SECTION 1: INTRODUCTION

ESE 430 – Feedback Control Systems

Introduction

Introduction

- In ESE 330, you learned how to ***model*** dynamic systems and ***simulate*** their responses
  - **Analysis** – how does a given system respond
  - **Design** (possibly) – tuning system parameters to achieve a desired response
- In ESE 430, you will learn how to design ***feedback control systems*** to improve the response of a given system in three primary areas:
  - ***Dynamic response***
  - ***Steady-state error***
  - ***Stability***

Introduction

- In this section of notes we will take a look at a simple motor-driven rack and pinion positioning system example to do the following:
  - Review dynamic system modeling fundamentals
    - Bond graphs
    - State-variable models
    - System poles/zeros
    - Transient response – step, impulse, …
    - Frequency response
  - Introduce feedback control
    - What is it?
    - How can it help us obtain a desired system response?

Dynamic System Modeling

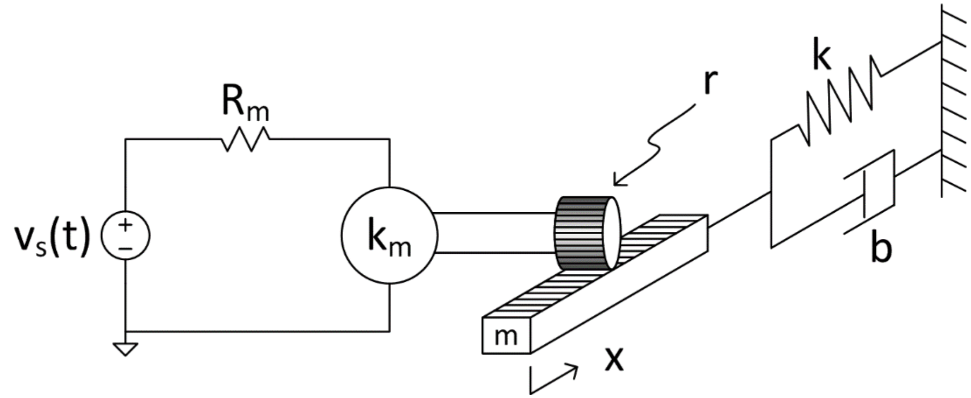
The design of a feedback control system requires first having a model of the system to be controlled.

This sub-section provides a review of dynamic system modeling and analysis fundamentals.

Bond-Graph Model

Rack and Pinion Positioning System

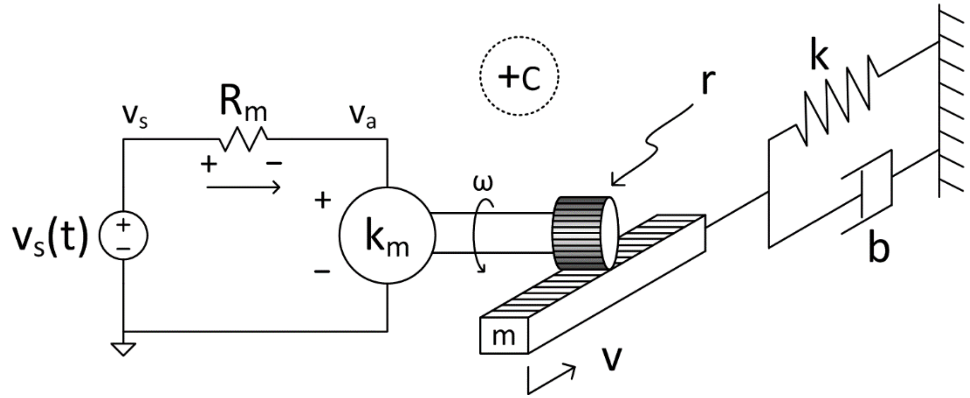
- Simplified rack and pinion positioning system
  - E.g., automated assembly equipment, print-head driver, etc.



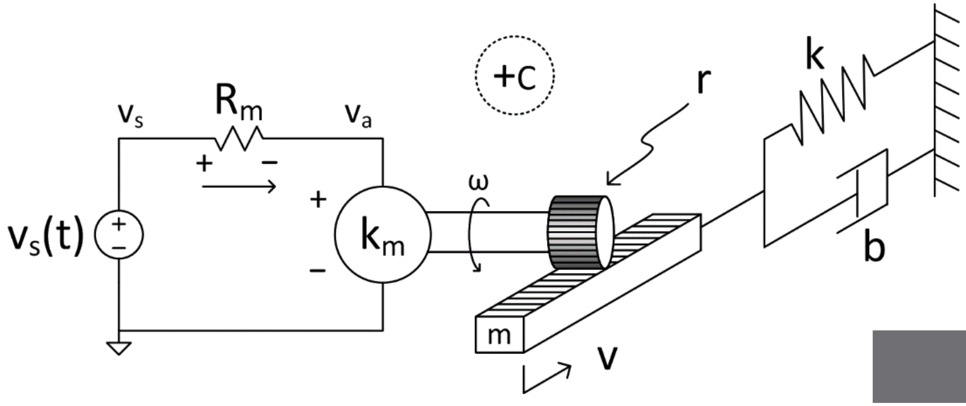
  - Voltage source drives a DC motor
    - Motor inductance neglected here
  - Motor turns shaft and pinion gear
  - As pinion turns, rack translates
    - The thing to be positioned is attached to the rack
    - Rack connection has both compliance and damping

Bond-Graph Model

- Create a bond-graph model
- First, ***annotate the schematic***
  - Label all node voltages
  - Indicate assumed positive voltage polarities and current direction
  - Label velocities at each mass and end of each spring and damper
  - Choose displacements of springs and dampers to be positive in either compression or tension



Bond-Graph Model



- Next, ***tabulate all one- and two-port elements***, along with relevant voltages or velocities
  - Bond orientation follows power convention
  - Include TF and GY equations

Element	Voltage/Velocity
$v_s(t): S_e \rightharpoonup$	v_s
$R_m: R \leftharpoonup$	$v_{R_m} = v_s - v_a$
$\rightharpoonup GY \rightharpoonup$ $\omega = 1/k_m \cdot v_a$	v_a ω
$\rightharpoonup TF \rightharpoonup$ $v = r \cdot \omega$	ω v
$m: I \leftharpoonup$	v
$b: R \leftharpoonup$	v
$1/k: C \leftharpoonup$	v

Bond-Graph Model

- Use the table to ***construct a bond graph***

Element	Voltage/Velocity
$v_s(t): S_e \rightharpoonup$	v_s
$R_m: R \leftharpoondown$	$v_{R_m} = v_s - v_a$
$\rightharpoonup GY \rightharpoonup$ $\omega = 1/k_m \cdot v_a$	v_a ω

Element	Voltage/Velocity
$\rightharpoonup TF \rightharpoonup$ $v = r \cdot \omega$	ω v
$m: I \leftharpoondown$	v
$b: R \leftharpoondown$	v
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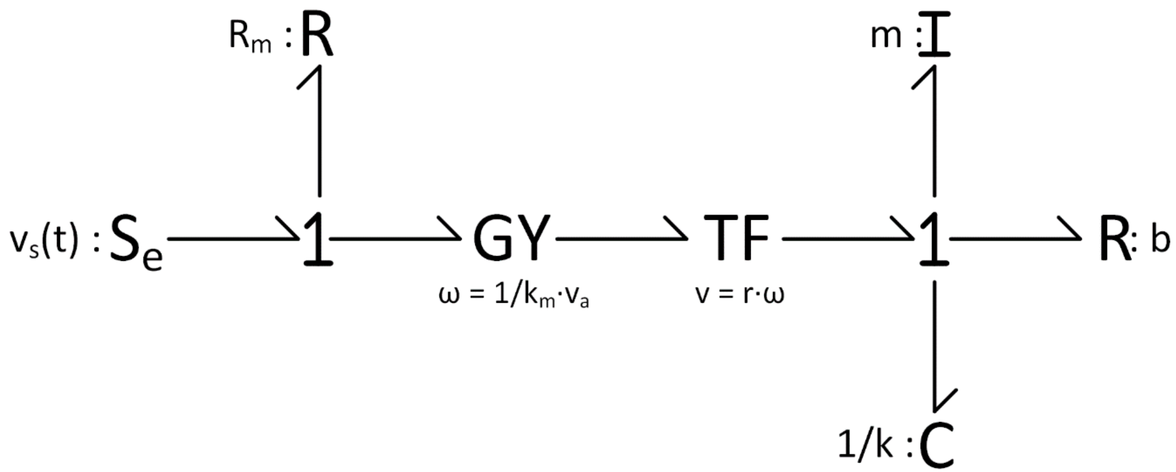
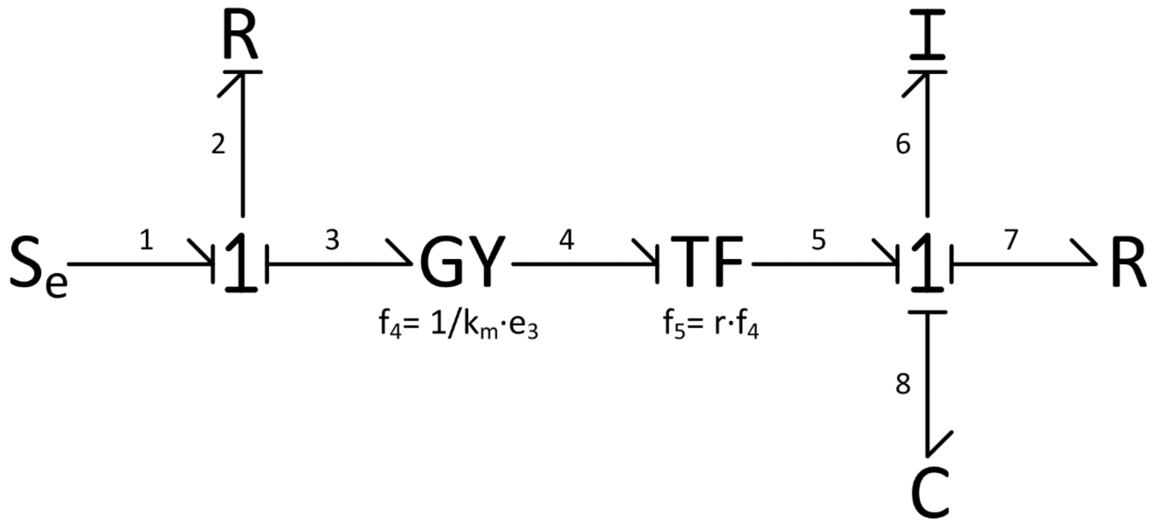


Bond-Graph Model

- Create ***computational bond graph*** and ***assign causality***



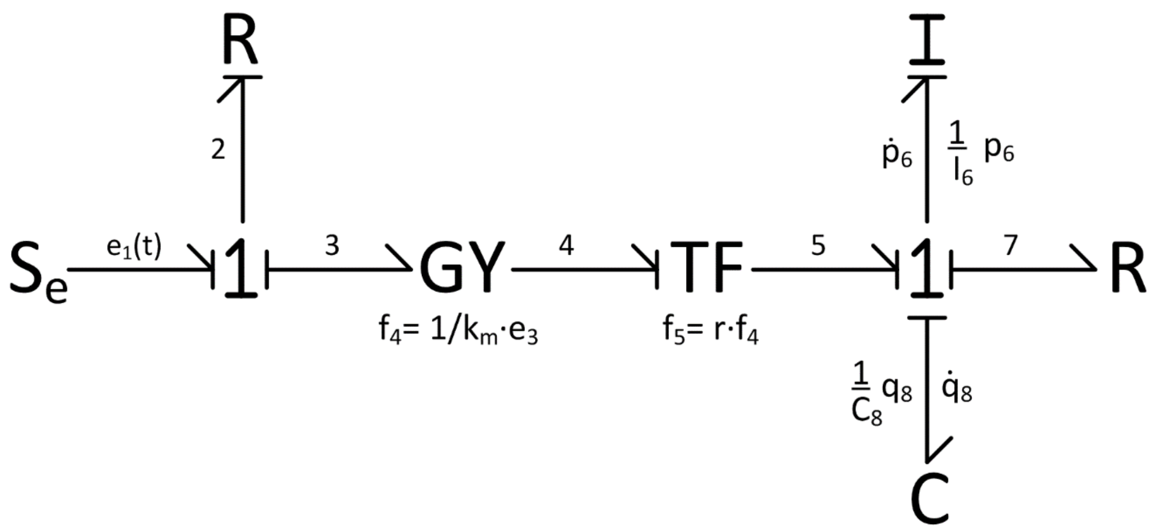
- I_6 and C_8 both have integral causality
  - Two independent energy-storage elements
  - A second-order system

Bond-Graph Model

- State variables:

$$\mathbf{x} = \begin{bmatrix} p_6 \\ q_8 \end{bmatrix}$$

- Annotate the bond graph in preparation for state equation derivation
  - Sources
  - State variable derivatives as effort/flow on independent I's and C's
  - Apply constitutive laws to annotate the other power variables



State-Variable Model

State-Variable System Model

- Use annotated bond graph to derive a state-variable model for the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x} + Du$$

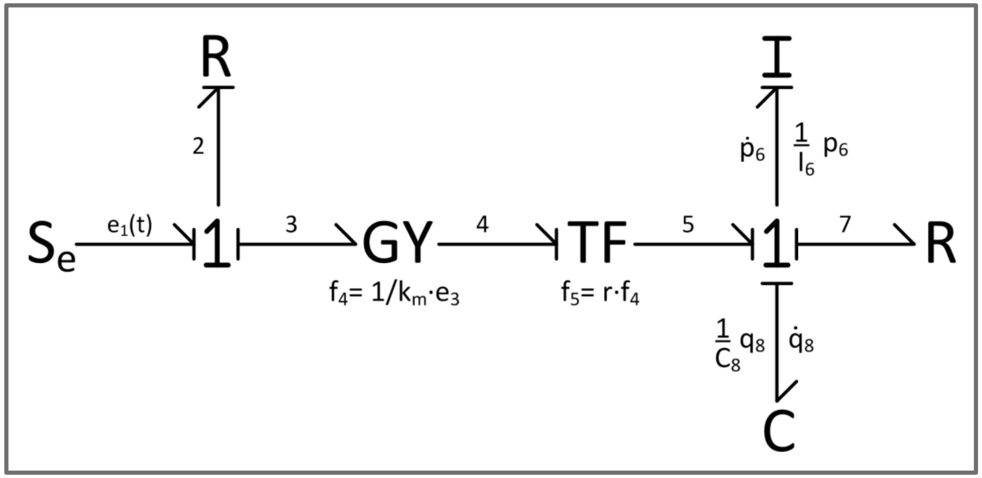
  - State derivatives are linear combinations of state variables and inputs
  - Output is a linear combination of states and inputs
  - This is a ***SISO system***
    - Single-input, single-output
    - u, y, and D are scalars

State Equation Derivation

- Follow causality through the bond graph to express state variable derivatives as linear combinations of states and inputs
  - Start with $\dot{p}_6$

$$\dot{p}_6 = e_6 = e_5 - e_7 - e_8$$

$$\dot{p}_6 = \frac{1}{r} e_4 - R_7 f_7 - \frac{1}{C_8} q_8$$



$$\dot{p}_6 = \frac{k_m}{r} f_3 - \frac{R_7}{I_6} p_6 - \frac{1}{C_8} q_8 \qquad (1)$$

$$f_3 = f_2 = \frac{1}{R_2} e_2 = \frac{1}{R_2}(e_1(t) - e_3) = \frac{1}{R_2} e_1(t) - \frac{k_m}{R_2} f_4$$

$$f_3 = \frac{1}{R_2} e_1(t) - \frac{k_m}{R_2}\frac{1}{r} f_5 = \frac{1}{R_2} e_1(t) - \frac{k_m}{R_2 r}\frac{1}{I_6} p_6 \qquad (2)$$

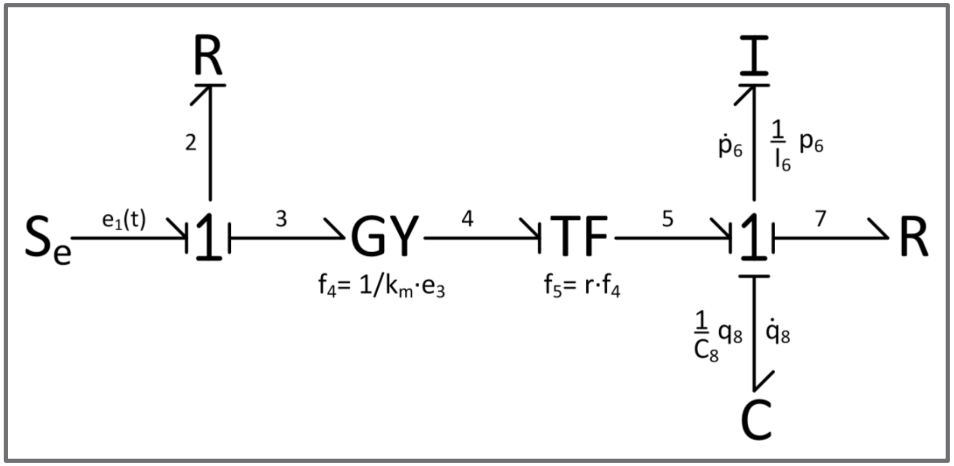
State Equation and Output Equation

- Substituting (2) into (1) give the first state equation:

$$\dot{p}_6 = -\frac{k_m^2 + R_7 r^2 R_2}{r^2 R_2 I_6} p_6 - \frac{1}{C_8} q_8 + \frac{k_m}{r R_2} e_1(t) \qquad (3)$$

- Next, move on to $\dot{q}_8$

$$\dot{q}_8 = f_8 = f_6 = \frac{1}{I_6} p_6$$



- The second state equation:

$$\dot{q}_8 = \frac{1}{I_6} p_6 \qquad (4)$$

- The output is the position of the rack, which is also the displacement of the spring

$$y = q_8 \qquad (5)$$

State-Variable System Model

- Equations (3) – (5) can be assembled in matrix form to give the state-variable system model:

$$\begin{bmatrix} \dot{p}_6 \\ \dot{q}_8 \end{bmatrix} = \begin{bmatrix} -\frac{k_m^2 + R_7 r^2 R_2}{r^2 R_2 I_6} & -\frac{1}{C_8} \\ \frac{1}{I_6} & 0 \end{bmatrix} \begin{bmatrix} p_6 \\ q_8 \end{bmatrix} + \begin{bmatrix} \frac{k_m}{r R_2} \\ 0 \end{bmatrix} e_1(t) \tag{6}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} p_6 \\ q_8 \end{bmatrix}$$

- Substituting in physical system parameters:

$$\begin{bmatrix} \dot{p} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} -\frac{k_m^2 + b r^2 R_m}{r^2 R_m m} & -k \\ \frac{1}{m} & 0 \end{bmatrix} \begin{bmatrix} p \\ x \end{bmatrix} + \begin{bmatrix} \frac{k_m}{r R_m} \\ 0 \end{bmatrix} e_1(t) \tag{7}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ x \end{bmatrix}$$

Transfer Function

Transfer Function

- The state-variable model is one of many possible mathematical models for the system
  - A ***time-domain*** model
- The ***transfer function*** is another model
  - A ***Laplace-domain*** model
- The ratio of the output to the input in the Laplace domain, assuming zero initial conditions:

$$G(s) = \frac{Y(s)}{U(s)}$$

  - Useful for determining the Laplace-domain output

$$Y(s) = G(s) \cdot U(s)$$

  - System poles/zeros are readily apparent
  - Substitute $s \rightarrow j\omega$ for frequency response function

Transfer Function

- A couple of ways to convert from the state-variable model to the transfer function
  - Calculate directly:

$$G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$$

    - Requires matrix inversion
  - Solve for the states using Cramer's rule, combine according to the output equation and solve algebraically for $G(s) = Y(s)/U(s)$
    - Matrix inversion is not required
- We'll step through both methods

State Space → Transfer Function – 1

- From the $\mathbf{A}\ \mathbf{B}\ \mathbf{C}\ \mathbf{D}$ matrices that define the state variable model:

$$G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$$

- For our example:

$$G(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s + \dfrac{k_m^2 + br^2R_m}{r^2R_m m} & k \\ -\dfrac{1}{m} & s \end{bmatrix}^{-1} \begin{bmatrix} \dfrac{k_m}{rR_m} \\ 0 \end{bmatrix}$$

- The inverse of the $(sI - A)$ matrix is

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{adj(s\mathbf{I} - \mathbf{A})}{|s\mathbf{I} - \mathbf{A}|} = \frac{adj(s\mathbf{I} - \mathbf{A})}{\Delta(s)}$$

- $\Delta(s)$ is the ***characteristic polynomial*** of the system

The Characteristic Polynomial

- The ***characteristic polynomial***

$$\Delta(s) = |sI - A| = \begin{vmatrix} s + \frac{k_m^2 + br^2R_m}{r^2R_m m} & k \\ -\frac{1}{m} & s \end{vmatrix}$$

$$\Delta(s) = s^2 + \frac{k_m^2 + br^2R_m}{r^2R_m m}s + \frac{k}{m}$$

- Recall
  - $\Delta(s)$ is the denominator of the Laplace transform of every state and the output
  - The roots of $\Delta(s)$, the poles of the system, determine the nature of the system response

State Space → Transfer Function – 1

$$(s\mathbf{I}-\mathbf{A})^{-1}=\frac{1}{\Delta(s)}\begin{bmatrix} s & -k \\ \frac{1}{m} & s+\frac{k_m^2+br^2R_m}{r^2R_m m}\end{bmatrix}$$

- Substituting back into the expression for $G(s)$

$$G(s)=\frac{1}{\Delta(s)}\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix} s & -k \\ \frac{1}{m} & s+\frac{k_m^2+br^2R_m}{r^2R_m m}\end{bmatrix}\begin{bmatrix}\frac{k_m}{rR_m} \\ 0\end{bmatrix}$$

$$G(s)=\frac{1}{\Delta(s)}\begin{bmatrix}\frac{1}{m} & s+\frac{k_m^2+br^2R_m}{r^2R_m m}\end{bmatrix}\begin{bmatrix}\frac{k_m}{rR_m} \\ 0\end{bmatrix}$$

- The system transfer function:

$$G(s)=\frac{{k_m}/{mR_m r}}{s^2+\frac{k_m^2+br^2R_m}{r^2R_m m}s+\frac{k}{m}}$$

State Space → Transfer Function – 2

- Alternatively, find $G(s)$ by Laplace transforming the state equation and applying ***Cramer's rule***
- The state equation in general form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathrm{u} \qquad (1)$$

- Apply the Laplace transform, assuming zero initial conditions

$$s\mathbf{X}(s) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}U(s)$$

- Collecting the transform of the state vector on the left-hand side

$$s\mathbf{X}(s) - \mathbf{A}\mathbf{X}(s) = \mathbf{B}U(s)$$

- Factoring out $\mathbf{X}(s)$

$$(s\mathbf{I} - \mathbf{A})\mathbf{X}(s) = \mathbf{B}U(s) \qquad (2)$$

State Space → Transfer Function – 2

- Can now apply ***Cramer's rule*** to solve for individual elements in $\mathbf{X}(s)$, i.e., the Laplace transform of individual states

$$X_i(s) = \frac{|(s\mathbf{I} - \mathbf{A})_i|}{|s\mathbf{I} - \mathbf{A}|}$$

- $(s\mathbf{I} - \mathbf{A})_i$ is the matrix formed by replacing the i^{th} column of $(s\mathbf{I} - \mathbf{A})$ with $\mathbf{B}U(s)$, the RHS of (2)
- Determine as many states as are required to calculate $Y(s)$

State Space → Transfer Function – 2

- State variable model for our system

$$\begin{bmatrix} \dot{p} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} -\frac{k_m^2 + br^2R_m}{r^2R_m m} & -k \\ \frac{1}{m} & 0 \end{bmatrix} \begin{bmatrix} p \\ x \end{bmatrix} + \begin{bmatrix} \frac{k_m}{rR_m} \\ 0 \end{bmatrix} e_1(t)$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ x \end{bmatrix}$$

- Here the output depends only on the displacement of the spring, $y(t) = x(t)$
  - To find $Y(s)$, apply Cramer's rule to the Laplace transformed state equation to find $X(s)$
    - NOTE: $X(s)$ is the Laplace transform of the displacement of the spring, $\mathbf{X}(s)$ is the Laplace transform of the state vector

State Space → Transfer Function – 2

$$X(s) = \frac{|(s\mathbf{I} - \mathbf{A})_2|}{|s\mathbf{I} - \mathbf{A}|} = \frac{1}{\Delta(s)} \begin{vmatrix} s + \dfrac{k_m^2 + br^2R_m}{r^2R_m m} & \dfrac{k_m}{R_m r}U(s) \\ -\dfrac{1}{m} & 0 \end{vmatrix} = \frac{{k_m}/{mR_m r}\,U(s)}{s^2 + \dfrac{k_m^2 + br^2R_m}{r^2R_m m}s + \dfrac{k}{m}}$$

- The Laplace transform of the output is

$$Y(s) = X(s) = \frac{{k_m}/{mR_m r}\,U(s)}{s^2 + \dfrac{k_m^2 + br^2R_m}{r^2R_m m}s + \dfrac{k}{m}}$$

- Dividing both sides by the input gives the transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{{k_m}/{mR_m r}}{s^2 + \dfrac{k_m^2 + br^2R_m}{r^2R_m m}s + \dfrac{k}{m}}$$

System Poles & Zeros

System Poles & Zeros

$$G(s) = \frac{k_m/_{mR_m r}}{s^2 + \frac{k_m^2 + br^2R_m}{r^2R_m m}s + \frac{k}{m}}$$

- ***Poles:*** values of s for which $G(s) = \infty$
  - The roots of the denominator, $\Delta(s)$
  - Solutions to the ***characteristic equation***, $\Delta(s) = 0$
  - Here, there are two poles
- ***Zeros:*** values of s for which $G(s) = 0$
  - The roots of the numerator polynomial
  - Here, there are none

Natural Frequency & Damping Ratio

- This second-order characteristic polynomial

$$\Delta(s) = s^2 + \frac{k_m^2 + br^2 R_m}{r^2 R_m m} s + \frac{k}{m}$$

can be re-written as

$$\Delta(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

- ζ is the ***damping ratio***

$$\zeta = \frac{k_m^2 + br^2 R_m}{2\sqrt{km} r^2 R_m}$$

- ω_n is the ***natural frequency***

$$\omega_n = \sqrt{\frac{k}{m}}$$

System Poles & Zeros

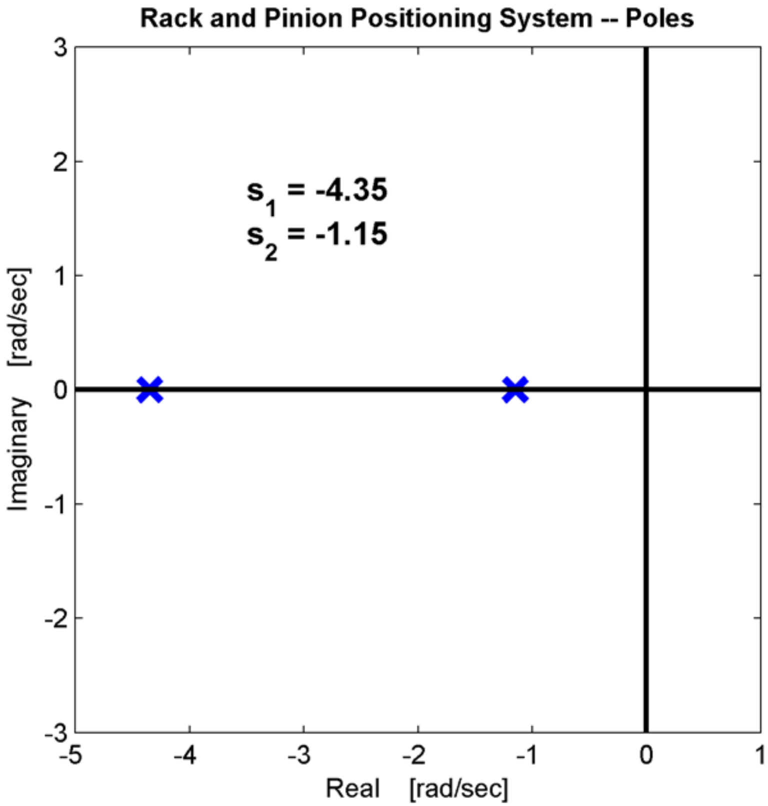
- The second-order system has two poles at

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

- The value of the damping ratio, ζ, determines the nature of the two poles:
  - $\boldsymbol{\zeta > 1}$: two real, distinct poles – ***over-damped***
  - $\boldsymbol{\zeta = 1}$: two real, identical poles – ***critically-damped***
  - $\boldsymbol{\zeta < 1}$: complex-conjugate pair poles – ***under-damped***
- Type of poles, and, therefore, the value of ζ, determines the nature of the response

System Poles & Zeros

- Assume the following system parameter values:
  - $R_m = 8\ \Omega$
  - $k_m = 0.02\ Nm/A$
  - $r = 0.01\ m$
  - $m = 0.1\ kg$
  - $k = 0.5\ N/m$
  - $b = 0.05\ Ns/m$
- Poles:
  - $s_1 = -1.15\ rad/sec$
  - $s_2 = -4.35\ rad/sec$



- $\zeta = 1.23 > 1$ – ***over-damped*** – distinct, real poles
  - Monotonic step response – ***no overshoot or ringing***

Dynamic System Response

Dynamic System Response

- Often characterize systems by their responses to particular classes of inputs, e.g.:
  - ***Impulse response:*** response to an impulse with zero initial conditions – time-domain response
  - ***Step response:*** response to a unit step with zero initial conditions – time-domain response
  - ***Frequency response:*** system response to sinusoidal inputs of varying frequency – system gain and phase as functions of frequency – frequency-domain response
- Additionally, we often want to simulate the system's response to arbitrary inputs

Impulse Response

- The system response in the Laplace domain is given by the product of the transfer function and the Laplace transform of the input

$$Y(s) = G(s) \cdot U(s)$$

- The Laplace transform of an impulse function is

$$\mathcal{L}\{\delta(t)\} = 1$$

therefore, a system's impulse response is the inverse Laplace transform of its transfer function

$$g(t) = \mathcal{L}^{-1}\{G(s)\}$$

Impulse Response

- For our rack and pinion positioning system:

$$g(t) = \mathcal{L}^{-1}\{G(s)\} = \mathcal{L}^{-1}\left\{\frac{2.5}{s^2 + 5.5s + 5}\right\}$$

- Inverse transform via ***partial fraction expansion***

$$G(s) = \frac{2.5}{s^2+5.5s+5} = \frac{r_1}{s+1.15} + \frac{r_2}{s+4.35} \qquad (1)$$

$$2.5 = r_1(s + 4.35) + r_2(s + 1.15)$$

$$2.5 = (r_1 + r_2)s + 4.35r_1 + 1.15r_2$$

- Equating coefficients and solving the resulting system of two equations gives the following ***residues***:

$$r_1 = 0.7809$$

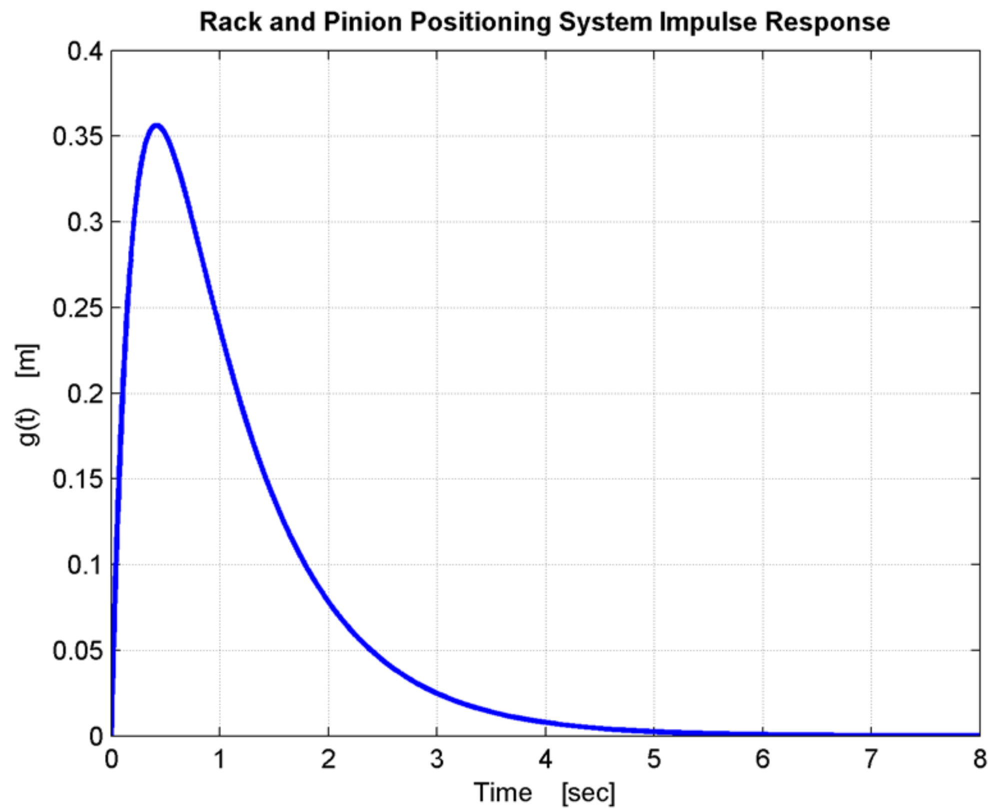
$$r_2 = -0.7809$$

Impulse Response

- Substituting the residues back into (1) gives

$$G(s) = \frac{0.7809}{s+1.15} - \frac{0.7809}{s+4.35}$$

- Inverse Laplace transforming gives the impulse response:



$$g(t) = 0.7809e^{-1.15t} - 0.7809e^{-4.35t}$$

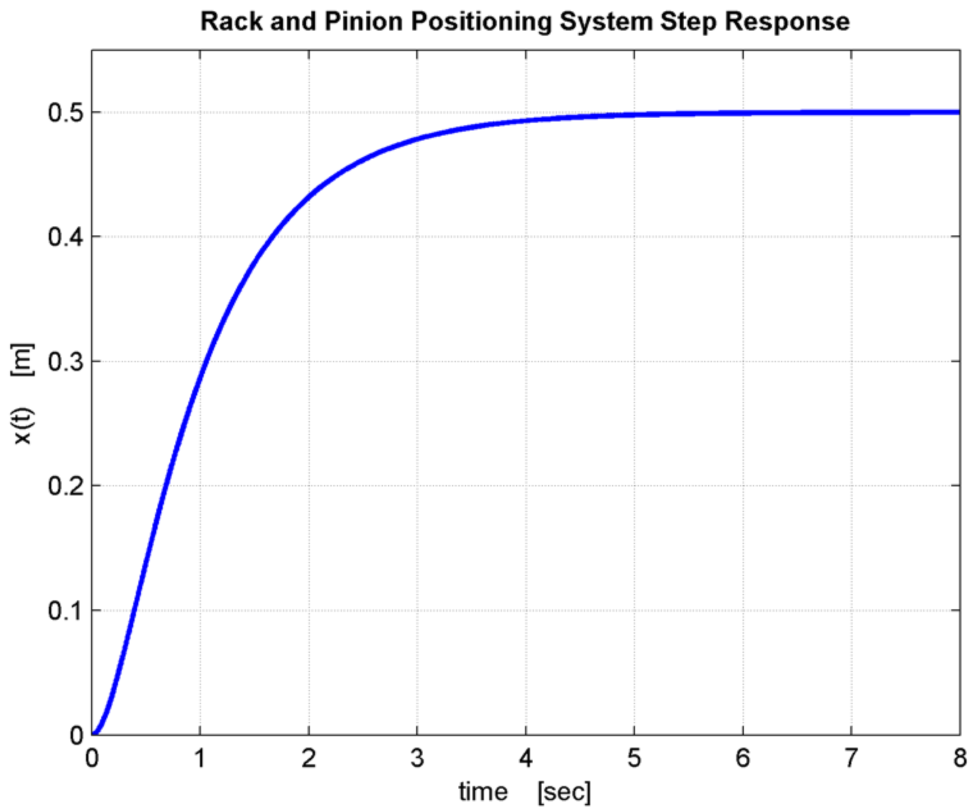
Step Response

- The step response in the Laplace domain is

$$Y(s) = U(s) \cdot G(s)$$

$$Y(s) = \frac{1}{s} \cdot \frac{2.5}{s^2 + 5.5s + 5}$$

- Inverse transforming gives the time-domain step response:



$$y(t) = 0.5 - 0.6795e^{-1.15t} + 0.1795e^{-4.35t}$$

Frequency Response

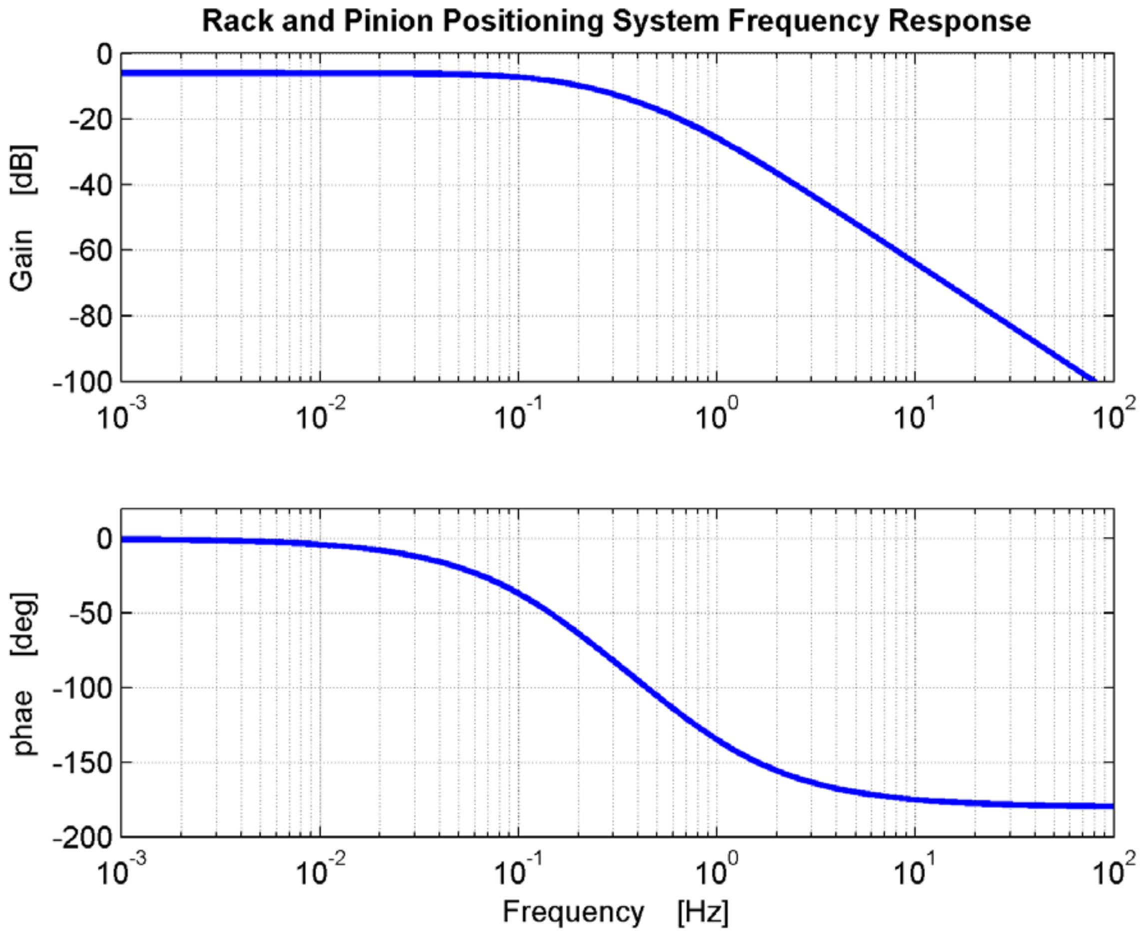
- The ***frequency response function*** or ***sinusoidal transfer function*** is obtained by substituting $j\omega$ for s in the transfer function

$$G(j\omega) = \frac{2.5}{(j\omega)^2 + 5.5(j\omega) + 5}$$

- This ***complex function of frequency*** can be evaluated to give the system's:
  - ***Gain:*** the ratio of the magnitudes of the system's (sinusoidal) output to input as function of frequency
  - ***Phase:*** the phase shift from the (sinusoidal) input to the output as a function of frequency

Frequency Response

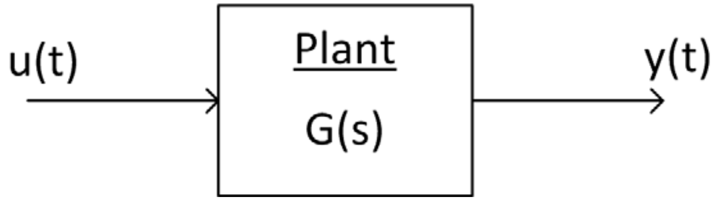
- Gain and phase are plotted as a ***Bode plot:***



Open-Loop System Response

Block Diagram Model

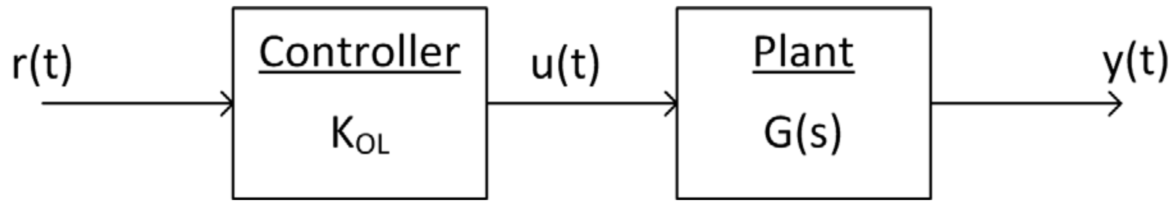
- Our positioning system, or ***plant***, can be represented in ***block diagram*** form as



- It has an input, $u(t)$, and an output, $y(t)$
  - Input/output relationship described by the plant model: transfer function, state variable model, etc.

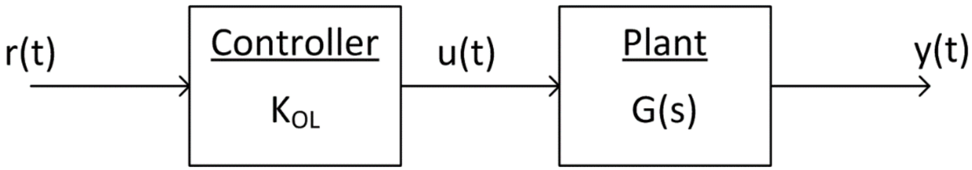
Open-Loop Configuration

- Say we want to command a 5 cm displacement from our plant
  - Plant input, $u(t)$, is a voltage applied to a motor
- We'd like the input to be the desired displacement, e.g. 5 cm
  - This desired output specified by the ***reference input***, $r(t)$
- Block diagram is now:



- Added a ***controller*** block
  - Constant gain, K_{OL}, to convert from $r(t)$ to $u(t)$
  - Value of K_{OL} depends on properties of the plant

Open-Loop Controller Gain



- How do we determine K_{OL}?
- The steady-state gain of the system is

$$G_{ss} = \lim_{s\to 0} G(s) = \lim_{s\to 0}\left(\frac{2.5}{s^2 + 5.5s + 5}\right) = 0.5\ m/V$$

- Set $K_{OL} = 1/G_{ss} = 2\ V/m$
- Say, for example, that we want a displacement of $5\ cm$

$$r(t) = 0.05\ m$$

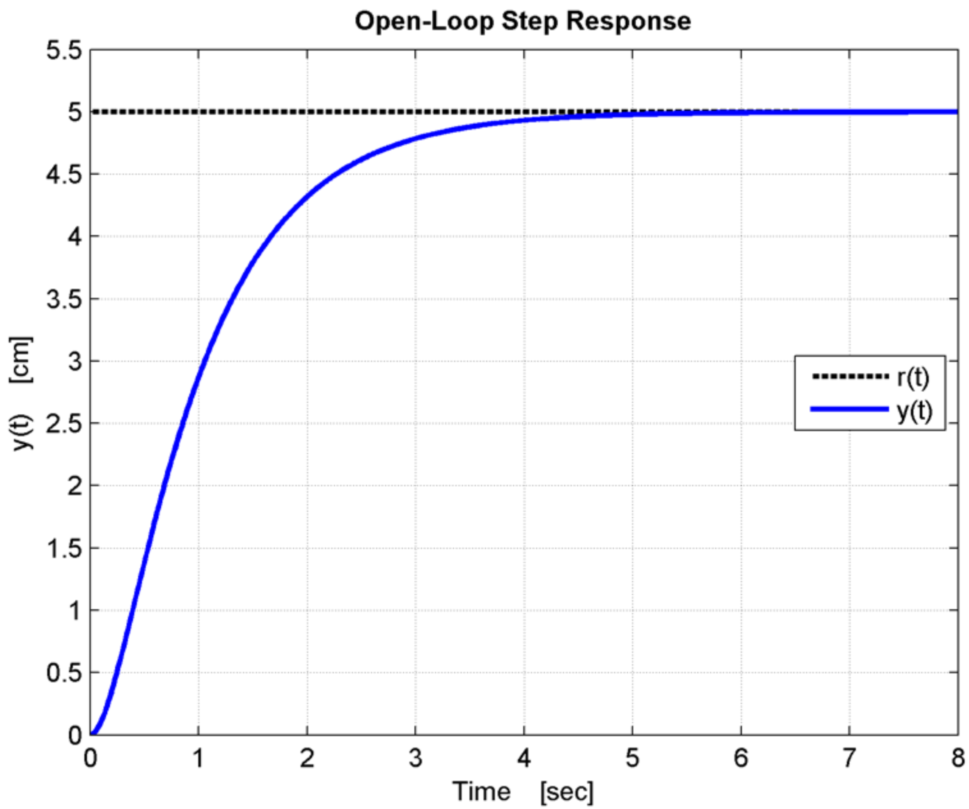
$$u(t) = K_{OL} \cdot r(t) = 2\ V/m \cdot 0.05\ m = 100\ mV$$

$$y_{ss} = u(t) \cdot G_{ss} = 100\ mV \cdot 0.5\ m/V = 5\ cm$$

Open-Loop Response

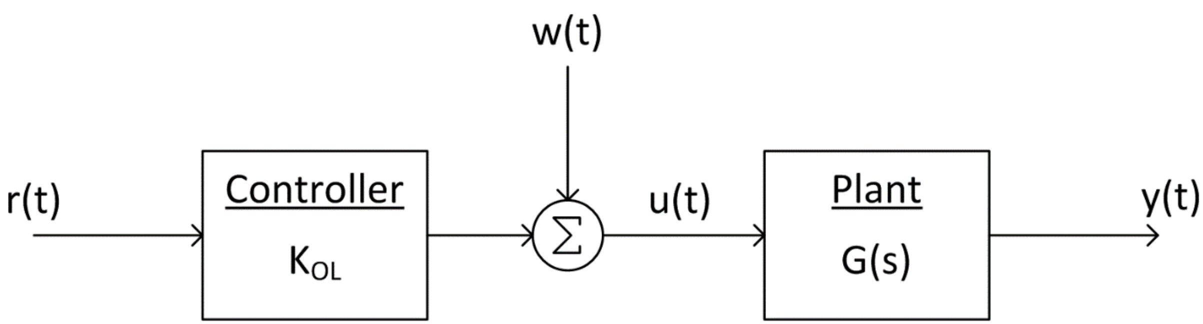
- The open-loop controller yields a steady-state output equal to the reference input

$$y_{ss} = r(t)$$



Disturbance Input

- Consider what happens if some external factor affects the system
  - Additional load
  - Increased drag due to part wear, etc.
- This is a ***disturbance***
  - Model as an additional input to the plant, $w(t)$:



Effect of Disturbance

- The open-loop controller (K_{OL}) was designed for the specific plant characteristics
  - Disturbance not accounted for
- Now,

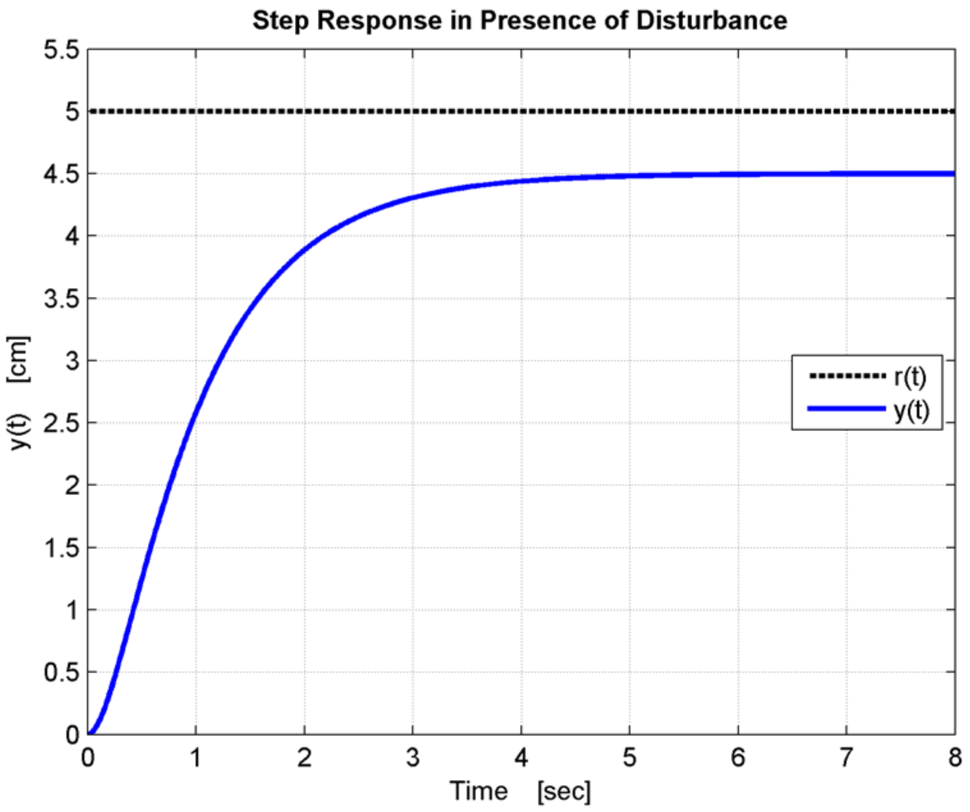
$$y_{ss} = [K_{OL} \cdot r(t) + w(t)]G_{ss}$$

$$y_{ss} = r(t) + w(t) \cdot G_{ss}$$

- ***Steady-state error*** results

$$y_{ss} \neq r(t)$$

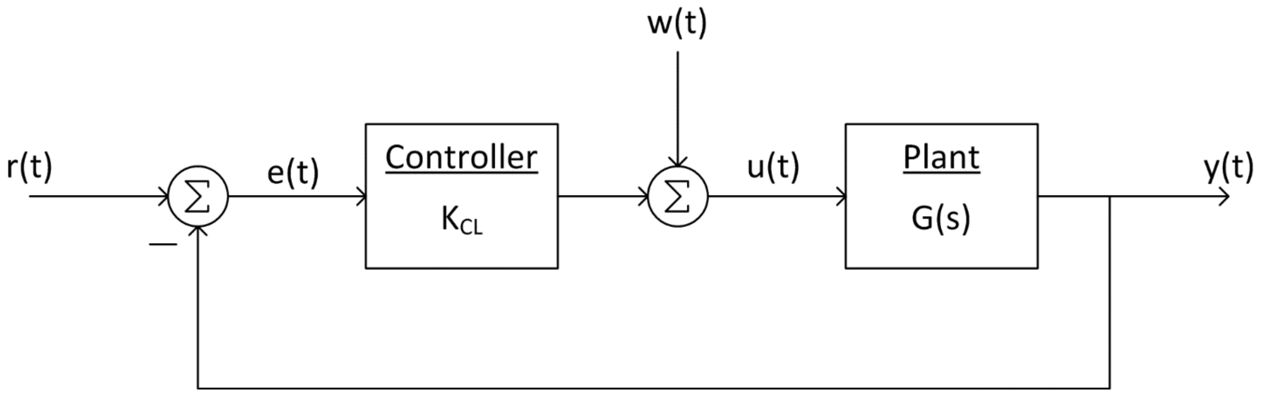
$$e_{ss} = r(t) - y_{ss}$$



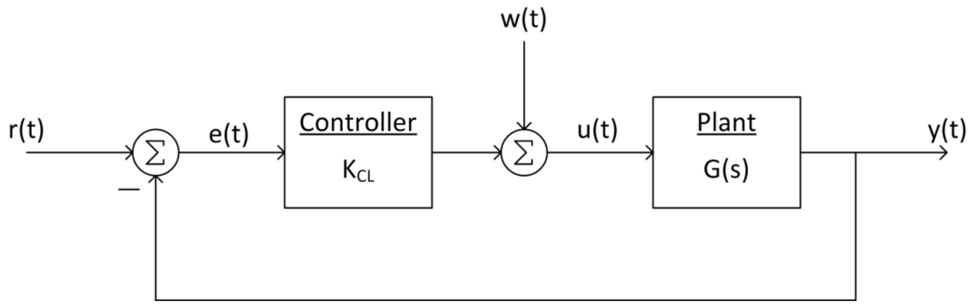
Closed-Loop Feedback Control

Feedback Control

- Steady-state error due to disturbance input can be addressed by adding ***feedback***
- The output is measured and ***fed back*** to the input
  - ***Subtracted*** from the reference input – ***negative feedback***



Feedback Control



- This is a ***closed-loop*** configuration
- Difference between reference (desired output), $r(t)$, and actual output, $y(t)$, is the ***error***, $e(t)$

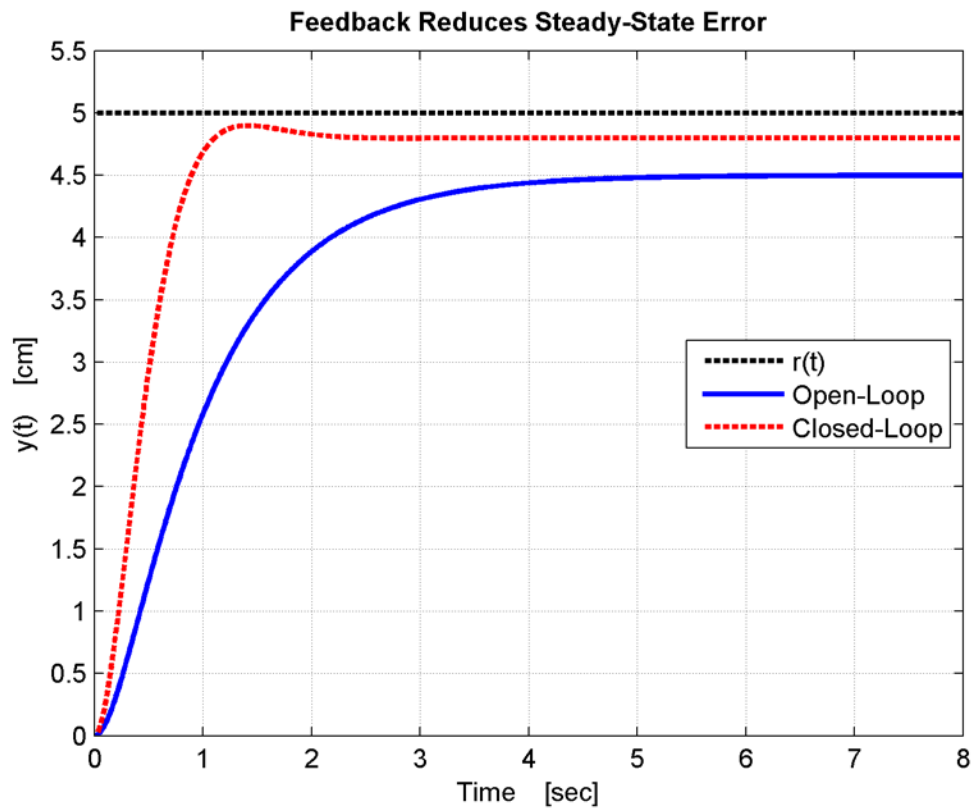
$$e(t) = r(t) - y(t)$$

- Error gets multiplied by the closed-loop controller gain, K_{CL}
- Input to the plant, $u(t)$, is the controller output plus the disturbance input

$$u(t) = K_{CL} \cdot e(t) + w(t)$$

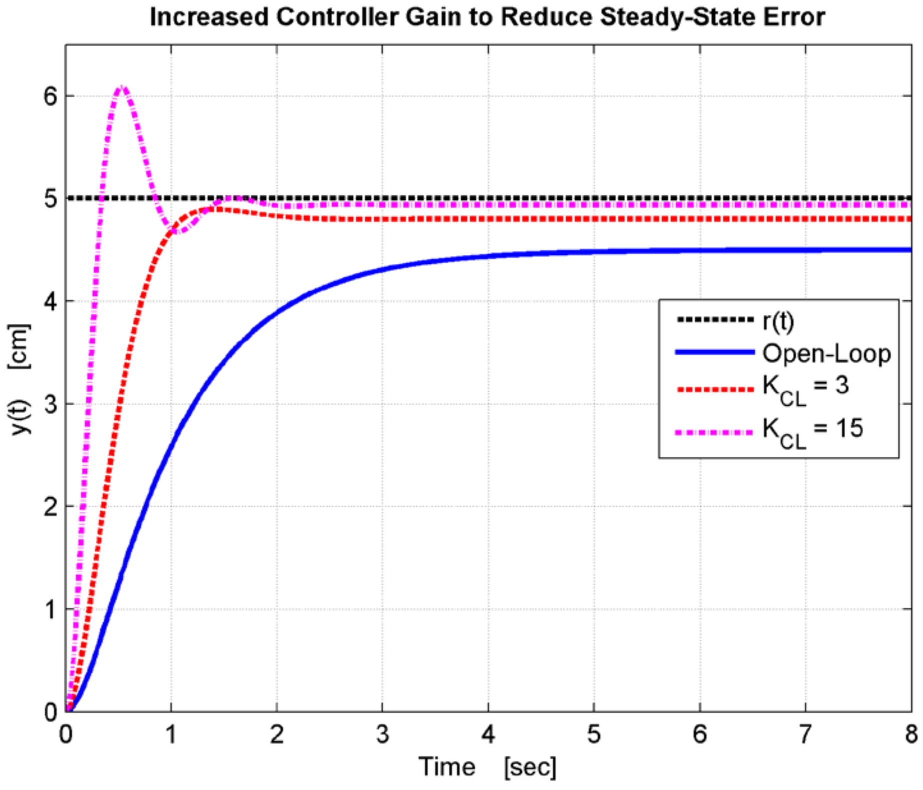
Closed-Loop Response

- Because $y(t)$ is fed back and used to generate $u(t)$, ***error is reduced***
  - Though not eliminated, in this case
- ***Dynamics of the closed-loop system differ from the open-loop system***



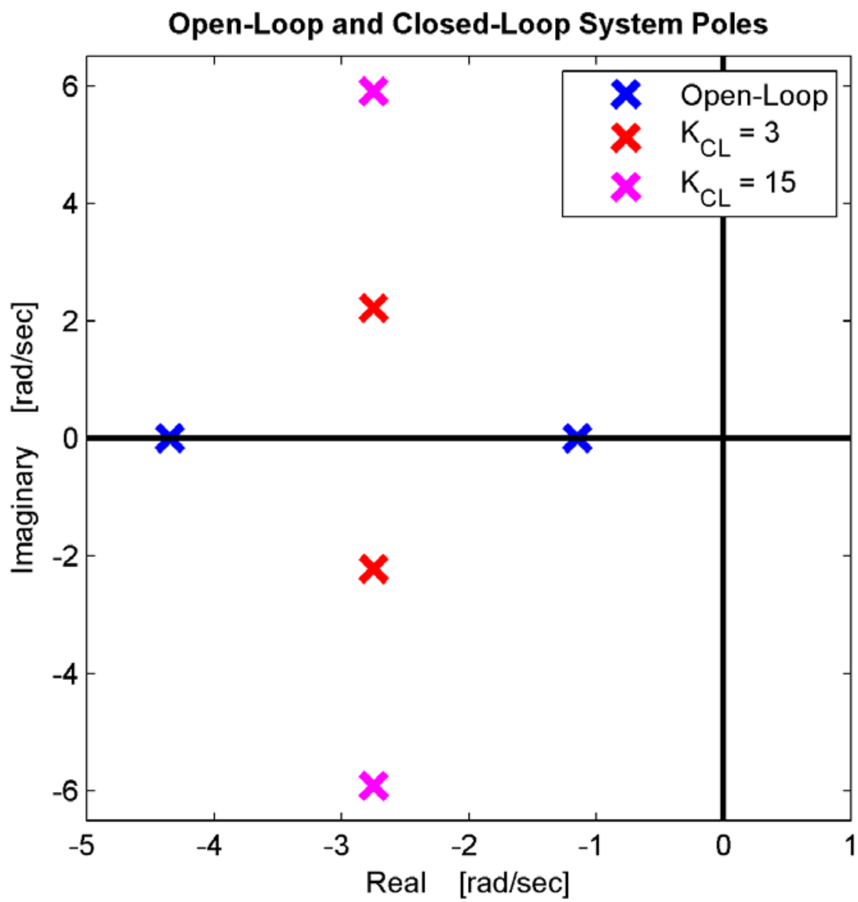
Reducing Steady-State Error

- Increasing controller gain can further reduce steady-state error
- Closed-loop system dynamics have changed a lot
  - Faster risetime, increased overshoot
  - Could this pose a problem?



Closed-Loop Poles

- Nature of open-loop and closed-loop responses differ
  - Closed-loop system poles differ from open-loop system poles
- ***Feedback moves poles***



Second-Order Under-Damped System Poles

- We'll see that feedback will allow us to move poles to desirable locations
- Second-order poles:

$$s_{1,2} = -\sigma \pm j\omega_d$$

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

- Damping ratio

$$\zeta = \frac{\sigma}{\omega_n} = \sin(\theta)$$

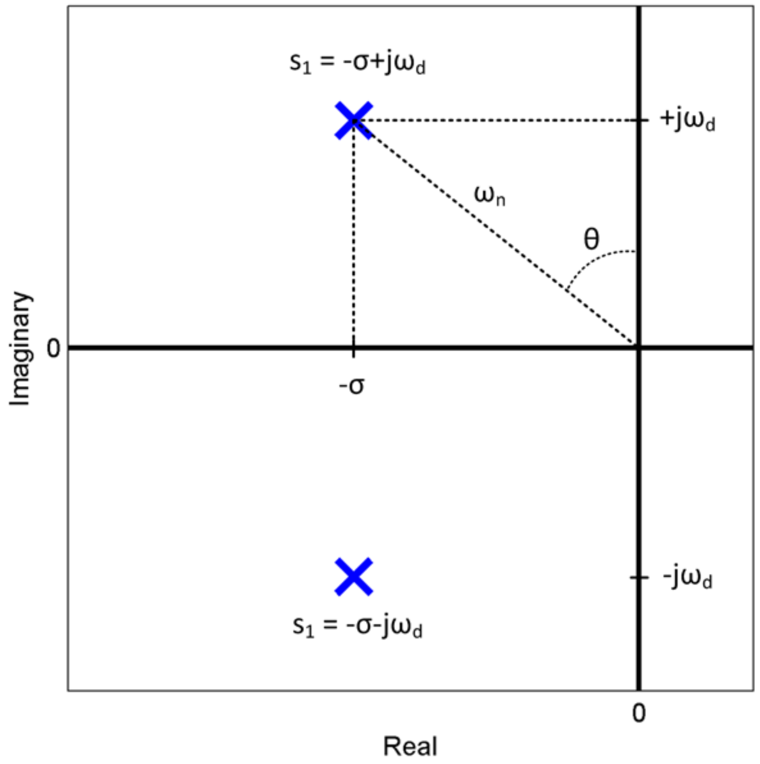
- Natural frequency

$$\omega_n = \sqrt{\sigma^2 + \omega_d^2}$$

- Damped natural frequency

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

Second-Order Pole Locations



Second-Order Under-Damped System Poles

- Pole location determines dynamic behavior
- Overshoot:

$$\%OS = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \cdot 100\%$$

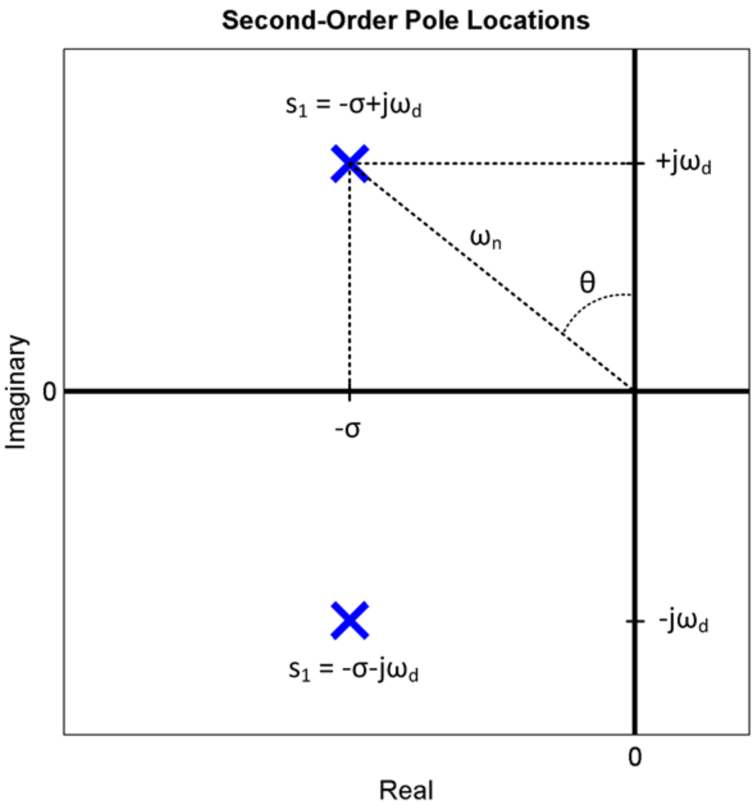
$$\zeta = -\frac{\ln(OS)}{\sqrt{\pi^2 + \ln^2(OS)}}$$

- Settling time ($\pm 1\%$) approximation:

$$t_s \approx \frac{4.6}{\sigma}$$

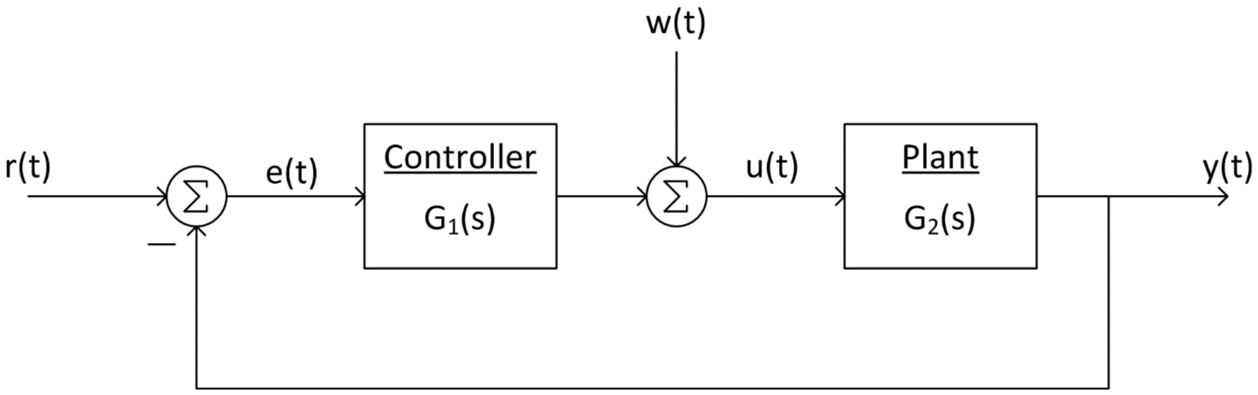
- Risetime approximation:

$$t_r \approx \frac{1.8}{\omega_n}$$



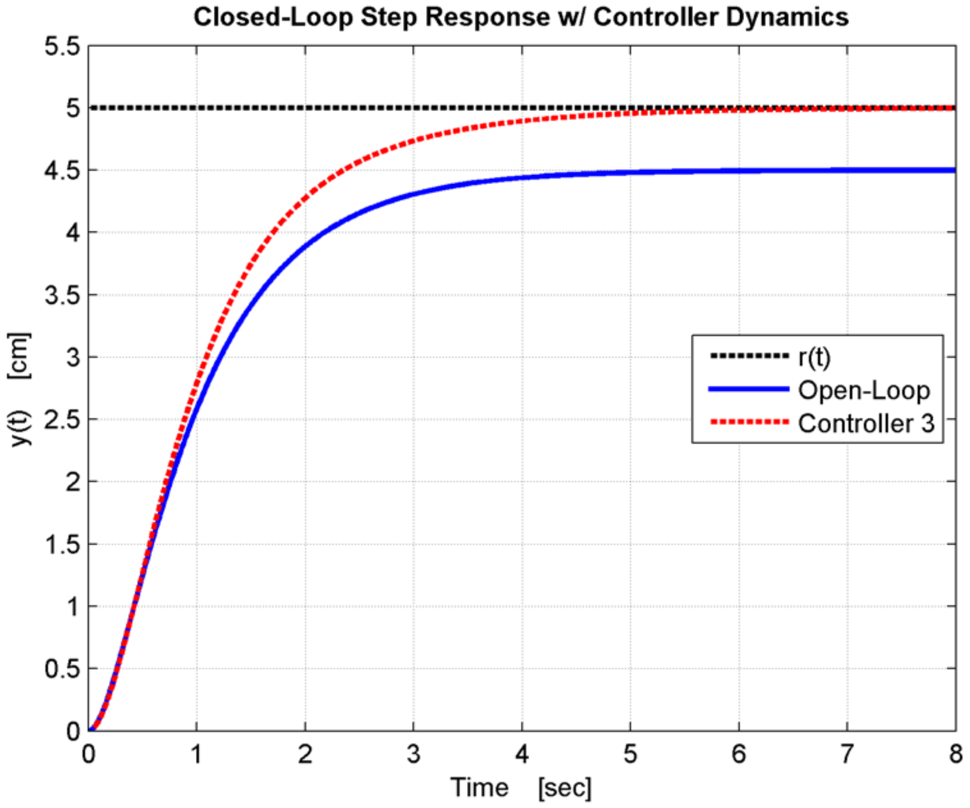
Adding Controller Dynamics

- Previous controller was a simple gain factor
  - ***Proportional*** control
- Controller could also be designed to have dynamics of its own – a ***compensator***
  - Controller transfer function may have poles and/or zeros
  - Allows for better control of closed-loop system response
    - Steady-state error – possible to eliminate
    - Transient response – risetime, overshoot, settling time



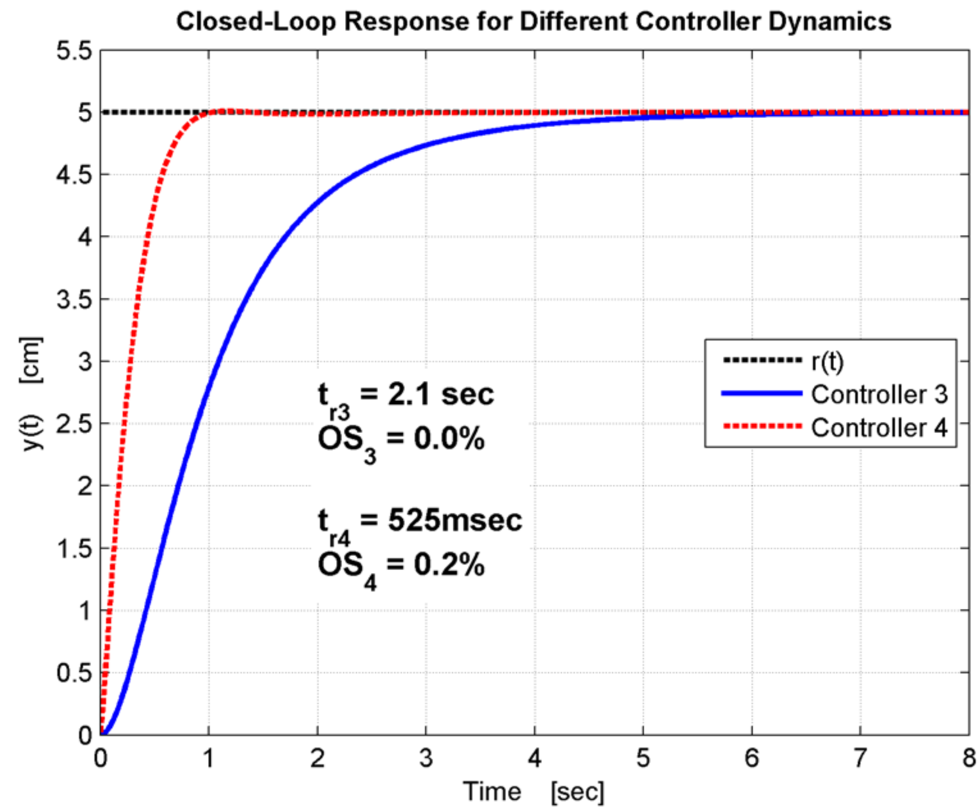
Closed-Loop Response

- Without getting into specifics, consider the effect of a controller that has a pole and two zeros
- Steady-state error has been eliminated
- Transient response nearly unchanged



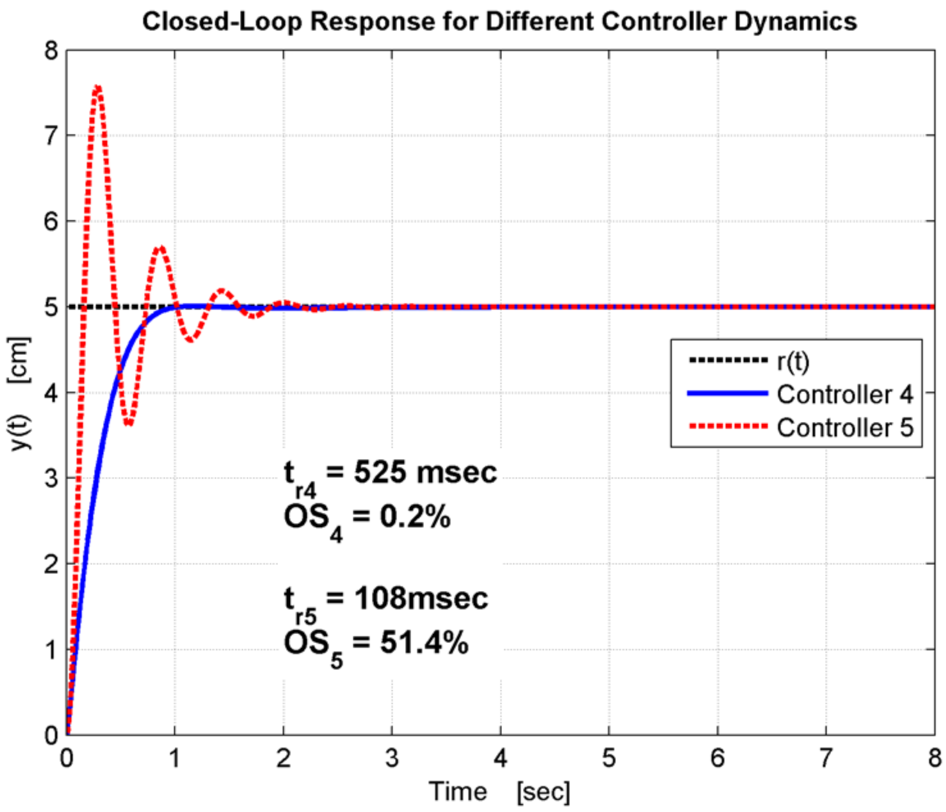
Closed-Loop Response

- Perhaps we want a faster response
- Alter closed-loop response by changing controller transfer function
  - Much faster risetime
  - Still almost no overshoot
  - Still no steady-state error

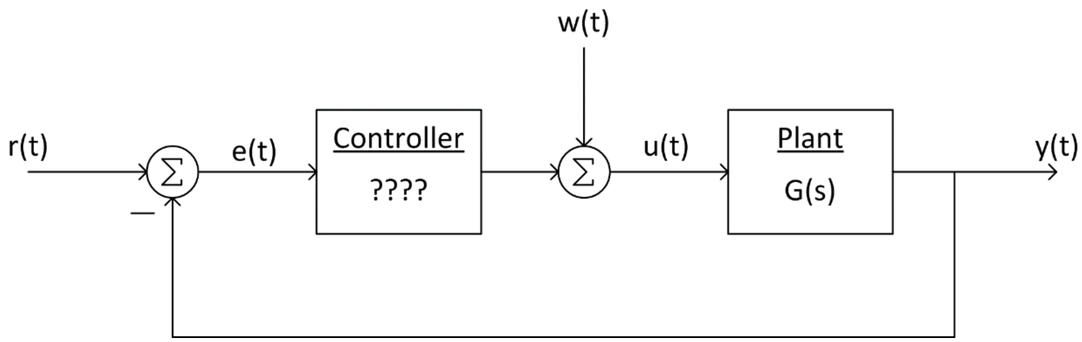


Closed-Loop Response

- Modify the controller again
  - Even faster risetime
  - Now, very large overshoot
  - Significant ringing
  - A desirable response? Perhaps not



Controller Design



- ***How do we determine the controller transfer function to yield the desired response?***
  - The topic of this course
- ***What is the controller?***
  - A block in a block diagram? Yes.
  - A mathematical function? Yes.
- ***But, how do we implement it?***
  - Electronics – digital computer or opamp circuits

ESE 430 Course Overview

1. Introduction
2. Block Diagrams & Signal Flow Graphs
3. Stability
4. Steady-State Error
5. Root-Locus Analysis
6. Root-Locus Design
7. Frequency-Response Analysis
8. Frequency-Response Design