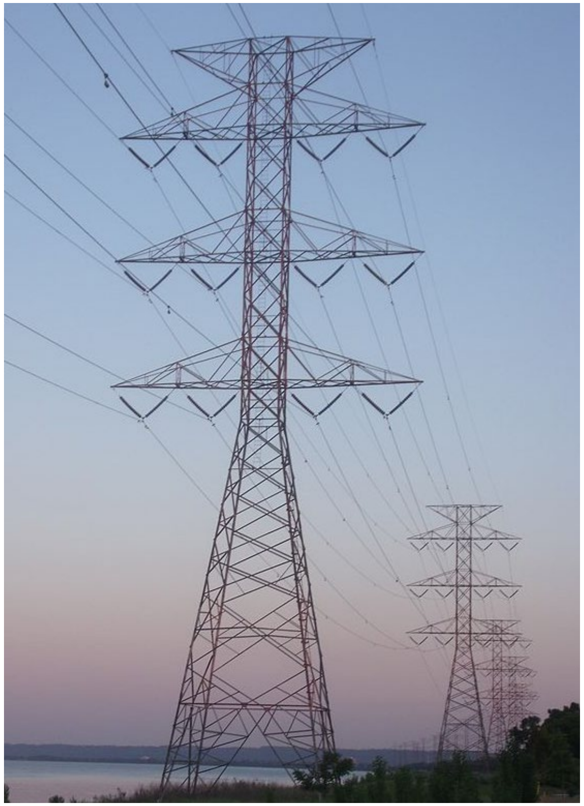
SECTION 4: TRANSMISSION LINES

ESE 470 – Energy Distribution Systems

Introduction

Transmission Lines

- Transmission and distribution of electrical power occurs over metal cables
  - Overhead AC or DC
  - Underground AC or DC
- In the U.S. nearly all transmission makes use of overhead AC lines
- These cables are good, but not perfect, conductors
  - Series impedance
  - Shunt admittance
- In this section of notes we'll look at how these are accounted for in equivalent circuit models



Electrical Properties of Transmission Lines

- ***Series resistance***
  - Voltage drop (IR) and real power loss (I^2R) along the line
  - Due to finite conductivity of the line
- ***Series inductance***
  - Series voltage drop, no real power loss
  - Only self inductance (no mutual inductance) in balanced systems
- ***Shunt conductance***
  - Real power loss (V^2G)
  - Leakage current due to corona effects or leakage at insulators
  - Typically neglected for overhead lines
- ***Shunt capacitance***
  - Capacitance to other conductors and to ground
  - Line-charging currents

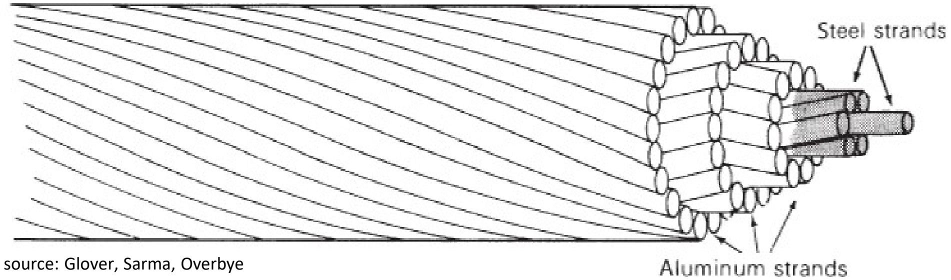
Conductors

Conductors

- Before getting into transmission line models, we'll take a look at the conductors themselves
- ***Aluminum*** is the most common conductor
  - Good conductivity
  - Light weight
  - Low cost
  - Plentiful supply
- Most common cable type combines aluminum and steel
  - ***Aluminum-conductor steel-reinforced*** (***ACSR***)
  - Bare, stranded cable
  - Core of steel strands provides strength
  - Outer aluminum strands provide good conductivity

ACSR Cables

- ACSR cables vary based on number of aluminum conductor strands and number of steel reinforcement strands
  - ACSR variants assigned bird code names, e.g.:
    - Dove: 26/7 Al/Steel stranding
    - Bluebird: 84/19 Al/Steel stranding



source: Glover, Sarma, Overbye

- Another increasingly popular cable type is ***all-aluminum-alloy conductor*** (***AAAC***)
  - Stronger
  - Lighter
  - Higher conductivity
  - More expensive

Cables

- Cables are sized to provide the required current-carrying capability or *ampacity*
  - Number of individual strands
  - Diameter of individual strands
- Strand and cable ***diameter*** commonly measured in ***mils***

$$1\ mil = 0.001"$$

- ***Cross-sectional area*** often measured in ***circular mils*** or ***cmil***
  - Area of a circle with a diameter of $d = 1\ mil = 0.001$"

$$1\ cmil = \pi\left(\frac{0.001}{2}\right)^2 = 785 \times 10^{-9}\ sq\ in$$

  - Area in $cmil$ of a cable with diameter $d\ mil$:

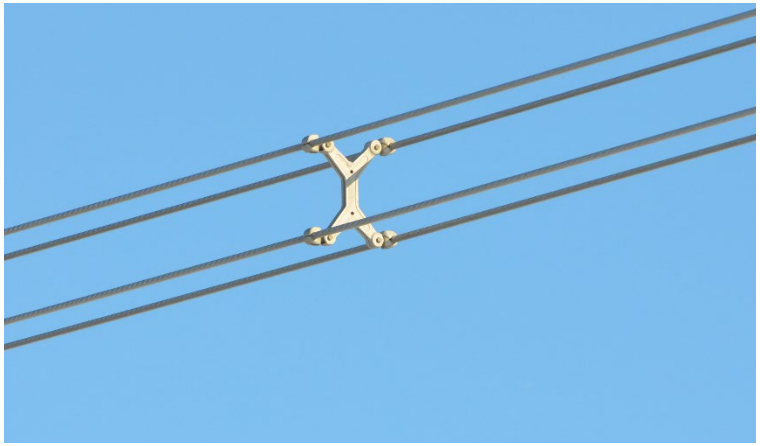
$$A = d^2$$

ACSR Cable

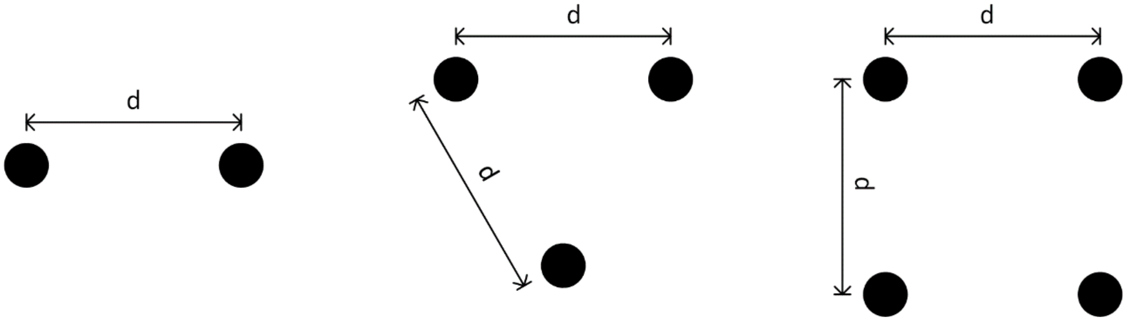
- Consider, for example, ***Falcon ACSR cable***
  - 54/19: 54 Al strands with a core of 19 steel strands
  - Al strand diameter: $172\ mil$
    - Al strand area: $(172\ mil)^2 = 29.584\ kcmil$
  - Steel strand diameter: $103\ mil$
    - Steel strand area: $(103\ mil)^2 = 10.609\ kcmil$
  - Cable diameter: 1.545“
    - Cable area: $(1545\ mil)^2 = 2387\ kcmil$
  - Ampacity: $1380\ A$
  - Weight: $10{,}777\ lb/mi$

Bundling

- In addition to increasing cable cross-sectional area, ampacity can be increased by adding additional cables to each phase – ***bundling***



- Two-, three-, and four-cable bundles are common:

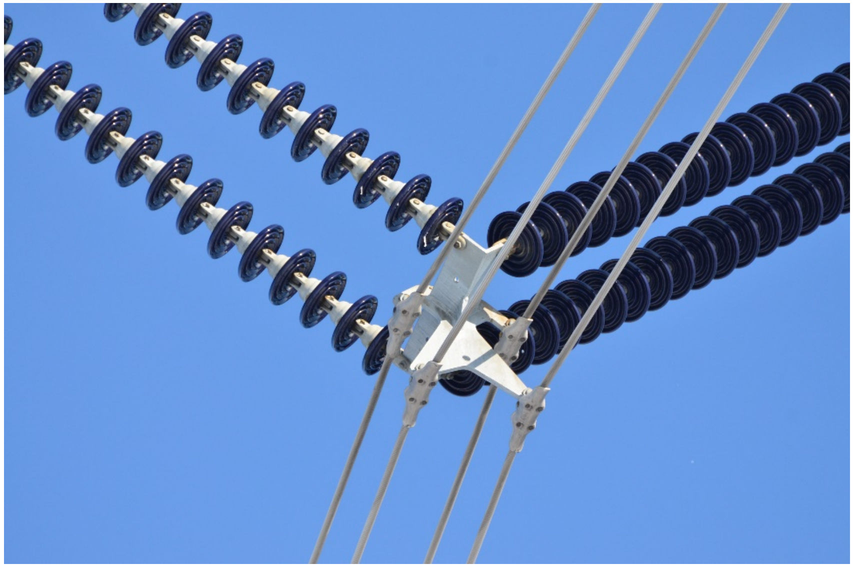


Bundling

- Typical bundling:
  - 345 kV: two conductors
  - 500 kV: three conductors
  - 765 kV: four conductors
- Advantages of bundling:
  - Lower resistance
  - Lower reactance (inductance)
  - Increased ampacity
  - Reduced electric field gradient surrounding phase conductor
    - Reduced corona
    - Reduced loss, noise, and RF interference
  - Improved heat dissipation

Insulators

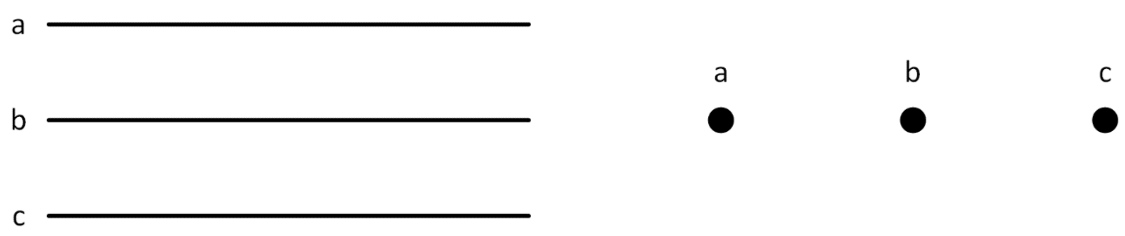
- Cables are supported by towers
  - Must connect, while retaining electrical isolation
- Connections are typically made through ceramic or glass insulators
- High-voltage lines suspended by strings of insulator discs



- One or two strings
  - Two prevents sway
- Number of discs dictated by line voltage, e.g.:
  - 4-6 for 69 kV
  - 30-35 for 765 kV

Transposition

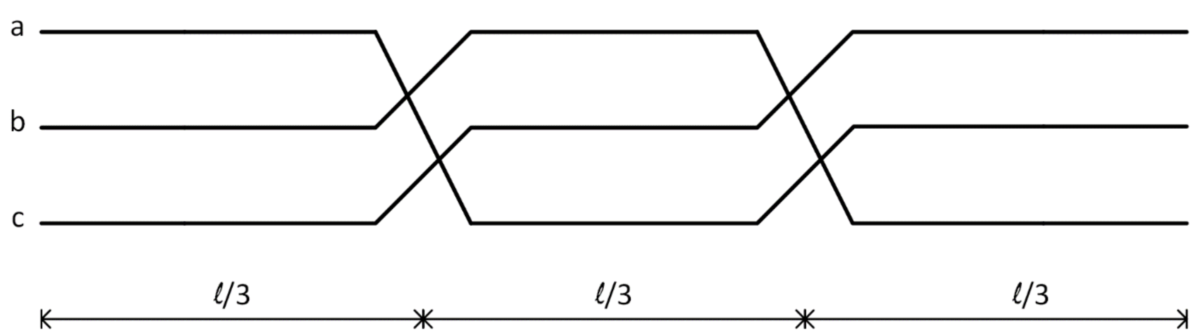
- Transmission-line inductance and capacitance determined by geometry
  - Cable size and relative spacing
- Consider three phases laid out side-by-side



- Phases a and c will have similar inductance and capacitance
- Inductance and capacitance of phase b will differ

Transposition

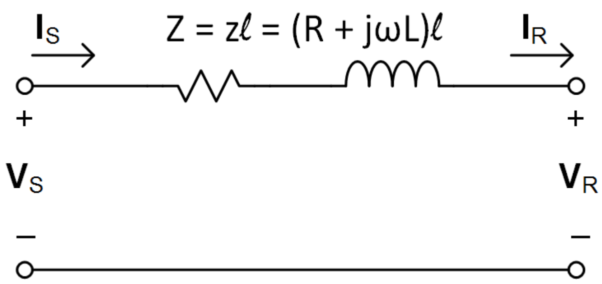
- ***Transposition***
  - Switch the position of each phase twice along the length of the line
  - Each phase occupies each position for one third of the line length
  - Line remains balanced



Medium- and Short-Line Models

Short-Line Model

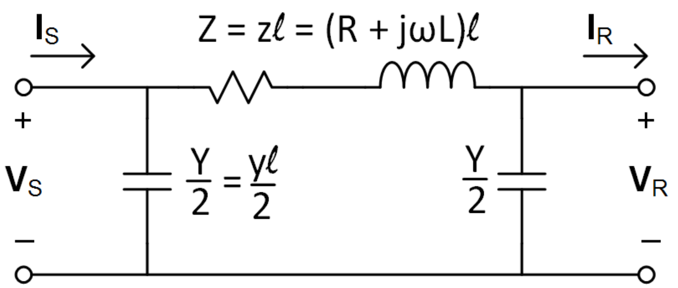
- How we choose to model the electrical characteristics of a transmission line depends on the length of the line
- **Short-line model**:
  - $< \sim 80\ km$
  - ***Lumped*** model
  - Account only for series impedance
  - Neglect shunt capacitance



- R and ωL are resistance and reactance per unit length, respectively
  - Each with units of Ω/m
- l is the length of the line

Medium-Line Model

- **<u>Medium-line model</u>** – **<u>nominal-π model</u>**:
  - $80\ km < l < 250\ km$
  - Lumped model
  - Now include shunt capacitance



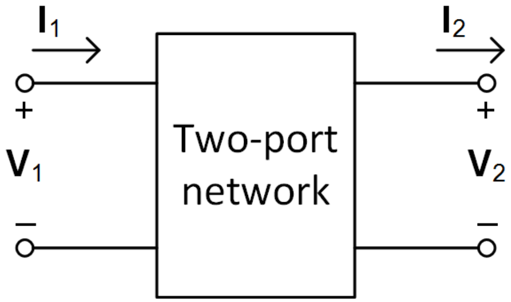
$$z = R + j\omega L\ \ \Omega/m \quad \text{and} \quad Z = zl\ \ \Omega$$
$$y = \omega C\ \ S/m \quad \text{and} \quad Y = yl\ \ S$$

- Still a ***lumped*** model
  - All impedances and admittances lumped into one or two circuit components

ABCD Parameters

Transmission Lines as Two-Port Networks

- Before moving on to a model for longer transmission lines, we'll look at an alternative tool for characterizing transmission line networks
- We can treat transmission lines as general two-port networks



- As two-port networks, we can characterize transmission lines with their ***ABCD parameters*** or ***chain parameters***

ABCD Parameters

- ABCD (or *chain* or *transmission* or *cascade*) parameters define the following two-port relationships

$$\boldsymbol{V}_1 = A\boldsymbol{V}_2 + B\boldsymbol{I}_2$$
$$\boldsymbol{I}_1 = C\boldsymbol{V}_2 + D\boldsymbol{I}_2$$

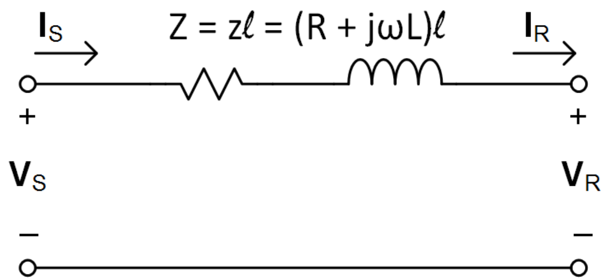
- In matrix form, the ***chain-parameter equations*** are

$$\begin{bmatrix} \boldsymbol{V}_1 \\ \boldsymbol{I}_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_2 \\ \boldsymbol{I}_2 \end{bmatrix}$$

- A, B, C, and D are, in general, ***complex numbers***
  - A and D are dimensionless
  - B is an impedance with units of Ω
  - C is an admittance with units of S
- V_1 and V_2 are ***line-to-neutral*** voltages
- If the network is ***reciprocal***, then $AD - BC = 1$
- If the network is ***symmetric***, then $A = D$

ABCD Parameters – Short-Line Model

- We'll now derive the ABCD parameters for the short-transmission-line model



- Applying KVL around the loop gives our first equation

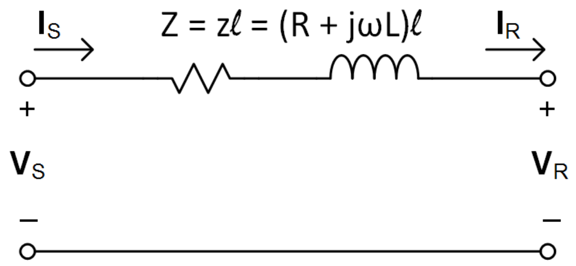
$$\boldsymbol{V}_S - \boldsymbol{I}_R Z - \boldsymbol{V}_R = 0$$

$$\boldsymbol{V}_S = \boldsymbol{V}_R + Z\boldsymbol{I}_R$$

So,

$$A = 1 \quad \text{and} \quad B = Z$$

ABCD Parameters – Short-Line Model



- Applying KCL gives the second equation

$$\boldsymbol{I}_s = \boldsymbol{I}_R$$

and

$$C = 0 \quad \text{and} \quad D = 1$$

- The short-line ABCD matrix is

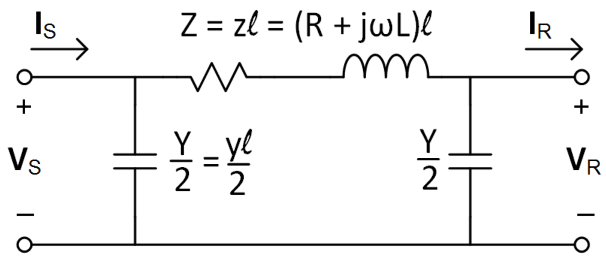
$$ABCD = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

- Note that, due to symmetry and reciprocity,

$$A = D \quad \text{and} \quad AD - BC = 1$$

ABCD Parameters – Medium-Line Model

- Next, for the medium-transmission-line model



- Applying KVL around the loop gives our first equation

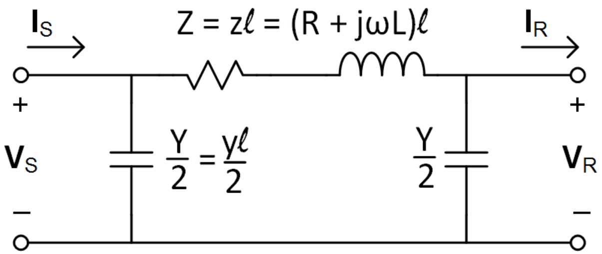
$$\boldsymbol{V}_S - \left(\boldsymbol{I}_R + \boldsymbol{V}_R \frac{Y}{2}\right) Z - \boldsymbol{V}_R = 0$$

$$\boldsymbol{V}_S = \left(1 + \frac{YZ}{2}\right) \boldsymbol{V}_R + Z\boldsymbol{I}_R$$

- This is the first chain parameter equation, where

$$A = \left(1 + \frac{YZ}{2}\right) \quad \text{and} \quad B = Z$$

ABCD Parameters – Medium-Line Model



- For the second equation, apply KCL at the sending end

$$\boldsymbol{I}_S - \boldsymbol{V}_S\frac{Y}{2} - \boldsymbol{I}_R - \boldsymbol{V}_R\frac{Y}{2} = 0$$

- Substituting in our previous expression for V_S

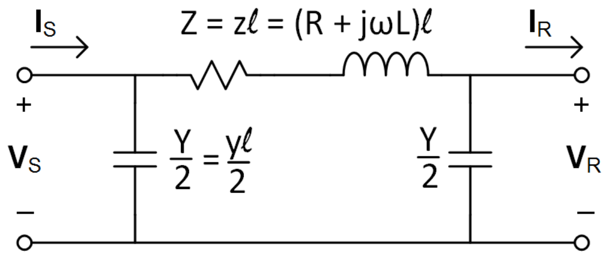
$$\boldsymbol{I}_S = \boldsymbol{V}_R\frac{Y}{2} + \boldsymbol{I}_R + \left(1 + \frac{YZ}{2}\right)\frac{Y}{2}\boldsymbol{V}_R + \frac{YZ}{2}\boldsymbol{I}_R$$

$$\boldsymbol{I}_S = \left(2 + \frac{YZ}{2}\right)\frac{Y}{2}\boldsymbol{V}_R + \left(1 + \frac{YZ}{2}\right)\boldsymbol{I}_R$$

- This is the second chain-parameter equation, where

$$C = \left(1 + \frac{YZ}{4}\right)Y \quad \text{and} \quad D = \left(1 + \frac{YZ}{2}\right)$$

ABCD Parameters – Medium-Line Model



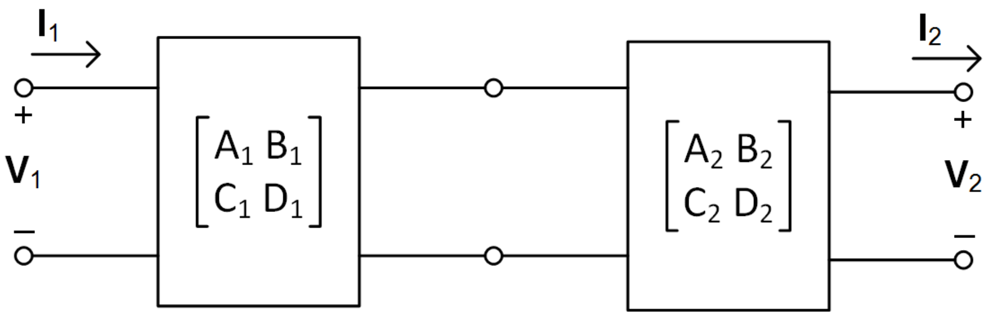
- The medium-line chain parameters are

$$ABCD = \begin{bmatrix} \left(1 + \frac{YZ}{2}\right) & Z \\ \left(1 + \frac{YZ}{4}\right) Y & \left(1 + \frac{YZ}{2}\right) \end{bmatrix}$$

- Again, note that, due to symmetry and reciprocity, $A = D$ and $AD - BC = 1$
- Also note that allowing $Y \rightarrow 0$ yields the chain parameters for the short-line model

Cascading Two-Port Networks

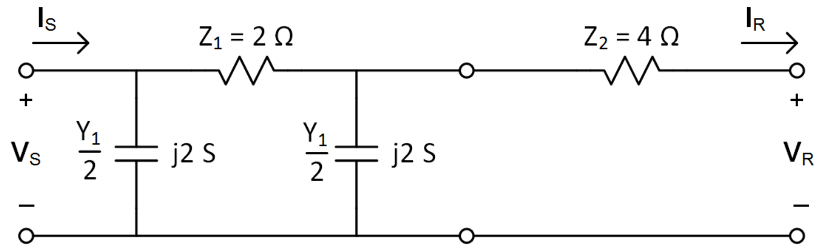
- ABCD parameters or chain parameters are also called ***cascade parameters***
- If we cascade multiple two-port networks, the ABCD parameter matrix for the cascade is the product of the individual ABCD parameter matrices



$$ABCD = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2 \end{bmatrix}$$

Cascaded Two-Ports - Example

- For example, consider the cascade of the following two two-port networks



- ABCD parameters for the first network are

$$ABCD_1 = \begin{bmatrix} \left(1 + \frac{Y_1 Z_1}{2}\right) & Z_1 \\ \left(1 + \frac{Y_1 Z_1}{4}\right) Y_1 & \left(1 + \frac{Y_1 Z_1}{2}\right) \end{bmatrix} = \begin{bmatrix} 1 + j4 & 2\ \Omega \\ -4 + j2\ S & 1 + j4 \end{bmatrix}$$

- And for the second network

$$ABCD_2 = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4\ \Omega \\ 0 & 1 \end{bmatrix}$$

- So the overall ABCD matrix is

$$ABCD = \begin{bmatrix} 1 + j4 & 6 + j16\ \Omega \\ -4 + j2\ S & -15 + j12 \end{bmatrix}$$

Cascaded Two-Ports - Example

- If a sending-end voltage of $\boldsymbol{V}_S = 120\angle 0°\ V$ is applied, and no load is connected, what is the receiving-end voltage?

$$\boldsymbol{V}_S = 120\angle 0°\ V \quad \text{and} \quad \boldsymbol{I}_R = 0\ A$$

$$\boldsymbol{V}_S = A\boldsymbol{V}_R + B\boldsymbol{I}_R$$

$$120\angle 0° = (1 + j4)V_R$$

- The no-load receiving-end voltage is

$$\boldsymbol{V}_R = \frac{120\angle 0°}{1 + j4} = 7.06 - j28.2\ V$$

$$\boldsymbol{V}_R = 29.1\angle - 75.96°\ V$$

Voltage Regulation

Voltage Regulation

- The voltage at the receiving end of a line will change depending on the load placed on the line
  - Magnitude of this change is quantified as ***voltage regulation***
- **Voltage regulation**:
  - Change in receiving-end voltage from no load to full load, expressed as a percentage of the full-load voltage

$$\%VR = \frac{|V_{RNL}| - |V_{RFL}|}{|V_{RFL}|} \cdot 100\%$$

  - Typically, transmission lines are designed to limit voltage regulation to about 10%
- As we've seen, the no-load voltage is given by

$$|V_{RNL}| = \frac{|\boldsymbol{V}_S|}{A}$$

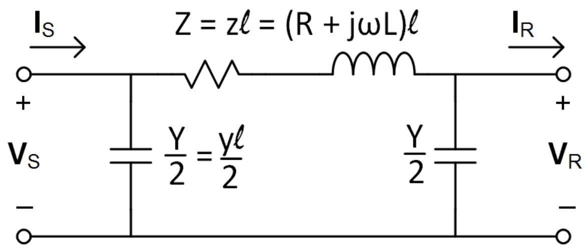
Voltage Regulation – Example 5.1

- Consider a three-phase, 60 Hz, 345 kV transmission line with the following properties
  - 200 km long
  - $z = 0.032 + j0.35\ \Omega/km,\ \ y = j4.2\ \mu S/km$
  - Full load is 700 MW at 95% of the rated voltage and a power factor of 0.99 leading

- Determine:
  - ABCD parameters for an appropriate transmission-line model
  - Phase shift between sending- and receiving-end voltages at full load
  - Percent voltage regulation

Voltage Regulation – Example 5.1

- Line is 200 km long, so a nominal-π model is appropriate



where

$$Z = z \cdot 200\ km = 6.4 + j70\ \Omega$$
$$Y = y \cdot 200\ km = j840\ \mu S$$

- The ABCD parameters are

$$ABCD = \begin{bmatrix} \left(1 + \frac{YZ}{2}\right) & Z \\ \left(1 + \frac{YZ}{4}\right)Y & \left(1 + \frac{YZ}{2}\right) \end{bmatrix} = \begin{bmatrix} 0.971 + j0.0027 & 6.4 + j70\ \Omega \\ -1.13 + j828\ \mu S & 0.971 + j0.0027 \end{bmatrix}$$

$$ABCD = \begin{bmatrix} 0.971\angle 0.159° & 70.3\angle 84.8°\ \Omega \\ 828\angle 90.08°\ \mu S & 0.971\angle 0.159° \end{bmatrix}$$

Voltage Regulation – Example 5.1

- At full load the line-to-line receiving-end voltage is

$$V_{RFL} = 345\ kV \cdot 0.95 = 327.8\ kV_{LL}$$

- And the line-to-neutral voltage is

$$V_{RFL} = \frac{327.8\ kV_{LL}}{\sqrt{3}} = 189.2\ kV_{LN}$$

- Using the receiving-end voltage as the reference, the receiving-end voltage phasor is

$$\boldsymbol{V}_R = 189.2\angle 0°\ kV$$

- Complex power to the load is

$$\boldsymbol{S_R} = \frac{P}{pf}\angle\theta = \frac{700\ MW}{0.99}\angle - \cos^{-1}(0.99)$$

$$\boldsymbol{S_R} = 707.1\angle - 8.1°\ MVA = 3 \cdot \boldsymbol{V_R I_R^*}$$

Voltage Regulation – Example 5.1

- The receiving-end current phasor is

$$\boldsymbol{I}_R = \left(\frac{\frac{\boldsymbol{S}_{\boldsymbol{R}}}{3}}{\boldsymbol{V}_{\boldsymbol{R}}}\right)^* = \frac{707.1\angle 8.1°\ MVA}{3 \cdot 189.2\angle 0°\ kV}$$

$$\boldsymbol{I}_R = 1.25\angle 8.1°\ kA$$

- To determine the phase shift from sending to receiving end, use chain parameters to determine $\boldsymbol{V}_S$ (line-to-neutral)

$$\boldsymbol{V}_S = A\boldsymbol{V}_R + B\boldsymbol{I}_R$$

$$\boldsymbol{V}_S = 0.971\angle 0.159° \cdot 189.2\angle 0°\ kV$$

$$+70.3\angle 84.8°\ \Omega \cdot 1.25\angle 8.1°\ kA$$

$$\boldsymbol{V}_S = 199.8\angle 26.1°\ kV_{\mathrm{LN}}$$

- So, the phase shift along the line is $-26.1°$

Voltage Regulation – Example 5.1

- The percent voltage regulation is given by

$$\%VR = \frac{|V_{RNL}| - |V_{RFL}|}{|V_{RFL}|} \cdot 100\%$$

- The line-to-neutral no-load voltage is

$$|V_{RNL}| = \left|\frac{\boldsymbol{V_S}}{A}\right| = \left|\frac{199.8\angle 26.1°}{0.971\angle 0.159°}\right| = 205.8\ kV$$

- The full-load line-to-neutral voltage was given to be

$$|V_{RFL}| = 189.2\ kV$$

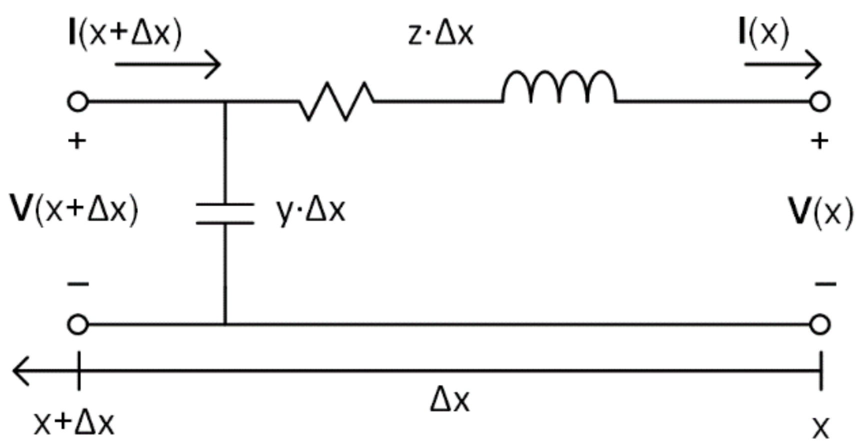
- So, the percent voltage regulation is

$$\%VR = \frac{205.8\ kV - 189.2\ kV}{189.2\ kV} \cdot 100\% = 8.7\%$$

Exact Transmission-Line Equations

Distributed Transmission Line Model

- The medium- and short-line models are ***lumped*** models
  - All series impedance lumped into one element
  - Shunt admittances lumped into two elements
- Real lines are ***distributed*** networks
  - Lumped models are inaccurate for long lines
- To treat a line as a distributed network, consider the impedance and admittance of a segment of ***differential length***, Δx



Transmission Line Differential Equations

- Apply KVL around the differential length of line

$$\boldsymbol{V}(x+\Delta x)=\boldsymbol{V}(x)+\boldsymbol{I}(x)z\Delta x$$

$$\frac{\boldsymbol{V}(x+\Delta x)-\boldsymbol{V}(x)}{\Delta x}=z\boldsymbol{I}(x) \qquad (1)$$

- If we let the length of the line segment, Δx, go to zero, we get

$$\frac{d\boldsymbol{V}(x)}{dx}=z\boldsymbol{I}(x) \qquad (2)$$

- A first-order differential equation
  - This is a second-order segment, so we need a second first-order differential equation to describe it completely
- Apply KCL at $(x+\Delta x)$

$$\boldsymbol{I}(x+\Delta x)=\boldsymbol{I}(x)+\boldsymbol{V}(x+\Delta x)y\Delta x$$

$$\frac{\boldsymbol{I}(x+\Delta x)-\boldsymbol{I}(x)}{\Delta x}=y\boldsymbol{V}(x+\Delta x) \qquad (3)$$

Transmission Line Differential Equations

- Again, letting $\Delta x \rightarrow 0$

$$\frac{d\boldsymbol{I}(x)}{dx} = y\boldsymbol{V}(x) \qquad (4)$$

- Our goal is a single differential equation in $\boldsymbol{V}(x)$ to describe the segment of transmission line
  - Must eliminate $\boldsymbol{I}(x)$
- Solving (2) for $\boldsymbol{I}(x)$ and differentiating gives

$$\frac{d\boldsymbol{I}(x)}{dx} = \frac{1}{Z}\frac{d^2\boldsymbol{V}(x)}{dx^2} \qquad (5)$$

- Substituting (5) into (4) yields the single second-order differential equation for the line segment

$$\frac{d^2\boldsymbol{V}(x)}{dx^2} - zy\boldsymbol{V}(x) = 0 \qquad (6)$$

Transmission Line Differential Equations

$$\frac{d^2V(x)}{dx^2} - zy\boldsymbol{V}(x) = 0 \qquad (6)$$

- This is a second-order, homogeneous, linear, constant-coefficient, ordinary differential equation
- Its characteristic equation is

$$s^2 - zy = 0$$

- The roots of the characteristic polynomial are

$$s = \pm\sqrt{zy} = \pm\gamma$$

where $\gamma = \sqrt{zy}$ is the ***propagation constant***, with units of m^{-1} (or rad/m)

- The solution to (6) is

$$\boldsymbol{V}(x) = K_1 e^{\gamma x} + K_2 e^{-\gamma x} \qquad (7)$$

where K_1 and K_2 are unknown constants to be determined through application of boundary conditions

Transmission Line Differential Equations

- We can get an expression for current by differentiating (7) and substituting back into (2)

$$\frac{dV(x)}{dx} = \gamma K_1 e^{\gamma x} - \gamma K_2 e^{-\gamma x} = z\boldsymbol{I}(x)$$

- Solving for $\boldsymbol{I}(x)$

$$\boldsymbol{I}(x) = \frac{K_1 e^{\gamma x} - K_2 e^{-\gamma x}}{z/\gamma} \tag{8}$$

- The term in the denominator of (8) is the ***characteristic impedance*** of the line, Z_c, with units of ohms (Ω)

$$Z_c = \frac{z}{\gamma} = \frac{z}{\sqrt{zy}} = \sqrt{\frac{z}{y}} \tag{9}$$

Transmission Line Differential Equations

- Using (9), (8) becomes

$$\boldsymbol{I}(x) = \frac{K_1 e^{\gamma x} - K_2 e^{-\gamma x}}{Z_c} \tag{10}$$

- We can now apply boundary conditions to determine the two unknown coefficients, K_1 and K_2
- At the receiving end of the line, which we'll define to be $x = 0$, we have

$$\boldsymbol{V}(0) = \boldsymbol{V}_R \quad \text{and} \quad \boldsymbol{I}(0) = \boldsymbol{I}_R$$

So,

$$\boldsymbol{V}(0) = K_1 + K_2 = \boldsymbol{V}_R$$

$$\boldsymbol{I}(0) = \frac{K_1 - K_2}{Z_c} = \boldsymbol{I}_R$$

Transmission Line Differential Equations

- Solving each equation for K_2

$$K_2 = \boldsymbol{V}_R - K_1 = K_1 - Z_c \boldsymbol{I}_R$$

- Solving for K_1, then back-substituting to solve for K_2 gives

$$K_1 = \frac{\boldsymbol{V}_R + Z_c \boldsymbol{I}_R}{2}$$

$$K_2 = \frac{\boldsymbol{V}_R - Z_c \boldsymbol{I}_R}{2}$$

- Substituting into (7) and (10)

$$\boldsymbol{V}(x) = \left(\frac{\boldsymbol{V}_R + Z_c \boldsymbol{I}_R}{2}\right) e^{\gamma x} + \left(\frac{\boldsymbol{V}_R - Z_c \boldsymbol{I}_R}{2}\right) e^{-\gamma x} \qquad (11)$$

$$\boldsymbol{I}(x) = \left(\frac{\boldsymbol{V}_R + Z_c \boldsymbol{I}_R}{2Z_c}\right) e^{\gamma x} - \left(\frac{\boldsymbol{V}_R - Z_c \boldsymbol{I}_R}{2Z_c}\right) e^{-\gamma x} \qquad (12)$$

Transmission Line Differential Equations

- Collecting V_R and I_R terms in (11) and (12)

$$\boldsymbol{V}(x) = \left(\frac{e^{\gamma x}+e^{-\gamma x}}{2}\right)\boldsymbol{V}_R + Z_c\left(\frac{e^{\gamma x}-e^{-\gamma x}}{2}\right)\boldsymbol{I}_R \qquad (13)$$

$$\boldsymbol{I}(x) = \frac{1}{Z_c}\left(\frac{e^{\gamma x}-e^{-\gamma x}}{2}\right)\boldsymbol{V}_R + \left(\frac{e^{\gamma x}+e^{-\gamma x}}{2}\right)\boldsymbol{I}_R \qquad (14)$$

- The terms in parentheses can be represented as hyperbolic functions

$$\boldsymbol{V}(x) = \cosh(\gamma x)\,\boldsymbol{V}_R + Z_c\sinh(\gamma x)\,\boldsymbol{I}_R \qquad (15)$$

$$\boldsymbol{I}(x) = \frac{1}{Z_c}\sinh(\gamma x)\,\boldsymbol{V}_R + \cosh(\gamma x)\,\boldsymbol{I}_R \qquad (16)$$

Transmission Line Differential Equations

- Equations (15) and (16) give the chain parameters for the two-port network between a point at location x along the line and the receiving end

$$ABCD(x) = \begin{bmatrix} \cosh(\gamma x) & Z_c \sinh(\gamma x) \\ \frac{1}{Z_c}\sinh(\gamma x) & \cosh(\gamma x) \end{bmatrix}$$

- For chain parameters between sending and receiving ends, we set $x = l$

$$ABCD = \begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \frac{1}{Z_c}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$

Propagation Constant

- We defined the ***propagation constant*** as

$$\gamma = \sqrt{zy}$$

- This is, in general, a ***complex*** value

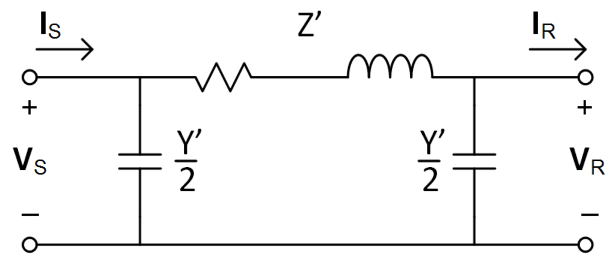
$$\gamma = \alpha + j\beta \tag{17}$$

- The real part, α, is the ***attenuation constant***
  - Represents ***loss*** along the line
  - Due to series resistance and/or shunt conductance
- The imaginary part, β, is the ***phase constant***
  - Represents ***change in phase*** along the line
  - Due to series reactance and/or shunt susceptance

Long-Line Equivalent Circuit

Long-Line Equivalent π Circuit

- Now that we have exact ABCD parameters for a distributed transmission line, we can create an equivalent π circuit



- Here we're using Z' and Y' to distinguish from $Z = zl$ and $Y = yl$ of the lumped, nominal π-circuit model
- Equating the ABCD parameters with those for the equivalent π circuit above

$$\begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \frac{1}{Z_c}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix} = \begin{bmatrix} 1 + \frac{Y'Z'}{2} & Z' \\ Y'\left(1 + \frac{Y'Z'}{4}\right) & 1 + \frac{Y'Z'}{2} \end{bmatrix}$$

Long-Line Equivalent π Circuit

- Equating the B parameters, we see that

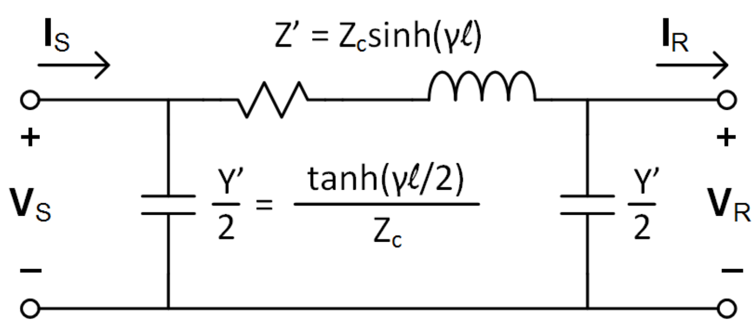
$$Z' = Z_c \sinh(\gamma l) \tag{18}$$

- Using (18) in the A-parameter equation gives

$$1 + \frac{Y'}{2} Z_c \sinh(\gamma l) = \cosh(\gamma l)$$

$$\frac{Y'}{2} = \frac{\cosh(\gamma l) - 1}{Z_c \sinh(\gamma l)} = \frac{\tanh\left(\frac{\gamma l}{2}\right)}{Z_c}$$

- The equivalent π circuit for long transmission lines (>250 km) is



Long-Line vs. Medium-Line Models

- We can compare this equivalent π circuit with the nominal π circuit used for medium-length lines, where

$$Z = zl \quad \text{and} \quad \frac{Y}{2} = y\frac{l}{2}$$

- Rewriting (18) using the definition for characteristic impedance,

$$Z' = \sqrt{\frac{z}{y}}\sinh(\gamma l) = zl\left(\sqrt{\frac{z}{y}}\frac{\sinh(\gamma l)}{zl}\right)$$

$$Z' = zl\frac{\sinh(\gamma l)}{\sqrt{zy}\,l}$$

$$Z' = Z\left(\frac{\sinh(\gamma l)}{\gamma l}\right) \tag{20}$$

- We see that the series impedance of the long-line model is equal to that of the medium-line model, multiplied by a correction factor

Long-Line vs. Medium-Line Models

- Doing the same for the shunt admittance, we have

$$\frac{Y'}{2} = \sqrt{\frac{y}{z}}\tanh\left({}^{\gamma l}/_{2}\right) = \frac{yl}{2}\left(\sqrt{\frac{y}{z}}\,\frac{\tanh\left({}^{\gamma l}/_{2}\right)}{{}^{yl}/_{2}}\right)$$

$$\frac{Y'}{2} = \frac{yl}{2}\frac{\tanh\left({}^{\gamma l}/_{2}\right)}{\sqrt{zy}\frac{l}{2}}$$

$$\frac{Y'}{2} = \frac{Y}{2}\frac{\tanh\left({}^{\gamma l}/_{2}\right)}{{}^{\gamma l}/_{2}}$$

- Again, we see a similar correction factor relating the admittance, Y, of the lumped, nominal π circuit to the admittance of the distributed, equivalent π circuit, Y'

Lossless Lines

Lossless Lines

- Transmission line models can be simplified significantly if we neglect loss
  - Sacrifice accuracy for the sake of simplicity
- Series resistance, R, and shunt conductance, G, are the model parameters accounting for loss
  - Let $R \to 0$ and $G \to 0$ – (we've already assumed $G = 0$)
- ***Propagation constant*** for a ***lossless*** line is

$$\gamma = j\beta$$

  - The ***attenuation constant*** is now zero, $\alpha \to 0$

$$\gamma = \sqrt{zy} = \sqrt{j\omega L \cdot j\omega C} = j\omega\sqrt{LC} = j\beta$$

$$\beta = \omega\sqrt{LC}$$

Lossless Lines – ABCD Parameters

- Using the propagation constant for a lossless line, the distributed model chain parameters become

$$A(x) = D(x) = \cosh(j\beta x) = \frac{e^{j\beta x} + e^{-j\beta x}}{2}$$

$$A(x) = D(x) = \cos(\beta x)$$

$$B(x) = Z_c \sinh(j\beta x) = Z_c \frac{e^{j\beta x} - e^{-j\beta x}}{2}$$

$$B(x) = jZ_c \sin(\beta x)$$

$$C(x) = \frac{1}{Z_c} \sinh(j\beta x) = \frac{1}{Z_c} \frac{e^{j\beta x} - e^{-j\beta x}}{2}$$

$$C(x) = j\frac{\sin(\beta x)}{Z_c}$$

Lossless Lines – ABCD Parameters

- Chain parameters at a distance x from the end of a lossless line are

$$ABCD(x) = \begin{bmatrix} \cos(\beta x) & jZ_c \sin(\beta x) \\ j\dfrac{\sin(\beta x)}{Z_c} & \cos(\beta x) \end{bmatrix}$$

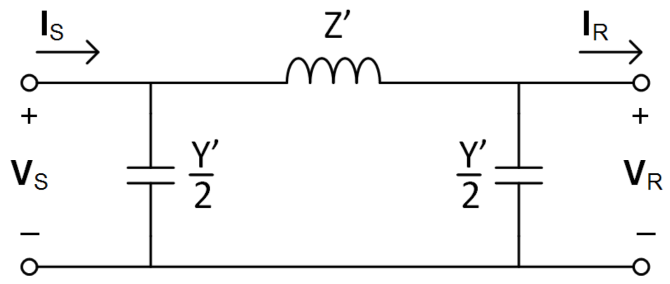
- And at the sending end of a line of length l, $x \rightarrow l$, and we have

$$ABCD = \begin{bmatrix} \cos(\beta l) & jZ_c \sin(\beta l) \\ j\dfrac{\sin(\beta l)}{Z_c} & \cos(\beta l) \end{bmatrix}$$

- The characteristic impedance of the lossless line is called the ***surge impedance***

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$$

Equivalent π Circuit – Lossless Line



- For the ***lossless line***

$$\gamma = j\beta$$

so,

$$Z' = Z_c \sinh(j\beta l) = j\sqrt{\frac{L}{C}}\sin(\beta l) = jX'$$

and,

$$\frac{Y'}{2} = \frac{\tanh\left(\frac{j\beta l}{2}\right)}{Z_c} = j\frac{\tan\left(\frac{\beta l}{2}\right)}{Z_c}$$

Wavelength

- The voltage along the lossless line is

$$\boldsymbol{V}(x) = A(x)\boldsymbol{V}_R + B(x)\boldsymbol{I}_R$$

$$\boldsymbol{V}(x) = \cos(\beta x)\,\boldsymbol{V}_R + jZ_c\sin(\beta x)\,\boldsymbol{I}_R$$

- A ***wavelength***, λ, is the distance required for a phase shift of $360°$ along the line
- There is a $360°$ phase shift when $x = \lambda$ and

$$\beta\lambda = 2\pi$$

- The wavelength is

$$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{1}{f\sqrt{LC}} = \frac{\nu}{f}}$$

where $\nu = 1/\sqrt{LC}$ is the ***propagation velocity*** along the line

Wavelength

- For *overhead* transmission lines,

$$v \approx c \approx 3 \times 10^8 m/s$$

- That is, electrical waves propagate along the line at roughly the speed of light
- At 60 Hz, the wavelength is

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8}{60} = 5000\ km$$

- This is approximately the distance across the U.S.
  - Most transmission lines are significantly shorter than a wavelength

Surge Impedance Loading

Surge Impedance Loading (SIL)

- ***Surge impedance loading (SIL)***
  - The power delivered by a transmission line to a resistive load whose impedance is equal to the surge impedance, Z_c, of that transmission line
- At SIL, the load current is

$$\boldsymbol{I}_R = \frac{\boldsymbol{V}_R}{Z_c}$$

- The voltage along the line is

$$\boldsymbol{V}(x) = \cos(\beta x)\,\boldsymbol{V}_R + jZ_c\sin(\beta x)\boldsymbol{I}_R$$

$$\boldsymbol{V}(x) = \cos(\beta x)\,\boldsymbol{V}_R + jZ_c\sin(\beta x)\,\frac{\boldsymbol{V}_R}{Z_c}$$

$$\boldsymbol{V}(x) = \boldsymbol{V}_R[\cos(\beta x) + j\sin(\beta x)]$$

$$\boldsymbol{V}(x) = \boldsymbol{V}_R\angle\beta x$$

- Note that at SIL, the *magnitude* of the voltage is constant along the line
  - A ***flat voltage profile***

Surge Impedance Loading (SIL)

- At SIL, the current along the line is given by

$$\boldsymbol{I}(x) = j\frac{\sin(\beta x)}{Z_c}\boldsymbol{V}_R + \cos(\beta x)\frac{\boldsymbol{V}_R}{Z_c}$$

$$\boldsymbol{I}(x) = \frac{\boldsymbol{V}_R}{Z_c}[\cos(\beta x) + j\sin(\beta x)]$$

$$\boldsymbol{I}(x) = \frac{\boldsymbol{V}_R}{Z_c}\angle\beta x$$

- The complex power along the line is

$$\boldsymbol{S}(x) = \boldsymbol{V}(x)\boldsymbol{I}(x)^* = (\boldsymbol{V}_R\angle\beta x)\left(\frac{\boldsymbol{V}_R}{Z_c}\angle\beta x\right)^*$$

$$\boldsymbol{S}(x) = \frac{|\boldsymbol{V}_R|^2}{Z_c} = P(x) + jQ(x)$$

- At SIL
  - Power flow is independent of position along the line
  - Reactive power is zero

Surge Impedance Loading (SIL)

- ***Surge impedance loading*** is typically defined in terms of a transmission line's rated voltage

$$SIL = \frac{V_{rated}^2}{Z_c}$$

- At SIL, we've seen that the ***voltage profile*** along a transmission line is flat
- At no load, $I_R = 0$, and the voltage is given by

$$\boldsymbol{V}(x) = \cos(\beta x)\,\boldsymbol{V}_{RNL}$$

- The source voltage is

$$\boldsymbol{V}_S = \cos(\beta l)\,\boldsymbol{V}_{RNL}$$

- So the receiving-end voltage in terms of the sending-end voltage is

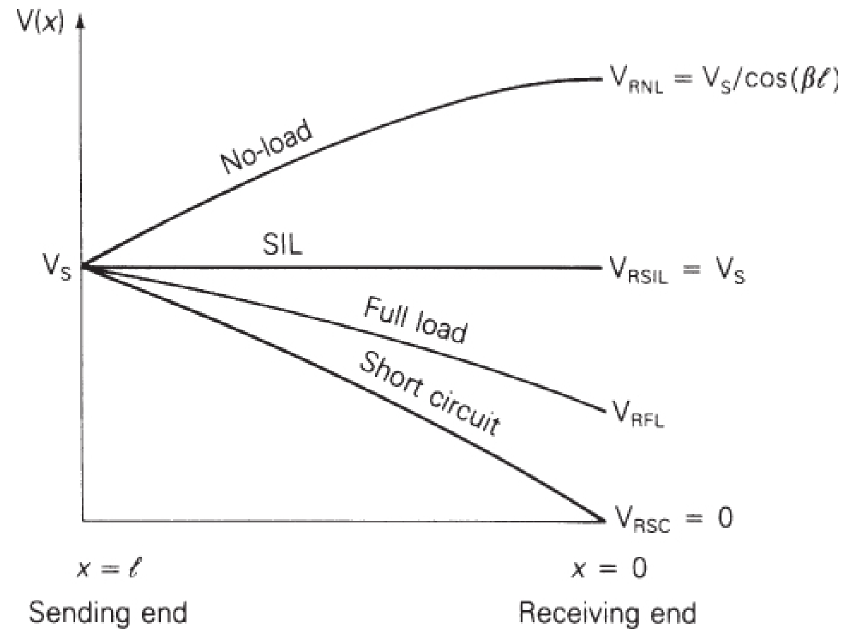
$$\boldsymbol{V}_{RNL} = \frac{\boldsymbol{V}_S}{\cos(\beta l)}$$

Surge Impedance Loading (SIL)

- The no-load receiving-end voltage is

$$\boldsymbol{V}_{RNL} = \frac{\boldsymbol{V}_S}{\cos(\beta l)}$$

- As long as $\beta l \leq \pi/2$, i.e. $l \leq \lambda/4$,
  - Voltage will increase along the length of the line
  - No-load receiving-end voltage is greater than the sending-end voltage
- Voltage regulation worsens with increasing line length



source: Glover, Sarma, Overbye

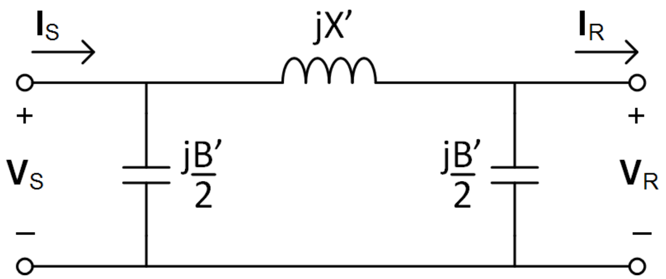
Steady-State Stability Limit

Real Power vs. Voltage Angle

- Assume a voltage angle between the sending and receiving ends of a lossless line of δ

$$\boldsymbol{V}_R = V_R\angle 0° \quad \text{and} \quad \boldsymbol{V}_S = V_S\angle\delta$$

- Using the equivalent π network for the lossless line, we can determine the receiving-end current



- Applying KCL at the receiving end

$$\boldsymbol{I}_R = \frac{\boldsymbol{V}_S - \boldsymbol{V}_R}{jX'} - j\frac{B'}{2}\boldsymbol{V}_R$$

$$\boldsymbol{I}_R = \frac{V_S\angle\delta - V_R\angle 0°}{jX'} - j\frac{B'}{2}V_R\angle 0°$$

Real Power vs. Voltage Angle

- The complex power at the load is

$$\boldsymbol{S}_R = \boldsymbol{V}_R \boldsymbol{I}_R^* = \frac{V_R V_S \angle -\delta - V_R^2}{-jX'} + j\frac{B'}{2}V_R^2$$

$$\boldsymbol{S}_R = j\,\frac{V_R V_S \angle -\delta}{X'} - j\frac{V_R^2}{X'} + j\frac{B'}{2}V_R^2$$

$$\boldsymbol{S}_R = j\frac{V_R V_S}{X'}[\cos(-\delta) + j\sin(-\delta)] - j\frac{V_R^2}{X'} + j\frac{B'}{2}V_R^2$$

$$\boldsymbol{S}_R = \frac{V_R V_S}{X'}\sin(\delta) + j\left[\frac{V_R V_S}{X'}\cos(\delta) - \frac{V_R^2}{X'} + \frac{B'}{2}V_R^2\right]$$

- The real power delivered is

$$P_R = P_S = \mathcal{R}e\{S_R\} = \frac{V_R V_S}{X'}\sin(\delta)$$

Power Flow – Lossless Lines

- The delivered power is a function of the voltage phase shift along the line, δ

$$P_R = \frac{V_R V_S}{X'} \sin(\delta)$$

- For the lossless line the series reactance is

$$X' = Z_c \sin(\beta l)$$

so,

$$P_R = \frac{V_R V_S}{Z_c \sin(\beta l)} \sin(\delta) = \frac{V_R V_S}{Z_c \sin\left(\frac{2\pi l}{\lambda}\right)} \sin(\delta)$$

Power Flow – Lossless Lines

- Converting V_R and V_S to per unit

$$P_R = \left(\frac{V_R}{V_{rated}}\right)\left(\frac{V_S}{V_{rated}}\right)\frac{V_{rated}^2}{Z_c \sin\left(\frac{2\pi l}{\lambda}\right)}\sin(\delta)$$

$$P_R = V_{R,pu}V_{S,pu}\left(\frac{V_{rated}^2}{Z_c}\right)\frac{\sin(\delta)}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

- The term in parentheses is SIL, so

$$P_R = V_{R,pu}V_{S,pu}SIL\frac{\sin(\delta)}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

- This provides a relationship between:
  - Power delivered over a transmission line
  - Voltage drop along the line
  - Power angle

Maximum Power Flow – Lossless Lines

$$P_R = \frac{V_R V_S}{Z_c \sin\left(\frac{2\pi l}{\lambda}\right)} \sin(\delta) = V_{R,pu} V_{S,pu} SIL \frac{\sin(\delta)}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

- The delivered power is a function of the voltage phase shift along the line
- Maximum power occurs when $\delta = 90°$

$$P_{max} = \frac{V_R V_S}{Z_c \sin\left(\frac{2\pi l}{\lambda}\right)} = \frac{V_{R,pu} V_{S,pu}\, SIL}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

- The ***steady-state stability limit*** of a line

Steady-State Stability Limit

$$P_{max} = \frac{V_R V_S}{Z_c \sin\left(\frac{2\pi l}{\lambda}\right)} = \frac{V_{R,pu} V_{S,pu}\, SIL}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

- This maximum power is the ***steady-state stability limit*** of a transmission line
- Loads exceeding this limit will result in a ***loss of synchronism*** at the receiving end
  - Synchronous machines at the sending and receiving ends will fall out of synchronization
- Steady-state stability limit proportional to
  - Inverse of line length
  - Square of the line voltage

Line Loadability

Transmission Line Loadability

- Three primary factors limit power flow over transmission lines:
  - Phase shift
  - Voltage drop
  - Thermal limit
- Relevant limit depends on line length

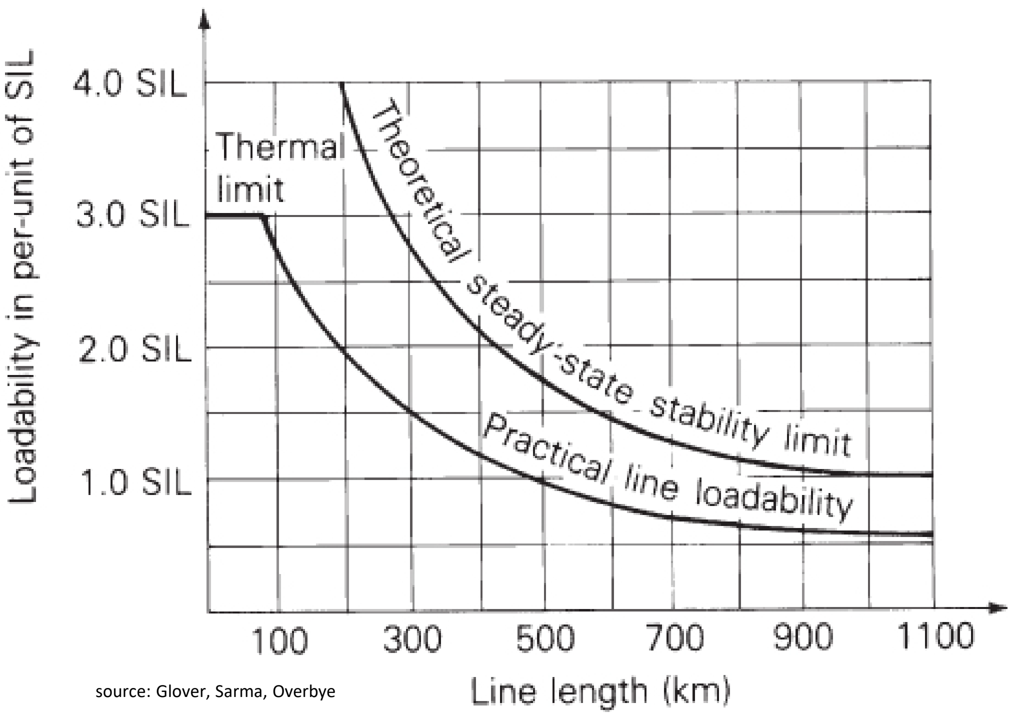
- **Phase shift**:
  - Proportional to ***line length*** and ***power flow***
  - Phase shift places a ***stability*** limit on power flow
  - Exceeding P_{max} ($\delta = 90°$) results in loss of synchronism
  - For satisfactory transient stability, typically $\delta \leq 30° \ldots 35°$
  - ***Stability*** limits the loadability of ***long transmission lines*** (>150 mi)

Transmission Line Loadability

- **Voltage drop**:
  - Voltage drop along a line is also proportional to ***line length*** and ***power flow***
  - Typically, voltage drop limited to 5% – 10%
  - ***Voltage drop*** limits power flow on ***medium-length lines*** (50mi – 150 mi)
- **Thermal limits**
  - As power flow increases, line temperature increases
  - As temperature increases, lines sag and loose tensile strength
  - A line's thermal limit is independent of line length
  - ***Thermal limits*** dominate for ***short lines*** (<50 mi)

Transmission Line Loadability

- Comparison of theoretical and practical loadability limits
- Practical limit assumes:
  - $V_R/V_s \geq 0.95$
  - $\delta \leq 30° \ldots 35°$



source: Glover, Sarma, Overbye

Practical Line Loadability – Example

- Determine how much power that can be transmitted over a 400 km, 500 kV transmission line, given the following:
  - Voltage drop along the line limited to 10%
  - Power angle limited to $\delta_{max} = 30°$
  - The characteristic impedance of the line is $Z_c = 280\Omega$
  - Assume $V_{S,pu} = 1.0\ p.u.$

- Power delivered to the receiving end of the line is

$$P_R = V_{R,pu} V_{S,pu} SIL \frac{\sin(\delta)}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

$$P_R = 0.9 \cdot 1.0 \cdot SIL \frac{\sin(30°)}{\sin\left(\frac{2\pi \cdot 400\ km}{5000\ km}\right)}$$

Practical Line Loadability – Example

- In terms of SIL, the power the line can deliver is

$$P_R = 0.934 \cdot SIL$$

- Surge impedance loading for the line is

$$SIL = \frac{V_{rated}^2}{Z_c} = \frac{(500\ kV)^2}{280\ \Omega} = 892.9\ MW$$

so,

$$P_R = 0.934 \cdot 892.9\ MW$$

$$\boxed{P_R = 834\ MW}$$

Example Problems

A 180 km, three-phase transmission line delivers 80 MW at 115 kV and a power factor of 0.96, lagging. The series impedance of the lines is $z = 0.03 + j0.3\ \Omega/\text{km}$, and the shunt admittance is $y = j4\ \mu\text{S/km}$.

a) Determine the appropriate set of chain parameters for the line.
b) How much power is delivered to the sending end of the line?

A 500 km transmission line with surge impedance of $Z_c = 270\ \Omega$ is used to deliver 1800 MW from a power plant to a load center. If the voltage drop along the line is limited to 6%, and the power angle is limited to 33°, what is the minimum rated voltage for the line?

A 400 km, 500 kV transmission line has a series impedance of $z = 0.03 + j0.35\ \Omega/km$ and a shunt admittance of $y = j4.4\ \mu S/km$. At full load, it delivers 1000 MW at 475 kV and unity power factor. Determine:

a) ABCD parameters
b) Sending-end voltage, current, power, and power factor
c) Full-load line losses

Reactive Compensation

Reactive Compensation

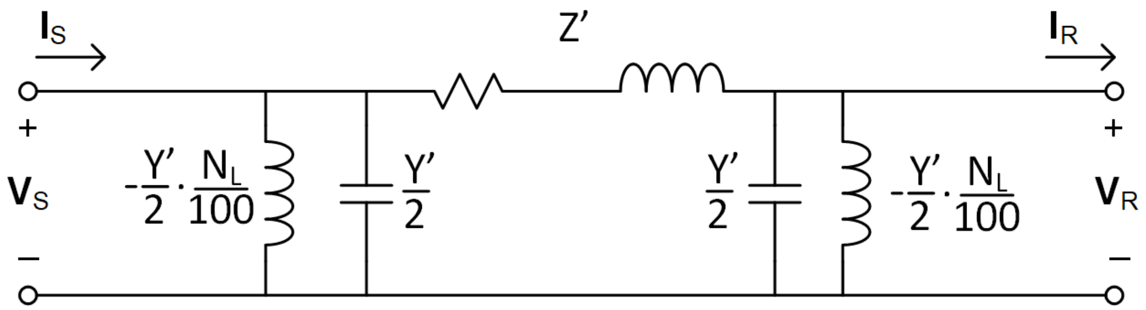
- Voltage profile and loadability of a transmission line depend on relative line and load impedances
  - By varying line impedance, we can affect voltage regulation and line loadability
  - Add shunt or series reactance to the line – ***reactive compensation***
- Types of reactive compensation
  - ***Shunt reactors*** (inductors)
    - Absorb reactive power
    - Reduce receiving-end voltage under light load
    - Must be removed under higher-load conditions
  - ***Shunt capacitors***
    - Supply reactive power
    - Increase receiving-end voltage at full load
    - Removed under light-load conditions

Reactive Compensation

- Types of reactive compensation (cont'd)
  - ***Series capacitors***
    - Reduce series line impedance
    - Reduce line voltage drops
    - Increase steady-state stability limit
  - ***Static VAR compensators*** (***SVCs***)
    - Thyristor-controlled shunt reactors and capacitors
    - Automatically adjust compensation depending on load

Reactive Compensation

- Amount of reactive compensation is typically expressed as a percentage of line impedance



- For example, the circuit above shows a transmission line with $NL\%$ shunt reactive compensation

Reactive Compensation – Example 1

- Consider a 300 km, 765 kV, three-phase transmission line with the following chain parameters:
  - $A = 0.9313\angle 0.209°$
  - $B = Z' = 97\angle 87.2°$
  - Shunt reactors, switched in during light-load conditions only, provide 75% compensation
  - Full-load current is 1.9 kA at 730 kV with unity power factor
  - The sending-end voltage, V_S, is constant
- Determine:
  - $\%VR$ of the uncompensated line
  - $\%VR$ of the compensated line

Reactive Compensation – Example 1

- Full-load, line-to-neutral, receiving-end voltage, using it as the 0° phase reference:

$$\boldsymbol{V}_{RFL} = \frac{730}{\sqrt{3}}\angle 0°\ kV = 421.5\angle 0°\ kV$$

- Use chain parameters to determine the sending-end voltage, $\boldsymbol{V}_S$

$$\boldsymbol{V}_S = A\boldsymbol{V}_{RFL} + B\boldsymbol{I}_{RFL}$$

$$\boldsymbol{V}_S = (0.9313\angle 0.209°)(421.5\angle 0°\ kV) + (97\angle 87.2°\ \Omega)(1.9\angle 0°\ kA)$$

$$\boldsymbol{V}_S = 442.3\angle 24.8°\ kV$$

- The no-load, line-to-neutral, receiving-end voltage is

$$\boldsymbol{V}_{RNL} = \frac{\boldsymbol{V}_S}{A} = \frac{442.3\angle 24.8°\ kV}{0.9313\angle 0.209°} = 474.9\angle 24.6°\ kV$$

- Percent voltage regulation for the uncompensated line is

$$\%VR = \frac{|V_{RNL}| - |V_{RFL}|}{|V_{RFL}|}\cdot 100\% = \frac{474.9\ kV - 421.5\ kV}{421.5\ kV}\cdot 100\%$$

$$\boxed{\%VR = 12.7\%}$$

Reactive Compensation – Example 1

- For the compensated line, we need to calculate new chain parameters
- Shunt admittance of the uncompensated line can be determined from the known chain parameters

$$A = 0.9313\angle 0.209° = 1 + \frac{Y'Z'}{2}$$

where

$$Z' = B = 97\angle 87.2\ \Omega$$

So,

$$Y' = \frac{(A-1)2}{Z'} = \frac{[(0.9313\angle 0.209°) - 1]2}{97\angle 87.2°\ \Omega}$$

$$Y' = 1.418 \times 10^{-3}\angle 89.97°\ S$$

$$Y' = 759 \times 10^{-9} + j1.42 \times 10^{-3}\ S$$

Reactive Compensation – Example 1

- After adding compensation, the equivalent shunt *susceptance* decreases by 75%

$$Y_{eq} = 759 \times 10^{-9} + j1.42 \times 10^{-3}\, S \cdot 0.25$$

$$Y_{eq} = 759 \times 10^{-9} + j355 \times 10^{-6}\, S$$

- Use Y_{eq} to calculate the A parameter for the compensated line

$$A_{eq} = 1 + \frac{Y_{eq} Z'}{2} = 0.983\angle 0.05°$$

- Note that shunt reactive compensation does not affect the series impedance, Z', and therefor does not affect B

Reactive Compensation – Example 1

- The no-load receiving-end voltage for the compensated line:

$$\boldsymbol{V}_{RNL} = \frac{\boldsymbol{V}_S}{A_{eq}} = \frac{442.3\angle 24.8° \; kV}{0.983\angle 0.05°}$$

$$\boldsymbol{V}_{RNL} = 449.9\angle 24.8° \; kV$$

- Percent voltage regulation for the compensated line is

$$\%VR = \frac{|\boldsymbol{V}_{RNL}| - |\boldsymbol{V}_{RFL}|}{|\boldsymbol{V}_{RFL}|} \cdot 100\%$$

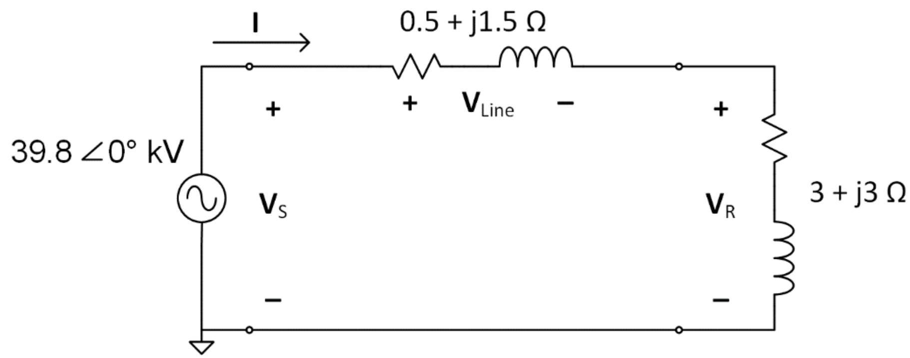
$$\%VR = \frac{449.9 \; kV - 421.5 \; kV}{421.5 \; kV} \cdot 100\%$$

$$\boxed{\%VR = 6.8\%}$$

- Reactive compensation has improved voltage regulation from 12.7% to 6.8%

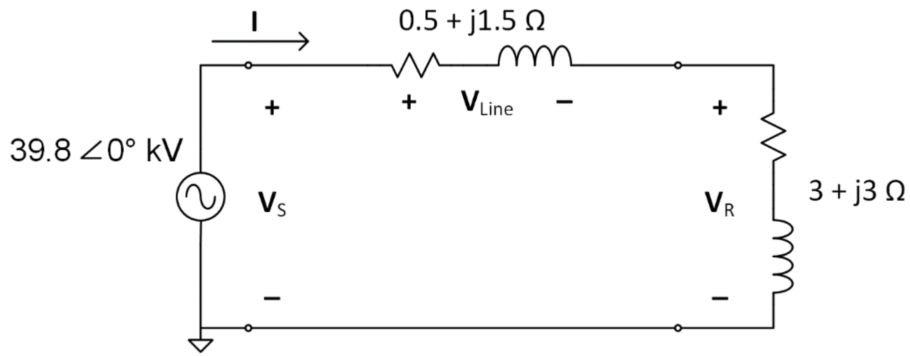
Reactive Compensation – Example 2

- In this example we will use phasor diagrams to illustrate the ***relationship between reactive power flow and line voltage***
- Consider a the following per-phase circuit
  - Could loosely represent a 69 kV subtransmission line
    - Values exaggerated for illustration purposes



- We will look at the effect of adding shunt capacitive compensation at the receiving end

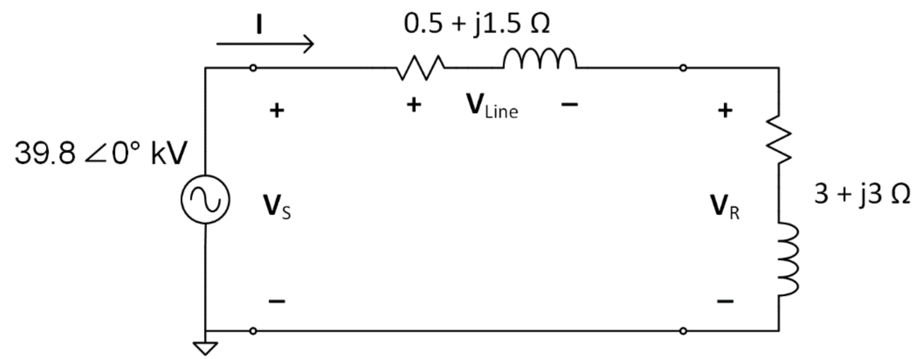
Reactive Compensation – Example 2



- Three scenarios considered:
  1. P_R = 145 MW; no compensation; pf = 0.707, lagging
  2. P_R = 145 MW; -j35 Ω shunt C; pf = 0.99, lagging
  3. P_R = 145 MW; -j15 Ω shunt C; pf = 0.95, leading
- Note that real power to the load is held constant
  - Equivalent load impedance adjusted to accomplish this
  - Again, power is exaggerated for illustration purposes

Reactive Compensation – Example 2

- **Scenario #1**:
  - No reactive compensation
  - $P_R = 145\ MW$
  - $pf = 0.707$, lagging

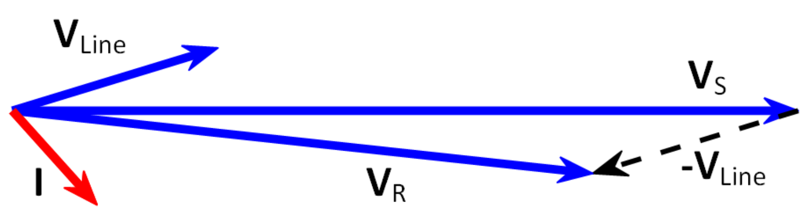


- Lagging current:

$$\mathbf{I} = 6.98\angle - 52°\ kA$$

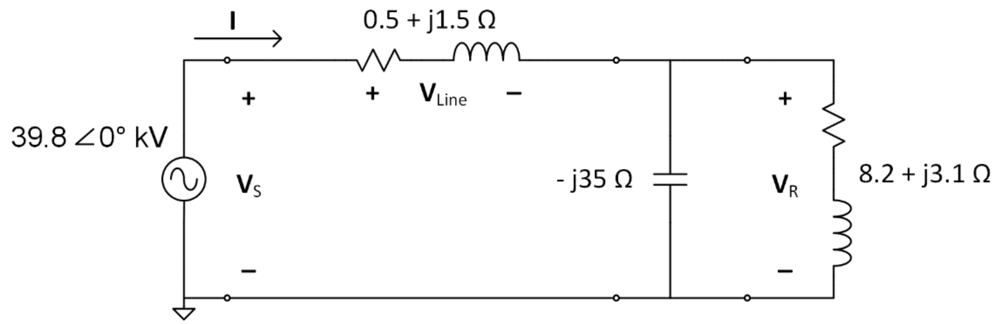
- Receiving end voltage:

$$\mathbf{V}_R = 29.6\angle - 7.1°\ kV$$



Reactive Compensation – Example 2

- **Scenario #2**:
  - $-j35\ \Omega$ shunt compensation
  - $P_R = 145\ MW$
  - $pf = 0.99$, lagging

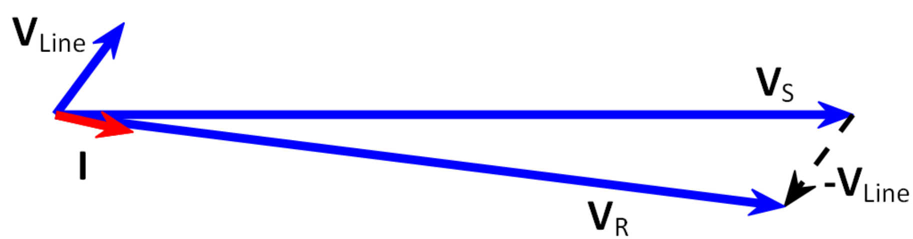


- Current magnitude and phase reduced:

$$\mathbf{I} = 3.97\angle -14.5°\ kA$$

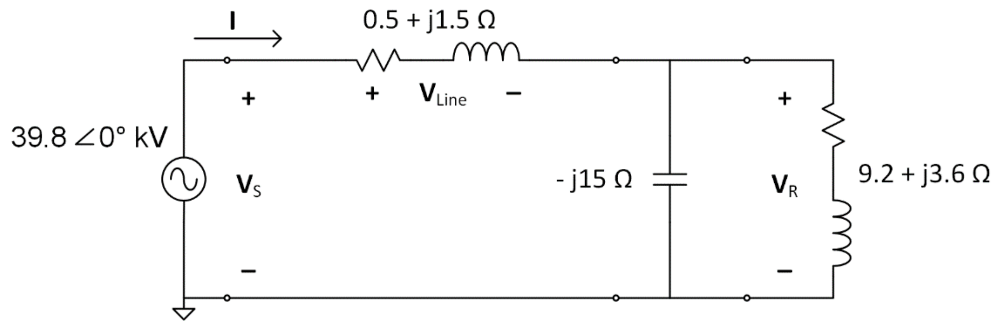
- Receiving end voltage increased:

$$\mathbf{V}_R = 36.8\angle -8.2°\ kV$$



Reactive Compensation – Example 2

- **Scenario #3**:
  - $-j15\ \Omega$ shunt compensation
  - $P_R = 145\ MW$
  - $pf\ = 0.95$, *leading*

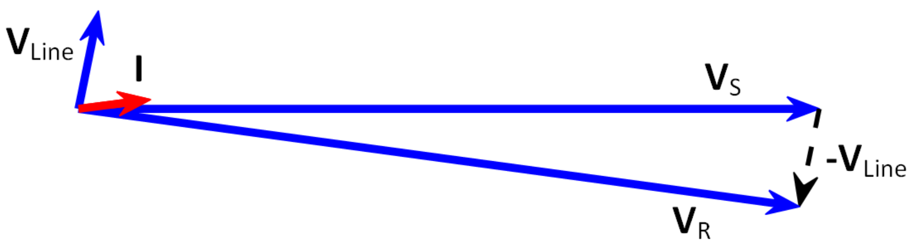


- Current now *leads* the source:

$$\mathbf{I} = 3.9\angle 8.4°\ kA$$

- Receiving end voltage increased further:

$$\mathbf{V}_R = 39.2\angle - 8.9°\ kV$$



Example Problems

Draw a phasor diagram indicating V_S, I, V_L, and V_R for the following circuit for a source power of

a) $\mathbf{S}_s = 10\angle -45°\ MVA$
b) $\mathbf{S}_s = 10\angle 0°\ MVA$
c) $\mathbf{S}_s = 10\angle 45°\ MVA$