SECTION 2: THREE-PHASE POWER FUNDAMENTALS

ESE 470 – Energy Distribution Systems

AC Circuits & Phasors

AC Electrical Signals

- AC electrical signals (voltages and currents) are ***sinusoidal***
  - Generated by rotating machinery
- Sinusoidal voltage (or current):

$$v(t) = V_p \cos(\omega t + \phi) \tag{1}$$

  - This is a ***time-domain*** or ***instantaneous*** form expression
- Characterized by three parameters
  - ***Amplitude***
  - ***Frequency***
  - ***Phase***

Amplitude

$$v(t) = V_p \cos(\omega t + \phi)$$

- V_p in the above expression is ***amplitude*** or ***peak voltage***
- We typically characterize power-system voltages and currents in terms of their ***root-mean-square*** (rms) values

$$V_{rms} = \left(\frac{1}{T}\int_0^T v(t)^2 dt\right)^{\frac{1}{2}} \quad (2)$$

- A signal delivers the same power to a resistive load as a DC signal equal to its rms value
- For ***sinusoids***:

$$V_{rms} = \frac{V_p}{\sqrt{2}} \quad (3)$$

Euler's Identity

- Euler's identity allows us to express sinusoidal signals as complex exponentials

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t) \tag{4}$$

so

$$e^{j(\omega t+\phi)} = \cos(\omega t + \phi) + j\sin(\omega t + \phi) \tag{5}$$

and

$$V_p \cos(\omega t + \phi) = V_p Re\left\{e^{j(\omega t+\phi)}\right\}$$

$$V_p \cos(\omega t + \phi) = \sqrt{2}\, V_{rms} Re\left\{e^{j(\omega t+\phi)}\right\} \tag{6}$$

Phasor Representation

- ***Phasor representation*** simplifies circuit analysis when dealing with sinusoidal signals
  - Drop the time-harmonic (oscillatory) portion of the signal representation
    - Known and constant
  - Represent with ***rms amplitude*** and ***phase*** only
- For example, consider the time-domain voltage expression

$$v(t) = \sqrt{2}\, V_{rms} \cos(\omega t + \phi)$$

- The phasor representation, in ***exponential*** form, is

$$\boldsymbol{V} = V_{rms} e^{j\phi}$$

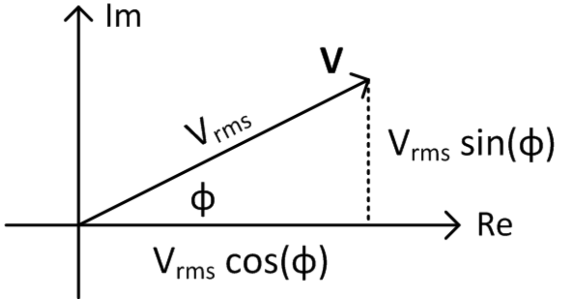
- Can also express in ***polar*** or ***Cartesian*** form

$$\boldsymbol{V} = V_{rms} \angle \phi = V_{rms} \cos(\phi) + jV_{rms} \sin(\phi)$$

- In these notes **bold** type will be used to distinguish phasors
- We'll always assume rms values for phasor magnitudes

Phasors

- Think of a phasor as a vector in the complex plane
  - Has ***magnitude*** and ***angle***



- Circuit analysis in the phasor domain is simplified
  - Derivative and integrals become algebraic expressions
- Consider the voltage across inductance and capacitance:

	Time Domain	Phasor Domain
Capacitor	$v(t) = \frac{1}{c}\int i(t)dt$	$\boldsymbol{V} = \frac{1}{j\omega C}\boldsymbol{I}$
Inductor	$v(t) = L\frac{di}{dt}$	$\boldsymbol{V} = j\omega L\boldsymbol{I}$
Resistor	$v(t) = i(t)R$	$\boldsymbol{V} = \boldsymbol{I}R$

Phasors

- In general, in the phasor domain

$$\boldsymbol{V} = \boldsymbol{I}Z \quad (7)$$

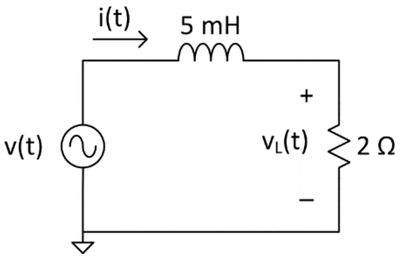
and

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{Z}$$

- Ohm's law
- Z is a complex impedance
  - Not a phasor, but also expressed in exponential, polar, or Cartesian form

Phasors - Example

- Determine $i(t)$ and $v_L(t)$ for the following circuit, driven by a $120\ V_{rms}$, $60\ Hz$ source



- At $60\ Hz$ the inductor impedance is

$$jX_L = j\omega L = j2\pi \cdot 60\ Hz \cdot 5\ mH = j1.88\ \Omega$$

- The total impedance seen by the source is

$$Z = R + jX_L = 2 + j1.88\ \Omega$$

- Converting to polar form

$$Z = |Z|\angle\theta$$

$$|Z| = \sqrt{R^2 + X^2} = 2.74\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right) = 43°$$

$$Z = 2.74\angle 43°\ \Omega$$

Phasors – Example

- The source voltage is

$$v(t) = \sqrt{2} \cdot 120V cos(2\pi \cdot 60Hz \cdot t)$$

- The source voltage phasor is

$$\boldsymbol{V} = 120\angle 0° \, V$$

- The current phasor is

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{Z} = \frac{120\angle 0° \, V}{2.74\angle 43° \, \Omega} = 43.7\angle -43° \, A$$

- We can use the current phasor to determine the phasor for the voltage across the resistor

$$\boldsymbol{V_L} = \boldsymbol{I}R = (43.7\angle -43°) \cdot 2\Omega$$

$$\boldsymbol{V_L} = 87.4\angle -43° \, V$$

Phasors – Example

- We have ***phasor*** representations for desired quantities

$$\boldsymbol{I} = 43.7\angle - 43° \ A$$

$$\boldsymbol{V_L} = 87.4\angle - 43° \ V$$

- We can now convert these to their time-domain expressions

$$i(t) = \sqrt{2} \cdot 43.7 \ A \cdot \cos(2\pi \cdot 60Hz \cdot t - 43°)$$

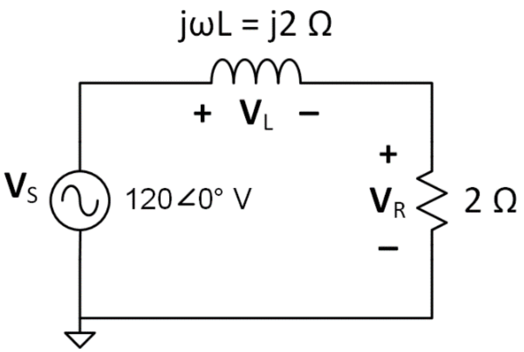
$$v(t) = \sqrt{2} \cdot 87.4 \ V \cdot \cos(2\pi \cdot 60Hz \cdot t - 43°)$$

Phasor Diagrams

Phasor Diagrams

- Phasors are complex values
  - Magnitude and phase
  - Vectors in the complex plane
  - Can represent graphically
- ***Phasor diagram***
  - Graphical representation of phasors in a circuit
  - KVL and Ohm's law expressed graphically

Phasor Diagram – Example 1



- Source voltage is the reference phasor

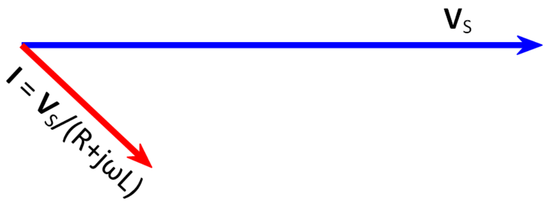
$$\boldsymbol{V_S} = 120\angle 0°\ V$$

  - Its phasor diagram:

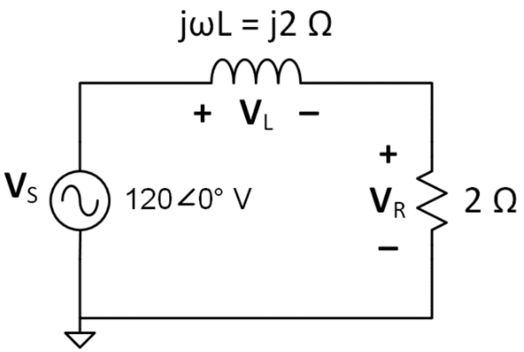
- Ohm's law gives the current

$$\boldsymbol{I} = \frac{\boldsymbol{V_S}}{2 + j2\ \Omega} = 42.2\angle - 45°\ A$$

  - Adding to the phasor diagram:



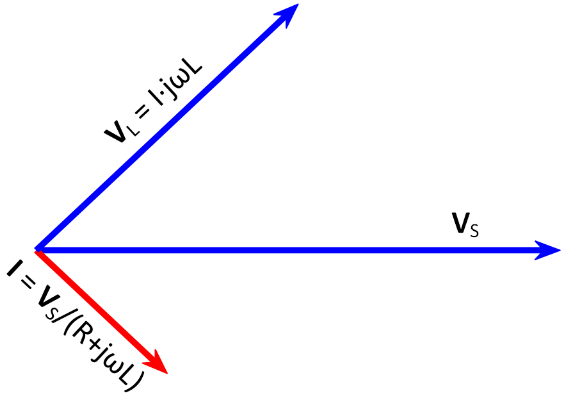
Phasor Diagram – Example 1



- Ohm's law gives the inductor voltage

$$\boldsymbol{V}_L = \boldsymbol{I} \cdot j\omega L = (42.2\angle -45°\ A) \cdot j2\ \Omega$$

$$\boldsymbol{V}_L = 85\angle 45°\ V$$

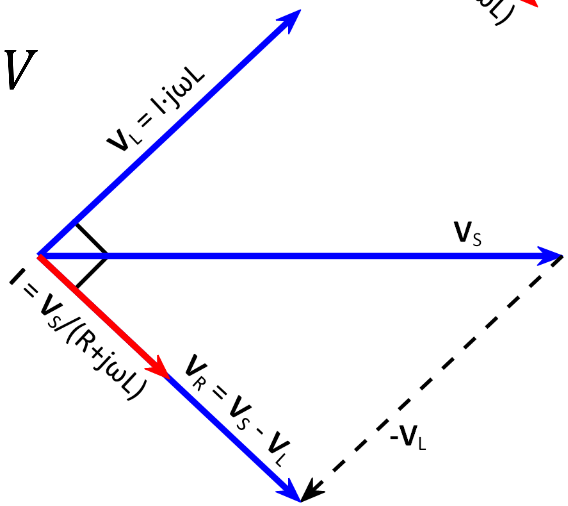


- Finally, KVL gives $\boldsymbol{V}_R$

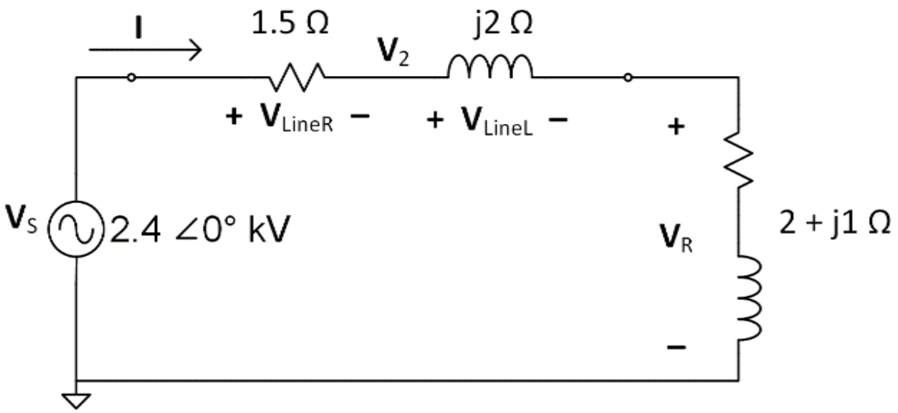
$$\boldsymbol{V}_R = \boldsymbol{V}_S - \boldsymbol{V}_L$$

$$\boldsymbol{V}_R = 120\angle 0°\ V - 85\angle 45°\ V$$

$$\boldsymbol{V}_R = 85\angle -45°$$



Phasor Diagram – Example 2

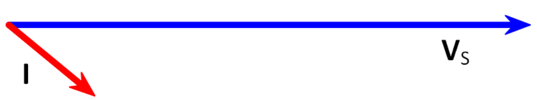


- Source voltage is the reference phasor

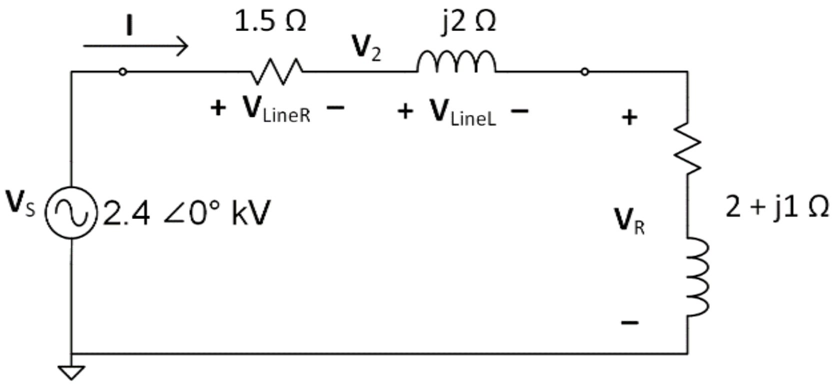
$$V_S = 2.4\angle 0°\ kV$$

- Ohm's law gives the current

$$I = \frac{V_S}{3.5 + j3\ \Omega} = 521\angle -41°\ A$$



Phasor Diagram – Example 2



- Ohm's law gives the resistor voltage

$$\boldsymbol{V}_{LineR} = \boldsymbol{I} \cdot R$$

$$\boldsymbol{V}_{LineR} = (521\angle-41°\ A) \cdot 1.5\ \Omega$$

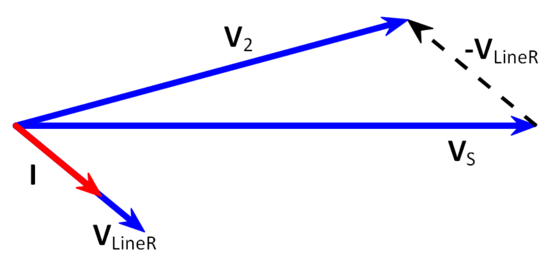
$$\boldsymbol{V}_{LineR} = 781\angle-41°\ V$$

- KVL gives $\boldsymbol{V}_2$

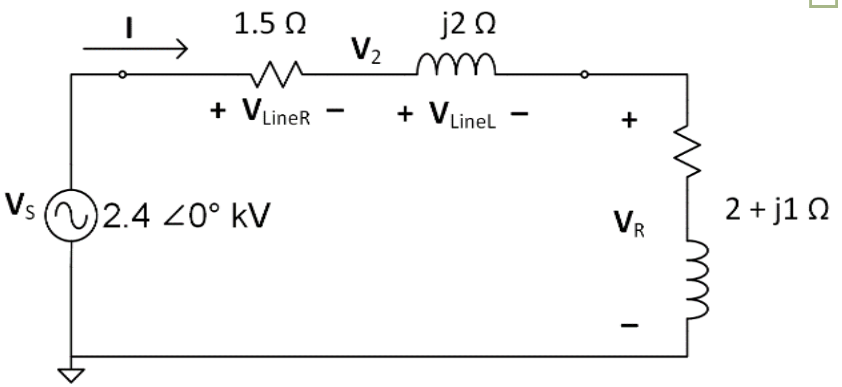
$$\boldsymbol{V}_2 = \boldsymbol{V}_S - \boldsymbol{V}_{LineR}$$

$$\boldsymbol{V}_2 = 2.4\angle 0°\ kV - 781\angle-41°\ V$$

$$\boldsymbol{V}_2 = 1.88\angle 15.7°\ kV$$



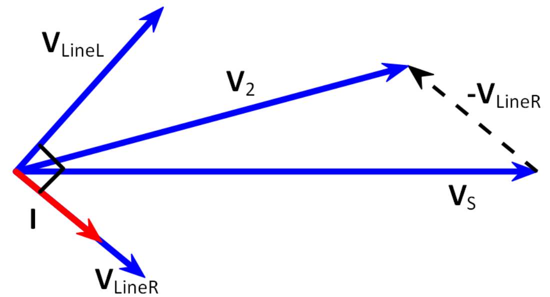
Phasor Diagram – Example 2



- Drop across the inductor:

$$\boldsymbol{V}_{LineL} = (521\angle -41°\ A)\cdot j2\ \Omega$$

$$\boldsymbol{V}_{LineL} = 1.04\angle 49°\ kV$$

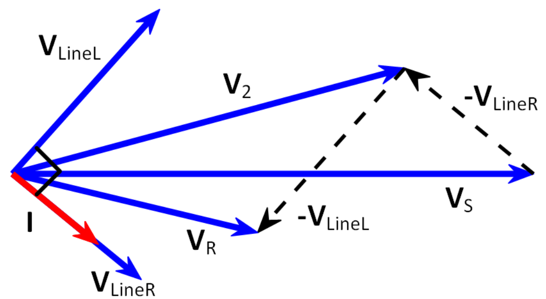


- KVL gives the voltage across the load

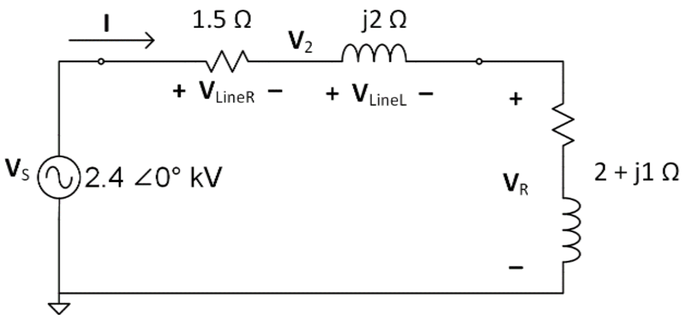
$$\boldsymbol{V}_R = \boldsymbol{V}_2 - \boldsymbol{V}_{LineL}$$

$$\boldsymbol{V}_R = 1.88\angle 15.7° kV - 1.04\angle 49°\ kV$$

$$\boldsymbol{V}_R = 1.16\angle -14°\ kV$$



Phasor Diagram – Example 2

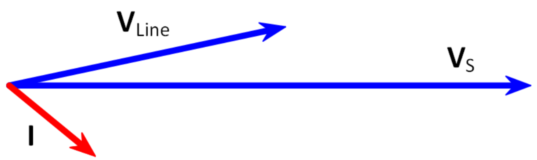


- Alternatively, treat the line as a single impedance

$$\boldsymbol{V}_{Line} = \boldsymbol{I} \cdot Z_{Line}$$

$$\boldsymbol{V}_{Line} = (521\angle-41°\ A) \cdot (1.5 + j2\ \Omega)$$

$$\boldsymbol{V}_{LineL} = 1.3\angle 12.5°\ kV$$

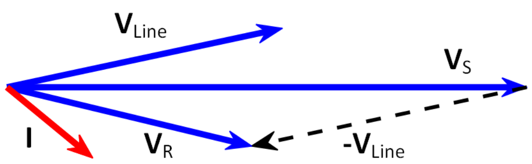


- KVL gives the voltage across the load

$$\boldsymbol{V}_R = \boldsymbol{V}_S - \boldsymbol{V}_{Line}$$

$$\boldsymbol{V}_R = 2.4\angle 0° kV - 1.3\angle 12.5°\ kV$$

$$\boldsymbol{V}_R = 1.16\angle-14°\ kV$$



Power – Real Power & Power Factor

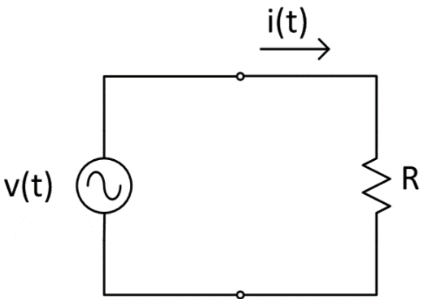
Power

- The overall goal of a power distribution network is to transfer power from a source to loads
- **Instantaneous power**:
  - Power supplied by a source or absorbed by a load or network element as a function of time

$$p(t) = v(t) \cdot i(t) \tag{8}$$

- The nature of this instantaneous power flow is determined by the impedance of the load
- Next, we'll look at the instantaneous power delivered to loads of different impedances

Instantaneous Power – Resistive Load



- The voltage across the resistive load is

$$v(t) = V_p\cos(\omega t + \delta)$$

- Current through the resistor is

$$i(t) = \frac{V_p}{R}\cos(\omega t + \delta)$$

- The instantaneous power absorbed by the resistor is

$$p_R(t) = v(t)\cdot i(t) = V_p\cos(\omega t + \delta)\cdot\frac{V_p}{R}\cos(\omega t + \delta)$$

$$p_R(t) = \frac{V_p^2}{R}\cos^2(\omega t + \delta) = \frac{V_p^2}{R}\frac{1}{2}[1 + \cos(2\omega t + 2\delta)]$$

Instantaneous Power – Resistive Load

$$p_R(t) = \frac{V_p^2}{2R}[1 + \cos(2\omega t + 2\delta)]$$

- Making use of the rms voltage

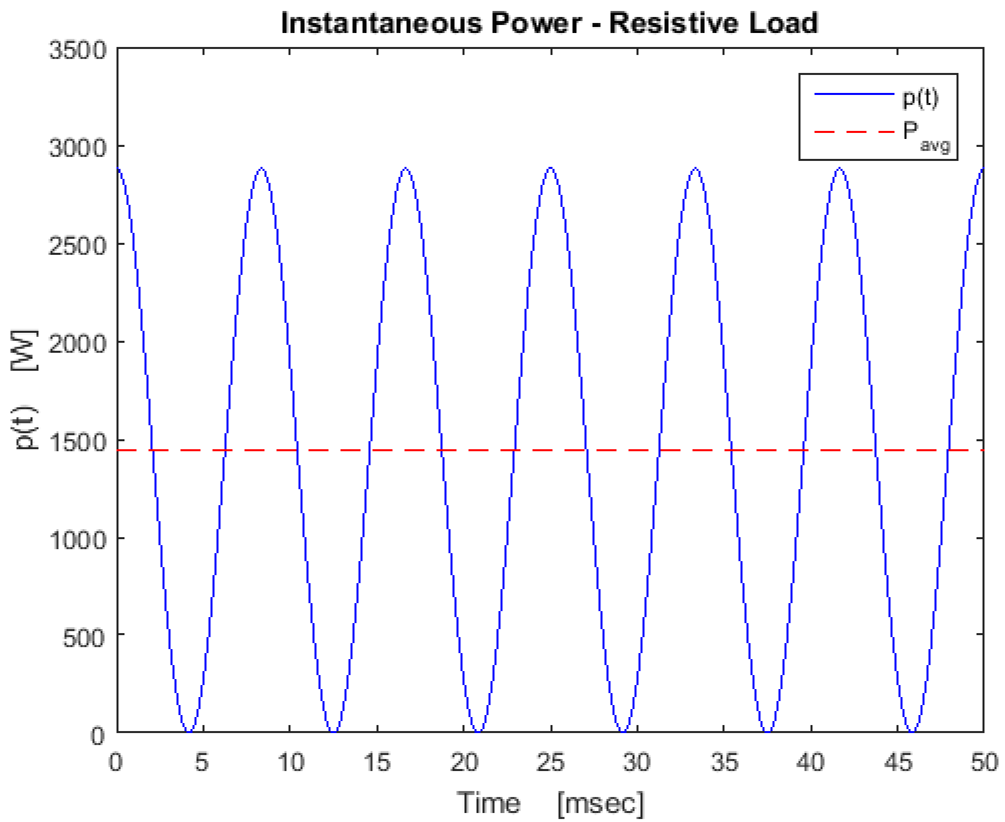
$$p_R(t) = \frac{\left(\sqrt{2}\,V_{rms}\right)^2}{2R}[1 + \cos(2\omega t + 2\delta)]$$

$$p_R(t) = \frac{V_{rms}^2}{R}[1 + \cos(2\omega t + 2\delta)] \qquad (9)$$

- The instantaneous power absorbed by the resistor has a non-zero average value and a double-frequency component

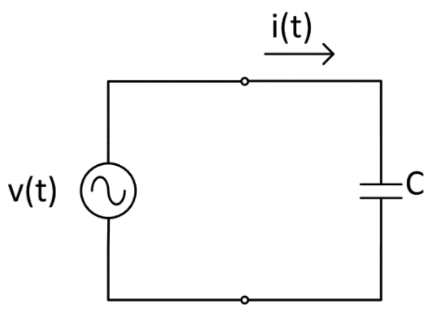
Instantaneous Power – Resistive Load

- Power delivered to the resistive load has a non-zero average value and a double-frequency component



Instantaneous Power – Capacitive Load

- Now consider the power absorbed by a purely capacitive load
  - Again, $v(t) = V_p \cos(\omega t + \delta)$



- The current flowing to the load is

$$i(t) = I_p \cos(\omega t + \delta + 90°)$$

where

$$I_p = \frac{V_p}{X_C} = \frac{V_p}{1/\omega C} = \omega C V_p$$

- The instantaneous power delivered to the capacitive load is

$$p_C(t) = v(t) \cdot i(t)$$

$$p_C(t) = V_p \cos(\omega t + \delta) \cdot \omega C V_p \cos(\omega t + \delta + 90°)$$

Instantaneous Power – Capacitive Load

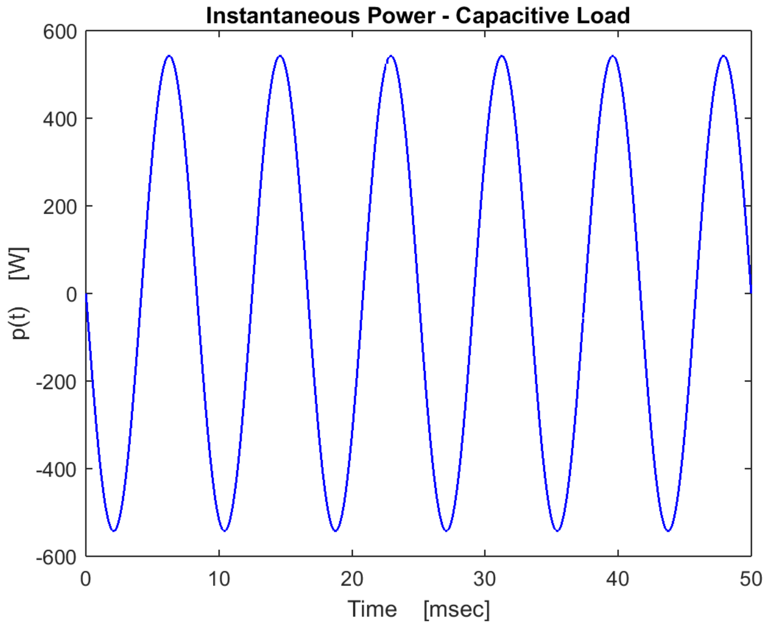
$$p_C(t) = \omega C V_p^2 \frac{1}{2}[\cos(-90°) + \cos(2\omega t + 2\delta + 90°)]$$

$$p_C(t) = \omega C \frac{V_p^2}{2} \cdot \cos(2\omega t + 2\delta + 90°)$$

- In terms of rms voltage

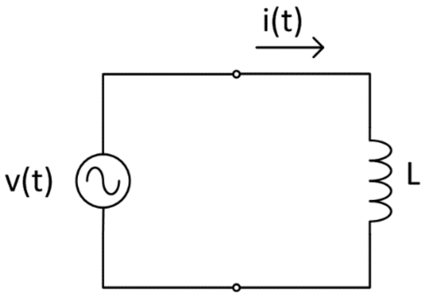
$$p_C(t) = \omega C V_{rms}^2 \cdot \cos(2\omega t + 2\delta + 90°) \qquad (10)$$

- This is a double frequency sinusoid, but, unlike for the resistive load, the average value is zero



Instantaneous Power – Inductive Load

- Now consider the power absorbed by a purely inductive load
- Now the load current *lags* by 90°



$$i(t) = I_p \cos(\omega t + \delta - 90°)$$

where

$$I_p = \frac{V_p}{X_L} = \frac{V_p}{\omega L}$$

- The instantaneous power delivered to the inductive load is

$$p_L(t) = v(t) \cdot i(t)$$

$$p_L(t) = V_p \cos(\omega t + \delta) \cdot \frac{V_p}{\omega L} \cos(\omega t + \delta - 90°)$$

Instantaneous Power – Inductive Load

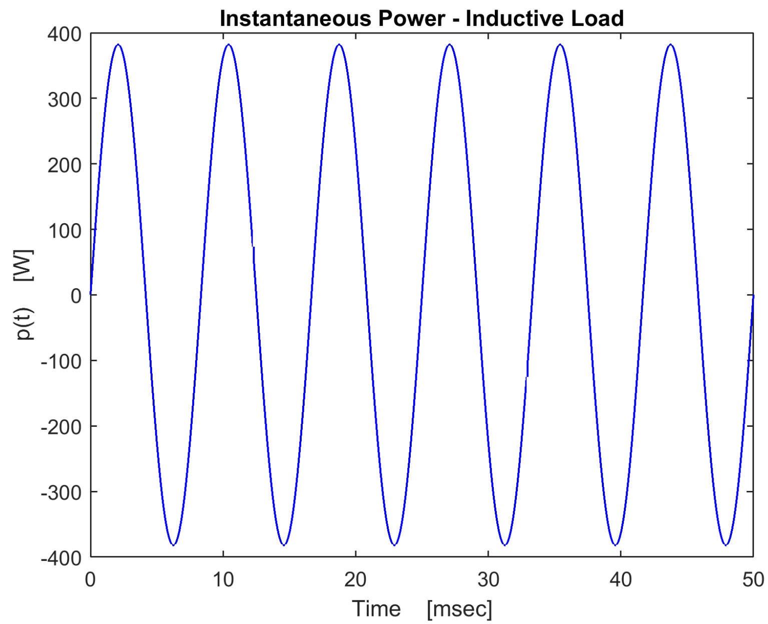
$$p_L(t) = \frac{V_p^2}{\omega L}\frac{1}{2}[\cos(90°) + \cos(2\omega t + 2\delta - 90°)]$$

$$p_L(t) = \frac{V_p^2}{2\omega L} \cdot \cos(2\omega t + 2\delta - 90°)$$

- In terms of rms voltage

$$p_L(t) = \frac{V_{rms}^2}{\omega L} \cdot \cos(2\omega t + 2\delta - 90°) \quad (11)$$

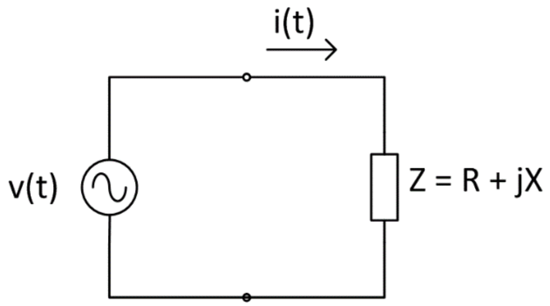
- As for the capacitive load, this is a double frequency sinusoid with an average value of zero



Instantaneous Power – General Impedance

- Finally, consider the instantaneous power absorbed by a general RLC load
- Phase angle of the current is determined by the angle of the impedance

$$i(t) = I_p \cos(\omega t + \beta)$$



- The instantaneous power is

$$p(t) = V_p \cos(\omega t + \delta) \cdot I_p \cos(\omega t + \beta)$$

$$p(t) = \frac{V_p I_p}{2}[\cos(\delta - \beta) + \cos(2\omega t + \delta + \beta)]$$

$$p(t) = V_{rms} I_{rms}\left[\cos(\delta - \beta) + \cos\big(2\omega t + 2\delta - (\delta - \beta)\big)\right]$$

Instantaneous Power – General Impedance

- Using the following trig identity

$$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

we get

$$p(t) = V_{rms} I_{rms} [\cos(\delta - \beta) + \cos(\delta - \beta)\cos(2\omega t + 2\delta) + \sin(\delta - \beta)\sin(2\omega t + 2\delta)]$$

and

$$p(t) = V_{rms} I_{rms} \cos(\delta - \beta)\,[1 + \cos(2\omega t + 2\delta)] + V_{rms} I_{rms} \sin(\delta - \beta)\sin(2\omega t + 2\delta)$$

Instantaneous Power – General Impedance

- Letting

$$I_R = I_{rms}\cos(\delta-\beta) \quad \text{and} \quad I_X = I_{rms}\sin(\delta-\beta)$$

we have

$$p(t) = V_{rms}I_R[1+\cos(2\omega t+2\delta)] + V_{rms}I_X\sin(2\omega t+2\delta) \qquad (12)$$

- There are two components to the power:

$$p_R(t) = V_{rms}I_R[1+\cos(2\omega t+2\delta)] \qquad (13)$$

is the power absorbed by the resistive component of the load, and

$$p_X(t) = V_{rms}I_X\sin(2\omega t+2\delta) \qquad (14)$$

is the power absorbed by the reactive component of the load

Real Power

- According to (9) and (13), power delivered to a resistance has a non-zero average value
  - Purely resistive load or a load with a resistive component
- This is ***real power***, ***average power***, or ***active power***

$$P = V_{rms} I_R$$

$$\boxed{P = V_{rms} I_{rms} \cos(\delta - \beta)} \quad (15)$$

- Real power has units of ***watts*** (W)
- Real power is power that results in work (or heat dissipation)

Power Factor

- The phase angle $(\delta - \beta)$ represents the phase difference between the voltage and the current
  - This is the ***power factor angle***
  - The angle of the load impedance
- Note that the *real power* is a function of the *cosine of the power factor angle*

$$P = V_{rms} I_{rms} \cos(\delta - \beta)$$

- This is the ***power factor***

$$p.f. = \cos(\delta - \beta) \qquad (16)$$

- For a purely resistive load, voltage and current are in phase

$$p.f. = \cos(\delta - \beta) = \cos(0°) = 1$$

$$P = V_{rms} I_{rms}$$

Power Factor

- For a purely capacitive load, current leads the voltage by $90°$

$$p.f. = \cos(\delta - \beta) = \cos(-90°) = 0$$

$$P = 0$$

  - This is referred to as a ***leading power factor***
  - Power factor is *leading* for loads with *capacitive* reactance
- For a purely inductive load, current lags the voltage by $90°$

$$p.f. = \cos(\delta - \beta) = \cos(90°) = 0$$

$$P = 0$$

  - Loads with inductive reactance have *lagging* power factors
- Note that power factor is defined to always be ***positive***

$$0 \le p.f. \le 1$$

Reactive & Complex Power

Reactive Power

- The other part of instantaneous power, as given by (12), is the power delivered to the reactive component of the load

$$p_X(t) = V_{rms} I_{rms} \sin(\delta - \beta) \sin(2\omega t + 2\delta)$$

- Unlike real power, this component of power has zero average value
- The *amplitude* is the ***reactive power***

$$Q = V_{rms} I_{rms} \sin(\delta - \beta) \;\; var$$

- Units are ***volts-amperes reactive***, or ***var***
- Power that flows to and from the load reactance
  - Does not result in work or heat dissipation

Complex Power

- ***Complex power*** is defined as the product of the rms voltage phasor and *conjugate* rms current phasor

$$\boxed{\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^*} \tag{18}$$

where the voltage has phase angle δ

$$\boldsymbol{V} = V_{rms}\angle\delta$$

and the current has phase angle β

$$\boldsymbol{I} = I_{rms}\angle\beta \quad \rightarrow \quad \boldsymbol{I}^* = I_{rms}\angle-\beta$$

- The complex power is

$$\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^* = (V_{rms}\angle\delta)(I_{rms}\angle-\beta)$$

$$\boldsymbol{S} = V_{rms}I_{rms}\angle(\delta-\beta) \tag{19}$$

Complex Power

- Complex power has units of ***volts-amperes*** (VA)
- The *magnitude* of complex power is ***apparent power***

$$S = V_{rms} I_{rms} \ \ VA \tag{20}$$

- Apparent power also has units of volts-amperes
- Complex power is the vector sum of real power (in phase with $\boldsymbol{V}$) and reactive power ($\pm 90°$ out of phase with $\boldsymbol{V}$)

$$\boldsymbol{S} = P + jQ \tag{21}$$

Complex Power

- ***Real power*** can be expressed in terms of complex power

$$P = Re\{\boldsymbol{S}\}$$

or in terms of ***apparent power***

$$P = S \cdot \cos(\delta - \beta) = S \cdot p.f.$$

- Similarly, ***reactive power***, is the imaginary part of complex power

$$Q = Im\{\boldsymbol{S}\}$$

and can also be related to ***apparent power***

$$Q = S \cdot \sin(\delta - \beta)$$

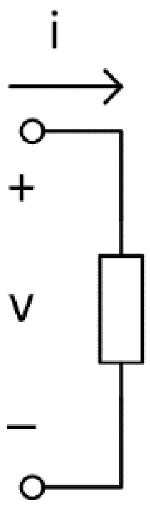
- And, ***power factor*** is the ***ratio*** between ***real power*** and ***apparent power***

$$p.f. = \cos(\delta - \beta) = \frac{P}{S}$$

Passive Sign Convention

Power Convention – Load Convention

- Applying a consistent sign convention allows us to easily determine whether network elements supply or absorb real and reactive power
- ***Passive sign convention*** or ***load convention***
  - Positive current defined to enter the positive voltage terminal of an element
- If $P > 0$ or $Q > 0$, then real or reactive power is ***absorbed*** by the element
- If $P < 0$ or $Q < 0$, then real or reactive power is ***supplied*** by the element



Power Absorbed by Passive Elements

- Complex power absorbed by a ***resistor***

$$\boldsymbol{S_R} = \boldsymbol{V}\boldsymbol{I_R^*} = (V\angle\delta)\left(\frac{V}{R}\angle-\delta\right)$$

$$\boldsymbol{S_R} = \frac{V^2}{R}$$

  - Positive and purely real
    - Resistors ***absorb real*** power
    - ***Reactive*** power is ***zero***

- Complex power absorbed by a ***capacitor***

$$\boldsymbol{S_C} = \boldsymbol{V}\boldsymbol{I_C^*} = (V\angle\delta)(-j\omega CV\angle-\delta)$$

$$\boldsymbol{S_C} = -j\omega CV^2$$

  - Negative and purely imaginary
    - Capacitors ***supply reactive*** power
    - ***Real*** power is ***zero***

Power Absorbed by Passive Elements

- Complex power absorbed by an ***inductor***

$$\boldsymbol{S_L} = \boldsymbol{V}\boldsymbol{I_L^*} = (V\angle\delta)\left(\frac{V}{-j\omega L}\angle - \delta\right)$$

$$\boldsymbol{S_L} = j\frac{V^2}{\omega L}$$

  - Positive and purely imaginary
    - Inductors ***absorb reactive*** power
    - ***Real*** power is ***zero***

- In summary:
  - Resistors absorb real power, zero reactive power
  - Capacitors supply reactive power, zero real power
  - Inductors absorb reactive power, zero real power

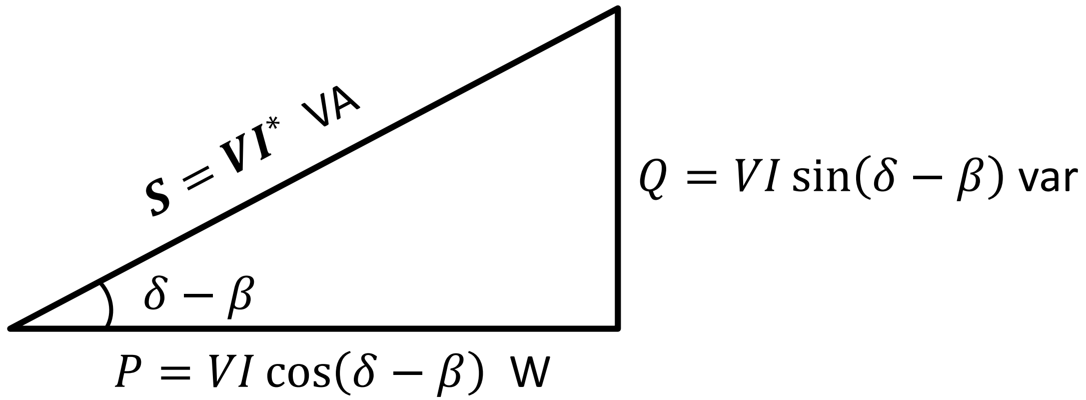
Power Triangle

Power Triangle

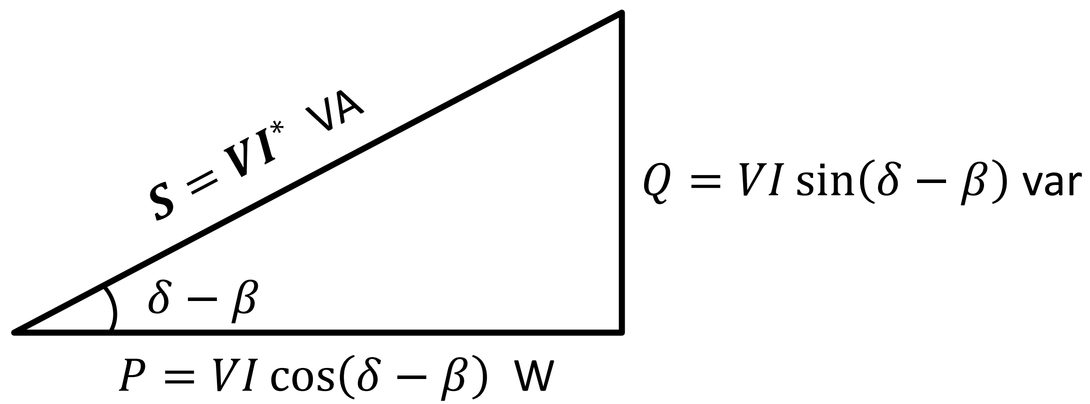
- Complex power is the vector sum of real power (in phase with $\boldsymbol{V}$) and reactive power ($\pm 90°$ out of phase with $\boldsymbol{V}$)

$$\boldsymbol{S} = P + jQ$$

- Complex, real, and reactive powers can be represented graphically, as a ***power triangle***



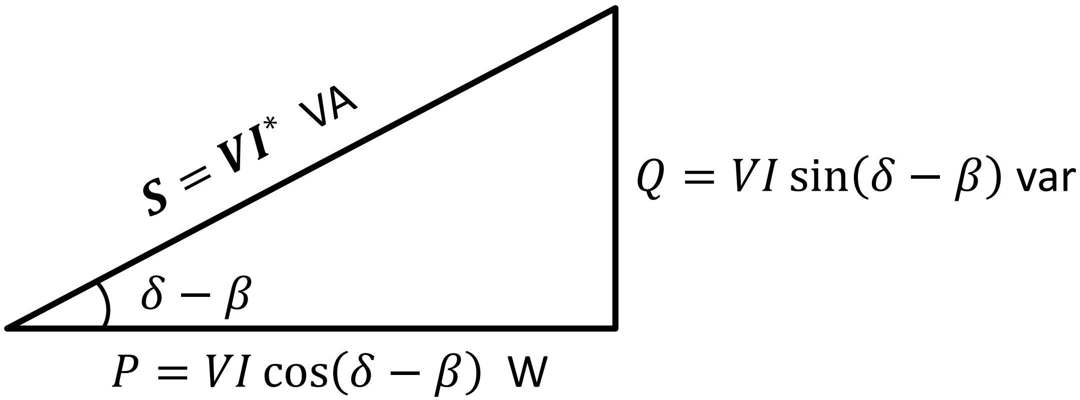
Power Triangle



- Quickly and graphically provides power information
  - ***Power factor*** and power factor angle
  - ***Leading*** or ***lagging*** power factor
  - Reactive nature of the load – ***capacitive*** or ***inductive***

Lagging Power Factor

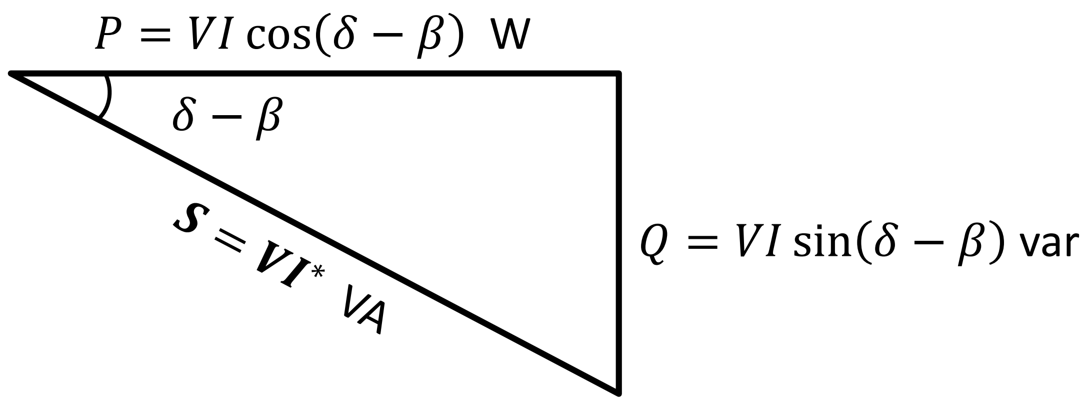
- For loads with ***inductive*** reactance
  - Impedance angle is positive
  - Power factor angle is positive
  - Power factor is ***lagging***



- Q is positive
  - The load ***absorbs*** reactive power

Leading Power Factor

- For loads with ***capacitive*** reactance
  - Impedance angle is negative
  - Power factor angle is negative
  - Power factor is ***leading***



- Q is negative
  - The load ***supplies*** reactive power

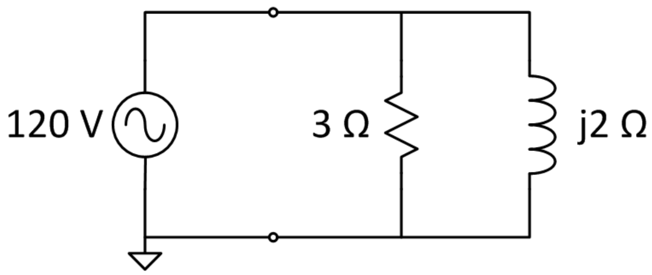
Power Factor Correction

Power Factor Correction

- The overall goal of power distribution is to supply power to do work
  - *Real* power
- Reactive power does not perform work, but
  - Must be supplied by the source
  - Still flows over the lines
- For a given amount of real power consumed by a load, we'd like to
  - Reduce reactive power, Q
  - Reduce S relative to P, that is
  - Reduce the p.f. angle, and
  - Increase the p.f.
- ***Power factor correction***

Power Factor Correction – Example

- Consider a source driving an inductive load



- Determine:
  - Real power absorbed by the load
  - Reactive power absorbed by the load
  - p.f. angle and p.f.
- Draw the power triangle

- Current through the resistance is

$$\boldsymbol{I_R} = \frac{120\ V}{3\ \Omega} = 40\ A$$

- Current through the inductance is

$$\boldsymbol{I_L} = \frac{120\ V}{j2\ \Omega} = 60\angle -90°\ A$$

- The total load current is

$$\boldsymbol{I} = \boldsymbol{I_R} + \boldsymbol{I_L} = (40 - j60)A = 72.1\angle -56.3°\ A$$

Power Factor Correction – Example

- The power factor angle is

$$\theta = (\delta - \beta) = 0° - (-56.3°)$$

$$\boxed{\theta = 56.3°}$$

- The power factor is

$$p.f. = \cos(\theta) = \cos(56.3°)$$

$$\boxed{p.f. = 0.55 \text{ lagging}}$$

- Real power absorbed by the load is

$$P = VI\cos(\theta) = 120\,V \cdot 72.1\,A \cdot 0.55$$

$$\boxed{P = 4.8\,kW}$$

- Alternatively, recognizing that real power is power absorbed by the resistance

$$P = VI_R = 120\,V \cdot 40\,A = 4.8\,kW$$

Power Factor Correction – Example

- Reactive power absorbed by the load is

$$Q = VI\sin(\theta) = 120\,V \cdot 72.1\,A \cdot 0.832$$

$$\boxed{Q = 7.2\,kvar}$$

- This is also the power absorbed by the load inductance

$$Q = VI_L = 120\,V \cdot 60\,A = 7.2\,kvar$$

- Apparent power is

$$S = VI = 120\,V \cdot 72.1\,A = 8.65\,kVA$$

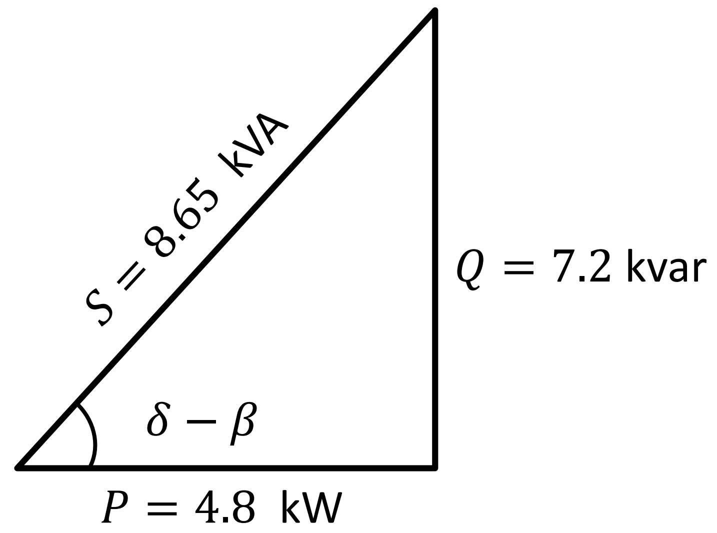
- Or, alternatively

$$S = \sqrt{P^2 + Q^2}$$

$$S = \sqrt{(4.8\,kW)^2 + (7.2\,kvar)^2} = 8.65\,kVA$$

Power Factor Correction – Example

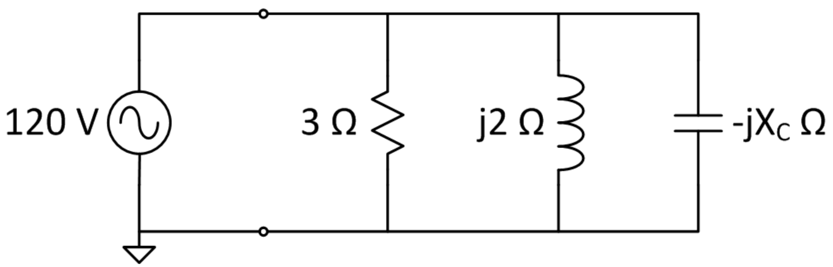
- The ***power triangle***:
- Here, the source is supplying 4.8 kW at a power factor of 0.55 lagging
- Let's say we want to reduce the apparent power supplied by the source
  - Deliver 4.8 kW at a p.f. of 0.9 lagging
- Add ***power factor correction***
- Add capacitors to *supply* reactive power



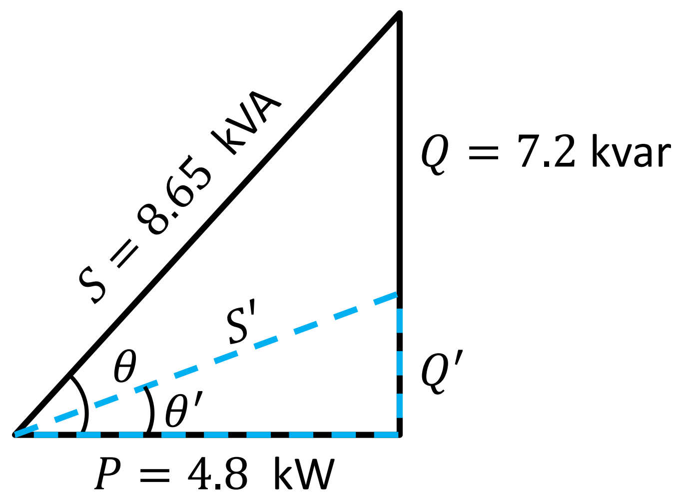
Power Factor Correction – Example

- For $p.f. = 0.9$, we need a power factor angle of

$$\theta' = \cos^{-1}(0.9) = 25.8°$$



- Power factor correction will help flatten the power triangle:



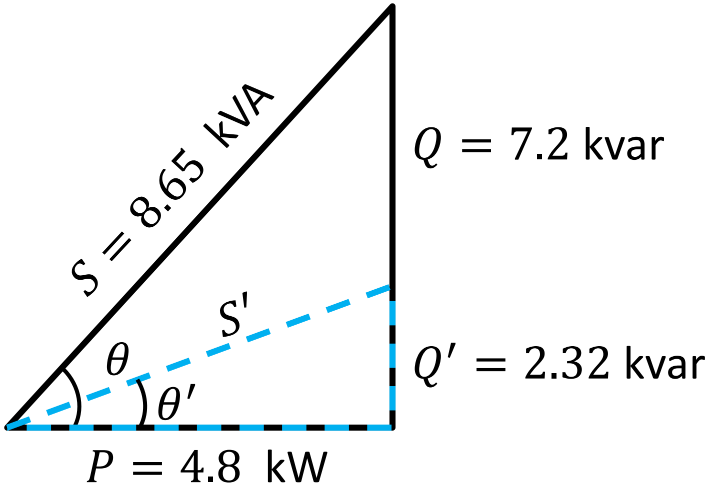
Power Factor Correction – Example

- Reactive power to the power-factor-corrected load is reduced from Q to Q'

$$Q' = P\tan(\theta')$$

$$Q' = 4.8\ kW \cdot \tan(25.8°)$$

$$Q' = 2.32\ kvar$$



- The required reactive power *absorbed* (negative, so it is *supplied*) by the capacitors is

$$Q_C = Q' - Q = 2.32\ kvar - 7.2\ kvar$$

$$Q_C = -4.88\ kvar$$

Power Factor Correction – Example

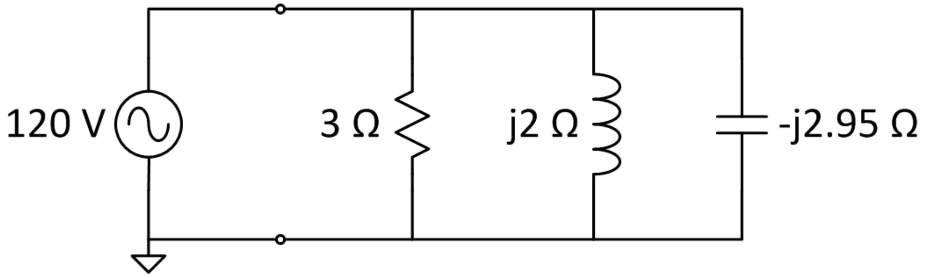
- Reactive power absorbed by the capacitor is

$$Q_C = \frac{V^2}{X_C}$$

- So the required capacitive reactance is

$$X_C = \frac{V^2}{Q_C} = \frac{(120\ V)^2}{-4.88\ kvar} = -2.95\ \Omega$$

- The addition of $-j2.95\ \Omega$ provides the desired power factor correction

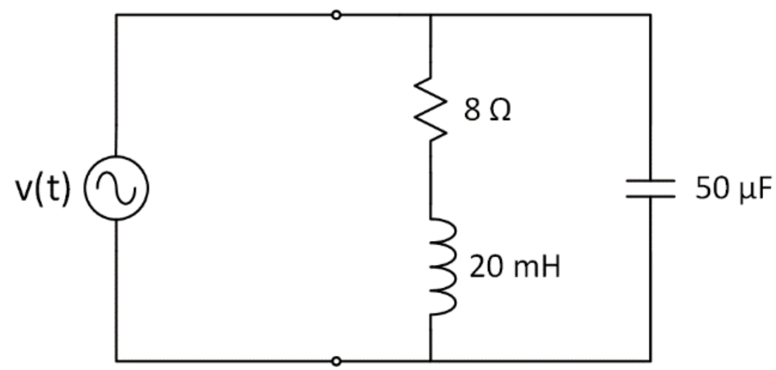


Example Problems

The source voltage in the circuit is

$$v(t) = \sqrt{2} \cdot 120V \cos(2\pi \cdot 60Hz \cdot t).$$

Determine the complex power delivered to the load.



Two three-phase load are connected in parallel:

- 50 kVA at a power factor of 0.9, leading
- 125 kW at a power factor of 0.85, lagging.

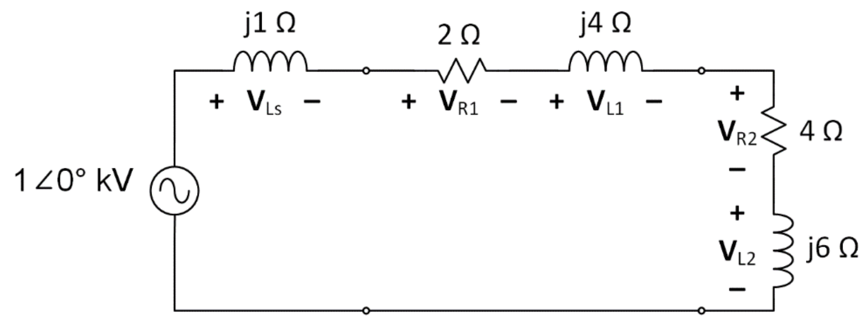
Draw the power triangle and determine the combined power factor.

Power is delivered to a single-phase load with an impedance of $Z_L = 3 + j2\ \Omega$ at 120 V. Add power factor correction in parallel with the load to yield a power factor of 0.95, lagging.

Determine the reactive power and impedance of the power factor correction component.

Draw a phasor diagram for the following circuit.

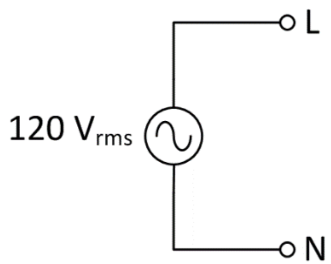
- Draw a phasor for the voltage across each component and for the current
- Apply KVL graphically. That is, add the individual component phasors together graphically to show that the result is equal to the source voltage phasor.



Balanced Three-Phase Networks

Balanced Three-Phase Networks

- We are accustomed to ***single-phase*** power in our homes and offices
  - A single ***line*** voltage referenced to a ***neutral***

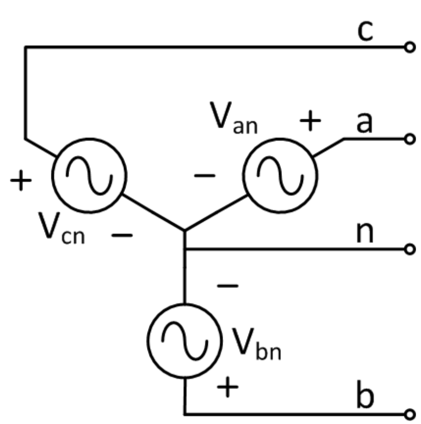


- Electrical power is generated, transmitted, and largely consumed (by industrial customers) as ***three-phase power***
  - Three individual line voltages and (possibly) a neutral
  - Line voltages all differ in phase by $\pm 120°$

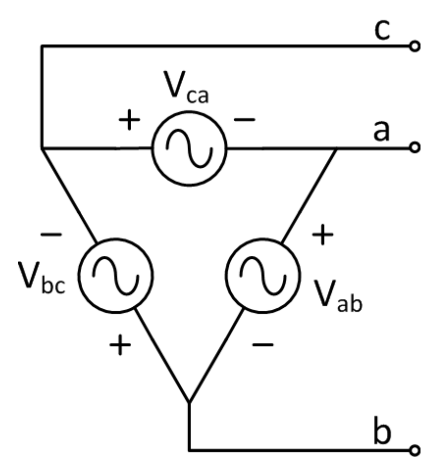
Δ- and Y-Connected Networks

- Two possible three-phase configurations
  - Applies to both ***sources*** and ***loads***

Y-Connected Source



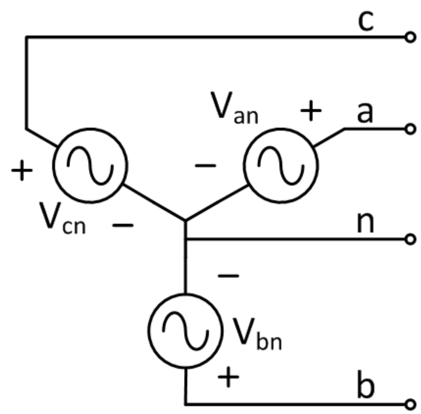
Δ-Connected Source



- Y-connected network has a neutral node
- Δ-connected network has no neutral

Line-to-Neutral Voltages

- In the Y network, voltages V_{an}, V_{bn}, and V_{cn} are ***line-to-neutral voltages***
- A three-phase source is ***balanced*** if
  - Line-to-neutral voltages have equal magnitudes
  - Line-to-neutral voltage are each $120°$ out of phase with one another

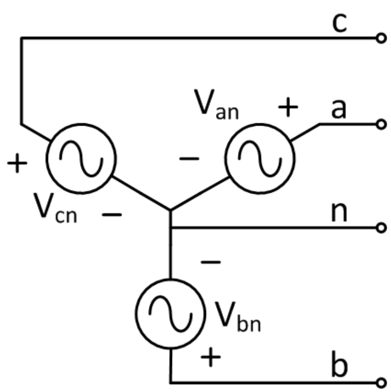


- A three-phase network is balanced if
  - Sources are balanced
  - The impedances connected to each phase are equal

Line-to-Neutral Voltages

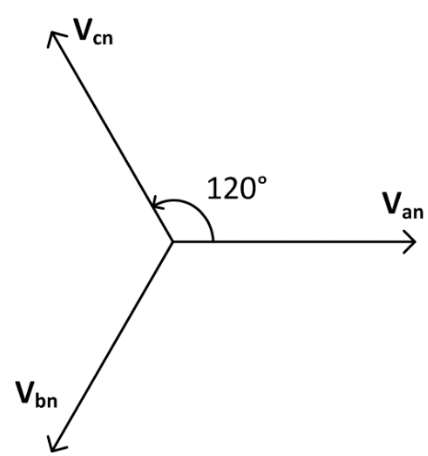
- The line-to-neutral voltages are

$$\boldsymbol{V_{an}} = V_{LN}\angle 0°$$
$$\boldsymbol{V_{bn}} = V_{LN}\angle -120°$$
$$\boldsymbol{V_{cn}} = V_{LN}\angle -240° = V_{LN}\angle +120°$$



- This is a ***positive-sequence*** or ***abc-sequence*** source
  - $\boldsymbol{V_{an}}$ leads $\boldsymbol{V_{bn}}$, which leads $\boldsymbol{V_{cn}}$
- Can also have a ***negative-*** or ***acb-sequence*** source
  - $\boldsymbol{V_{an}}$ leads $\boldsymbol{V_{cn}}$, which leads $\boldsymbol{V_{bn}}$
- We'll always assume *positive*-sequence sources

Positive-Sequence Phasor Diagram:



Line-to-Line Voltages

- The voltages between the three phases are ***line-to-line voltages***
- Apply KVL to relate line-to-line voltages to line-to-neutral voltages

$$\boldsymbol{V_{ab}} - \boldsymbol{V_{an}} + \boldsymbol{V_{bn}} = \boldsymbol{0}$$

$$\boldsymbol{V_{ab}} = \boldsymbol{V_{an}} - \boldsymbol{V_{bn}}$$

- We know that

$$\boldsymbol{V_{an}} = V_{LN}\angle 0°$$

and

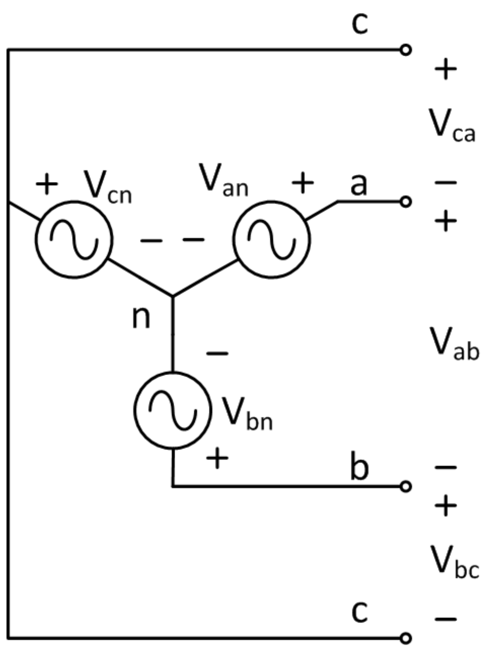
$$\boldsymbol{V_{bn}} = V_{LN}\angle -120°$$

so

$$\boldsymbol{V_{ab}} = V_{LN}\angle 0° - V_{LN}\angle -120° = V_{LN}(1\angle 0° - 1\angle -120°)$$

$$\boldsymbol{V_{ab}} = V_{LN}\left[1 - \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right] = V_{LN}\left[\frac{3}{2} + j\frac{\sqrt{3}}{2}\right]$$

$$\boxed{\boldsymbol{V_{ab}} = \sqrt{3}V_{LN}\angle 30°}$$



Line-to-Line Voltages

- Again applying KVL, we can find $\boldsymbol{V_{bc}}$

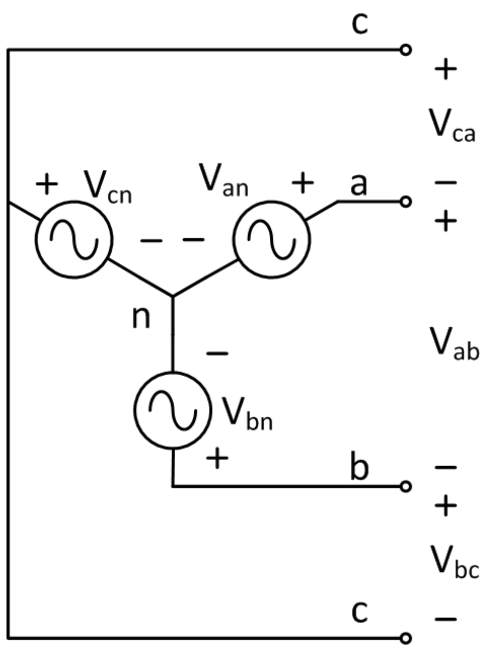
$$\boldsymbol{V_{bc}} = \boldsymbol{V_{bn}} - \boldsymbol{V_{cn}}$$

$$\boldsymbol{V_{bc}} = V_{LN}\angle -120° - V_{LN}\angle 120°$$

$$\boldsymbol{V_{bc}} = V_{LN}\left[\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right]$$

$$\boldsymbol{V_{bc}} = V_{LN}\left(-j\sqrt{3}\right)$$

$$\boxed{\boldsymbol{V_{bc}} = \sqrt{3}V_{LN}\angle -90°}$$



- Similarly,

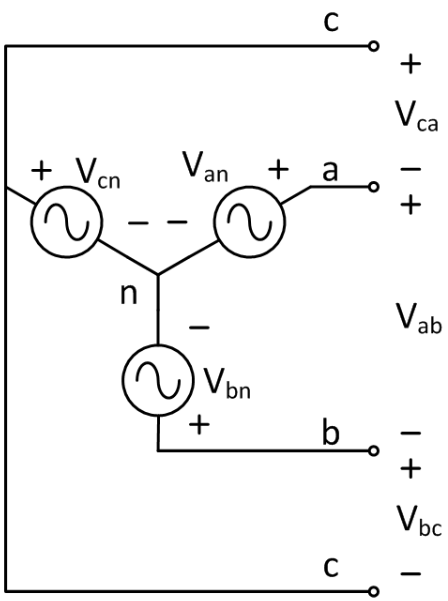
$$\boxed{\boldsymbol{V_{ca}} = \sqrt{3}V_{LN}\angle 150°}$$

Line-to-Line Voltages

- The line-to-line voltages, with V_{an} as the reference:

$$\boldsymbol{V_{ab}} = \sqrt{3}V_{LN}\angle 30°$$
$$\boldsymbol{V_{bc}} = \sqrt{3}V_{LN}\angle -90°$$
$$\boldsymbol{V_{ca}} = \sqrt{3}V_{LN}\angle 150°$$

- Line-to-line voltages are $\sqrt{3}$ times the line-to-neutral voltage



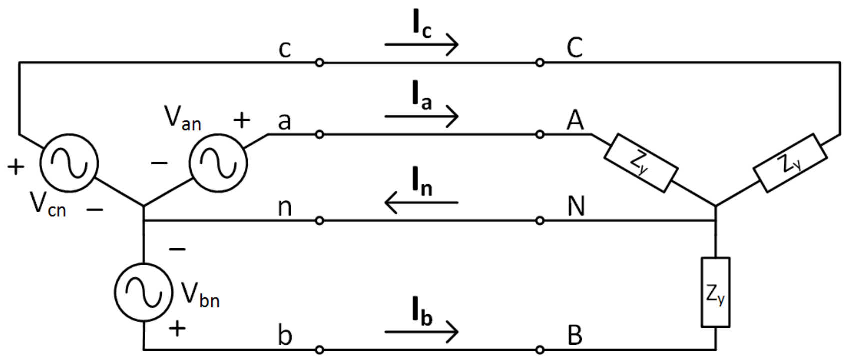
- Can also express in terms of individual line-to-neutral voltages:

$$\boldsymbol{V_{ab}} = \sqrt{3}\boldsymbol{V_{an}}\angle 30°$$
$$\boldsymbol{V_{bc}} = \sqrt{3}\boldsymbol{V_{bn}}\angle 30°$$
$$\boldsymbol{V_{ca}} = \sqrt{3}\boldsymbol{V_{cn}}\angle 30°$$

Currents in Three-Phase Networks

Line Currents in Balanced 3ϕ Networks

- We can use the line-to-neutral voltages to determine the line currents
  - Y-connected source and load
  - Balanced load – all impedances are equal: Z_Y



$$\boldsymbol{I_a} = \frac{\boldsymbol{V_{AN}}}{Z_Y} = \frac{V_{LN}\angle 0°}{Z_Y}$$

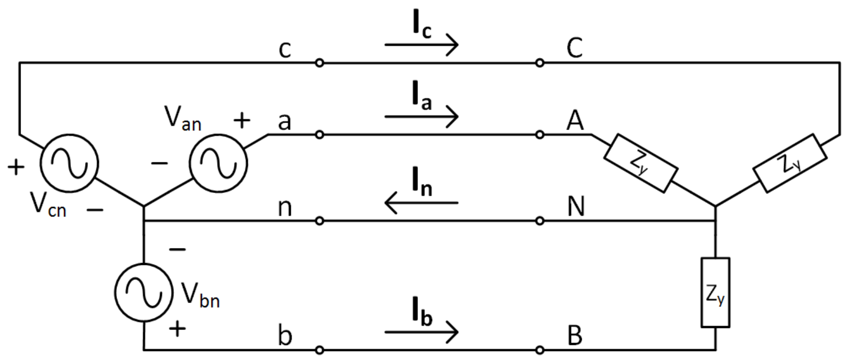
$$\boldsymbol{I_b} = \frac{\boldsymbol{V_{BN}}}{Z_Y} = \frac{V_{LN}\angle -120°}{Z_Y}$$

$$\boldsymbol{I_c} = \frac{\boldsymbol{V_{CN}}}{Z_Y} = \frac{V_{LN}\angle +120°}{Z_Y}$$

- Line currents are balanced as long as the source and load are balanced

Neutral Current in Balanced 3ϕ Networks

- Apply KCL to determine the neutral current



$$\boldsymbol{I_n} = \boldsymbol{I_a} + \boldsymbol{I_b} + \boldsymbol{I_c}$$

$$\boldsymbol{I_n} = \frac{V_{LN}}{Z_Y}[1\angle 0° + 1\angle - 120° + 1\angle 120°]$$

$$\boldsymbol{I_n} = \frac{V_{LN}}{Z_Y}\left[1 + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right]$$

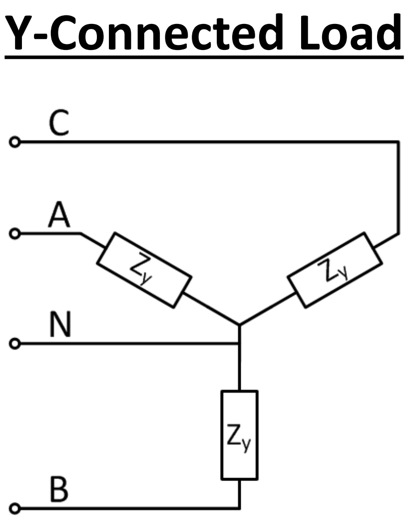
$$\boldsymbol{I_n} = 0$$

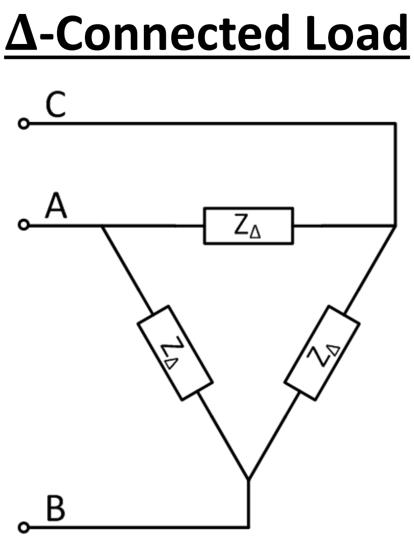
- The neutral conductor carries no current in a balanced three-phase network

Y- and Δ-connected Loads

Three-Phase Load Configurations

- As for sources, three-phase loads can also be connected in two different configurations

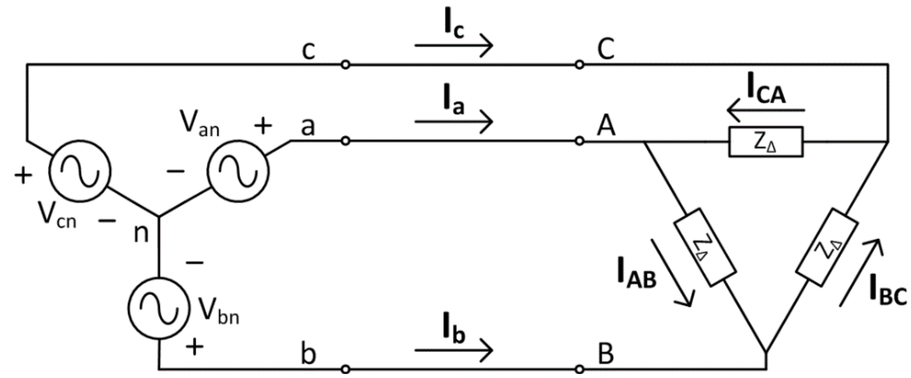




- The Y load has a neutral connection, but the Δ load does not
- Currents in a Y-connected load are the line currents we just determined
- Next, we'll look at currents in a Δ-connected load

Balanced Δ-Connected Loads

- We can use line-to-line voltages to determine the currents in Δ-connected loads



$$\boldsymbol{I_{AB}} = \frac{\boldsymbol{V_{AB}}}{Z_\Delta} = \frac{\sqrt{3}\boldsymbol{V_{AN}}\angle 30°}{Z_\Delta} = \frac{\sqrt{3}V_{LN}\angle 30°}{Z_\Delta}$$

$$\boldsymbol{I_{BC}} = \frac{\boldsymbol{V_{BC}}}{Z_\Delta} = \frac{\sqrt{3}\boldsymbol{V_{BN}}\angle 30°}{Z_\Delta} = \frac{\sqrt{3}V_{LN}\angle -90°}{Z_\Delta}$$

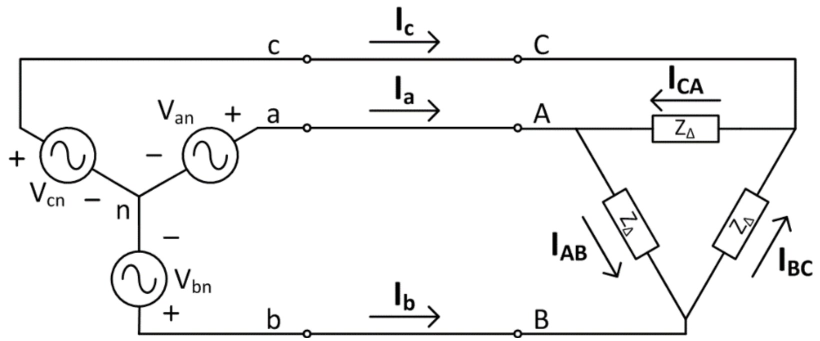
$$\boldsymbol{I_{CA}} = \frac{\boldsymbol{V_{CA}}}{Z_\Delta} = \frac{\sqrt{3}\boldsymbol{V_{CN}}\angle 30°}{Z_\Delta} = \frac{\sqrt{3}V_{LN}\angle 150°}{Z_\Delta}$$

Balanced Δ-Connected Loads

- Applying KCL, we can determine the line currents

$$\boldsymbol{I_a} = \boldsymbol{I_{AB}} - \boldsymbol{I_{CA}}$$

$$\boldsymbol{I_a} = \frac{\sqrt{3}V_{LN}}{Z_\Delta}[1\angle 30° - 1\angle 150°]$$



$$\boldsymbol{I_a} = \frac{\sqrt{3}V_{LN}}{Z_\Delta}\left[\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) - \left(-\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)\right] = \frac{\sqrt{3}V_{LN}}{Z_\Delta}[\sqrt{3}] = \frac{3V_{LN}}{Z_\Delta}$$

- The other line currents can be found similarly:

$$\boldsymbol{I_a} = \frac{3V_{LN}\angle 0°}{Z_\Delta} = \sqrt{3}\boldsymbol{I_{AB}}\angle -30°$$

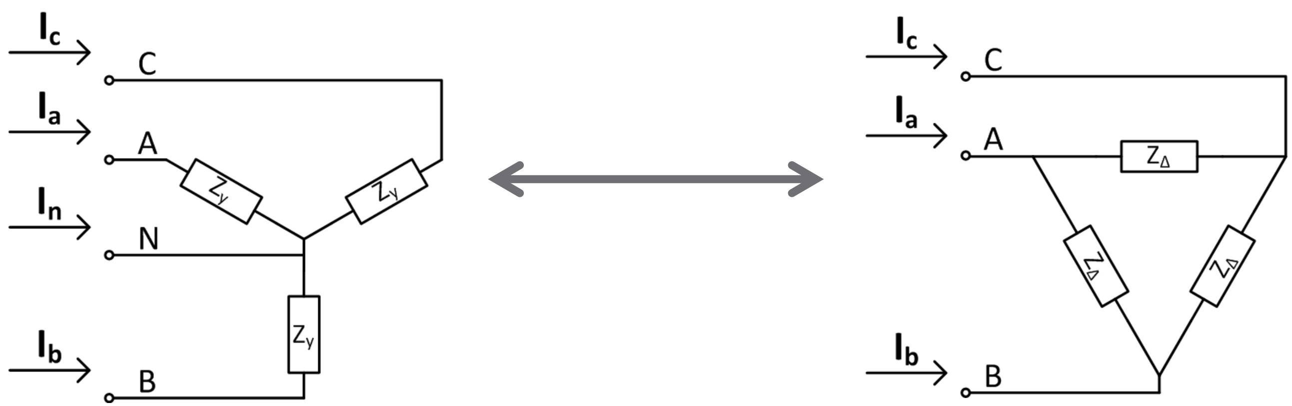
$$\boldsymbol{I_b} = \frac{3V_{LN}\angle -120°}{Z_\Delta} = \sqrt{3}\boldsymbol{I_{BC}}\angle -30°$$

$$\boldsymbol{I_c} = \frac{3V_{LN}\angle 120°}{Z_\Delta} = \sqrt{3}\boldsymbol{I_{CA}}\angle -30°$$

Δ-Y Conversion

$\Delta - Y$ Conversion

- Analysis is often simpler when dealing with Y-connected loads
  - Would like a way to convert Δ loads to Y loads (and vice versa)



- For a Y load and a Δ load to be equivalent, they must result in equal line currents

$\Delta - Y$ Conversion

- Line currents for a Y-connected load:

$$\boldsymbol{I_a} = \frac{V_{LN}\angle 0°}{Z_Y}$$

$$\boldsymbol{I_b} = \frac{V_{LN}\angle -120°}{Z_Y}$$

$$\boldsymbol{I_c} = \frac{V_{LN}\angle 120°}{Z_Y}$$

- For a Δ-connected load:

$$\boldsymbol{I_a} = \frac{3V_{LN}\angle 0°}{Z_\Delta}$$

$$\boldsymbol{I_b} = \frac{3V_{LN}\angle -120°}{Z_\Delta}$$

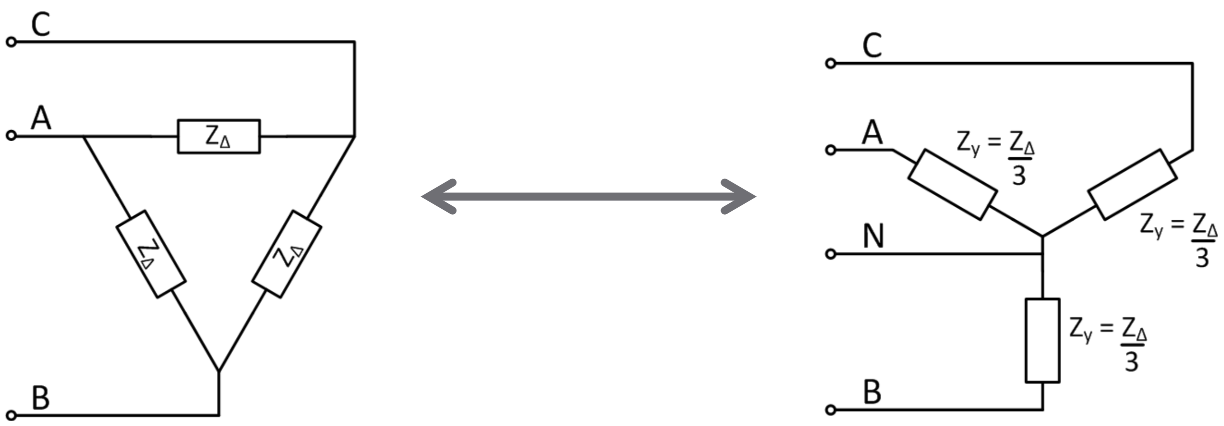
$$\boldsymbol{I_c} = \frac{3V_{LN}\angle 120°}{Z_\Delta}$$

$\Delta - Y$ Conversion

- Equating any of the three line currents, we can determine the impedance relationship

$$\frac{V_{LN}\angle 0°}{Z_Y} = \frac{3V_{LN}\angle 0°}{Z_\Delta}$$

$$\boxed{Z_Y = \frac{Z_\Delta}{3}} \quad \text{and} \quad \boxed{Z\Delta = 3Z_Y}$$

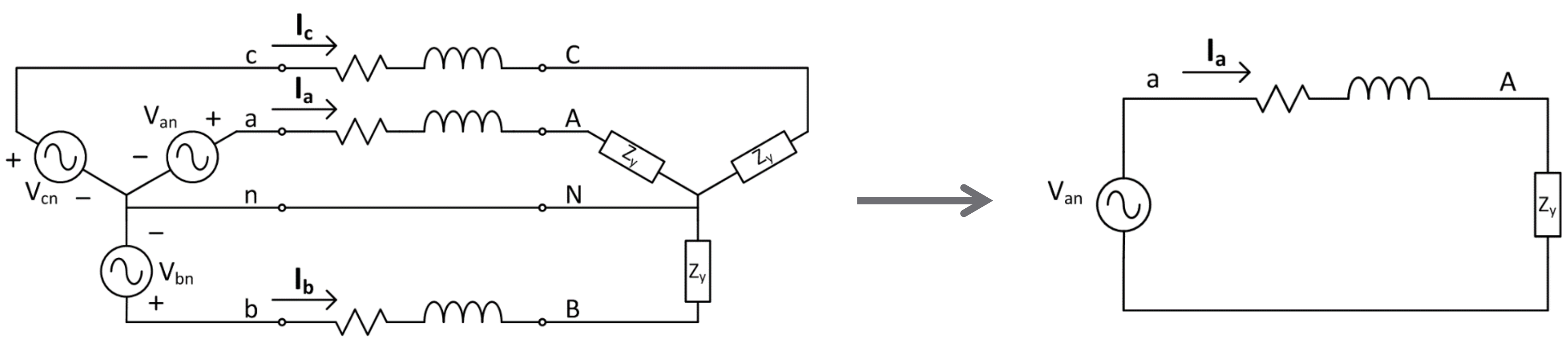


Per-Phase Analysis

Line-to-Neutral Schematics

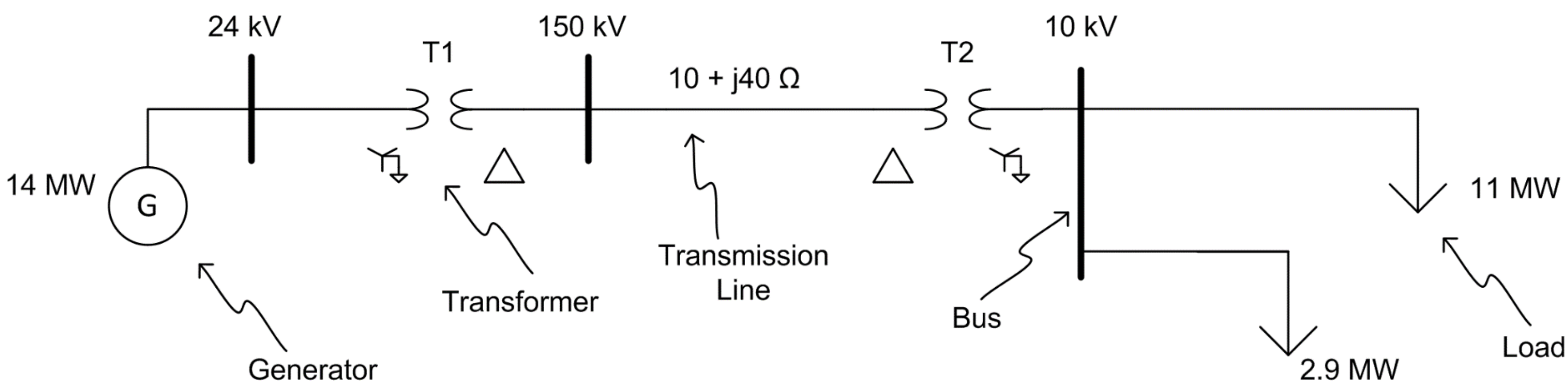
- For balanced networks, we can simplify our analysis by considering only a single phase
  - A ***per-phase analysis***
  - Other phases are simply shifted by $\pm 120°$

- For example, a balanced Y-Y circuit:



One-Line Diagrams

- Power systems are often depicted using ***one-line diagrams*** or ***single-line diagrams***
  - Not a schematic – not all wiring is shown
- For example:



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Example Problems

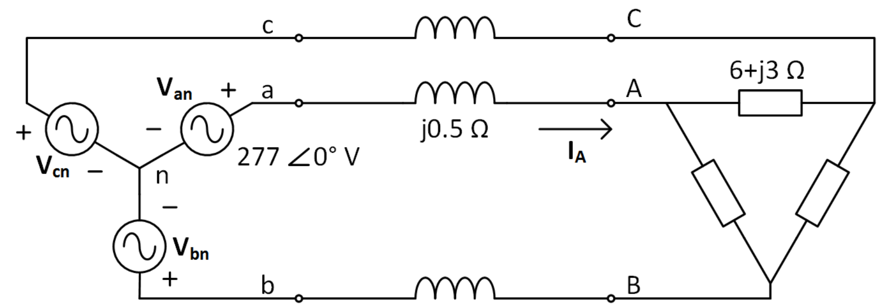
- Given the following balanced 3-ϕ quantities:

$$\mathbf{V}_{BC} = 480\angle 15° \text{ and } \mathbf{I}_B = 21\angle -28°$$

- Find:
  1) $\mathbf{V}_{AB}$
  2) $\mathbf{V}_{AN}$
  3) $\mathbf{I}_A$
  4) $\mathbf{I}_C$

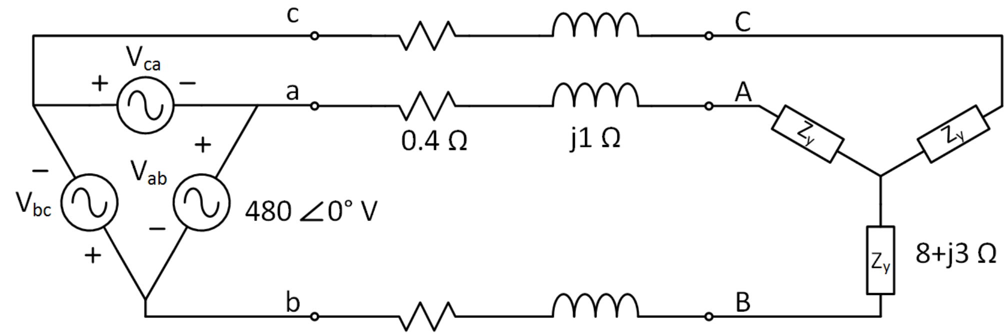
Find:

- Per-phase circuit
- Line current, $\mathbf{I}_A$
- Load voltage



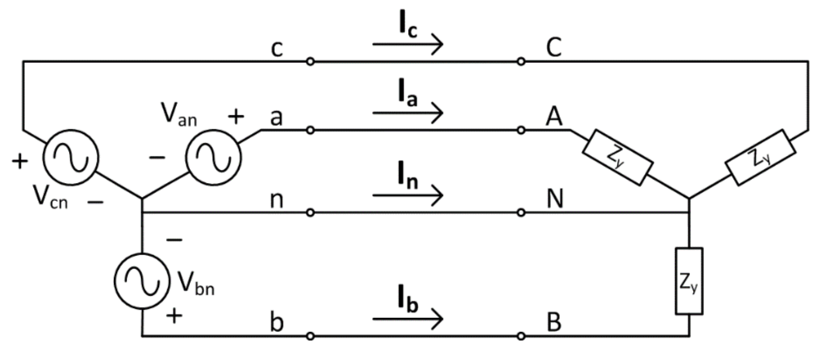
Find:

- Per-phase circuit
- Line current, $\mathbf{I}_A$
- L-L and L-N load voltages



Power in Balanced 3ϕ Networks

Instantaneous Power



- We'll first determine the instantaneous power supplied by the source
  - Neglecting line impedance, this is also the power absorbed by the load
- The phase a line-to-neutral voltage is

$$v_{an}(t) = \sqrt{2} V_{LN} \cos(\omega t + \delta)$$

- The phase a current is

$$i_a(t) = \sqrt{2} I_L \cos(\omega t + \beta)$$

where β depends on the load impedance

Instantaneous Power

- The instantaneous power delivered out of phase a of the source is

$$p_a(t) = v_{an}(t) i_a(t)$$

$$p_a(t) = 2V_{LN} I_L \cos(\omega t + \delta) \cos(\omega t + \beta)$$

$$p_a(t) = V_{LN} I_L \cos(\delta - \beta) + V_{LN} I_L \cos(2\omega t + \delta + \beta)$$

- The b and c phases are shifted by $\pm 120°$
  - Power from each of these phases is

$$p_b(t) = V_{LN} I_L \cos(\delta - \beta) + V_{LN} I_L \cos(2\omega t + \delta + \beta - 240°)$$

$$p_c(t) = V_{LN} I_L \cos(\delta - \beta) + V_{LN} I_L \cos(2\omega t + \delta + \beta + 240°)$$

Instantaneous Power

- The *total* power delivered by the source is the sum of the power from each phase

$$p_{3\phi}(t) = p_a(t) + p_b(t) + p_c(t)$$

$$\begin{aligned} p_{3\phi}(t) = 3V_{LN}I_L\cos(\delta-\beta) \\ +V_{LN}I_L[\cos(2\omega t+\delta+\beta) \\ +\cos(2\omega t+\delta+\beta-240°) \\ +\cos(2\omega t+\delta+\beta+240°)] \end{aligned}$$

- Everything in the square brackets cancels, leaving

$$\boxed{p_{3\phi}(t) = 3V_{LN}I_L\cos(\delta-\beta) = P_{3\phi}}$$

- ***Power in a balanced 3ϕ network is constant***
- In terms of line-to-line voltages, the power is

$$\boxed{P_{3\phi} = \sqrt{3}V_{LL}I_L\cos(\delta-\beta)}$$

Complex Power

- The ***complex power*** delivered by phase a is

$$\boldsymbol{S_a} = \boldsymbol{V_{an}}\boldsymbol{I_a^*} = V_{LN}\angle\delta(I_L\angle\beta)^*$$

$$\boldsymbol{S_a} = V_{LN}I_L\angle(\delta-\beta)$$

$$\boldsymbol{S_a} = V_{LN}I_L\cos(\delta-\beta) + jV_{LN}I_L\sin(\delta-\beta)$$

- For phase b, complex power is

$$\boldsymbol{S_b} = \boldsymbol{V_{bn}}\boldsymbol{I_b^*} = V_{LN}\angle(\delta-120°)\big(I_L\angle(\beta-120°)\big)^*$$

$$\boldsymbol{S_b} = V_{LN}I_L\angle(\delta-\beta)$$

$$\boldsymbol{S_b} = V_{LN}I_L\cos(\delta-\beta) + jV_{LN}I_L\sin(\delta-\beta)$$

- This is equal to $\boldsymbol{S_a}$ and also to phase $\boldsymbol{S_c}$

Complex Power

- The ***total complex power*** is

$$\boldsymbol{S_{3\phi}} = \boldsymbol{S_a} + \boldsymbol{S_b} + \boldsymbol{S_c}$$

$$\boldsymbol{S_{3\phi}} = 3V_{LN}I_L\angle(\delta - \beta)$$

$$\boldsymbol{S_{3\phi}} = 3V_{LN}I_L\cos(\delta - \beta) + j3V_{LN}I_L\sin(\delta - \beta)$$

- The ***apparent power*** is the magnitude of the complex power

$$S_{3\phi} = 3V_{LN}I_L$$

Complex Power

- Complex power can be expressed in terms of the real and reactive power

$$\boldsymbol{S_{3\phi}} = P_{3\phi} + jQ_{3\phi}$$

- The ***real power***, as we've already seen is

$$P_{3\phi} = 3V_{LN}I_L\cos(\delta - \beta)$$

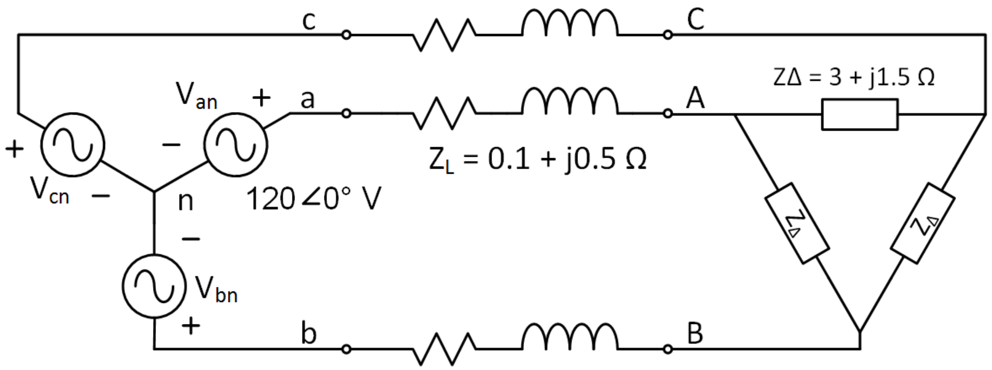
- The ***reactive power*** is

$$Q_{3\phi} = 3V_{LN}I_L\sin(\delta - \beta)$$

Advantages of Three-Phase Power

- Advantages of three-phase power:
  - For a given amount of power, ***half the amount of wire required*** compared to single-phase
    - No return current on neutral conductor
  - ***Constant real power***
    - Constant motor torque
    - Less noise and vibration of machinery

Three-Phase Power – Example



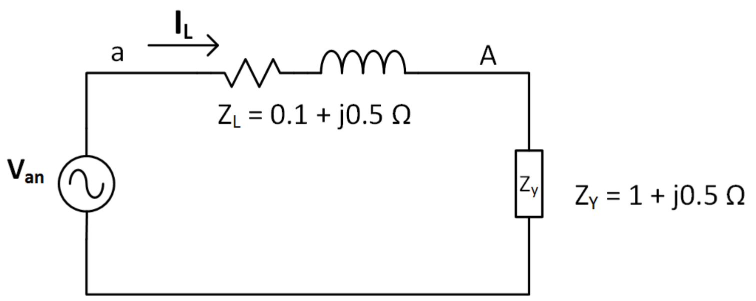
- Determine
  - Load voltage, $\boldsymbol{V_{AB}}$
  - Power triangle for the load
  - Power factor at the load

- We'll do a per-phase analysis, so first convert the Δ load to a Y load

$$Z_Y = \frac{Z_\Delta}{3} = 1 + j0.5\ \Omega$$

Three-Phase Power – Example

- The per-phase circuit:



- The line current is

$$\boldsymbol{I_L} = \frac{\boldsymbol{V_{an}}}{Z_L + Z_Y} = \frac{120\angle 0°\ V}{1.1 + j1\ \Omega} = \frac{120\angle 0°\ V}{1.45\angle 42.3°\ \Omega}$$

$$\boldsymbol{I_L} = 80.7\angle -42.3°\ A$$

- The line-to-neutral voltage at the load is

$$\boldsymbol{V_{AN}} = \boldsymbol{I_L} Z_Y = (80.7\angle -42.3°\ A)(1 + j0.5\ \Omega)$$

$$\boldsymbol{V_{AN}} = (80.7\angle -42.3°\ A)(1.12\angle 26.6°\ \Omega)$$

$$\boldsymbol{V_{AN}} = 90.25\angle -15.71°\ \mathrm{V}$$

Three-Phase Power – Example

- The line-to-line load voltage is

$$\boldsymbol{V_{AB}} = \sqrt{3}\boldsymbol{V_{AN}}\angle 30°$$

$$\boxed{\boldsymbol{V_{AB}} = 156\angle 14.3°\ V}$$

- Alternatively, we could calculate line-to-line voltage from phase A and phase B line-to-neutral voltages

$$\boldsymbol{V_{AB}} = \boldsymbol{V_{AN}} - \boldsymbol{V_{BN}}$$

$$\boldsymbol{V_{AB}} = 90.25\angle - 15.71°\ \text{V} - 90.25\angle - 135.71°\ \text{V}$$

$$\boldsymbol{V_{AB}} = 156\angle 14.3°\ V$$

Three-Phase Power – Example

- The complex power absorbed by the load is

$$\boldsymbol{S}_{3\phi} = 3\boldsymbol{S}_A = 3\boldsymbol{V}_{AN}\boldsymbol{I}_L^*$$

$$\boldsymbol{S}_{3\phi} = 3(90.25\angle -15.71°\,V)(80.7\angle -42.3°\,A)^*$$

$$\boldsymbol{S}_{3\phi} = 21.85\,\angle 26.6°\,kVA$$

$$\boldsymbol{S}_{3\phi} = 19.53 + j9.78\,kVA$$

- The apparent power:

$$S_{3\phi} = 21.85\,kVA$$

- Real power:

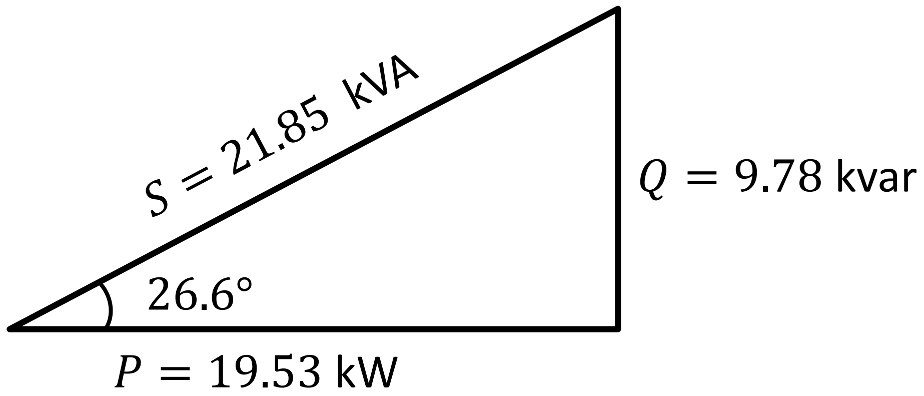
$$P = 19.53\,kW$$

- Reactive power:

$$Q = 9.78\,kvar$$

Three-Phase Power – Example

- The power triangle at the load:



- The power factor at the load is

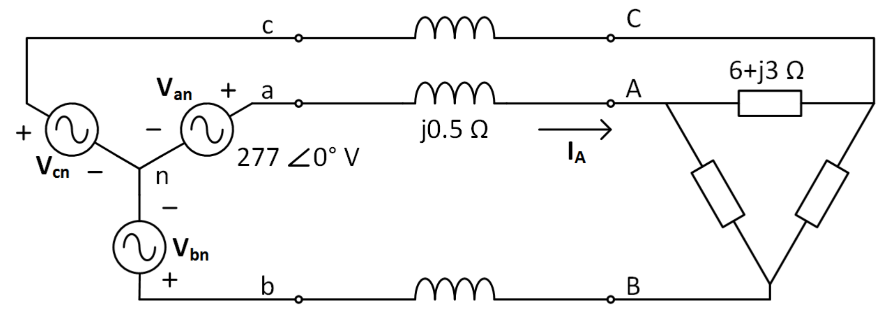
$$p.f. = \cos(26.6°) = \frac{P}{S} = \frac{19.53\ kW}{21.85\ kVA}$$

$$\boxed{p.f. = 0.89\ lagging}$$

Example Problems

Find:

- Source power
- Source power factor
- Load power
- Load power factor



Find:

- Source power
- Load power
- Power *lost* in lines