SECTION 3: POWER TRANSFORMERS

ESE 470 – Energy Distribution Systems

Power Transformers

- Transformers are used throughout the electrical grid
  - Step voltages up and down for transmission, distribution, and consumption
  - Located at power stations, substations, along distribution feeders, and at industrial customers

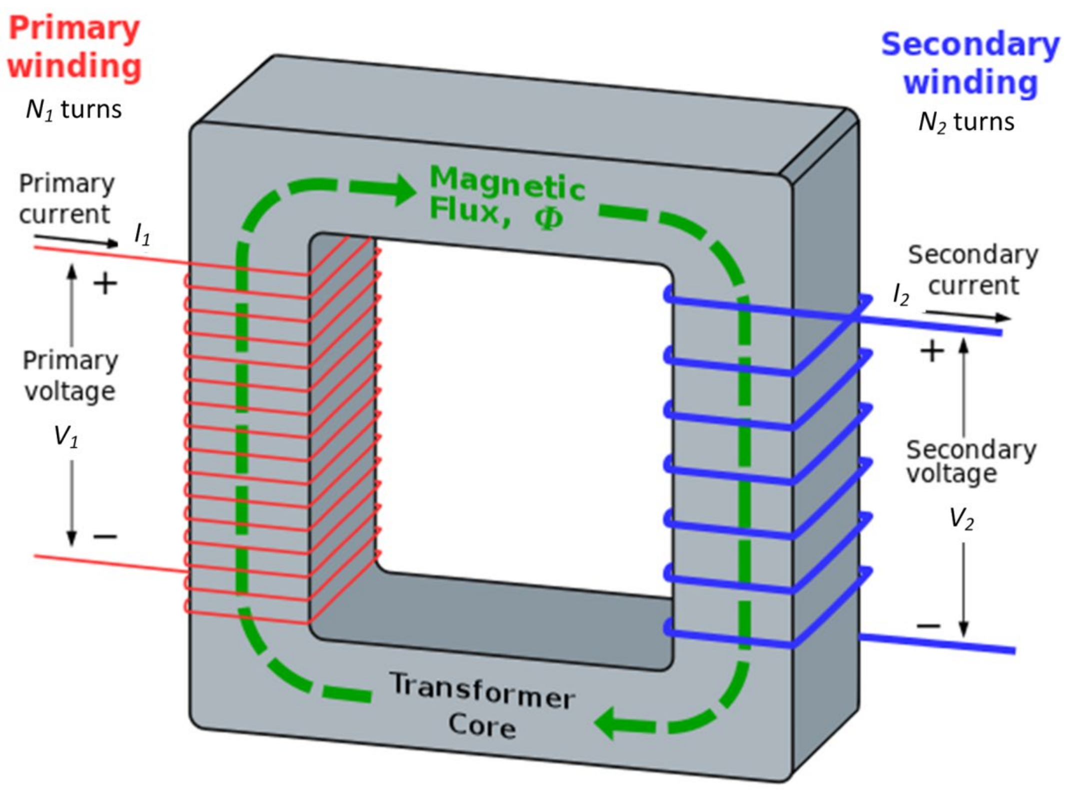
www.exportersindia.com

- We'll first review the fundamentals of ideal transformers, then look at how we can model real transformers for analysis within the electrical grid

Ideal Transformers

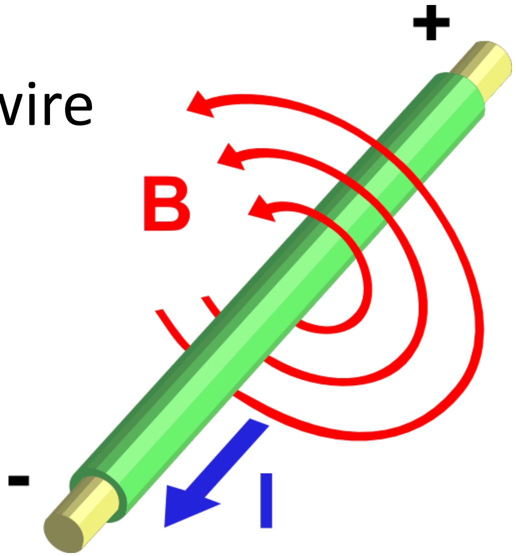
Ideal Transformers

- A single-phase transformer consists of two coils of wire wound around a magnetic core
- Used for ***stepping voltages up or down***
  - Stepped up for transmission
  - Stepped down for distribution and consumption
- To understand transformers, we must review two laws of electromagnetics
  - Ampere's law
  - Faraday's law



Ampere's Law

- Electrical current flowing through a wire generates a magnetic field encircling that wire
- Direction of field given by right-hand rule
  - Thumb points in direction of current
  - Fingers curl in direction of field



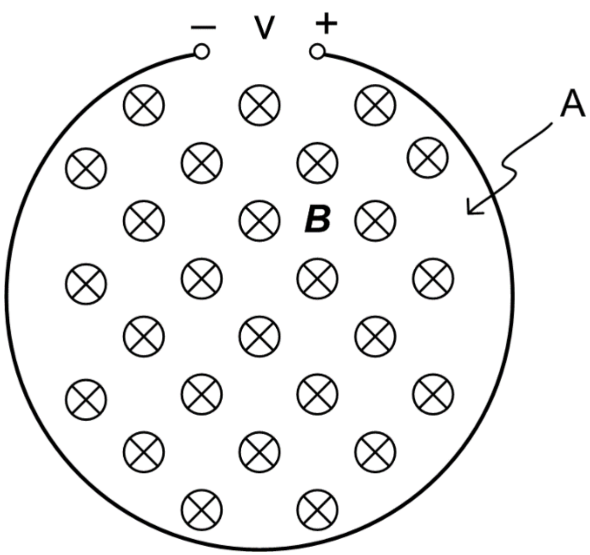
- ***Ampere's law***

$$\oint \boldsymbol{H} \cdot dl = \frac{1}{\mu} \oint \boldsymbol{B} \cdot dl = I \qquad (1)$$

  - $\boldsymbol{H}$ is the magnetic field intensity, $\boldsymbol{B}$ is the magnetic flux density, μ is permeability, and I is current
- Ampere's law says:
  - *Integrating the magnetic field around a closed contour gives the total current enclosed by that contour*

Faraday's Law

- A time-varying magnetic flux through a loop or coil of wire will produce a voltage across that loop or coil
- ***Faraday's law*** gives the voltage produced across an N-turn coil



$$v(t) = -N\frac{d\phi}{dt} \qquad (2)$$

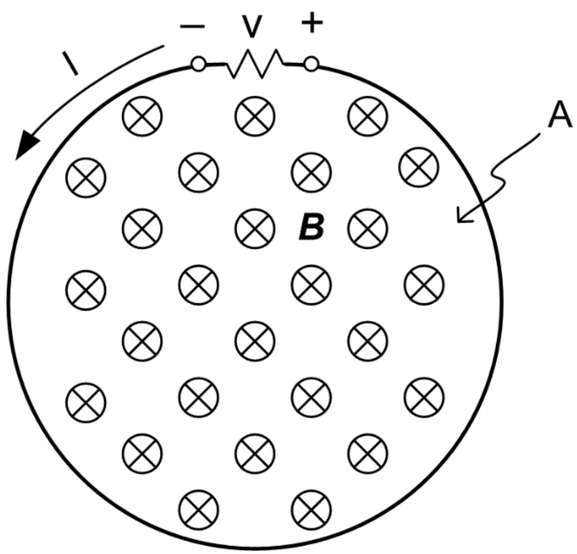
- ϕ is the magnetic flux penetrating the coil:

$$\phi = B \cdot A \qquad (3)$$

where A is the cross-sectional area of the coil

Lenz's Law

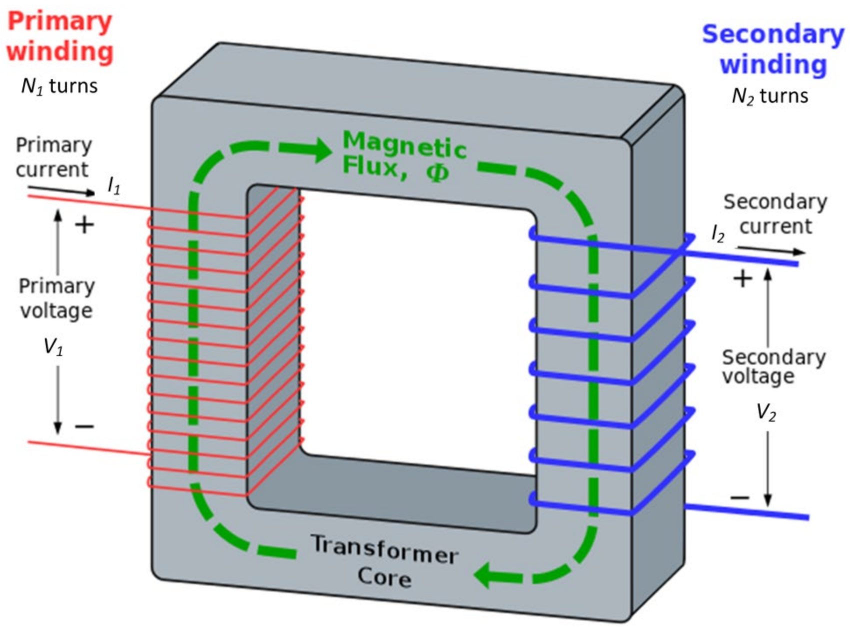
$$v(t) = -N\frac{d\phi}{dt}$$



- The negative sign in Faraday's law gives the voltage polarity
  - Close the loop with an external resistance
  - Current flows and generates a magnetic field
  - Magnetic field opposes the original change in magnetic flux
- This is ***Lenz's law***
- Often see Faraday's law written without the negative sign

Ideal Transformers

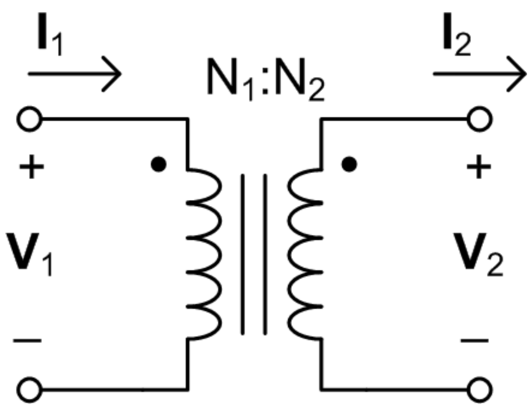
- Current flow in the primary winding generates a magnetic flux in the core
  - Ampere's law
- Flux in the core penetrates the secondary winding



- If that flux is time-varying, a voltage is induced across the secondary
  - Faraday's law

Ideal Transformers

- For ***ideal transformers***, we assume the following:
  1. Windings have zero resistance – no losses in windings
  2. Permeability of the core is infinite, $\mu = \infty$, and the reluctance of the core is zero, $R = 0$
  3. All flux is entirely confined to the core – no *leakage flux*
  4. No core losses – no *hysteresis* or *eddy currents*



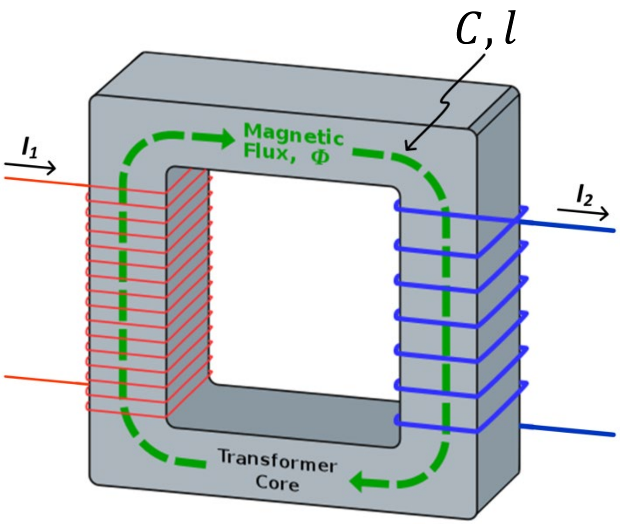
- Dots on symbol indicate polarity
  - Current enters one dotted terminal, current leaves the other
  - Positive voltage at one dotted terminal, positive voltage at the other

Transformer Current Relationships

Ideal Transformers – Current Relationships

- Evaluate Ampere's law around a closed contour, C
  - Through center of core
  - Length l
  - Everywhere tangential to the $\boldsymbol{H}$-field

$$\oint \boldsymbol{H} \cdot dl = I$$



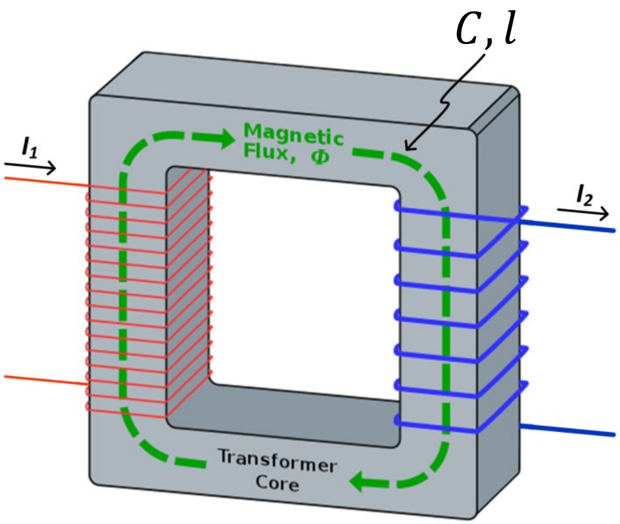
- The magnetic field is tangential to C, so

$$\oint \boldsymbol{H} \cdot dl = \boldsymbol{H}l \qquad (4)$$

- Contour encloses N_1 turns of the primary winding and N_2 turns of the secondary in the opposite direction
  - Total enclosed current:

$$\boldsymbol{I} = N_1\boldsymbol{I}_1 - N_2\boldsymbol{I}_2 \qquad (5)$$

Ideal Transformers – Current Relationships



- Ampere's law gives us

$$\boldsymbol{H}l = N_1\boldsymbol{I_1} - N_2\boldsymbol{I_2} \qquad (6)$$

- We can relate magnetic field intensity to flux density

$$\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu} \qquad (7)$$

- And, multiplying by the cross-sectional area of the core, we get the flux

$$\boldsymbol{\Phi} = \boldsymbol{B}A = \mu\boldsymbol{H}A \quad \rightarrow \quad \boldsymbol{H} = \frac{\boldsymbol{\Phi}}{\mu A} \qquad (8)$$

- Substituting (8) into (6), we have

$$\left(\frac{l}{\mu A}\right)\boldsymbol{\Phi} = N_1\boldsymbol{I_1} - N_2\boldsymbol{I_2} \qquad (9)$$

- The term in parentheses is the ***reluctance*** of the core

$$R = \left(\frac{l}{\mu A}\right) \qquad (10)$$

Ideal Transformers – Current Relationships

- Using (10) in (9) gives

$$N_1\boldsymbol{I_1} - N_2\boldsymbol{I_2} = R\boldsymbol{\Phi} \qquad (11)$$

- Recall that, for an ideal transformer, $R = 0$, so

$$N_1\boldsymbol{I_1} = N_2\boldsymbol{I_2} \qquad (12)$$

or

$$\boxed{\boldsymbol{I_1} = \frac{N_2}{N_1}\boldsymbol{I_2} \quad \text{and} \quad \boldsymbol{I_2} = \frac{N_1}{N_2}\boldsymbol{I_1}} \qquad (13)$$

- We define the ***turns ratio*** as the ratio of the number of turns on the primary winding to the number of turns on the secondary winding

$$a_t = \frac{N_1}{N_2} \qquad (14)$$

- Using the turns ratio, the current relationships are

$$\boxed{\boldsymbol{I_1} = \frac{1}{a_t}\boldsymbol{I_2} \quad \text{and} \quad \boldsymbol{I_2} = a_t\boldsymbol{I_1}} \qquad (15)$$

Transformer Voltage Relationships

Ideal Transformers – Voltage Relationships

- Faraday's law relates the voltage at each winding to the flux through that winding

$$\boldsymbol{V_1} = -N_1 \frac{d\boldsymbol{\Phi}}{\mathrm{dt}} \quad \text{and} \quad \boldsymbol{V_2} = -N_2 \frac{d\boldsymbol{\Phi}}{dt} \qquad (16, 17)$$

- Dividing the (16) by (17) gives

$$\frac{\boldsymbol{V_1}}{\boldsymbol{V_2}} = \frac{N_1}{N_2}$$

So,

$$\boxed{\boldsymbol{V_1} = a_t \boldsymbol{V_2} \quad \text{and} \quad \boldsymbol{V_2} = \frac{1}{a_t} \boldsymbol{V_1}} \qquad (18)$$

Ideal Transformers

- To summarize current and voltage relationships:

$$\boldsymbol{I_2} = a_t \boldsymbol{I_1} \tag{15}$$

$$\boldsymbol{V_2} = \frac{1}{a_t} \boldsymbol{V_1} \tag{18}$$

- ***Step-up transformer***
  - $a_t < 1,\ N_1 < N_2$
  - Voltage increases from primary to secondary
  - Current decreases
- ***Step-down transformer***
  - $a_t > 1,\ N_1 > N_2$
  - Voltage decreases from primary to secondary
  - Current increases

Transformer Power & Impedance

Ideal Transformers - Power

- The complex power entering the primary side of the transformer is

$$\boldsymbol{S_1} = \boldsymbol{V_1}\boldsymbol{I_1^*} \tag{19}$$

- And, the complex power delivered out of the secondary side is

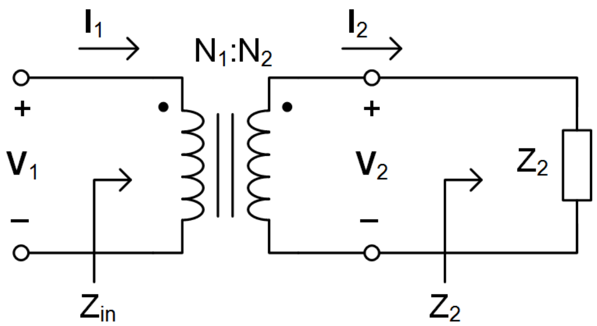
$$\boldsymbol{S_2} = \boldsymbol{V_2}\boldsymbol{I_2^*} \tag{20}$$

- Using the transformer voltage and current relationships, (19) becomes

$$\boldsymbol{S_1} = a_t\boldsymbol{V_2}\frac{1}{a_t}\boldsymbol{I_2^*} = \boldsymbol{V_2}\boldsymbol{I_2^*} = \boldsymbol{S_2}$$

- We see that power is conserved in an ideal transformer
  - As expected, since we've assumed there are no losses

Ideal Transformers - Impedance



- By definition, the impedance seen looking into the primary side of a transformer is

$$Z_{in} = \frac{\boldsymbol{V_1}}{\boldsymbol{I_1}} \quad (21)$$

- An impedance Z_2, connected to the secondary side dictates

$$Z_2 = \frac{\boldsymbol{V_2}}{\boldsymbol{I_2}} \quad (22)$$

- Using the I/V relationships, we get

$$Z_{in} = \frac{a_t V_2}{1/a_t I_2} = a_t^2 \frac{V_2}{I_2} = a_t^2 Z_2$$

$$\boxed{Z_{in} = a_t^2 Z_2 = Z_2'} \quad (23)$$

- The impedance seen looking into the primary side of a transformer is the impedance connected to the secondary side multiplied by the turns ratio squared
  - The ***reflected load impedance***

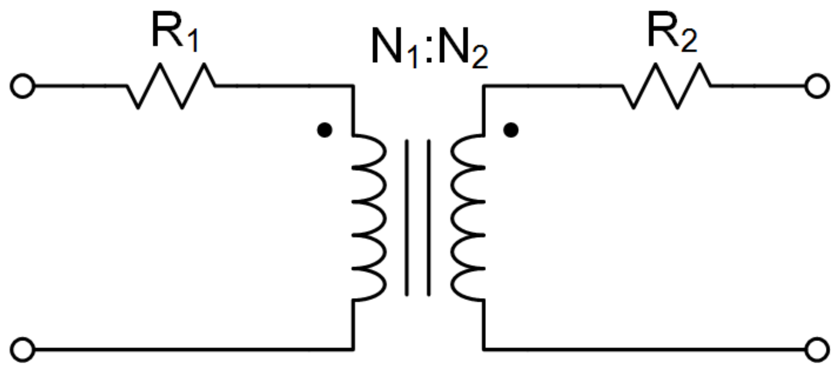
Real Transformer Models

Real Transformers

- In practice, transformers are not ideal
  - Winding losses
  - Leakage flux
  - Finite core permeability – non-zero reluctance
  - Core losses
- Need an ***equivalent circuit model*** to account for these non-idealities

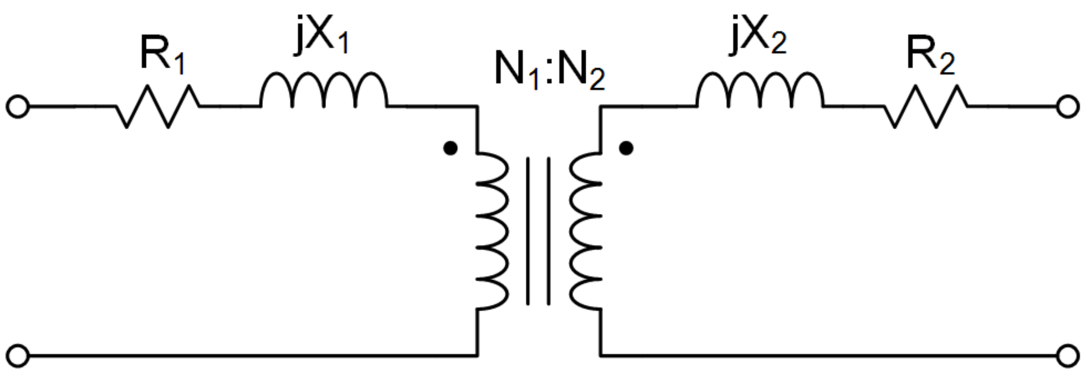
Winding Losses

- Wires of the primary and secondary windings have non-zero resistance
  - Results in losses in the windings
  - Add series resistance to each side of the ideal transformer model



Leakage Flux

- Not all flux generated by the primary side links the secondary winding
  - ***Leakage flux***
- Results in a voltage drop that leads the current by 90°
  - Winding inductance



Finite Core Permeability

- Going back to (11), we have

$$N_1 \boldsymbol{I_1} - N_2 \boldsymbol{I_2} = R\boldsymbol{\Phi} \tag{11}$$

where we'll now account for non-zero reluctance

- Faraday's law in phasor form tells us

$$\boldsymbol{V} = Nj\omega\boldsymbol{\Phi}$$

so

$$\boldsymbol{\Phi} = \frac{\boldsymbol{V}}{Nj\omega} \tag{23}$$

- Substituting (23) into (11) we have

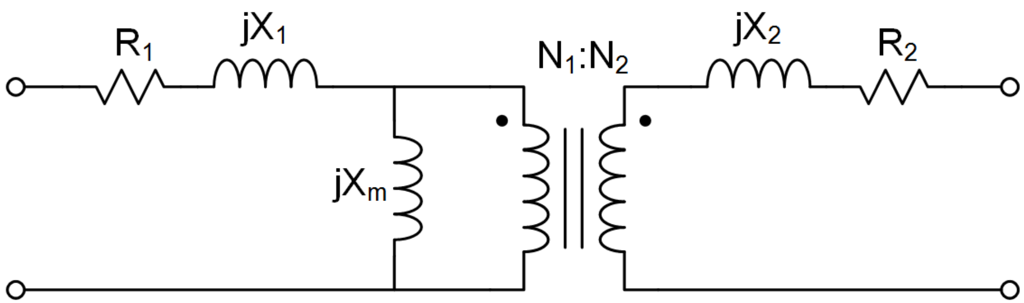
$$N_1 \boldsymbol{I_1} - N_2 \boldsymbol{I_2} = -jR\frac{\boldsymbol{V_1}}{N_1\omega}$$

$$\boldsymbol{I_1} - \frac{N_2}{N_1}\boldsymbol{I_2} = -j\frac{R}{N_1^2\omega}\boldsymbol{V_1}$$

Finite Core Permeability

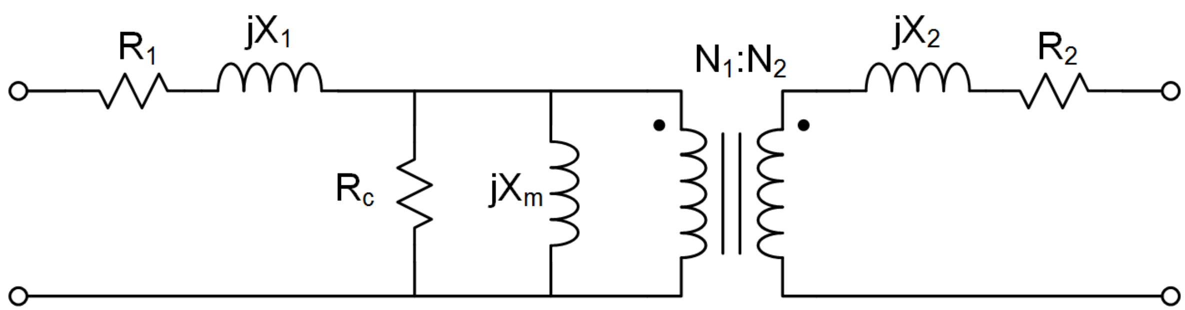
$$\boldsymbol{I_1} - \frac{N_2}{N_1}\boldsymbol{I_2} = -j\,\frac{\mathcal{R}}{N_1^2\omega}\boldsymbol{V_1} \tag{24}$$

- These are all *currents*
- The term on the right-hand side is the ***magnetizing current***
  - Due to non-zero reluctance
  - *Lags* $\boldsymbol{V_1}$ by $90°$
  - Model as a ***shunt inductor***



Core Losses

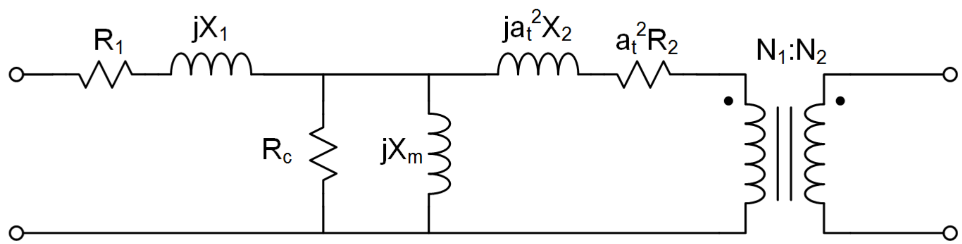
- In addition to losses in the windings, real transformers have losses in the core
  - Hysteresis in the $\boldsymbol{B/H}$ relationship
  - Eddy currents
- Modeled as ***shunt resistance*** on the primary side, R_m



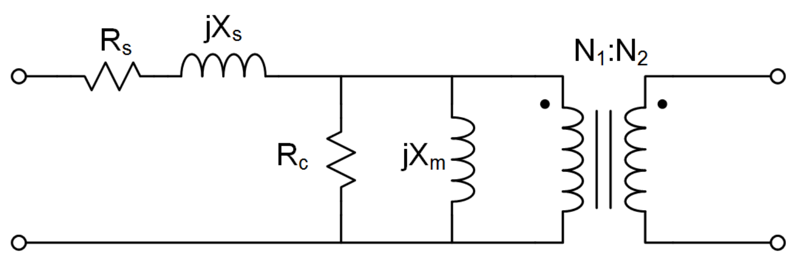
- ***Laminated cores*** are used to limit eddy current losses

Real Transformer Model

- The model can be simplified by referring the secondary-side impedances to the primary side



- We can make a further simplifying approximation by combining series impedances

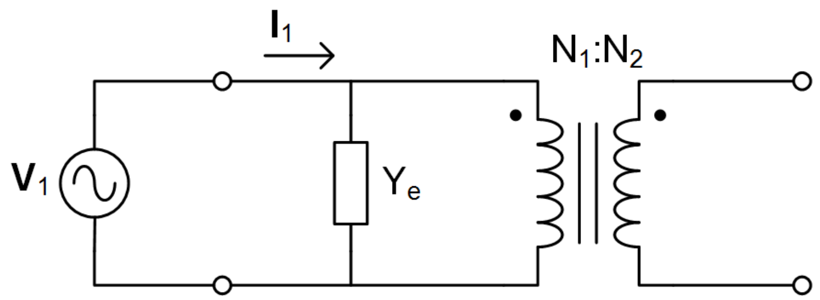


- The transformers themselves in these models are *ideal*

Identifying Model Parameters

Identifying Model Parameters

- Open-circuit test
  - Rated voltage applied to the primary
  - Secondary is open – no load
  - Measure current and power loss at the primary
  - Neglect series impedances



- Here, Y_e is the excitation current admittance

$$Y_e = \frac{1}{R_c} + \frac{1}{jX_m} = G_c - jB_m$$

Open-Circuit Test

- Measuring the primary-side voltage and current allows for calculation of the *magnitude* of the excitation admittance

$$|Y_e| = \frac{I_{1,oc}}{V_{1,oc}} = |G_c - jB_m|$$

- Measured power loss allows for calculation of G_c

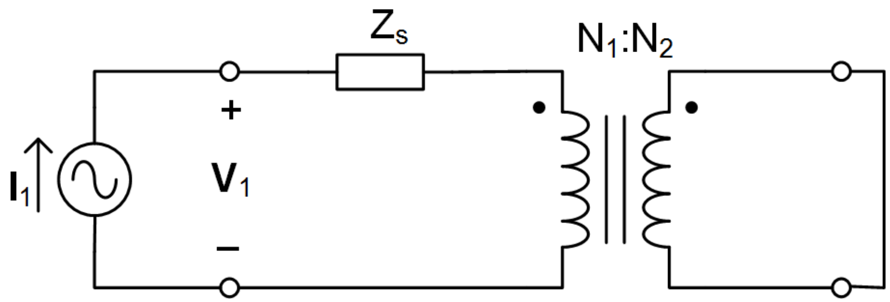
$$G_c = \frac{P_{oc}}{V_{1,oc}^2}$$

- Finally, calculate B_m from $|Y_e|$ and G_c

$$B_m = \sqrt{|Y_e|^2 - G_c^2}$$

Short-Circuit Test

- Short-circuit test
  - Rated *current* applied at the primary side
  - Secondary side is shorted
  - Measure voltage and power loss at the primary side
  - Neglect shunt admittance



- Here, Z_s is the equivalent *series* impedance

$$Z_s = R_s + jX_s$$

Short-Circuit Test

- Determine $|Z_s|$ from the measured primary-side voltage and current

$$|Z_s| = \frac{V_{1,sc}}{I_{1,sc}} = |R_s + jX_s|$$

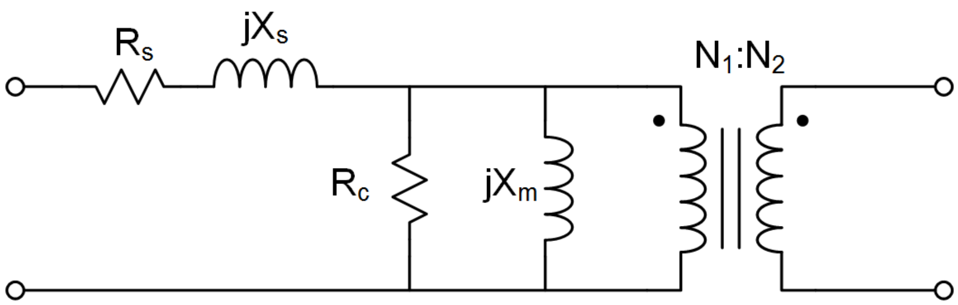
- Use the power loss measurement to determine R_s

$$R_s = \frac{P_{sc}}{I_{1,sc}^2}$$

- Finally, calculate X_s from $|Z_s|$ and R_s

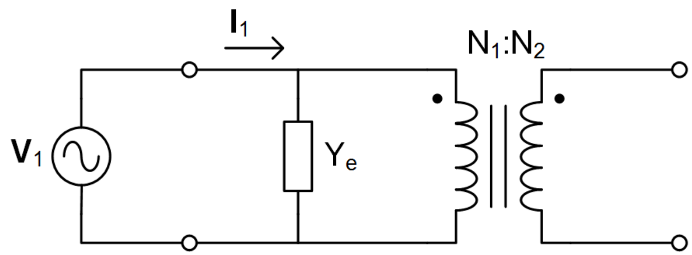
$$X_s = \sqrt{|Z_s|^2 - R_s^2}$$

Transformer Model – Example



- A single-phase, 100 kVA, 480/120 V transformer is subjected to short-circuit and open-circuit tests to determine model parameters
- The results:
  - Open circuit: $I_{1,oc} = 0.05\ A\ ,\quad P_{oc} = 0.1\ W$
  - Short circuit: $V_{1,sc} = 80\ V\ ,\quad P_{sc} = 10\ kW$
- Determine model parameters: R_s, X_s, R_c, and X_m

Transformer Model – Example



- From the open-circuit test:

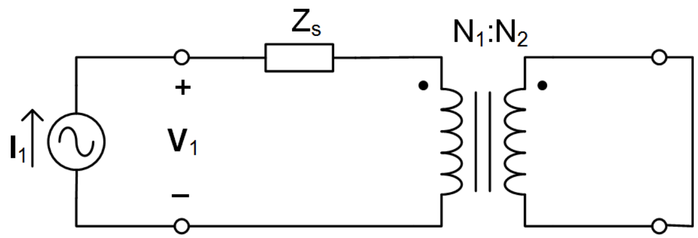
$$|Y_e| = \frac{I_{1,oc}}{V_{1,oc}} = \frac{0.05\ A}{480\ V} = 104\ \mu S$$

$$G_c = \frac{P_{oc}}{V_{1,oc}^2} = \frac{0.1\ W}{(480\ V)^2} = 434\ nS$$

$$B_m = \sqrt{|Y_e|^2 - G_c^2} = 104\ \mu S$$

$$R_c = \frac{1}{G_c} = 2.3\ M\Omega$$ and $$X_m = \frac{1}{B_m} = 9.62\ k\Omega$$

Transformer Model – Example



- From the short-circuit test:

$$|Z_s| = \frac{V_{1,sc}}{I_{1,sc}} = \frac{V_{1,sc}}{I_{1,rated}}$$

- Here, the applied current is the *rated current*
  - Determine from the nameplate power rating

$$I_{1,rated} = \frac{S_{rated}}{V_{1,rated}} = \frac{100\ kVA}{480\ V} = 208.3\ A$$

- The magnitude of the series impedance is

$$|Z_s| = \frac{80\ V}{208.3\ A} = 384\ m\Omega$$

Transformer Model – Example

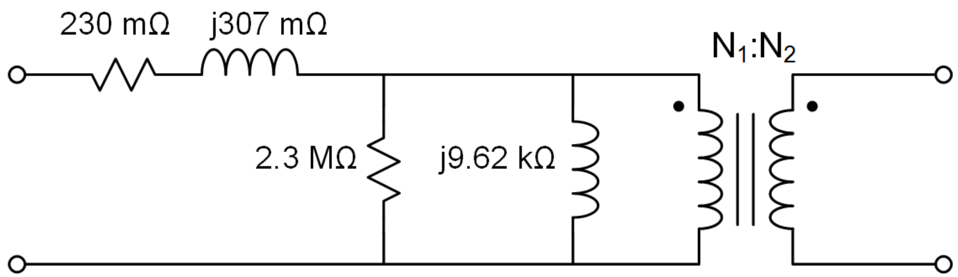
- Using the measured power and primary-side current, we can calculate series resistance

$$R_s = \frac{P_{1,sc}}{I_{1,sc}^2} = \frac{10\ kW}{(208\ A)^2} = 230\ m\Omega$$

- The equivalent series reactance is

$$X_s = \sqrt{|Z_s|^2 - R_s^2} = 307\ m\Omega$$

- The equivalent circuit model is:



The Per-Unit System

The Per-Unit System

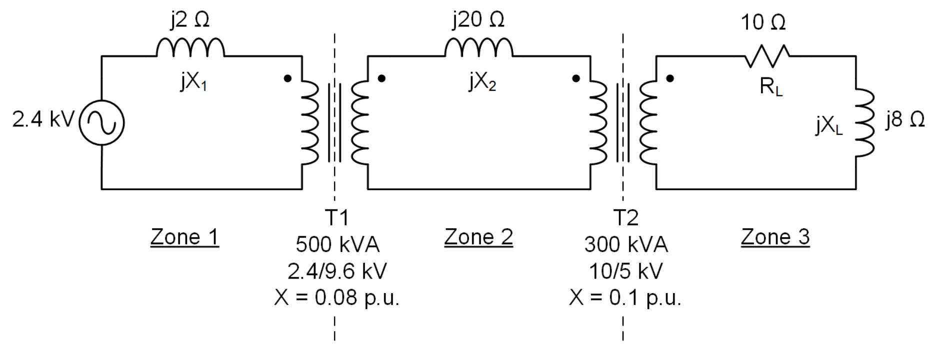
- Power systems contain many, many transformers
- System analysis would require always referring impedances from one side of transformers to the other
  - Tedious and error prone
- Instead, we use a system of normalized voltages, currents, power, and impedance
  - The ***per-unit system***
- In per-unit, all quantities are normalized by base values – for example:

$$V_{pu} = \frac{V}{V_{base}} \tag{1}$$

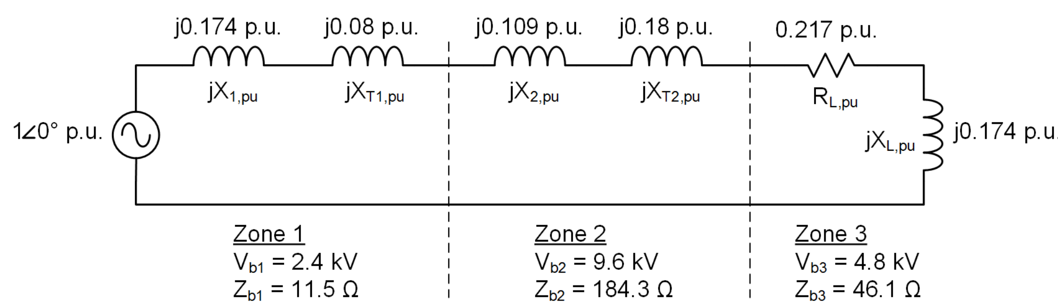
- The selected base values are all related by transformer turns ratios
  - ***Transformers are eliminated*** from the per-unit schematic

The Per-Unit System - Preview

- Initial schematic:



- Per-unit schematic:
  - Transformers have been eliminated
  - All values are in per-unit (p.u.)



- To understand how to interpret per-unit schematics, we'll see how to convert both single- and three-phase circuits to per-unit

Per-Unit Conversion – Single-Phase

- To convert a single-phase circuit to per-unit:
  1. Re-draw the circuit
     - Eliminate all transformers
     - Clearly delineate voltage zones, as defined by the transformers
     - Do not label any numerical voltage, current, impedance, or power values
  2. Select a single power base value, $S_b = P_b = Q_b$
  3. Select a voltage base, V_b, for each voltage level
     - Must be related by transformer turns ratios
  4. Calculate the impedance base at each voltage level

$$Z_b = \frac{V_b^2}{S_b} = R_b = X_b \qquad (2)$$

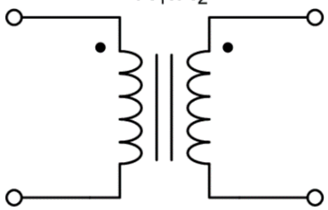
  5. Calculate the current base at each voltage level

$$I_b = \frac{V_b}{Z_b} = \frac{S_b}{V_b} \qquad (3)$$

  6. Convert all actual values to per-unit values

Transformer Base Impedance

- Transformer impedances are typically specified in per-unit or as percent of base impedance values
- For example:



- Here, the reactance accounting for leakage flux (series inductance) is specified
  - 10% of base impedance value for the transformer
- Base impedance given by nameplate voltage and power ratings
- At the primary side

$$Z_{b1} = \frac{V_{b1}^2}{S_b} = \frac{(240\ V)^2}{30\ kVA} = 1.92\ \Omega$$

Transformer Impedance

- Per-unit reactance is calculated from actual reactance as

$$X_{pu} = \frac{X}{Z_b}$$

- So the actual leakage reactance is

$$X = X_{pu} \cdot Z_b = 0.1 \cdot 1.92\ \Omega = 192\ m\Omega$$

- The per-unit leakage reactance is specified on the base dictated by the transformer's nameplate ratings
  - System base voltages and impedances at primary and secondary side may differ from the intrinsic transformer base
  - Per-unit reactance can be ***converted to a new base*** when necessary

Per-Unit Base Conversion

- Convert an old per-unit reactance, $X_{pu,old}$, to a per-unit reactance on a new base, $X_{pu,new}$

$$X_{pu,new} = \frac{X}{Z_{b,new}} = X\frac{S_{b,new}}{V_{b,new}^2} \tag{4}$$

- We know we can express the actual reactance as

$$X = X_{pu,old} \cdot Z_{b,old} = X_{pu,old}\left(\frac{V_{b,old}^2}{S_{b,old}}\right) \tag{5}$$

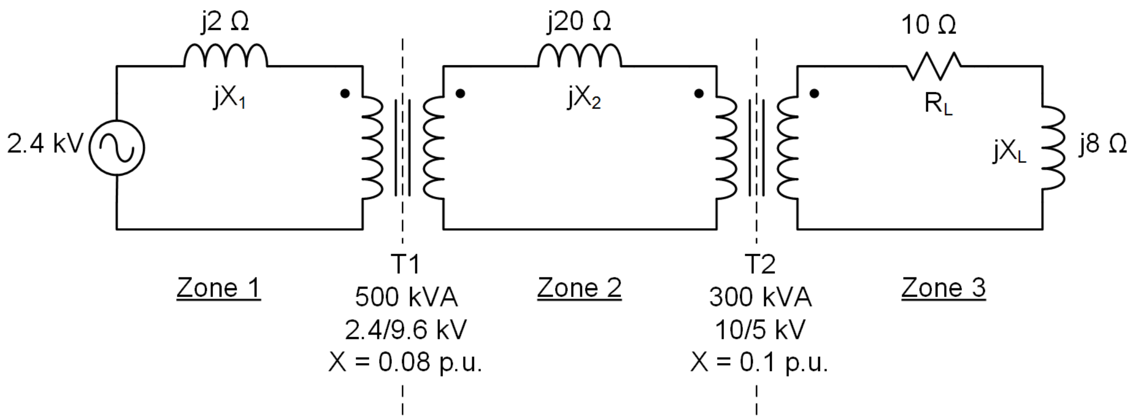
- Substituting (5) into (4)

$$X_{pu,new} = X_{pu,old}\left(\frac{V_{b,old}^2}{S_{b,old}}\right)\left(\frac{S_{b,new}}{V_{b,new}^2}\right)$$

- This provides a general formula for converting from one base to another

$$\boxed{Z_{pu,new} = Z_{pu,old}\left(\frac{V_{b,old}^2}{S_{b,old}}\right)\left(\frac{S_{b,new}}{V_{b,new}^2}\right)} \tag{6}$$

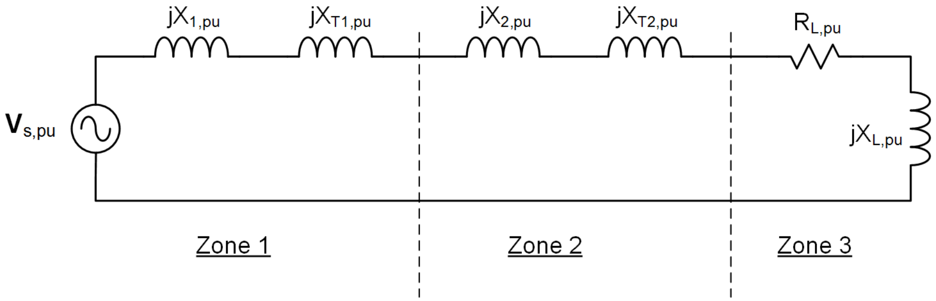
Single-Phase Per-Unit Conversion – Example



- Convert the circuit above to a per-unit circuit
  - Note that transformer reactances are already specified as per-unit values
  - Three zones – three voltage bases

Single-Phase Per-Unit Conversion – Example

- First, re-draw the schematic in per-unit form



  - We'll fill in the various per-unit values as we calculate them

- Next, arbitrarily select a single power base for the network

$$S_b = 500\ kVA$$

Single-Phase Per-Unit Conversion – Example

- Next, select a voltage base and calculate impedance and current base values for each voltage level
  - Zone 1:

$$V_{b1} = 2.4\ kV$$

$$Z_{b1} = \frac{V_{b1}^2}{S_b} = \frac{(2.4\ kV)^2}{500\ kVA} = 11.5\ \Omega$$

$$I_{b1} = \frac{V_{b1}}{Z_{b1}} = \frac{2.4\ kV}{11.5\ \Omega} = 208.3\ A$$

  - Zone 2:

$$V_{b2} = 9.6\ kV$$

$$Z_{b2} = \frac{V_{b2}^2}{S_b} = \frac{(9.6\ kV)^2}{500\ kVA} = 184.3\ \Omega$$

$$I_{b2} = \frac{V_{b2}}{Z_{b2}} = \frac{9.6\ kV}{184.3\ \Omega} = 52.1\ A$$

Single-Phase Per-Unit Conversion – Example

- Zone 3:

$$V_{b3} = 4.8\ kV$$

$$Z_{b3} = \frac{V_{b3}^2}{S_b} = \frac{(4.8\ kV)^2}{500\ kVA} = 46.1\ \Omega$$

$$I_{b3} = \frac{V_{b3}}{Z_{b3}} = \frac{4.8\ kV}{46.1\ \Omega} = 104.1\ A$$

- Next, calculate the per-unit circuit impedances, starting with zone 1:

$$X_{1,pu} = \frac{X_1}{Z_{b1}} = \frac{2\ \Omega}{11.5\ \Omega} = 0.174\ p.u.$$

- For transformer T1, the base voltages are the same as the nameplate rating for the transformer
  - Per-unit leakage reactance is the same as the nameplate value

$$X_{T1,pu} = 0.08\ p.u.$$

Single-Phase Per-Unit Conversion – Example

- In zone 2:

$$X_{2,pu} = \frac{20\ \Omega}{184.3\ \Omega} = 0.109\ p.u.$$

- For transformer T2, the base voltages differ from the rated nameplate values
  - $X_{T2,pu}$ must be converted to the system base

$$X_{T2,pu} = X_{T2,pu,old}\left(\frac{V_{b,old}^2}{S_{b,old}}\right)\left(\frac{S_b}{V_b^2}\right)$$

$$X_{T2,pu} = 0.1\ p.u.\left(\frac{(10\ kV)^2}{300\ kVA}\right)\left(\frac{500\ kVA}{(9.6\ kV)^2}\right)$$

$$X_{T2,pu} = 0.18\ p.u.$$

Single-Phase Per-Unit Conversion – Example

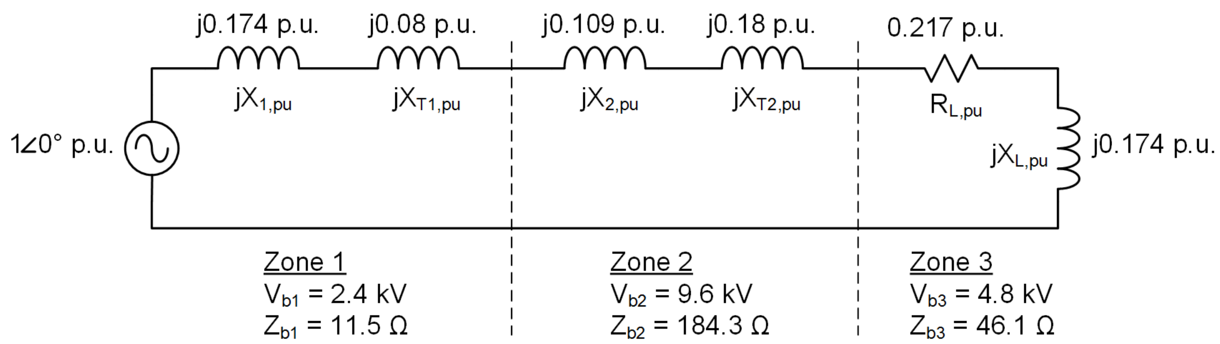
- In zone 3, at the load:

$$R_{L,pu} = \frac{10\ \Omega}{46.1\ \Omega} = 0.217\ p.u.$$

and

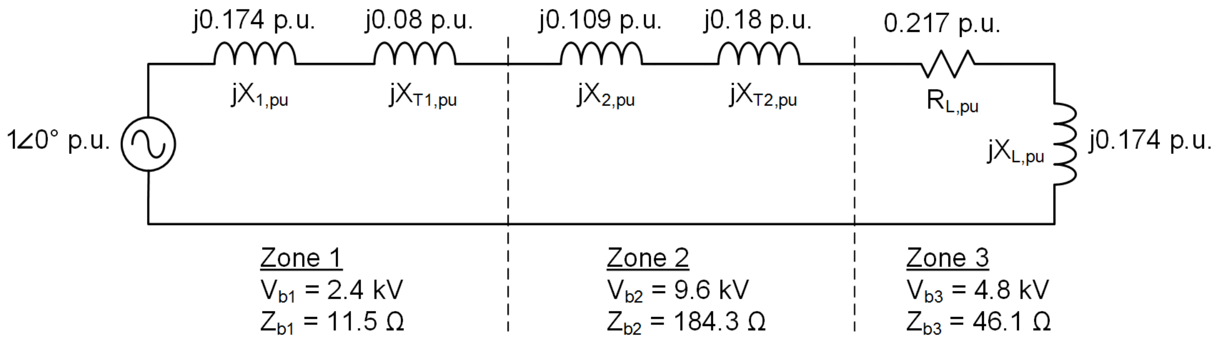
$$X_{L,pu} = \frac{8\ \Omega}{46.1\ \Omega} = 0.174\ p.u.$$

- The resulting per-unit circuit:



- Note that there are no ideal transformers in the per-unit circuit

Per-Unit Circuit Analysis – Example



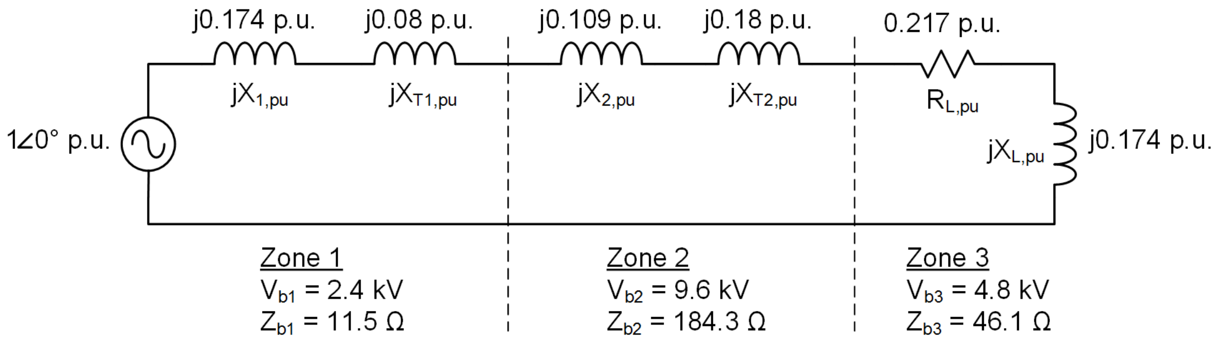
- For the circuit from the previous example, determine the per-unit and actual load current
- Very simple when using the per-unit circuit

$$\boldsymbol{I}_{L,pu} = \frac{V_{s,pu}}{R_L + j\left(X_{1,pu} + X_{T1,pu} + X_{2,pu} + X_{T2,pu} + X_{L,pu}\right)}$$

$$\boldsymbol{I}_{L,pu} = \frac{1\angle 0^\circ}{0.217 + j0.717\ p.u.} = 1.33\angle -73.2^\circ\ p.u.$$

$$\boxed{\boldsymbol{I}_{L,pu} = 1.33\angle -73.2^\circ\ p.u.}$$

Per-Unit Circuit Analysis – Example



- The current base at the load is

$$\boldsymbol{I}_{b3} = \frac{V_{b3}}{Z_{b3}} = \frac{4.8\ kV}{46.1\ \Omega} = 104.1\text{ A}$$

- So, the actual load current is

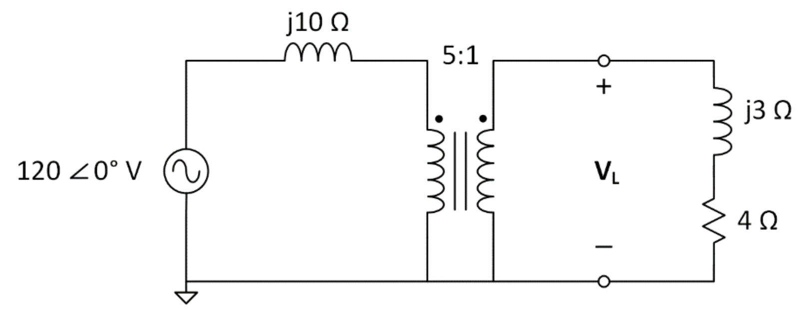
$$\boldsymbol{I}_L = \boldsymbol{I}_{L,pu} \cdot I_{b3} = 104.1\ A \cdot 1.33\angle -73.2°$$

$$\boxed{\boldsymbol{I}_L = 139\angle -73.2°\ A}$$

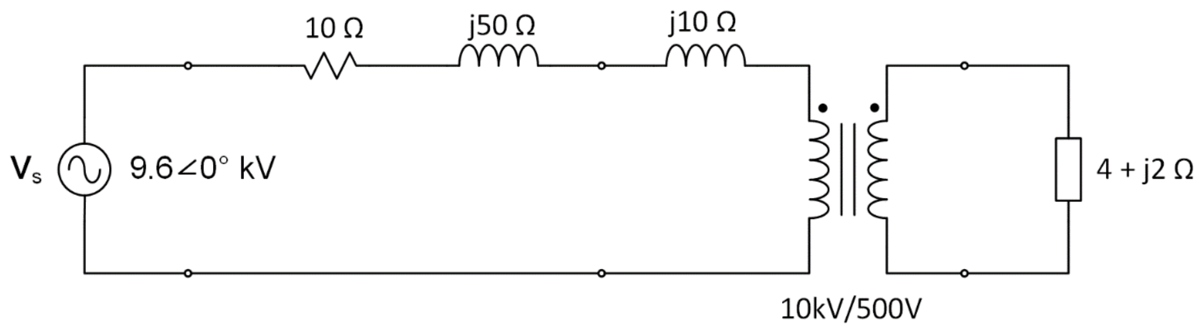
Example Problems

Find:

- Load voltage
- Load power



Draw a per-unit schematic for the following circuit, and determine the load voltage and power in both per-unit and actual values. Use a power base of 200 kVA.



Three-Phase Per-Unit System

Three-Phase Per-Unit Circuits

- Balanced three-phase circuits can be represented in per-unit as well
  - Use a 3-ϕ power base
  - Use line-to-line voltages

- Procedure for conversion to per-unit:
  1. Re-draw the circuit
     - Eliminate all transformers
     - Clearly delineate voltage zones, as defined by the transformers
     - Do not label any numerical voltage, current, impedance, or power values
  2. Select a single 3-ϕ power base, $S_{b3\phi}$
  3. Select line-to-line voltage bases, $V_{b,LL}$ for each voltage level
     - Must be related by transformer turns ratios

Three-Phase Per-Unit Circuits

4. Calculate the impedance base at each voltage level

$$Z_b = \frac{V_{b,LL}^2}{S_{b3\phi}} = \frac{\left(\sqrt{3}V_{b,LN}\right)^2}{3S_{b1\phi}} = \frac{V_{b,LN}^2}{S_{b1\phi}} \qquad (7)$$

- Note that this is identical to (2), the single-phase impedance base

5. Calculate the current base at each voltage level

$$I_b = \frac{S_{b3\phi}}{\sqrt{3}V_{b,LL}} = \frac{3S_{b1\phi}}{\sqrt{3}\sqrt{3}V_{b,LN}} = \frac{S_{b1\phi}}{V_{b,LN}}$$

or (8)

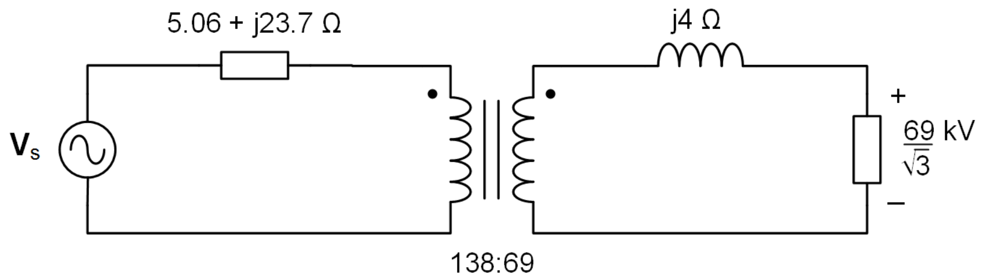
$$I_b = \frac{V_{b,LL}}{\sqrt{3}Z_b} = \frac{V_{b,LN}}{Z_b}$$

- Again, identical to the single-phase impedance base, (3)

6. Convert all actual values to per-unit values

Three-Phase Per-Unit Analysis - Example

- The following is a per-phase schematic for a balanced three-phase circuit



- The power delivered to the load is

$$\boldsymbol{S}_L = 100 + j50\ MVA$$

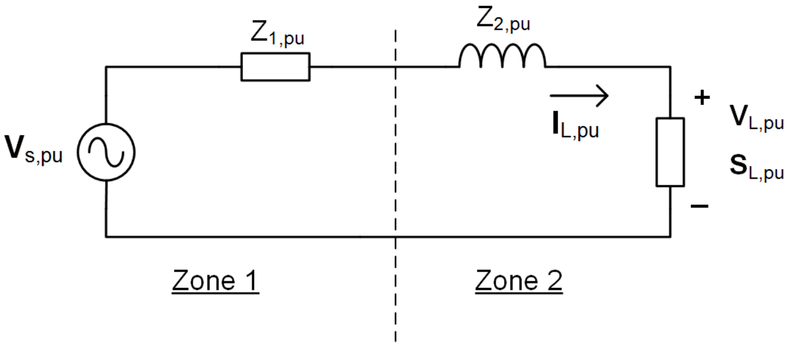
- The line-to-line voltage at the load is

$$\boldsymbol{V}_{L,LL} = 69\angle 0°\ kV$$

- Determine:
  - Supply current
  - Complex power delivered from the supply

Three-Phase Per-Unit Analysis - Example

- We'll first convert the circuit to a per-unit circuit
- Begin by drawing the per-unit circuit
  - Eliminate ideal transformers
  - Don't yet know the per-unit values



- Next, arbitrarily select a ***three-phase power base***

$$S_{b3\phi} = 150\ MVA$$

Three-Phase Per-Unit Analysis - Example

- Select ***line-to-line voltage bases*** for each voltage level
  - At the source: $V_{b,LL1} = 138\ kV$
  - At the load: $V_{b,LL2} = 69\ kV$

- Use the power and voltage bases to calculate the ***impedance base*** at each voltage level

$$Z_{b1} = \frac{V_{b,LL1}^2}{S_{b3\phi}} = \frac{(138\ kV)^2}{150\ MVA} = 126.96\ \Omega$$

$$Z_{b2} = \frac{V_{b,LL2}^2}{S_{b3\phi}} = \frac{(69\ kV)^2}{150\ MVA} = 31.74\ \Omega$$

Three-Phase Per-Unit Analysis - Example

- Calculate the current base at each voltage level

$$I_{b1} = \frac{V_{b,LL1}}{\sqrt{3}Z_{b1}} = \frac{138\ kV}{\sqrt{3}\cdot 126.96\ \Omega} = 627.6\ A$$

$$I_{b2} = \frac{V_{b,LL2}}{\sqrt{3}Z_{b2}} = \frac{69\ kV}{\sqrt{3}\cdot 31.74\ \Omega} = 1255\ A$$

- Next, convert the circuit impedances to per-unit
  - At the source:

$$Z_{1,pu} = \frac{5.06 + j23.7\ \Omega}{126.96\ \Omega} = 0.19\angle 77.9°\ p.u.$$

  - At the load side:

$$Z_{2,pu} = \frac{4\angle 90°\ \Omega}{31.74\ \Omega} = 0.126\angle 90°\ p.u.$$

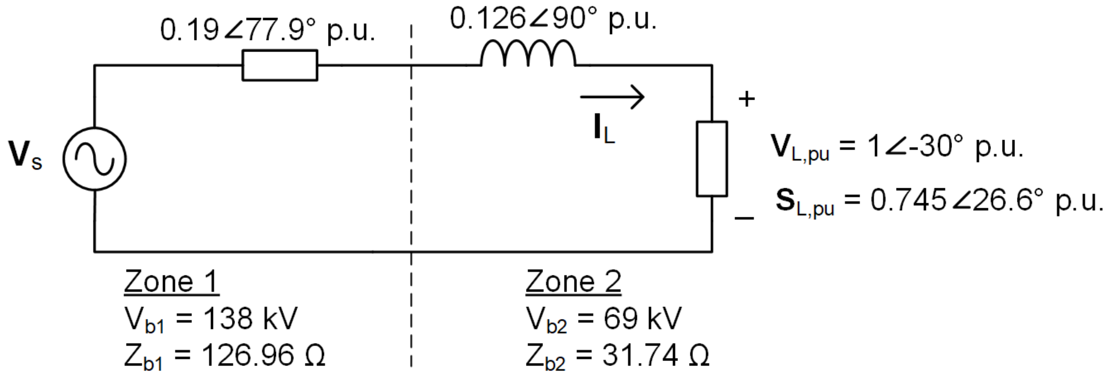
Three-Phase Per-Unit Analysis - Example

- The per-unit, three-phase power to the load is

$$\boldsymbol{S}_{L,pu} = \frac{100 + j50\ MVA}{S_{b3\phi}} = \frac{111.8\angle 26.6°\ MVA}{150\ MVA}$$

$$\boldsymbol{S}_{L,pu} = 0.745\angle 26.6°\ p.u.$$

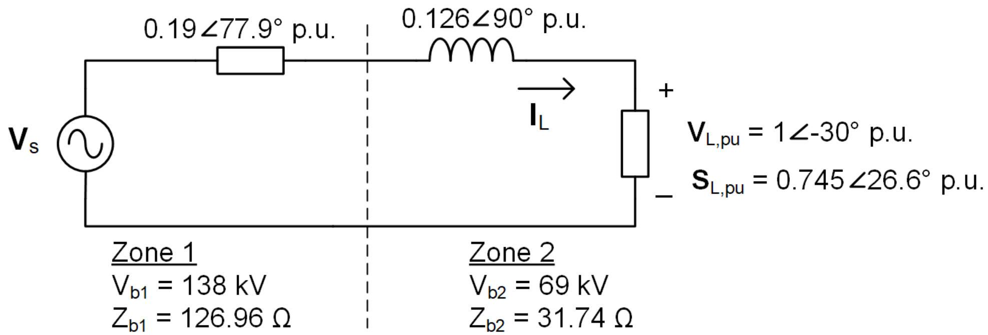
- We can now complete the per-unit, per-phase circuit:



- Note that that, even though we are using a L-L voltage base, voltages in the per-phase circuit are still L-N
    - Load voltage phase agrees with line-to-line 0° phase reference

Three-Phase Per-Unit Analysis - Example

- Now, use the per-unit circuit to determine the per-unit load current



$$\boldsymbol{I}_{L,pu} = \left(\frac{\boldsymbol{S}_{L,pu}}{\boldsymbol{V}_{L,pu}}\right)^* = \left(\frac{0.745\angle 26.6°}{1\angle -30°}\right)^*$$

$$\boldsymbol{I}_{L,pu} = 0.745\angle -56.6°\, p.u. = \boldsymbol{I}_{s,pu}$$

- The source voltage is

$$\boldsymbol{V}_{s,pu} = 1\angle -30° + \boldsymbol{I}_{L,pu}(0.19\angle 77.9° + 0.126\angle 90°)$$

$$\boldsymbol{V}_{s,pu} = 1.147\angle -20.24°$$

Three-Phase Per-Unit Analysis - Example

- The per-unit source power is

$$\boldsymbol{S}_{s,pu} = \boldsymbol{V}_{s,pu}\boldsymbol{I}_{s,pu}^{*} = 1.147\angle -20.24° \cdot 0.745\angle 56.6°$$

$$\boldsymbol{S}_{s,pu} = 0.855\angle 36.4°$$

- We can now use the per-unit values to determine actual quantities
- The actual supply voltage is (note the $+30°$ phase shift due to conversion to L-L)

$$\boldsymbol{V}_{s} = \boldsymbol{V}_{s,pu} \cdot V_{b1} \cdot 1\angle 30° = 1.147\angle -20.24° \cdot 138\ kV \cdot 1\angle 30°$$

$$\boxed{\boldsymbol{V}_{s} = 158.3\angle 9.76\ kV}$$

- The actual supply current is

$$\boldsymbol{I}_{s} = \boldsymbol{I}_{s,pu} \cdot I_{b1} = 0.745\angle -26.6° \cdot 627.6\ A$$

$$\boxed{\boldsymbol{I}_{s} = 467.6\angle -26.6°\ A}$$

- And, the actual supply power is

$$S_{s} = S_{s,pu} \cdot S_{b3\phi} = 0.855\angle 36.4° \cdot 150\ MVA$$

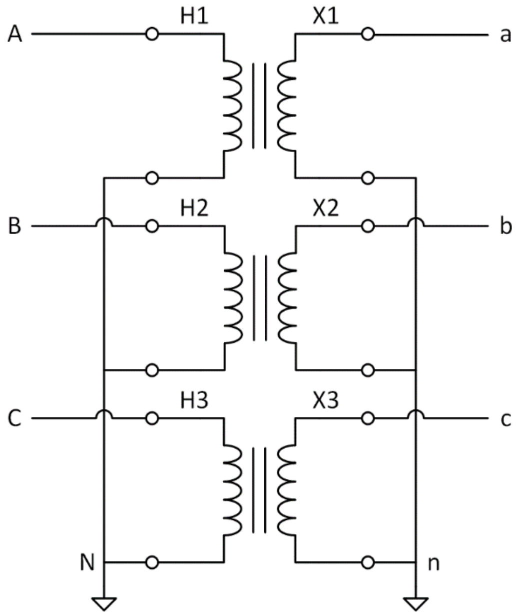
$$\boxed{S_{s} = 128.18\angle 36.4°\ MVA = 103.2 + j76.1\ MVA}$$

3-ϕ Transformers – Y/Y, Δ/Δ

Three-Phase Transformers

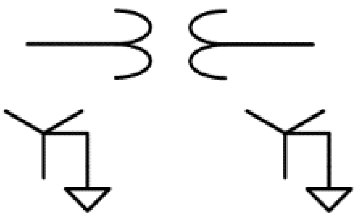
- Three-phase transformers are constructed from banks of single-phase transformers
- Primary and secondary sides can each be connected as a Y or a Δ configuration
- Four Possible configurations:
  - Y-Y
  - Δ-Δ
  - Y-Δ
  - Δ-Y

Y-Y Transformer

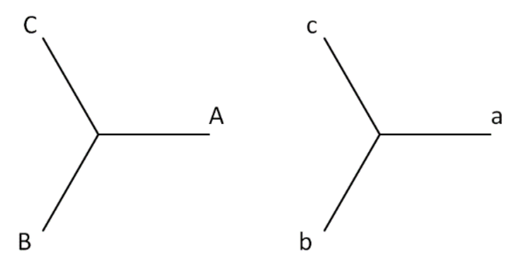


- Note nomenclature:
  - High-voltage terminals: H_1, H_2, H_3
  - Low-voltage terminals: X_1, X_2, X_3
  - Terminal labels serve as dots
  - High-voltage phases: A, B, C
  - Low-voltage phases: a, b, c

One-line diagram:

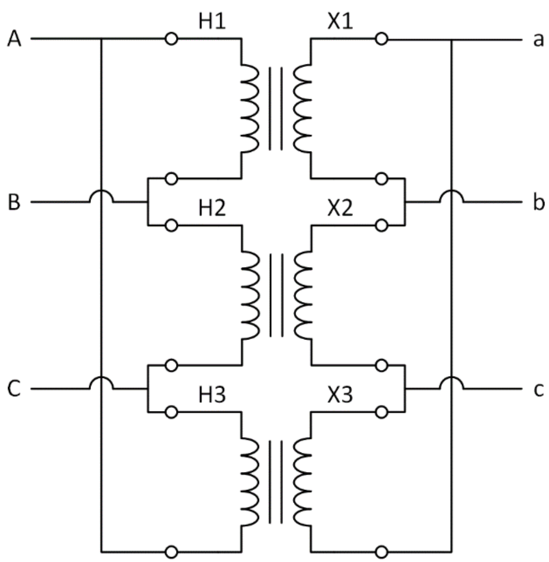


Phasor diagram:



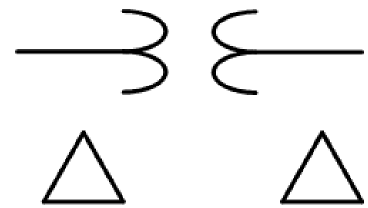
- No phase shift through the transformer

Δ-Δ Transformer

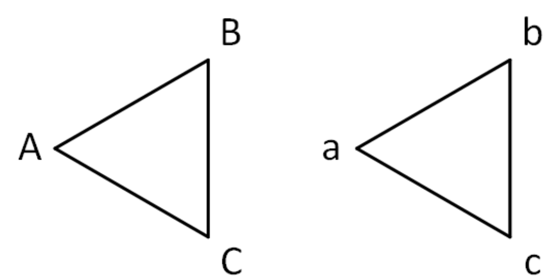


- Voltages across windings are line-to-line voltages
- No phase shift through the transformer

One-line diagram:

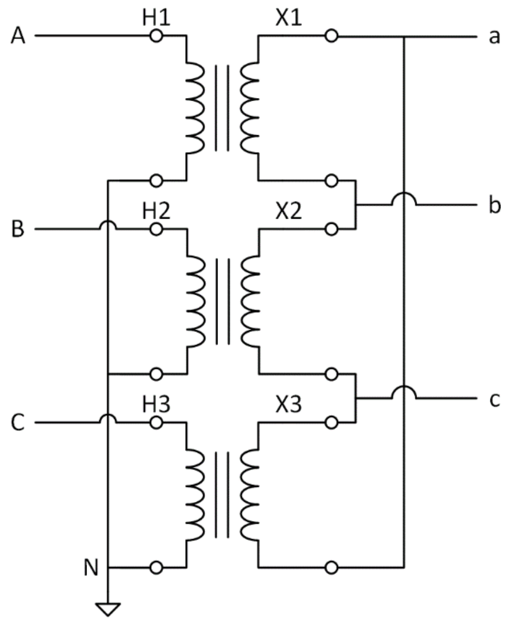


Phasor diagram:

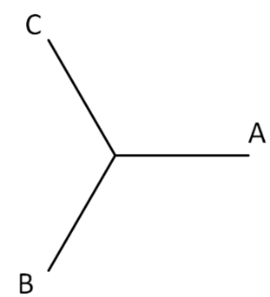


3-ϕ Transformers – Y/Δ, Δ/Y

Y-Δ (or Δ-Y) Transformer



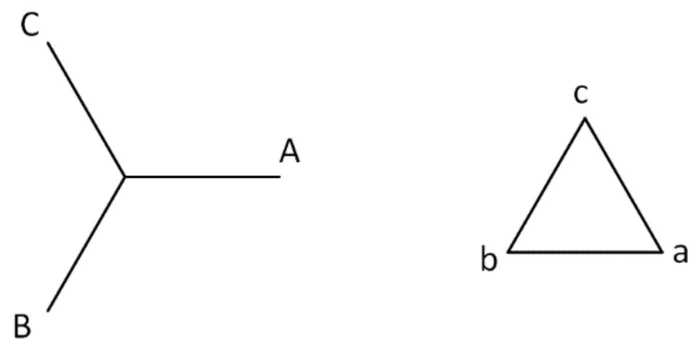
Input Phasor diagram:



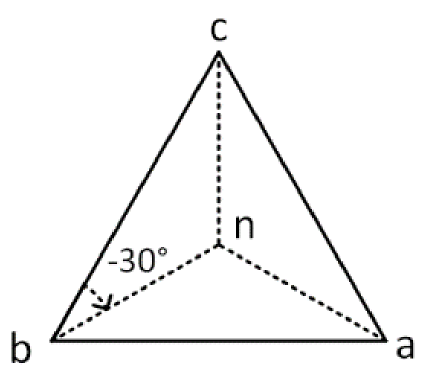
- Here the phase relationships are a bit more complicated
- Each primary voltage is in phase with its corresponding secondary voltage
  - $\boldsymbol{V}_{AN}$ is in phase with $\boldsymbol{V}_{ab}$
  - $\boldsymbol{V}_{BN}$ is in phase with $\boldsymbol{V}_{bc}$
  - $\boldsymbol{V}_{CN}$ is in phase with $\boldsymbol{V}_{ca}$

Y-Δ (Δ-Y) Transformer

- The Y- and Δ-side phasor diagrams:



- Note that there is a phase shift of $-30°$ between line-to-neutral voltages on the Y side and line-to-neutral voltages on the Δ side



Y-Δ (Δ-Y) Transformer – Voltage Relationship

- Transformer turns ratio, a_t, relates line-to-neutral voltage on the Y side to line-to-line voltage on the Δ side

$$\boldsymbol{V}_{ab} = \frac{1}{a_t}\boldsymbol{V}_{AN}$$

- On the Δ side

$$\boldsymbol{V}_{ab} = \sqrt{3}\boldsymbol{V}_{an}\angle 30°$$

so

$$\boxed{\boldsymbol{V}_{an} = \frac{1}{\sqrt{3}a_t}\boldsymbol{V}_{AN}\angle -30°}$$

- The turns ratio is effectively increased by a factor of $\sqrt{3}$ and there is a phase shift of $-30°$

Y-Δ (Δ-Y) Transformer – Current Relationship

- For current, we have

$$\boldsymbol{I}_{ab} = a_t \boldsymbol{I}_A$$

- On the Δ side, we know

$$\boldsymbol{I}_a = \sqrt{3}\boldsymbol{I}_{ab}\angle -30°$$

or

$$\boldsymbol{I}_{ab} = \frac{\boldsymbol{I}_a}{\sqrt{3}}\angle 30°$$

- So the current relationship is

$$\boxed{\boldsymbol{I}_a = \sqrt{3}a_t \boldsymbol{I}_A\angle -30°}$$

- Again, this shows that the effective turns ratio is

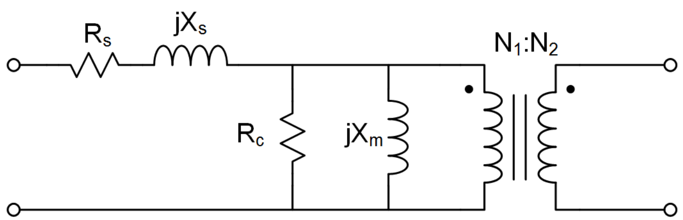
$$\sqrt{3}a_t$$

  - And, there is a phase shift through the transformer of $-30°$

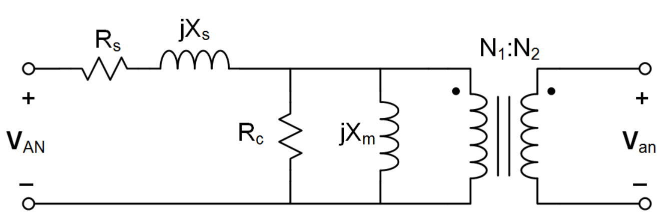
Per-Phase, Per-Unit Models

Per-Phase, Per-Unit Transformer Models

- Each of the three single-phase transformers in the three-phase bank can be modeled as

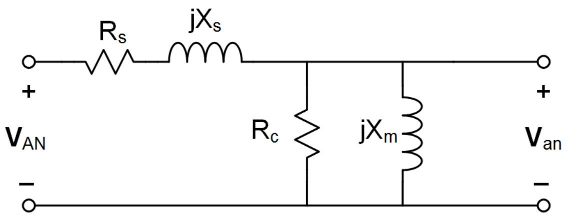


- In the per-phase model, we need the equivalent line-to-neutral impedances
  - Unchanged for Y connections
  - Divided by 3 for Δ connections
- The Y-Y or Δ-Δ equivalent ***per-phase*** circuit is the same as above:

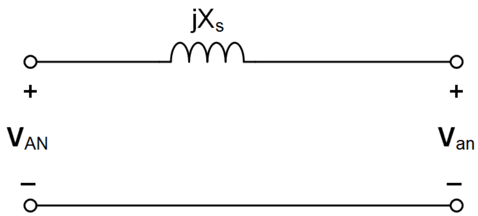


Per-Phase, Per-Unit Transformer Models

- The transformer disappears from the ***per-unit*** equivalent circuit:



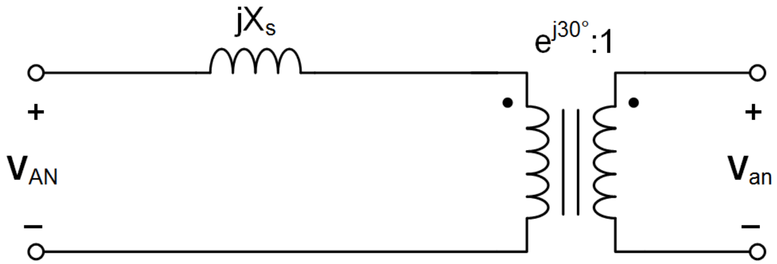
- It is common to neglect the exciting current and winding losses and account only for leakage flux



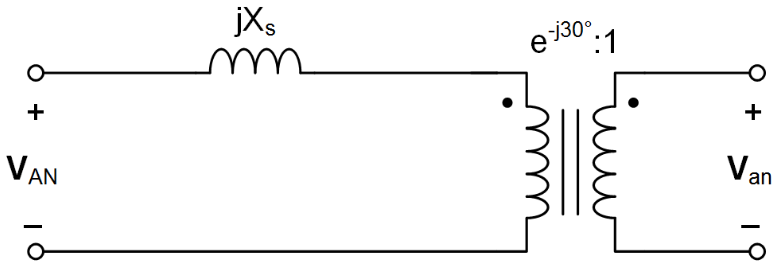
- For an ideal Y-Y or Δ-Δ transformer, the per-unit circuit is

Per-Phase, Per-Unit Transformer Models

- For the Y-Δ or Δ-Y transformer, we must account for the phase shift
  - Per-unit includes a conceptual phase-shifting transformer
  - Voltage bases must be related by the effective turns ratio – must include the $\sqrt{3}$ factor
- Simplified Y-Δ per-unit circuit:



- Simplified Δ-Y per-unit circuit:



Power Transformer Miscellany

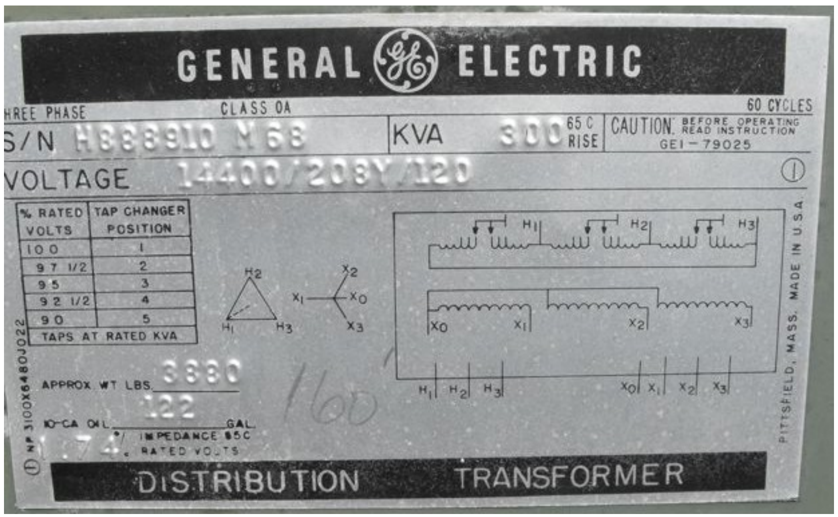
Transformer Uses

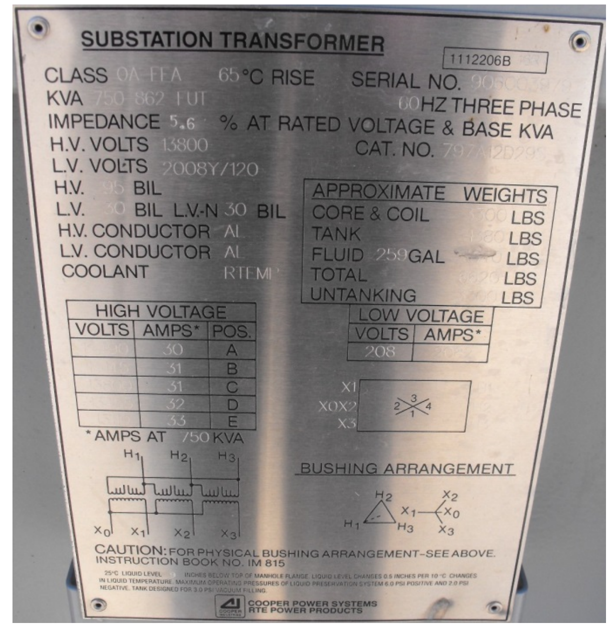
- Δ connection advantages:
  - Third harmonic current confined to the core
    - Due to non-linear B-H characteristics of the core
- Y connection advantages
  - Neutral point simplifies grounding
  - Reduced insulation requirement

- **Δ-Y transformers**
  - Most common type of transformers
  - Step-up/step-down with Y connection at high-voltage side to reduce insulation requirements
  - Δ winding confines third harmonic currents
- **Δ-Δ transformers**
  - Attractive from repair/maintenance standpoint
    - Can remove one transformer and still deliver (reduced) 3ϕ power
- **Y-Y transformers**
  - Not commonly used due to problems with third harmonic currents

Transformer Nameplate Ratings

- Transformer properties and ratings are specified on the transformer ***nameplate***:

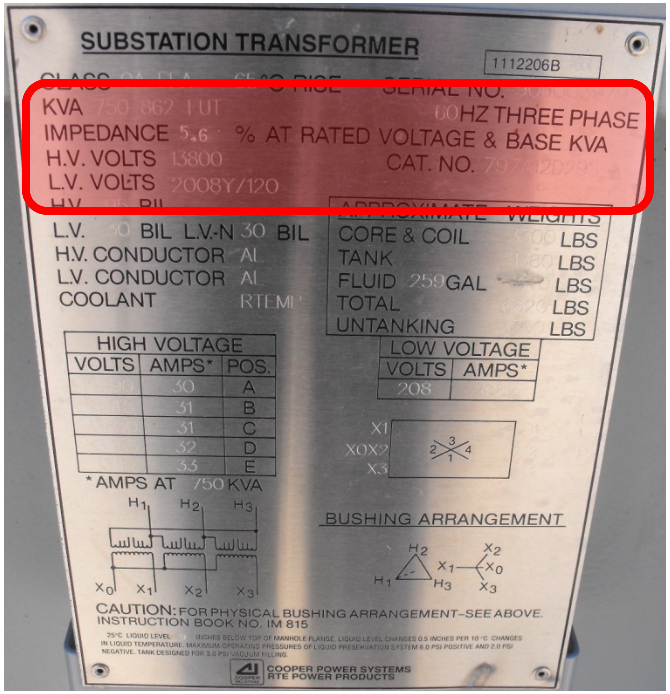




- Includes, among other specs:
  - Power rating
  - Voltage ratings – turns ratio
  - Configuration – delta or Y
  - Impedance

Transformer Impedance

- Impedance specified on the nameplate as a percentage
  - Per-unit value multiplied by 100%
  - Typically stamped on the nameplate – determined through testing
  - Assume this is ***series impedance magnitude***
- For example:
  - $S_{rated} = 750\ kVA$
  - $V_{rated} = 13.8\ kV$
  - $\%Z = 5.6\%$ or $0.056\ p.u.$



- The impedance base at the primary:

$$Z_b = \frac{V_{rated}^2}{S_{rated}} = \frac{(13.8\ kV)^2}{750\ kVA} = 253.9\ \Omega$$

- The actual impedance magnitude is

$$Z_s = \%Z \cdot Z_b = 0.056 \cdot 253.9\ \Omega$$

$$Z_s = 14.2\ \Omega$$

Transformer Impedance

- Another way to understand transformer impedance:
  - ***Voltage drop at the rated load due to transformer impedance, expressed as a percentage of the rated voltage***
- For the previous example, the rated current is

$$I_{rated} = \frac{S_{rated}}{V_{rated}} = \frac{750\ kVA}{13.8\ kV} = 54.3\ A$$

- We determined that the actual impedance was $14.2\ \Omega$, so

$$V_{drop} = I_{rated} \cdot Z_s = 54.3\ A \cdot 14.2\ \Omega = 771.7\ V$$

- Expressed as a percentage of the rated voltage, we have

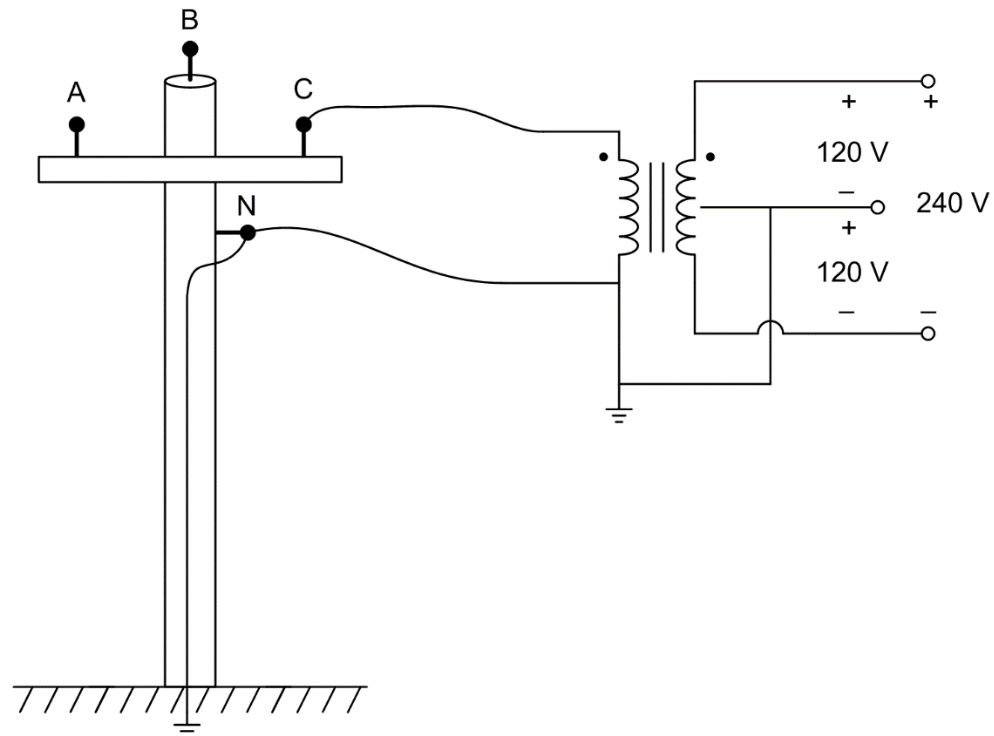
$$\%Z = \frac{V_{drop}}{V_{rated}} \cdot 100\% = \frac{771.7\ V}{13.8\ kV} \cdot 100\% = 5.6\%$$

Distribution Transformers

- Neighborhoods are typically in the range of 7.2 kV, line-to-neutral
- Pole-mounted ***distribution transformers*** step voltage down
  - Single phase 120/240 V
- ***Pad-mounted*** or ***vault*** (underground) distribution transformers are also common

Distribution Transformers

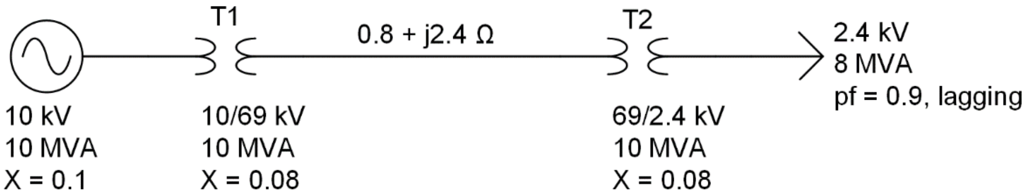
- Single-phase distribution transformers tap off of a single phase
- Neutral center tap on secondary
- Single-phase 120 V and 240 V service to homes and businesses



Example Problems

Using the ***single-phase*** per-unit conversion procedure,

a) Draw a per-phase, per-unit schematic
b) Determine the generator voltage in per-unit and in volts
c) Determine the power delivered by the generator in p.u. and VA



Using the ***three-phase*** per-unit conversion procedure,

a) Draw a per-phase, per-unit schematic
b) Determine the generator voltage in per-unit and in volts
c) Determine the power delivered by the generator in p.u. and VA