \, dt$$

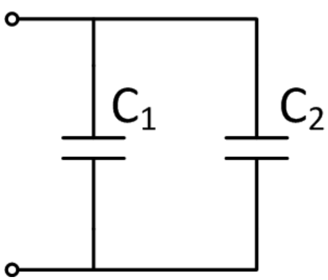
  - Capacitors in integral causality integrate applied flow to determine effort



Derivative Causality

- ***Derivative Causality*** – ***dependent*** energy-storage elements (I's and C's)
  - A component in derivative causality will either:
    - Differentiate effort to determine flow, or
    - Differentiate flow to determine effort
- ***Dependent*** energy-storage elements:
  - Energy storage directly tied to – algebraically related to – another energy-storage element
  - Dependent energy storage elements:



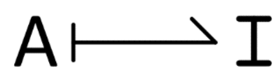


Derivative Causality

- ***Inertia***

$$e = \frac{dp}{dt} = I\frac{df}{dt}$$

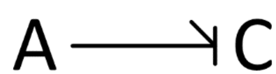
  - Inertias in derivative causality differentiate applied flow to determine effort
  - Flow determined by associated inertia in integral causality



- ***Capacitor***

$$f = \frac{dq}{dt} = C\frac{de}{dt}$$

  - Capacitors in derivative causality differentiate applied effort to determine flow
  - Effort determined by associated capacitor in integral causality

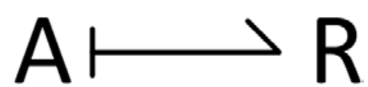


Arbitrary Causality

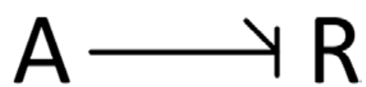
- ***<u>Arbitrary Causality</u>*** – resistors
  - Causality assigned to resistors is determined by the rest of the system
  - Constitutive law for resistors

$$e = f \cdot R \qquad \text{or} \qquad f = \frac{1}{R} e$$

  - Resistors can determine effort from an applied flow



  - Or, determine flow from an applied effort



Assigning Causality

Assigning Causality

- Starting with a simplified bond graph system model, assign causality to each element
  - Causality indicated by the addition of a ***causal stroke*** to each bond
- Follow a ***sequential causality assignment procedure***
  - Procedure is complete once a causal stroke has been assigned to all bonds in the model

Assigning Causality – Procedure

1) Pick a source and assign its required causality
   a) Follow through with any implicated restricted causal assignments (i.e. at 0-jct., 1-jct., TF, GY), extending these through the bond graph as far as possible
   b) Repeat for all unassigned sources

2) Pick an energy-storage element (I or C) and assign integral (i.e. preferred) causality
   a) Follow through with any implicated restricted causal assignments (i.e. at 0-jct., 1-jct., TF, GY), extending these through the bond graph as far as possible
   b) Repeat for all unassigned energy-storage elements

Assigning Causality – Procedure

- Often, the procedure is complete following step 2
  - If not, proceed to step 3:

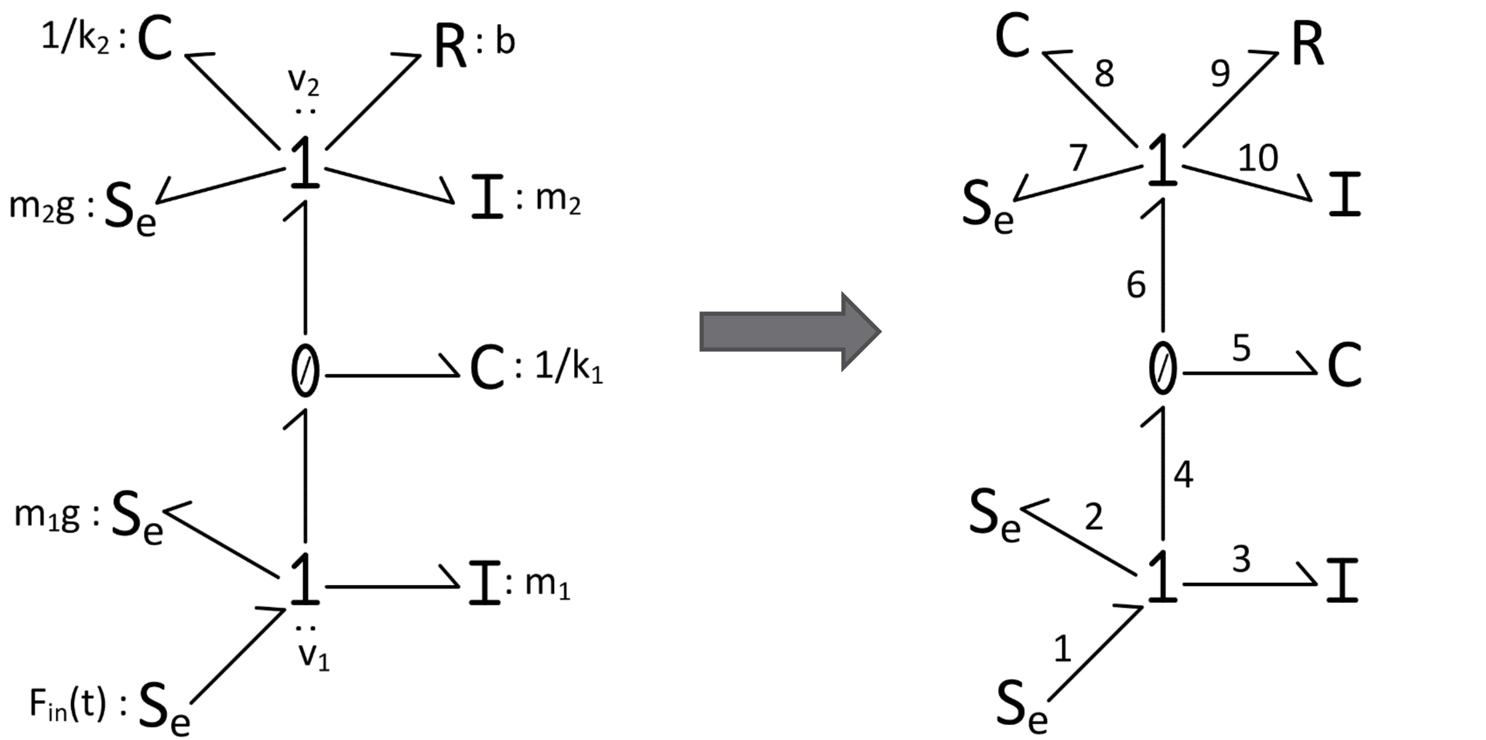
3) Pick an unassigned resistor, and arbitrarily assign causality
   a) Follow through with any implicated restricted causal assignments (i.e. at 0-jct., 1-jct., TF, GY), extending these through the bond graph as far as possible
   b) Repeat for all unassigned resistors

Causality Assignment – Results

- Four possible scenarios :
  1) All energy-storage elements in integral causality
     - All causality assigned following step 2
  2) Causality assignment completed by arbitrarily assigning causality of some R-elements
     - Indicates the presence of ***algebraic loops*** or ***resistor fields***
  3) Some energy-storage elements forced into derivative causality in step 2
     - This scenario referred to as ***derivative causality***
  4) Combination of 2 and 3, algebraic loops and derivative causality

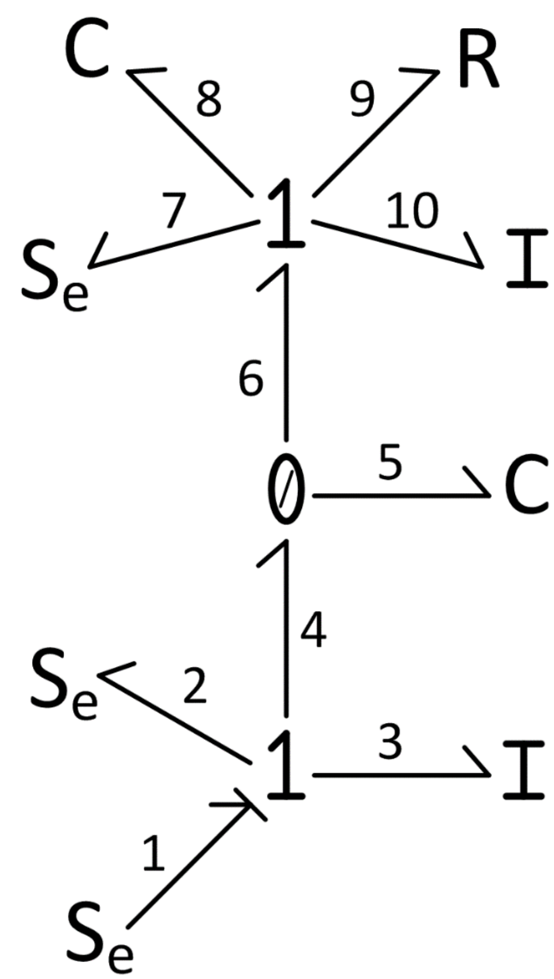
Assigning Causality – Example 1

- Mechanical system from the beginning of the section
- First, generate a ***computational bond graph***
  - Arbitrarily number the bonds
  - Drop the physical values associated with each element



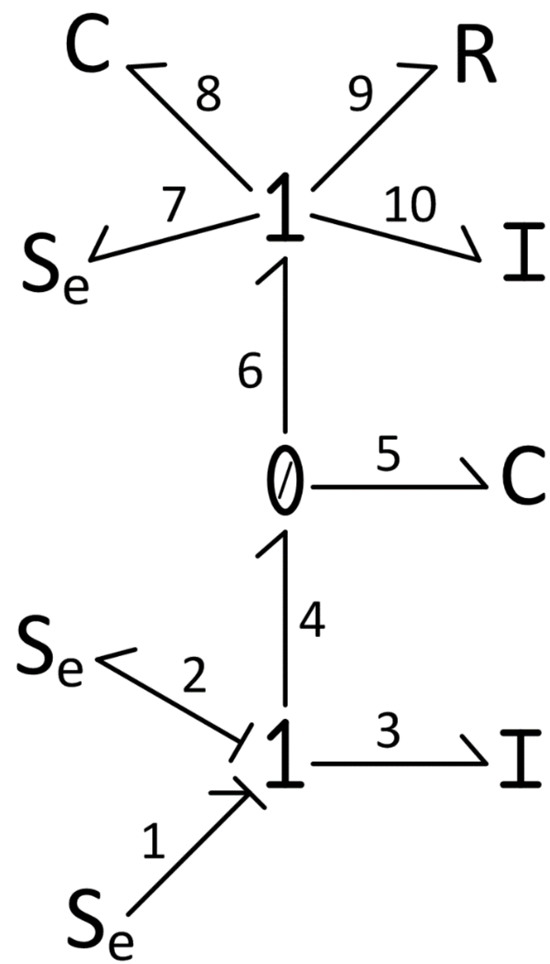
Assigning Causality – Example 1

- Assign causality to the computational bond graph
- **Step 1**: pick a source and assign the required causality
  - S_{e1} is an effort source
  - Causal stroke away from the source
  - Can have multiple causal strokes at the 1-jct, so can't go any further



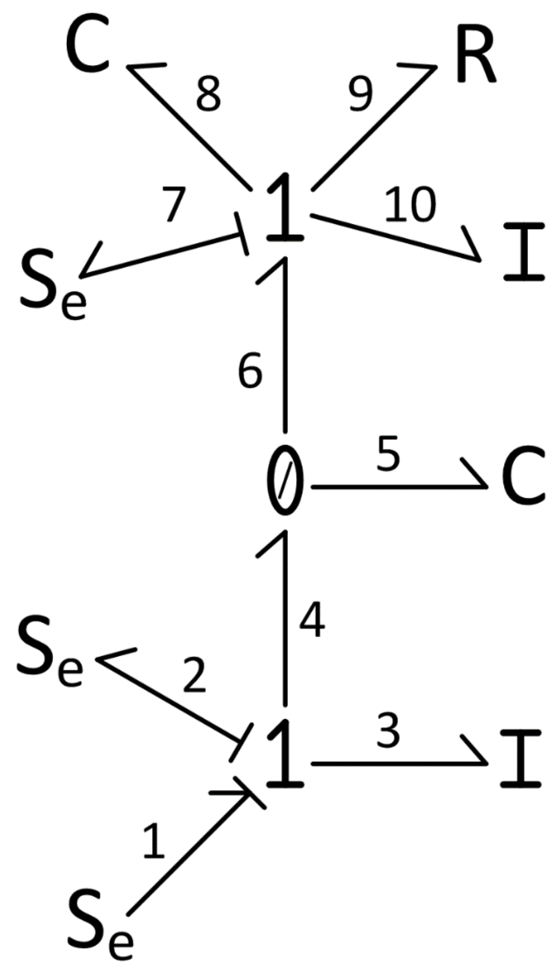
Assigning Causality – Example 1

- Pick an unassigned source and assign the required causality
  - Gravitational effort source acting on m_1, S_{e2}
  - Causal stroke at 1-jct side of the bond
  - Still two unassigned bonds at 1-jct
    - Only one will set the flow for the 1-jct, but don't yet know which one
    - Can't proceed any further



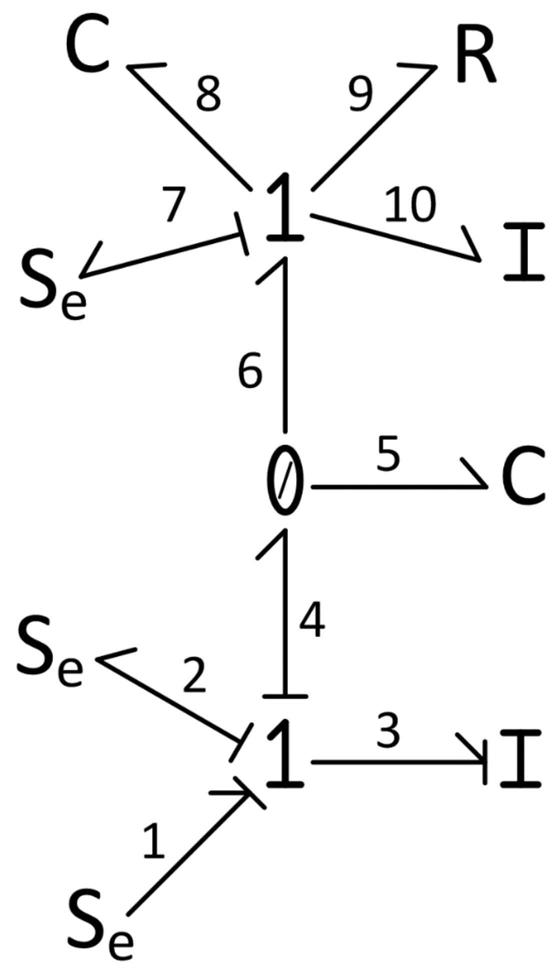
Assigning Causality – Example 1

- Pick an unassigned source and assign the required causality
  - Gravitational effort source acting on m_2, S_{e7}
  - Causal stroke at 1-jct side of the bond
  - Again, can't proceed any further
  - Causality of all sources assigned
  - Proceed to step 2



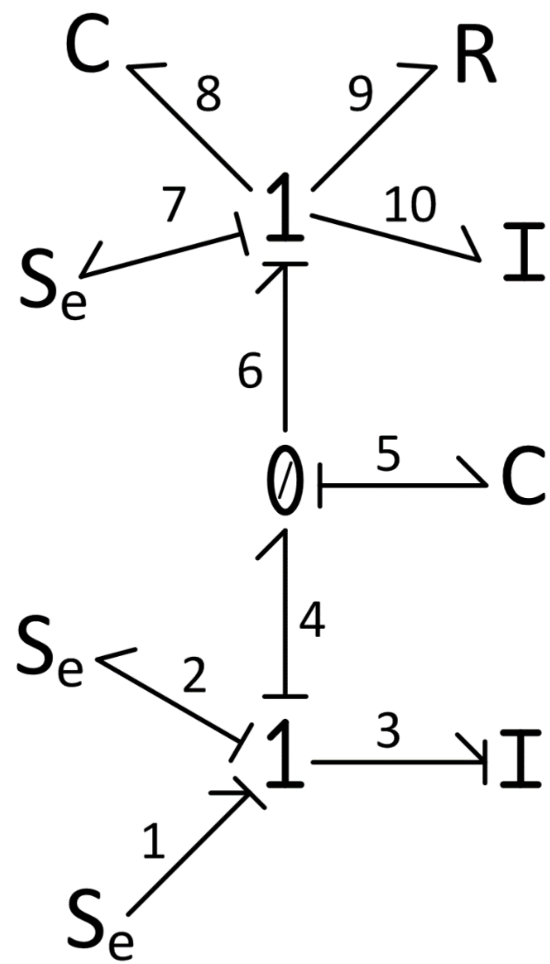
Assigning Causality – Example 1

- **<u>Step 2</u>**: pick an energy-storage element and assign integral causality
  - Inertia, I_3
  - Causal stroke near I_3
  - I_3 sets the flow for its 1-jct
  - Bond 4 cannot determine flow for the 1-jct
  - Causal stroke on bond 4 near the 1-jct
  - Can't proceed any further



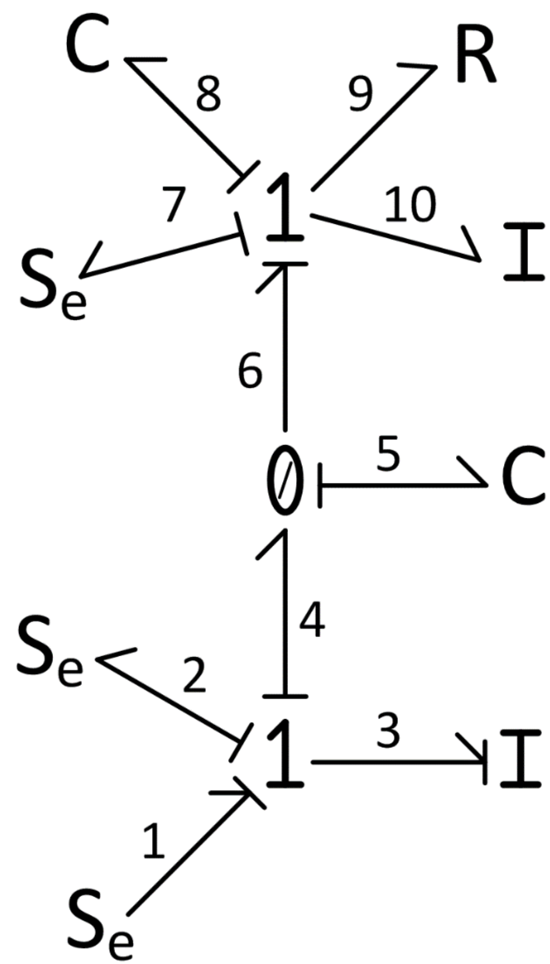
Assigning Causality – Example 1

- Pick an unassigned energy-storage element and assign integral causality
  - Capacitor C_5
  - Causal stroke away from C_5
  - C_5 sets the effort for the 0-jct
  - Bond 6 cannot determine effort for the 0-jct
  - Causal stroke on bond 6 near its 1-jct
  - Can't proceed any further



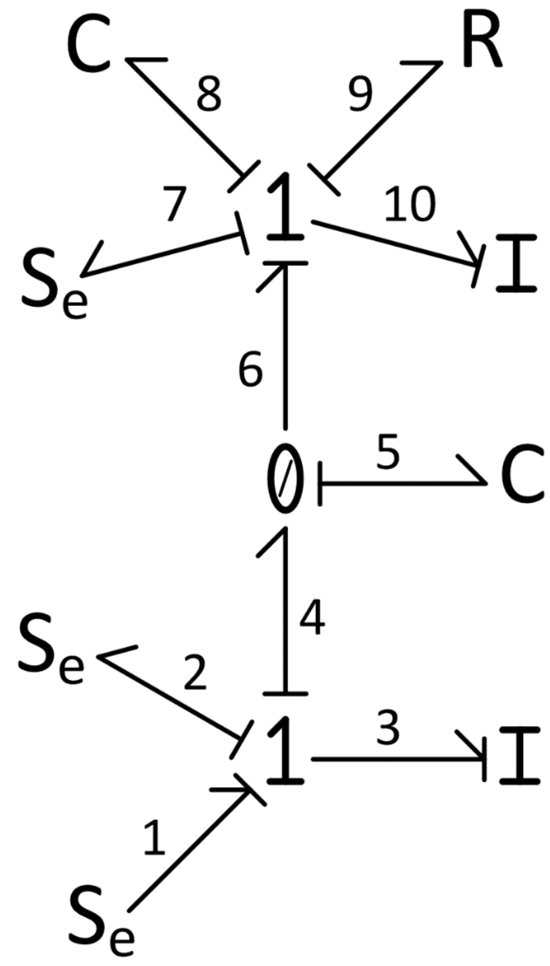
Assigning Causality – Example 1

- Pick an unassigned energy-storage element and assign integral causality
  - Capacitor C_8
  - Causal stroke away from C_8
  - Still don't know what element determines the flow for the v_2 1-jct
    - Could be R_9 or I_{10}
  - Move on to the next energy storage element



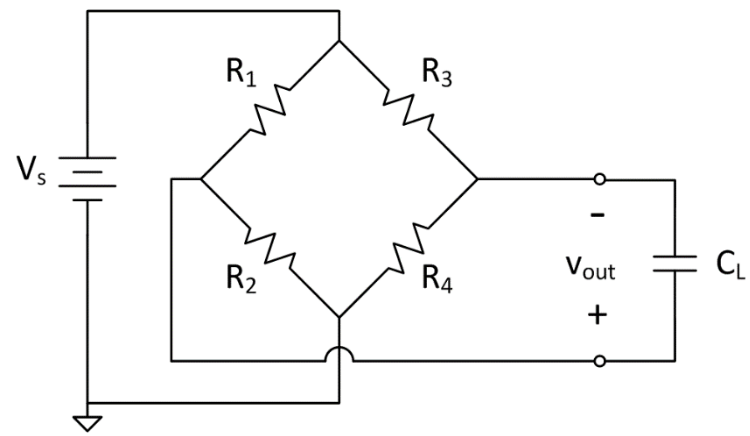
Assigning Causality – Example 1

- Pick an unassigned energy-storage element and assign integral causality
  - Inertia I_{10}
  - Causal stroke near from I_{10}
  - I_{10} sets the flow for its 1-jct
  - R_9 cannot set the flow for the 1-jct
  - Causal stroke away from R_9
- ***Causality assignment complete following step 2***

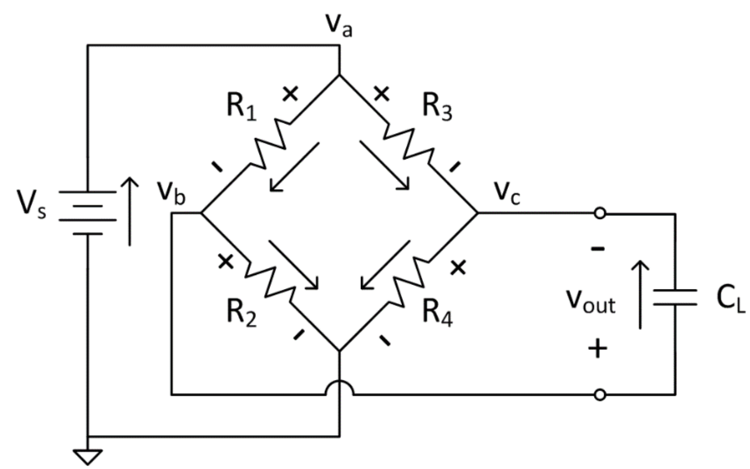


Assigning Causality – Example 2

- Consider a Wheatstone bridge circuit driving a capacitive load
  - Generate the bond graph and assign causality



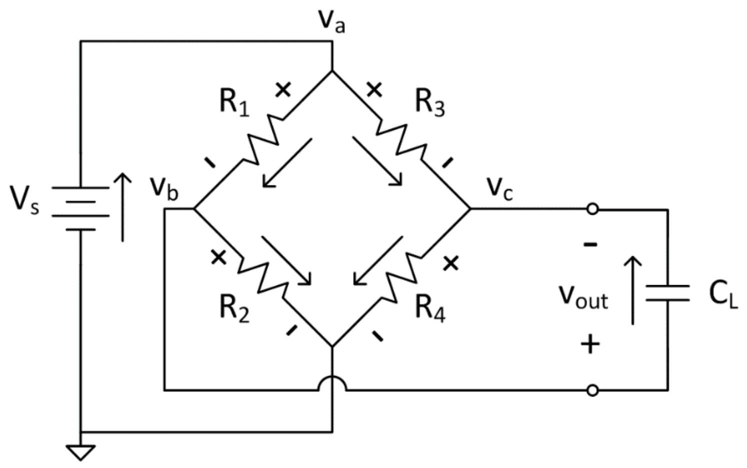
- First, identify and label all distinct node voltages on the schematic
  - Indicate voltage polarities and current directions



Assigning Causality – Example 2

- List all one and two-port elements along with their relevant voltages

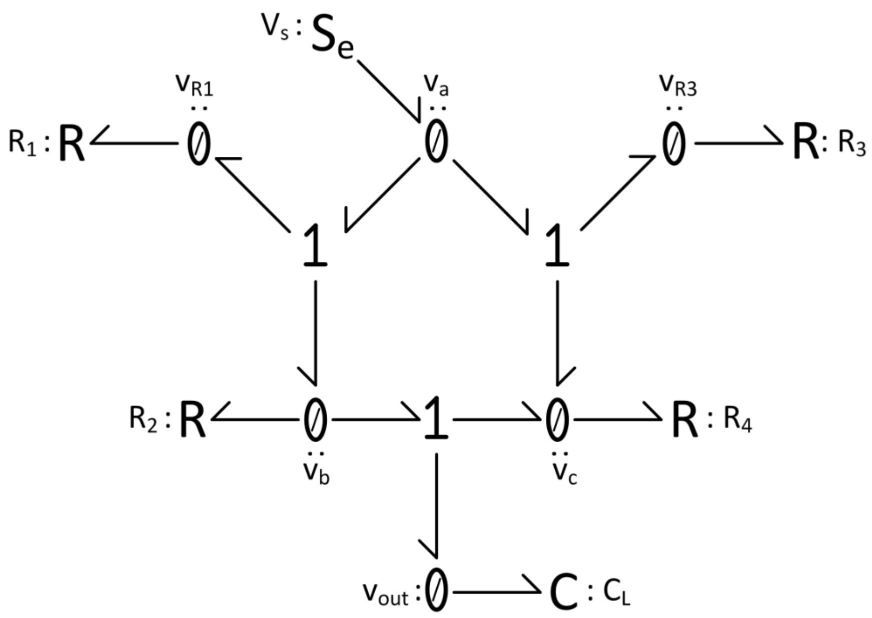
Element	Voltage
$V_s: S_e \rightharpoonup$	v_a
$R_1: R \leftharpoondown$	$v_{R1} = v_a - v_b$
$R_2: R \leftharpoondown$	v_b
$R_3: R \leftharpoondown$	$v_{R3} = v_a - v_c$
$R_4: R \leftharpoondown$	v_c
$C_L: C \leftharpoondown$	$v_{out} = v_b - v_c$



Assigning Causality – Example 2

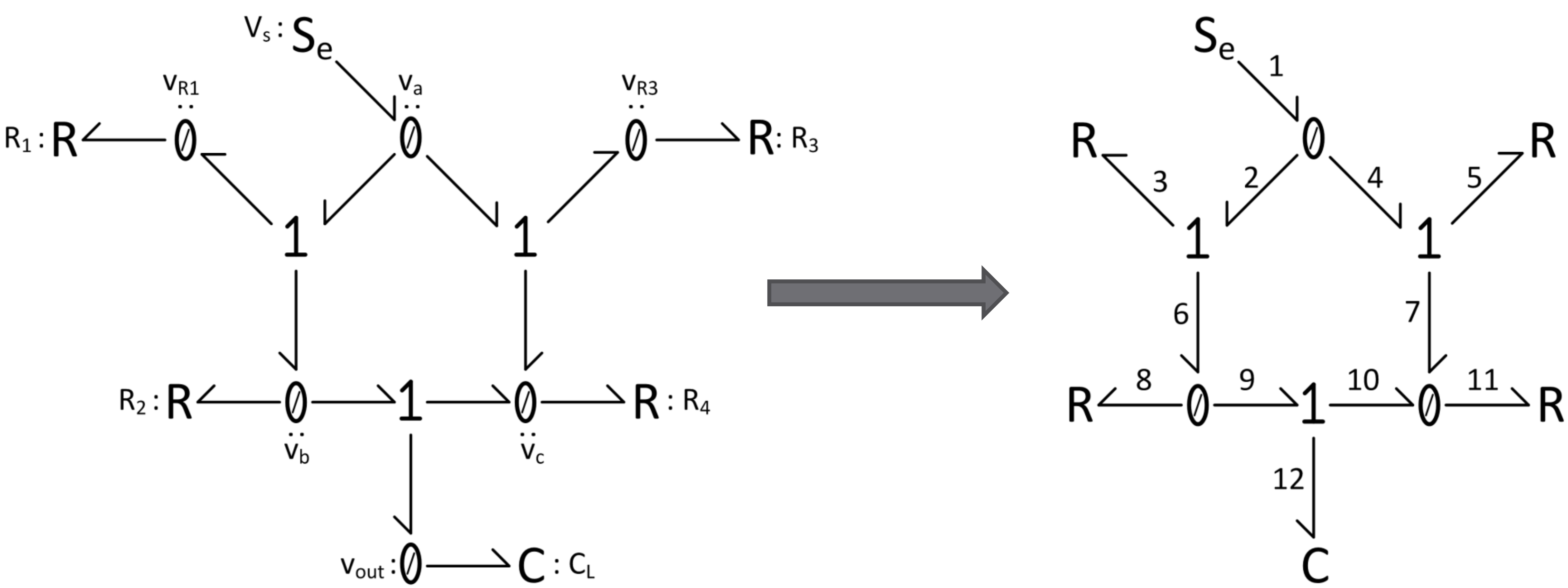
- Using the list of elements and voltages, generate the bond graph model for the circuit

Element	Voltage
$V_s\!:S_e \rightharpoonup$	v_a
$R_1\!:R \leftharpoondown$	$v_{R1} = v_a - v_b$
$R_2\!:R \leftharpoondown$	v_b
$R_3\!:R \leftharpoondown$	$v_{R3} = v_a - v_c$
$R_4\!:R \leftharpoondown$	v_c
$C_L\!:C \leftharpoondown$	$v_{out} = v_b - v_c$



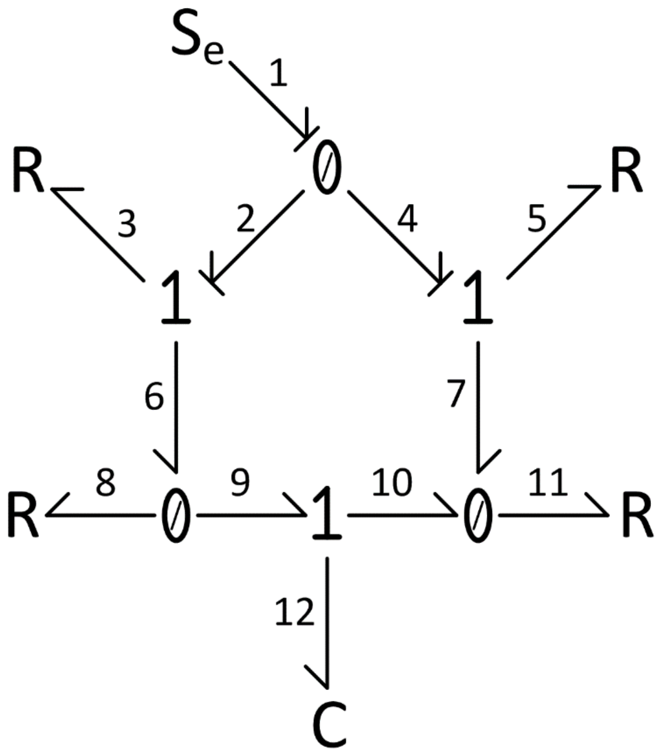
Assigning Causality – Example 2

- Simplify and create the computational bond graph



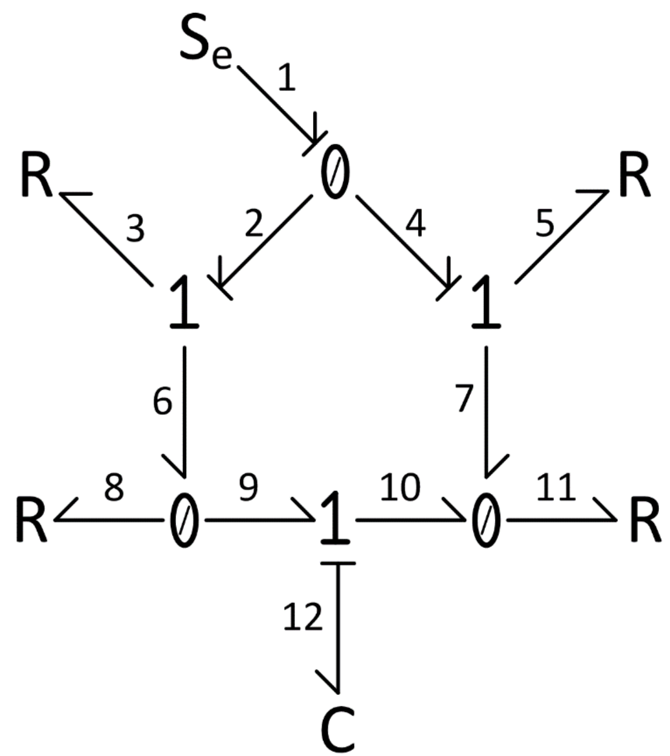
Assigning Causality – Example 2

- Assign causality to the computational bond graph
- **Step 1**: pick a source and assign the required causality
  - S_{e1} is an effort source
  - Causal stroke away from the source
  - S_{e1} sets the effort on its attached zero junction
    - Causal strokes on bonds 2 and 4 are near their respective 1-junctions
  - Can't proceed any further
    - Move on to step 2



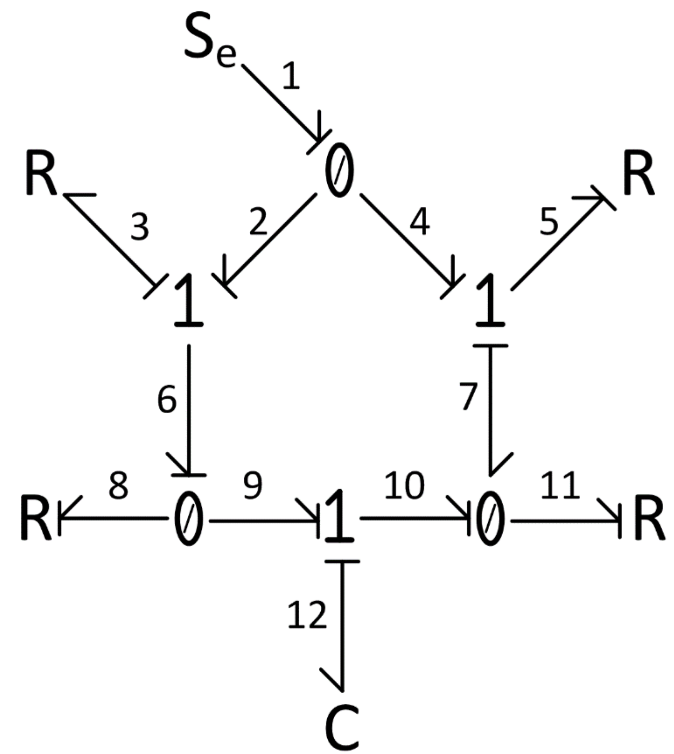
Assigning Causality – Example 2

- **Step 2**: pick an energy-storage element and assign integral causality
  - C_{12} is the only energy-storage element
  - Causal stroke away capacitor for integral causality
  - Can have more than one causal stroke near the attached 1-jct
  - Can't proceed any further
    - Move on to step 3



Assigning Causality – Example 2

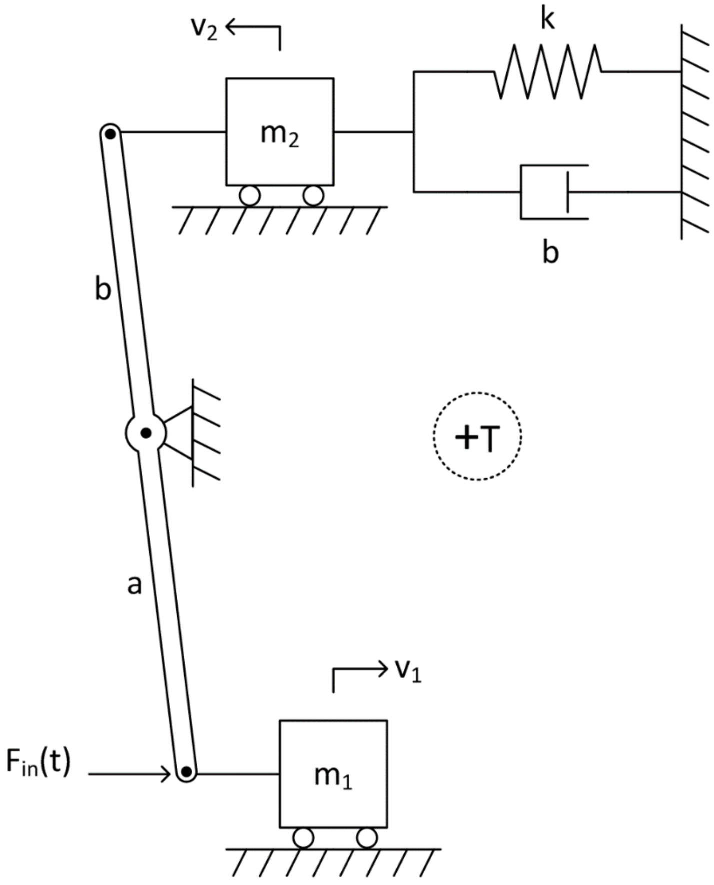
- **<u>Step 3</u>**: pick a resistor and arbitrarily assign causality
  - Start with R_3
    - Choosing R_3 to determine effort means bond 6 must set the flow on the attached 1-jct
  - Bond 6 sets the effort on its 0-jct
    - Bonds 8 and 9 cannot – their causal strokes are away from the 0-jct
  - Bonds 9 and 12 determine effort on their 1-jct
    - Bond 10 must determine flow
  - Bond 10 sets the effort for its 0-jct
    - Causal stroke on bonds 7 and 11 are away from the 0-jct
  - Bond 7 determines effort on its 1-jct
    - Bond 5 must set the flow for that 1-jct



- ***Causality assignment required arbitrary assignment of resistor causality***
  - ***Algebraic Loops are present***

Assigning Causality – Example 3

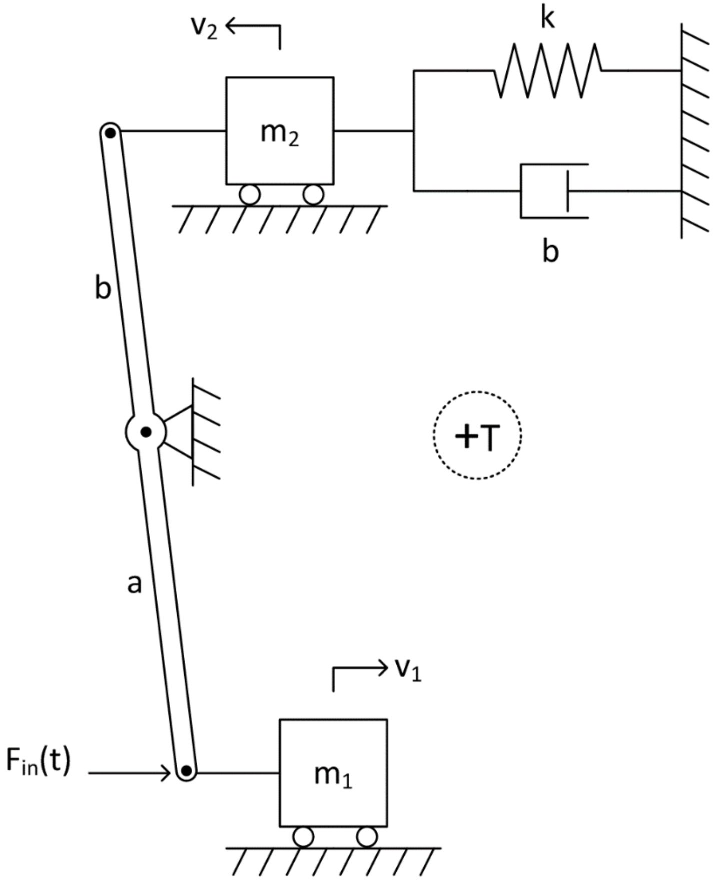
- Spring/mass/damper system
- Really only translational
  - No elements exist in the rotational domain
  - Massless, frictionless lever
- First, label all distinct non-zero velocities and select positive relative velocity reference for springs and dampers (tension, here)



Assigning Causality – Example 3

- Next, tabulate all one- and two-port elements and their corresponding velocities

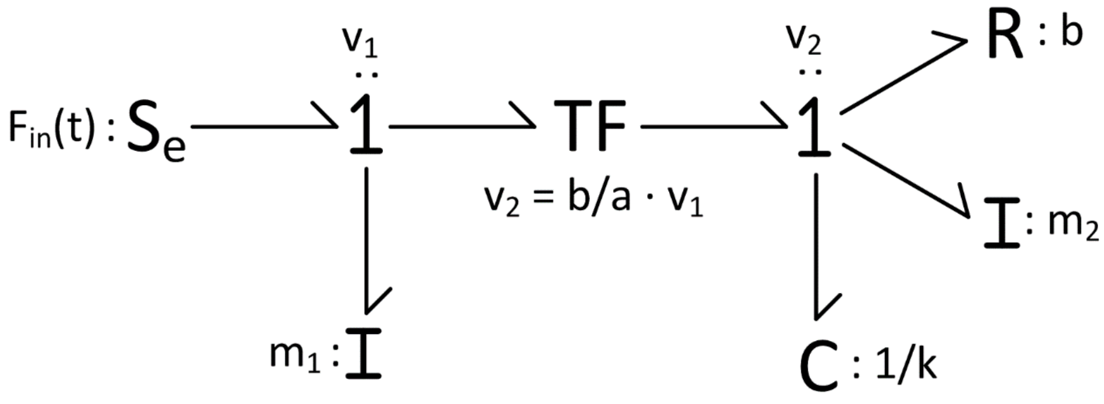
Element	Velocity
$m_1\!:I \leftharpoondown$	v_1
$m_2\!:I \leftharpoondown$	v_2
$1/k\!:C \leftharpoondown$	v_2
$b\!:R \leftharpoondown$	v_2
$F_{in}(t) : S_e \rightharpoonup$	v_1
$\rightharpoonup TF \rightharpoonup$ $v_2 = b/a \cdot v_1$	v_1 v_2



Assigning Causality – Example 3

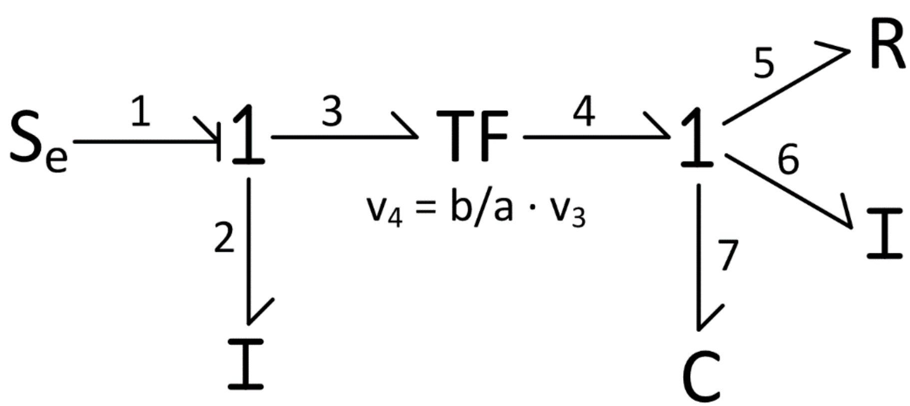
- Generate the bond graph
  - As always, annotate with the TF equation

Element	Velocity
$m_1 : I \leftharpoondown$	v_1
$m_2 : I \leftharpoondown$	v_2
$1/k : C \leftharpoondown$	v_2
$b : R \leftharpoondown$	v_2
$F_{in}(t) : S_e \rightharpoonup$	v_1
$\rightharpoonup TF \rightharpoonup$ $v_2 = b/a \cdot v_1$	v_1 v_2



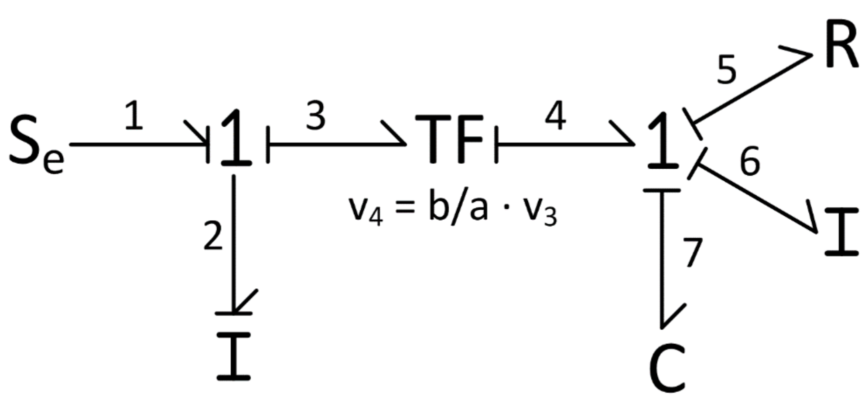
Assigning Causality – Example 3

- Generate a computational bond graph and begin assigning causality
- **Step 1**: pick a source and assign the required causality
  - S_{e1} is an effort source
  - Causal stroke away from the source
  - S_{e1} applies effort to its attached one junction
    - Bonds 2 or 3 could also apply effort to the 1-jct
  - Can’t proceed any further
    - Move on to step 2



Assigning Causality – Example 3

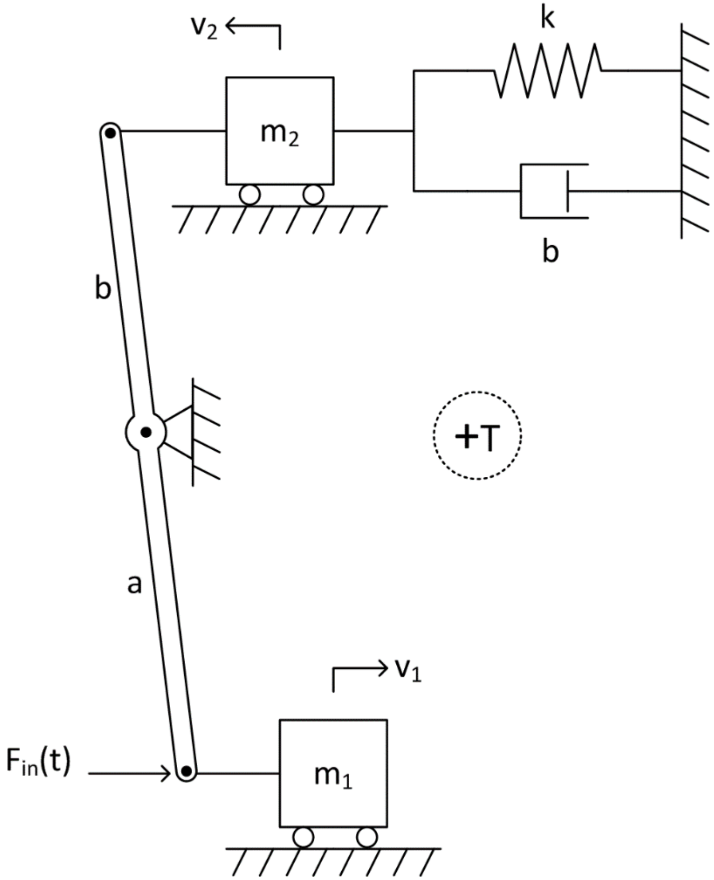
- **<u>Step 2</u>**: pick an energy-storage element and assign integral causality
  - Inertia element I_2
  - Causal stroke near I_2
  - I_2 determines the flow on its 1-jct
    - Bond 3 must apply effort to the 1-jct
  - Bond 3 determines flow at the transformer
    - Bond 4 must determine effort at the transformer
    - Bond 4 sets the flow on its 1-jct
    - Bonds 5, 6, and 7 must all apply effort to the 1-jct



- I_6 is in ***derivative causality***
  - I_2 and I_6 (m_1 and m_2) are ***not independent***

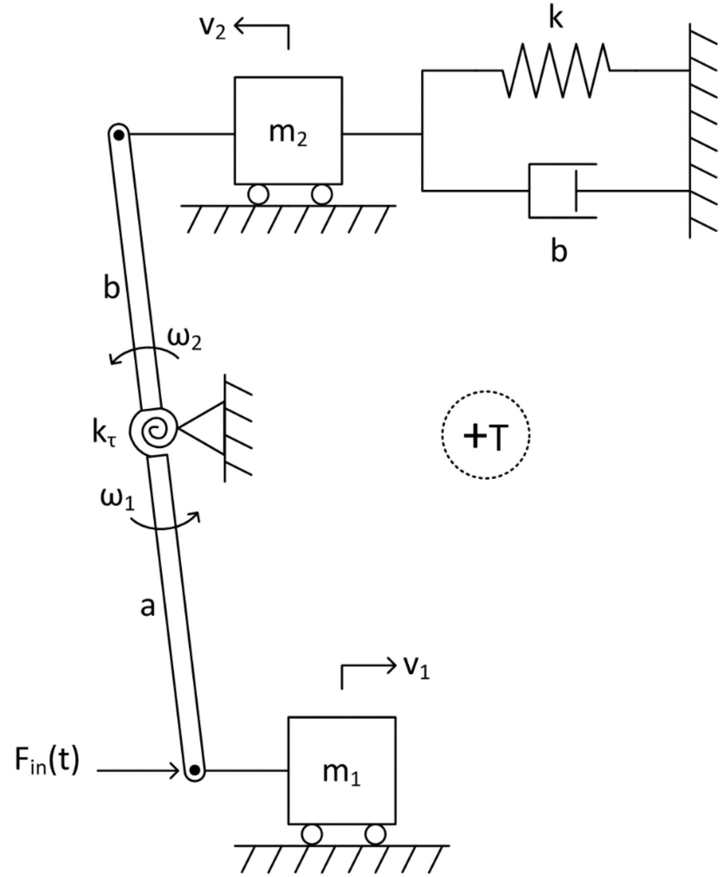
Assigning Causality – Example 3

- The physical model resulted in a bond graph with ***derivative causality***
- Presence of derivative causality is due to a modeling decision
  - Lever was assumed to be perfectly rigid



Assigning Causality – Example 4

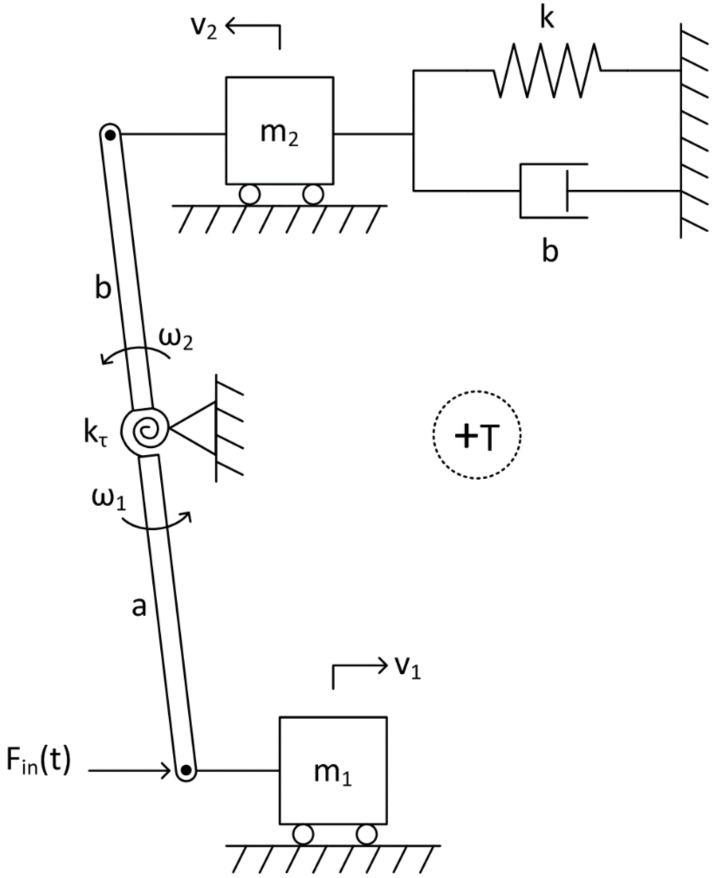
- Let's say we want to model some compliance of the lever
  - Add a torsional spring at the fulcrum
- Now the system includes both ***translational*** and ***rotational*** components
  - Must include angular velocities, ω_1 and ω_2
- m_1 and m_2 are now ***independent inertias***
  - v_1 and v_2 are independent



Assigning Causality – Example 4

- Capacitor added to the model to account for lever compliance
- Transformers translate between translational and rotational domains

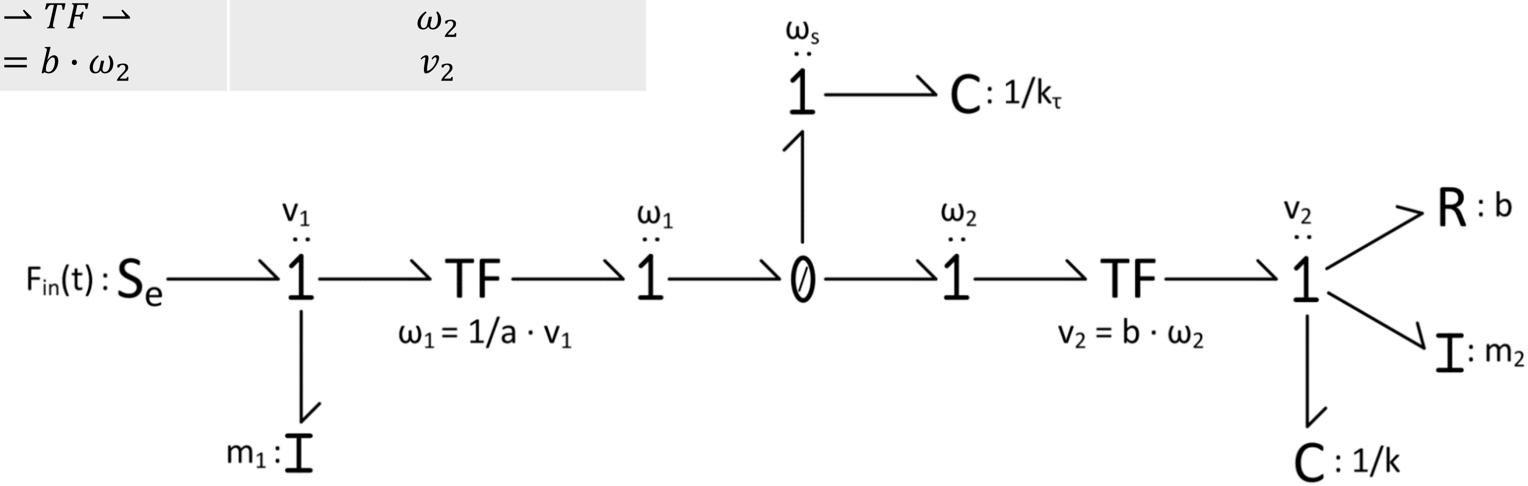
Element	Velocity
$m_1 : I \leftharpoondown$	v_1
$m_2 : I \leftharpoondown$	v_2
$1/k : C \leftharpoondown$	v_2
$b : R \leftharpoondown$	v_2
$F_{in}(t) : S_e \rightharpoonup$	v_1
$1/k_\tau : C \leftharpoondown$	$\omega_s = \omega_1 - \omega_2$
$\rightharpoonup TF \rightharpoonup$ $\omega_1 = 1/a \cdot v_1$	v_1 ω_1
$\rightharpoonup TF \rightharpoonup$ $v_2 = b \cdot \omega_2$	ω_2 v_2



Assigning Causality – Example 4

Element	Velocity
$m_1 : I \leftharpoondown$	v_1
$m_2 : I \leftharpoondown$	v_2
$1/k_\tau : C \leftharpoondown$	$\omega_s = \omega_1 - \omega_2$
$1/k : C \leftharpoondown$	v_2
$b : R \leftharpoondown$	v_2
$F_{in}(t) : S_e \rightharpoondown$	v_1
$\rightharpoondown TF \rightharpoondown$ $\omega_1 = 1/a \cdot v_1$	v_1 ω_1
$\rightharpoondown TF \rightharpoondown$ $v_2 = b \cdot \omega_2$	ω_2 v_2

- Generate the bond graph
  - Capacitor added in the rotational domain



Assigning Causality – Example 4

- Simplify, generate a computational bond graph, and assign causality
- A few more iterations of step 2 (assigning causality to energy-storage elements) are required
- Result now is a bond graph model where all energy storage elements are in ***integral causality***
  - ***All energy-storage elements are independent***