Introduction
Power System Faults

- **Faults** in three-phase power systems are **short circuits**
  - Line-to-ground
  - Line-to-line

- Result in the flow of excessive current
  - Damage to equipment
    - Heat – burning/melting
    - Structural damage due to large magnetic forces

- **Bolted** short circuits
  - True short circuits – i.e., zero impedance

- In general, fault impedance may be non-zero

- Faults may be **opens** as well
  - We’ll focus on short circuits
Types of Faults

- Type of faults from most to least common:
  - Single line-to-ground faults
  - Line-to-line faults
  - Double line-to-ground faults
  - Balanced three-phase (symmetrical) faults

- We’ll look first at the least common type of fault – the symmetrical fault – due to its simplicity
Subtransient Fault Current
Faults occur nearly instantaneously
- Lightening, tree fall, arcing over insulation, etc.

Step change from steady-state behavior
- Like throwing a switch to create the fault at $t = 0$

Consider an unloaded synchronous generator
- Equivalent circuit model:

- $R$: generator resistance
- $L$: generator inductance
- $i(t) = 0$ for $t < 0$
- Source phase, $\alpha$, determines voltage at $t = 0$
  - Short circuit can occur at any point in a 60 Hz cycle

\[ v(t) = \sqrt{2}V_0 \sin(\omega t + \alpha) \]
Fault Current

- The governing differential equation for $t > 0$ is

$$\frac{di}{dt} + i(t) \frac{R}{L} = \frac{\sqrt{2}V_G}{L} \sin(\omega t + \alpha)$$

- The solution gives the fault current

$$i(t) = \frac{\sqrt{2}V_G}{Z} \left[ \sin(\omega t + \alpha - \theta) - \sin(\alpha - \theta) e^{-t\frac{R}{L}} \right]$$

where $Z = \sqrt{R^2 + (\omega L)^2}$ and $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

- This total fault current is referred to as the asymmetrical fault current
  - It has a **steady-state** component

$$i_{ac}(t) = \frac{\sqrt{2}V_G}{Z} \sin(\omega t + \alpha - \theta)$$

  - And a **transient** component

$$i_{dc}(t) = -\frac{\sqrt{2}V_G}{Z} \sin(\alpha - \theta) e^{-t\frac{R}{L}}$$
Fault Current

- **Magnitude of the transient fault current,** \( i_{dc} \), **depends on** \( \alpha \)
  - \( i_{dc}(0) = 0 \) **for** \( \alpha = \theta \)
  - \( i_{dc}(0) = \sqrt{2}I_{ac} \) **for** \( \alpha = \theta - 90^\circ \)
  - \( I_{ac} = V_G/Z \) is the rms value of the steady-state fault current

- **Worst-case fault current** occurs for \( \alpha = \theta - 90^\circ \)

\[
i(t) = \frac{\sqrt{2}V_G}{Z} \left[ \sin\left(\frac{\omega t - \pi}{2}\right) + e^{-tR/L} \right]
\]
Fault Current

- Important points here:
  - Total fault current has both steady-state and transient components – asymmetrical
  - Magnitude of the asymmetry (transient component) depends on the phase of the generator voltage at the time of the fault
  - In this class, we will use the steady-state current component, \( I_{ac} \), as our primary fault current metric
The reactance of the generator was assumed constant in the previous example.

Physical characteristics of real generators result in a time-varying reactance following a fault.

Time-dependence modeled with three reactance values:

- $X''_d$: subtransient reactance
- $X'_d$: transient reactance
- $X_d$: synchronous reactance

Reactance increases with time, such that $X''_d < X'_d < X_d$. 
Sub-Transient Fault Current

- Transition rates between reactance values are dictated by two time constants:
  - \( \tau_{d''} \): short-circuit subtransient time constant
  - \( \tau_{d'} \): short-circuit transient time constant

- Neglecting generator resistance, i.e. assuming \( \theta = 90^\circ \), the synchronous portion of the fault current is

\[
i_{ac}(t) = \sqrt{2}V_G \left[ \left( \frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-t/\tau_{d''}} + \left( \frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/\tau_d} + \frac{1}{X_d} \right] \sin \left( \omega t + \alpha - \frac{\pi}{2} \right)
\]

- At the instant of the fault, \( t = 0 \), the rms synchronous fault current is

\[
I''_F = \frac{V_G}{X_d''}
\]

- This is the rms \textit{subtransient fault current}, \( I''_F \)
- This will be our primary metric for assessing fault current
Symmetrical Three-Phase Short Circuits
Symmetrical 3-\( \phi \) Short Circuits

- Next, we’ll calculate the subtransient fault current resulting from a balanced three-phase fault.

- We’ll make the following simplifying assumptions:
  - Transformers modeled with leakage reactance only
    - Neglect winding resistance and shunt admittances
    - Neglect \( \Delta-Y \) phase shifts
  - Transmission lines modeled with series reactance only
  - Synchronous machines modeled as constant voltage sources in series with subtransient reactances
    - Generators and motors
  - Induction motors are neglected or modeled as synchronous motors
  - Non-rotating loads are neglected
Symmetrical 3-φ Short Circuits

- We’ll apply superposition to determine three-phase subtransient fault current.
- Consider the following power system:

![Diagram]

- Assume there is a balanced three-phase short of bus 1 to ground at \( t = 0 \)
Symmetrical 3-Φ Short Circuits

- The instant of the fault can be modeled by the switch closing in the following line-to-neutral schematic.

- The short circuit (closed switch) can be represented by two back-to-back voltage sources, each equal to $V_F$. 
Symmetrical 3-ϕ Short Circuits

Applying superposition, we can represent this circuit as the sum of two separate circuits:

**Circuit 1**

**Circuit 2**
Assume that the value of the fault-location source, $V_F$, is the **pre-fault voltage** at that location.

- Circuit 1, then, represents the **pre-fault circuit**, so

  \[ I''_{F1} = 0 \]

- The $V_F$ source can therefore be removed from circuit 1.
Symmetrical 3-ϕ Short Circuits

- The current in circuit 1, $I_L$, is the pre-fault line current.

- Superposition gives the fault current:
  \[ I''_F = I''_{F1} + I''_{F2} = I''_{F2} \]

- The generator fault current is
  \[ I''_G = I''_{G1} + I''_{G2} \]
  \[ I''_G = I_L + I''_{G2} \]

- The motor fault current is
  \[ I''_M = I''_{M1} + I''_{M2} \]
  \[ I''_M = -I_L + I''_{M2} \]
Symmetrical 3-ϕ Fault – Example

- For the simple power system above:
  - Generator is supplying rated power
  - Generator voltage is 5% above rated voltage
  - Generator power factor is 0.95 lagging

- A bolted three-phase fault occurs at bus 1

- Determine:
  - Subtransient fault current
  - Subtransient generator current
  - Subtransient motor current
Symmetrical 3-φ Fault – Example

- First convert to per-unit
  - Use \( S_b = 100 \text{ MVA} \)
- Base voltage in the transmission line zone is
  \[ V_{b,tl} = 138 \text{ kV} \]
- Base impedance in the transmission line zone is
  \[ Z_{b,tl} = \frac{V_{b,tl}^2}{S_b} = \frac{(138 \text{ kV})^2}{100 \text{ MVA}} = 190.4 \text{ Ω} \]
- The per-unit transmission line reactance is
  \[ X_{tl} = \frac{20 \text{ Ω}}{190.4 \text{ Ω}} = 0.105 \text{ p.u.} \]
Symmetrical 3-ϕ Fault – Example

- The two per-unit circuits are

- These can be simplified by combining impedances
Symmetrical 3-φ Fault – Example

- Using circuit 2, we can calculate the subtransient fault current

\[ I''_F = \frac{1.05\angle0\degree}{j0.116} = 9.079\angle - 90\degree \text{ p.u.} \]

- To convert to kA, first determine the current base in the generator zone

\[ I_{b,G} = \frac{S_b}{\sqrt{3}V_{b,G}} = \frac{100 \text{ MVA}}{\sqrt{3} \cdot 13.8 \text{ kV}} = 4.18 \text{ kA} \]

- The **subtransient fault current** is

\[ I''_F = (9.079\angle - 90\degree) \cdot 4.18 \text{ kA} \]

\[ I''_F = 37.98\angle - 90\degree \text{ kA} \]
Symmetrical 3-ϕ Fault – Example

The **pre-fault line current** can be calculated from the pre-fault generator voltage and power

$$I_L = \frac{S_G}{\sqrt{3}V''_G} = \frac{100\ MVA}{\sqrt{3} \cdot 1.05 \cdot 13.8\ kV} \angle - \cos^{-1}(0.95)$$

$$I_L = 3.98\angle - 18.19^\circ\ kA$$

Or, in per-unit:

$$I_L = \frac{3.98\angle - 18.19^\circ\ kA}{4.18\ kA} = 0.952\angle - 18.19^\circ\ p.u.$$  

This will be used to find the generator and motor fault currents
The generator’s contribution to the fault current is found by applying current division

\[ I''_{G2} = I''_F \frac{0.505}{0.505 + 0.15} = 7.0 \angle -90^\circ \text{ p.u.} \]

Adding the pre-fault line current, we have the \textit{subtransient generator fault current}

\[ I''_G = I_L + I''_{G2} \]

\[ I''_G = 0.952 \angle -18.19^\circ + 7.0 \angle -90^\circ \]

\[ I''_G = 7.35 \angle -82.9^\circ \text{ p.u.} \]

Converting to kA

\[ I''_G = (7.35 \angle -82.9^\circ) \cdot 4.18 \text{ kA} \]

\[ I''_G = 30.74 \angle -82.9^\circ \text{ kA} \]
Symmetrical 3-ϕ Fault – Example

- Similarly, for the motor
  \[ I''_{M2} = I''_F \frac{0.15}{0.505 + 0.15} = 2.08\angle - 90^\circ \text{ p.u.} \]

- Subtracting the pre-fault line current gives the subtransient motor fault current
  \[ I''_M = -I_L + I''_{M2} \]
  \[ I''_M = -0.952\angle - 18.19^\circ + 2.08\angle - 90^\circ \]
  \[ I''_M = 2.0\angle - 116.9^\circ \]

- Converting to kA
  \[ I''_M = (2.0\angle - 116.9^\circ) \cdot 4.18 \text{ kA} \]
  \[ I''_M = 8.36\angle - 116.9^\circ \text{ kA} \]
Symmetrical Components
Symmetrical Components

- In the previous section, we saw how to calculate subtransient fault current for balanced three-phase faults.
- Unsymmetrical faults are much more common.
  - Analysis is more complicated.
- We’ll now learn a tool that will simplify the analysis of unsymmetrical faults.
  - The *method of symmetrical components*
Symmetrical Components

- The method of symmetrical components:
  - Represent an asymmetrical set of $N$ phasors as a sum of $N$ sets of symmetrical component phasors.
  - These $N$ sets of phasors are called sequence components.

- Analogous to:
  - Decomposition of electrical signals into differential and common-mode components.
  - Decomposition of forces into orthogonal components.

- For a three-phase system ($N = 3$), sequence components are:
  - Zero sequence components
  - Positive sequence components
  - Negative sequence components
Sequence Components

- **Zero sequence components**
  - Three phasors with equal magnitude and equal phase
  - $V_{a0}$, $V_{b0}$, $V_{c0}$

- **Positive sequence components**
  - Three phasors with equal magnitude and $\pm 120^\circ$, positive-sequence phase
  - $V_{a1}$, $V_{b1}$, $V_{c1}$

- **Negative sequence components**
  - Three phasors with equal magnitude and $\pm 120^\circ$, negative-sequence phase
  - $V_{a2}$, $V_{b2}$, $V_{c2}$
Sequence Components

- Note that the absolute phase and the magnitudes of the sequence components is not specified.
  - Magnitude and phase define a unique set of sequence components.

- Any set of phasors – balanced or unbalanced – can be represented as a sum of sequence components.

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c \\
\end{bmatrix} = \begin{bmatrix}
V_{a0} \\
V_{b0} \\
V_{c0} \\
\end{bmatrix} + \begin{bmatrix}
V_{a1} \\
V_{b1} \\
V_{c1} \\
\end{bmatrix} + \begin{bmatrix}
V_{a2} \\
V_{b2} \\
V_{c2} \\
\end{bmatrix}
\]  (1)
Sequence Components

- The phasors of each sequence component have a fixed phase relationship
  - If we know one, we know the other two
  - Assume we know phase $a$ – use that as the reference

- For the zero sequence components, we have
  \[ V_0 = V_{a0} = V_{b0} = V_{c0} \]  
  \hspace{1cm} (2)

- For the positive sequence components,
  \[ V_1 = V_{a1} = (1\angle120^\circ) \cdot V_{b1} = (1\angle240^\circ) \cdot V_{c1} \]  
  \hspace{1cm} (3)

- And, for the negative sequence components,
  \[ V_2 = V_{a2} = (1\angle240^\circ) \cdot V_{b2} = (1\angle120^\circ) \cdot V_{c2} \]  
  \hspace{1cm} (4)

- Note that we’re using phase $a$ as our reference, so
  \[ V_0 = V_{a0}, \quad V_1 = V_{a1}, \quad V_2 = V_{a2} \]
Next, we define a complex number, $a$, that has unit magnitude and phase of 120°

$$a = 1\angle120°$$  \hspace{1cm} (5)

- Multiplication by $a$ results in a rotation (a phase shift) of 120°
- Multiplication by $a^2$ yields a rotation of 240° = −120°

Using (5) to rewrite (3) and (4)

$$V_1 = V_{a1} = aV_{b1} = a^2V_{c1}$$  \hspace{1cm} (6)

$$V_2 = V_{a2} = a^2V_{b2} = aV_{c2}$$  \hspace{1cm} (7)
**Sequence Components**

- Using (2), (6), and (7), we can rewrite (1) in a simplified form

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & 1 \\
1 & a & a^2
\end{bmatrix}
\begin{bmatrix}
V_0 \\
V_1 \\
V_2
\end{bmatrix}
\]

(8)

- The vector on the left is the vector of phase voltages, \(V_p\)
- The vector on the right is the vector of (phase \(a\)) sequence components, \(V_s\)
- We’ll call the \(3 \times 3\) transformation matrix \(A\)

- We can rewrite (8) as

\[
V_p = AV_s
\]

(9)
We can express the sequence voltages as a function of the phase voltages by inverting the transformation matrix

\[ V_s = A^{-1} V_p \]  

(10)

where

\[ A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \]  

(11)

So

\[ \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \]  

(12)
Sequence Components

- The same relationships hold for three-phase currents
- The phase currents are
  \[ I_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \]
- And, the sequence currents are
  \[ I_s = \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \]
Sequence Components

- The transformation matrix, $A$, relates the phase currents to the sequence currents

$$I_p = AI_s$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \quad (13)$$

- And vice versa

$$I_s = A^{-1}I_p$$

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (14)$$
Before applying sequence components to unbalanced systems, let’s first look at the sequence components for a balanced, positive-sequence, three-phase system.

For a balanced system, we have

\[
V_b = V_a \cdot 1 \angle -120° = a^2 V_a
\]

\[
V_c = V_a \cdot 1 \angle 120° = a V_a
\]

The sequence voltages are given by (12)

- The zero sequence voltage is

\[
V_0 = \frac{1}{3} [V_a + V_b + V_c] = \frac{1}{3} [V_a + a^2 V_a + a V_a]
\]

\[
V_0 = \frac{1}{3} V_a [1 + a^2 + a]
\]

- Applying the identity \(1 + a^2 + a = 0\), we have

\[
V_0 = 0
\]
Sequence Components – Balanced System

- The positive sequence component is given by

\[ V_1 = \frac{1}{3} [V_a + aV_b + a^2 V_c] \]

\[ V_1 = \frac{1}{3} [V_a + a \cdot a^2 V_a + a^2 \cdot aV_a] \]

\[ V_1 = \frac{1}{3} [V_a + a^3 V_a + a^3 V_a] \]

- Since \( a^3 = 1 \angle 0^\circ \), we have

\[ V_1 = \frac{1}{3} [3V_a] \]

\[ V_1 = V_a \]
The negative sequence component is given by

\[ V_2 = \frac{1}{3} [V_a + a^2 V_b + aV_c] \]

\[ V_2 = \frac{1}{3} [V_a + a^2 \cdot a^2 V_a + a \cdot a V_a] \]

\[ V_2 = \frac{1}{3} [V_a + a^4 V_a + a^2 V_a] \]

Again, using the identity \(1 + a^2 + a = 0\), along with the fact that \(a^4 = a\), we have

\[ V_2 = 0 \]
So, for a positive-sequence, balanced, three-phase system, the sequence voltages are

\[ V_0 = 0, \quad V_1 = V_a, \quad V_2 = 0 \]

Similarly, the sequence currents are

\[ I_0 = 0, \quad I_1 = I_a, \quad I_2 = 0 \]

This is as we would expect

- No zero- or negative-sequence components for a positive-sequence balanced system
- Zero- and negative-sequence components are only used to account for imbalance
Sequence Components

- We have just introduced the concept of *symmetric components*
  - Allows for decomposition of, possibly unbalanced, three-phase phasors into *sequence components*

- We’ll now apply this concept to power system networks to develop *sequence networks*
  - *Decoupled networks* for each of the sequence components
  - Sequence networks become *coupled only at the point of imbalance*
  - Simplifies the analysis of unbalanced systems
Sequence Networks
Sequence Networks

- **Power system components each have their own set of sequence networks**
  - Non-rotating loads
  - Transmission lines
  - Rotating machines – generators and motors
  - Transformers

- Sequence networks for overall systems are interconnections of the individual sequence network

- Sequence networks become coupled in a particular way at the fault location depending on type of fault
  - Line-to-line
  - Single line-to-ground
  - Double line-to-ground

- Fault current can be determined through simple analysis of the coupled sequence networks
Sequence Networks – Non-Rotating Loads
Consider a balanced Y-load with the neutral grounded through some non-zero impedance.

Applying KVL gives the phase-\(a\)-to-ground voltage:

\[ V_{ag} = Z_y I_a + Z_n I_n \]

\[ V_{ag} = Z_y I_a + Z_n (I_a + I_b + I_c) \]

\[ V_{ag} = (Z_y + Z_n) I_a + Z_n I_b + Z_n I_c \]  \hspace{1cm} (15)

For phase \(b\):

\[ V_{bg} = Z_y I_b + Z_n I_n = Z_y I_b + Z_n (I_a + I_b + I_c) \]

\[ V_{bg} = Z_n I_a + (Z_y + Z_n) I_b + Z_n I_c \]  \hspace{1cm} (16)

Similarly, for phase \(c\):

\[ V_{ag} = Z_n I_a + Z_n I_b + (Z_y + Z_n) I_c \]  \hspace{1cm} (17)
Putting (15) – (17) in matrix form

\[
\begin{bmatrix}
V_{ag} \\
V_{bg} \\
V_{cg}
\end{bmatrix} =
\begin{bmatrix}
(Z_y + Z_n) & Z_n & Z_n \\
Z_n & (Z_y + Z_n) & Z_n \\
Z_n & Z_n & (Z_y + Z_n)
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]

or

\[V_p = Z_p I_p\]  \hspace{1cm} (18)

where \(V_p\) and \(I_p\) are the phase voltages and currents, respectively, and \(Z_p\) is the *phase impedance matrix*

We can use (9) and (13) to rewrite (18) as

\[AV_s = Z_p AI_s\]

Solving for \(V_s\)

\[V_s = A^{-1}Z_p AI_s\]

or

\[V_s = Z_s I_s\]  \hspace{1cm} (19)
Sequence Networks – Non-Rotating Loads

\[ V_s = Z_s I_s \]  \hspace{1cm} (19)

where \( Z_s \) is the sequence impedance matrix

\[ Z_s = A^{-1} Z_p A = \begin{bmatrix} (Z_y + 3Z_n) & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix} \]  \hspace{1cm} (20)

- Equation (19) then becomes a set of three uncoupled equations

\[ V_0 = (Z_y + 3Z_n)I_0 = Z_0I_0 \]  \hspace{1cm} (21)
\[ V_1 = Z_y I_1 = Z_1I_1 \]  \hspace{1cm} (22)
\[ V_2 = Z_y I_2 = Z_2I_2 \]  \hspace{1cm} (23)
Equations (21) – (23) describe the uncoupled sequence networks

- **Zero-sequence network:**

- **Positive-sequence network:**

- **Negative-sequence network:**
We can develop similar sequence networks for a balanced Δ-connected load

- \( Z_y = Z_\Delta / 3 \)
- There is no neutral point for the Δ-network, so \( Z_n = \infty \) - an open circuit

**Zero-sequence network:**

**Positive-sequence network:**

**Negative-sequence network:**
Sequence Networks – 3-ϕ Lines
Balanced, three-phase lines can be modeled as

\[
\begin{align*}
\begin{bmatrix}
V_{aa'} \\
V_{bb'} \\
V_{cc'}
\end{bmatrix} &=
\begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} \\
Z_{ba} & Z_{bb} & Z_{bc} \\
Z_{ca} & Z_{cb} & Z_{cc}
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix} =
\begin{bmatrix}
V_{an} - V_{a'n} \\
V_{bn} - V_{b'n} \\
V_{cn} - V_{c'n}
\end{bmatrix}
\end{align*}
\]
Sequence Networks – 3-\(\phi\) Lines

- Writing (24) in compact form

\[
V_p - V_{p'} = Z_p I_p
\]  

(25)

- \(Z_p\) is the phase impedance matrix
  - Self impedances along the diagonal
  - Mutual impedances elsewhere
  - Symmetric
  - Diagonal, if we neglect mutual impedances
Sequence Networks – 3-ϕ Lines

- We can rewrite (25) in terms of sequence components

\[
AV_s - AV_s' = Z_p AI_s
\]

\[
V_s - V_s' = A^{-1} Z_p AI_s
\]

\[
V_s - V_s' = Z_s I_s
\]

(26)

where \( Z_s \) is the sequence impedance matrix

\[
Z_s = A^{-1} Z_p A
\]

(27)

- \( Z_s \) is diagonal so long as the system impedances are balanced, i.e.

  - Self impedances are equal: \( Z_{aa} = Z_{bb} = Z_{cc} \)
  - Mutual impedances are equal: \( Z_{ab} = Z_{ac} = Z_{bc} \)
Sequence Networks – 3-\(\phi\) Lines

- For balanced lines, \(Z_s\) is diagonal
  \[
  Z_s = \begin{bmatrix}
  Z_{aa} + 2Z_{ab} & 0 & 0 \\
  0 & Z_{aa} - Z_{ab} & 0 \\
  0 & 0 & Z_{aa} - Z_{ab}
  \end{bmatrix} = \begin{bmatrix}
  Z_0 & 0 & 0 \\
  0 & Z_1 & 0 \\
  0 & 0 & Z_2
  \end{bmatrix}
  \]

- Because \(Z_s\) is diagonal, (26) represents three uncoupled equations
  \[
  V_0 - V_0' = Z_0 I_0 \\
  V_1 - V_1' = Z_1 I_1 \\
  V_2 - V_2' = Z_2 I_2
  \]
Equations (28) – (30) describe the voltage drop across three uncoupled sequence networks.

- Zero-sequence network:

- Positive-sequence network:

- Negative-sequence network:
Sequence Networks – Rotating Machines
Consider the following model for a *synchronous generator*

- Similar to the Y-connected load
  - Generator includes *voltage sources* on each phase
- Voltage sources are *positive sequence*
  - Sources will appear only in the *positive-sequence network*
Sequence Networks – Synchronous Generator

- Sequence networks for Y-connected synchronous generator
  - Zero-sequence network:
    - Circuit diagram
  - Positive-sequence network:
    - Circuit diagram
  - Negative-sequence network:
    - Circuit diagram
Sequence Networks – Motors

- **Synchronous motors**
  - Sequence networks identical to those for synchronous generators
  - Reference current directions are reversed

- **Induction motors**
  - Similar sequence networks to synchronous motors, except source in the positive sequence network set to zero
Sequence Networks – Transformers
Sequence Networks - Y-Y Transformers

- Per-unit sequence networks for transformers
  - Simplify by neglecting transformer shunt admittances
- Consider a Y-Y transformer

- Similar to the Y-connected load, the voltage drops across the neutral impedances are $3I_0Z_N$ and $3I_0Z_n$
  - $3Z_N$ and $3Z_n$ each appear in the zero-sequence network
  - Can be combined in the per-unit circuit as long as shunt impedances are neglected
Impedance accounting for leakage flux and winding resistance for each winding can be referred to the primary.

- Add together into a single impedance, \( Z_s \), in the per-unit model.

**Y-Y transformer sequence networks**

- **Zero-sequence network:**

  \[ + \quad 3(Z_N + Z_n) \quad Z_s \quad + \]
  \[ V_{h0} \quad - \quad V_{x0} \quad - \]

- **Positive-sequence network:**

  \[ + \quad Z_s \quad + \]
  \[ V_{h1} \quad - \quad V_{x1} \quad - \]

- **Negative-sequence network:**

  \[ + \quad Z_s \quad + \]
  \[ V_{h2} \quad - \quad V_{x2} \quad - \]
Sequence Networks – Y-Δ Transformers

- Y-Δ transformers differ in a couple of ways
  
  - Must account for phase shift from primary to secondary
    - For positive-sequence network, Y-side voltage and current lead Δ-side voltage and current
    - For negative-sequence network, Y-side voltage and current lag Δ-side voltage and current
  
  - No neutral connection on the Δ side
    - Zero-sequence current cannot enter or leave the Δ winding
Sequence Networks – Y-ΔTransformers

- Sequence networks for Y-ΔTransformers

  - Zero-sequence network:

  ![Zero-sequence network diagram]

  - Positive-sequence network:

  ![Positive-sequence network diagram]

  - Negative-sequence network:

  ![Negative-sequence network diagram]
Sequence Networks – $\Delta$-$\Delta$ Transformers

- $\Delta$-$\Delta$ transformers
  - Like Y-Y transformers, no phase shift
  - No neutral connections
    - Zero-sequence current cannot flow into or out of either winding
Sequence Networks – Δ-Δ Transformers

- **Sequence networks for Δ-Δ Transformers**

- **Zero-sequence network:**

- **Positive-sequence network:**

- **Negative-sequence network:**
Power in Sequence Networks
Power in Sequence Networks

- We can relate the power delivered to a system’s sequence networks to the three-phase power delivered to that system.

- We know that the complex power delivered to a three-phase system is the sum of the power at each phase

\[ S_p = V_{an}I_a^* + V_{bn}I_b^* + V_{cn}I_c^* \]

- In matrix form, this looks like

\[
S_p = [V_{an} \quad V_{bn} \quad V_{cn}][I_a^* \\
I_b^* \\
I_c^*] \\
S_p = V_p^T I_p^* \quad (28)
\]
Power in Sequence Networks

- Recall the following relationships

\[ V_p = AV_s \] 
\[ I_p = AI_s \] 

- Using (9) and (13) in (28), we have

\[ S_p = (AV_s)^T (AI_s)^* \]
\[ S_p = V_s^T A^T A^* I_s^* \] 

- Computing the product in the middle of the right-hand side of (29), we find

\[
A^T A^* = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3I_3
\]

where \( I_3 \) is the 3×3 identity matrix.
Equation (29) then becomes

\[ S_p = V_s^T 3I_3 I_s^* \]
\[ S_p = 3V_s^T I_s^* \]
\[ S_p = 3[V_0 \quad V_1 \quad V_2] \begin{bmatrix} I_0^* \\ I_1^* \\ I_2^* \end{bmatrix} \]
\[ S_p = 3(V_0 I_0^* + V_1 I_1^* + V_2 I_2^*) \]

The total power delivered to a three-phase network is three times the sum of the power delivered to the three sequence networks.

The three sequence networks represent only one of the three phases – recall, we chose to consider only phase \( a \).
Example Problems
A bolted, symmetric, three-phase fault occurs 60% of the way from bus 1 to bus 2. Determine the subtransient fault current in per-unit and in amperes. The load is consuming rated power at rated voltage and unity power factor.
Determine the sequence components for the following unbalanced set of three-phase voltage phasors:

\[
\begin{align*}
V_a &= 1 \angle 0^\circ \text{ p.u.} \\
V_b &= 0.5 \angle -60^\circ \text{ p.u.} \\
V_c &= 2 \angle 200^\circ \text{ p.u.}
\end{align*}
\]
Determine the phase components for the following set of sequence components:

\[ V_0 = 1\angle 60^\circ \text{ p.u.} \]
\[ V_1 = 1\angle 0^\circ \text{ p.u.} \]
\[ V_2 = 0 \text{ p.u.} \]
Unsymmetrical Faults
Unsymmetrical Faults

- The majority of faults that occur in three-phase power systems are unsymmetrical
  - Not balanced
  - Fault current and voltage differ for each phase

- The method of symmetrical components and sequence networks provide us with a tool to analyze these unsymmetrical faults

- We’ll examine three types of unsymmetrical faults
  - Single line-to-ground (SLG) faults
  - Line-to-line (LL) faults
  - Double line-to-ground (DLG) faults
Unsymmetrical Fault Analysis - Procedure

- Basic procedure for fault analysis:
  1. Generate sequence networks for the system
  2. Interconnect sequence networks appropriately at the fault location
  3. Perform circuit analysis on the interconnected sequence networks
Unsymmetrical Fault Analysis

- To simplify our analysis, we’ll make the following assumptions
  1. System is balanced before the instant of the fault
  2. Neglect pre-fault load current
     - All pre-fault machine terminal voltages and bus voltages are equal to $V_F$
  3. Transmission lines are modeled as series reactances only
  4. Transformers are modeled with leakage reactances only
  5. Non-rotating loads are neglected
  6. Induction motors are either neglected or modeled as synchronous motors
Unsymmetrical Fault Analysis

- Each sequence network includes all interconnected power-system components
  - Generators, motors, lines, and transformers

- Analysis will be simplified if we represent each sequence network as its Thévenin equivalent
  - From the perspective of the fault location

- For example, consider the following power system:
Unsymmetrical Fault Analysis – Sequence Networks

- The sequence networks for the system are generated by interconnecting the sequence networks for each of the components.

- The zero-sequence network:

- The positive-sequence network:
  - Assuming the generator is operating at the rated voltage at the time of the fault.
Unsymmetrical Fault Analysis – Sequence Networks

- The negative sequence network:

Now, let’s assume there is some sort of fault at bus 1
- Determine the Thévenin equivalent for each sequence network from the perspective of bus 1
- Simplifying the zero-sequence network to its Thévenin equivalent
Unsymmetrical Fault Analysis – Sequence Networks

- The positive-sequence network simplifies to the following circuit with the following Thévenin equivalent:

  ![Positive-sequence network diagram]

- Similarly, for the negative-sequence network, we have:

  ![Negative-sequence network diagram]

- Next, we’ll see how to interconnect these networks to analyze different types of faults:
Single-Line-to-Ground Fault
Unsymmetrical Fault Analysis – SLG Fault

- The following represents a generic three-phase network with terminals at the fault location:

- If we have a single-line-to-ground fault, where phase $a$ is shorted through $Z_f$ to ground, the model becomes:
Unsymmetrical Fault Analysis – SLG Fault

- The **phase-domain fault conditions**:
  
  \[ I_a = \frac{V_{ag}}{Z_f} \]  
  
  \[ I_b = I_c = 0 \]  

- Transforming these phase-domain currents to the sequence domain:
  
  \[
  \begin{bmatrix}
  I_0 \\
  I_1 \\
  I_2
  \end{bmatrix} = \frac{1}{3}
  \begin{bmatrix}
  1 & 1 & 1 \\
  1 & a & a^2 \\
  1 & a^2 & a
  \end{bmatrix}
  \begin{bmatrix}
  V_{ag}/Z_f \\
  0 \\
  0
  \end{bmatrix} = \frac{1}{3}
  \begin{bmatrix}
  V_{ag}/Z_f \\
  V_{ag}/Z_f \\
  V_{ag}/Z_f
  \end{bmatrix}
  \]  

- This gives one of our **sequence-domain fault conditions**:
  
  \[ I_0 = I_1 = I_2 \]
We know that

\[ I_a = \frac{V_{ag}}{Z_f} = I_0 + I_1 + I_2 \]  \hspace{1cm} (5)

and

\[ V_{ag} = V_0 + V_1 + V_2 \]  \hspace{1cm} (6)

Using (5) and (6) in (1), we get

\[ I_0 + I_1 + I_2 = \frac{1}{Z_f} (V_0 + V_1 + V_2) \]

Using (4), this gives our second sequence-domain fault condition

\[ I_0 = I_1 = I_2 = \frac{1}{3Z_f} (V_0 + V_1 + V_2) \]  \hspace{1cm} (7)
Unsymmetrical Fault Analysis – SLG Fault

- The sequence-domain fault conditions are satisfied by connecting the sequence networks in series along with three times the fault impedance.

- We want to find the phase domain fault current, $I_F$

\[
I_F = I_\alpha = I_0 + I_1 + I_2 = 3I_1
\]

\[
I_1 = \frac{V_F}{Z_0 + Z_1 + Z_2 + 3Z_F}
\]

\[
I_F = \frac{3V_F}{Z_0 + Z_1 + Z_2 + 3Z_F}
\]
Returning to our example power system

The interconnected sequence networks for a *bolted fault* at bus 1:
SLG Fault - Example

- The fault current is

\[ I_F = \frac{3V_F}{Z_0+Z_1+Z_2+3Z_F} \]

\[ I_F = \frac{3.0\angle30^\circ}{j0.487} = 6.16\angle - 60^\circ \text{ p. u.} \]

- The current base at bus 1 is

\[ I_b = \frac{S_b}{\sqrt{3}V_{b1}} = \frac{150 \text{ MVA}}{\sqrt{3} 230 \text{ kV}} = 376.5 \text{ A} \]

- So the fault current in kA is

\[ I_F = (6.16\angle - 60^\circ)(376.5 \text{ A}) \]

\[ I_F = 2.32\angle - 60^\circ \text{ kA} \]
Line-to-Line Fault
Unsymmetrical Fault Analysis – LL Fault

- Now consider a line-to-line fault between phase \(b\) and phase \(c\) through impedance \(Z_F\).

- Phase-domain fault conditions:

\[
I_a = 0 \quad (9)
\]

\[
I_b = -I_c = \frac{V_{bg} - V_{cg}}{Z_F} \quad (10)
\]

- Transforming to the sequence domain

\[
\begin{bmatrix}
I_0 \\
I_1 \\
I_2
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix} \begin{bmatrix}
0 \\
I_b \\
-I_b
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
0 \\
(a - a^2)I_b \\
(a^2 - a)I_b
\end{bmatrix} \quad (11)
\]

- So, the first two sequence-domain fault conditions are

\[
I_0 = 0 \quad (12)
\]

\[
I_2 = -I_1 \quad (13)
\]
Unsymmetrical Fault Analysis – LL Fault

To derive the remaining sequence-domain fault condition, rearrange (10) and transform to the sequence domain

\[ V_{bg} - V_{cg} = I_b Z_F \]

\[ (V_0 + a^2V_1 + aV_2) - (V_0 + aV_1 + a^2V_2) = (I_0 + a^2I_1 + aI_2)Z_F \]

\[ a^2V_1 + aV_2 - aV_1 - a^2V_2 = (a^2 - a)I_1Z_F \]

\[ (a^2 - a)V_1 - (a^2 - a)V_2 = (a^2 - a)I_1Z_F \]

The last sequence-domain fault condition is

\[ V_1 - V_2 = I_1Z_F \] (14)
Unsymmetrical Fault Analysis – LL Fault

- **Sequence-domain fault conditions**
  
  \[ I_0 = 0 \]  
  
  \[ I_2 = -I_1 \]  
  
  \[ V_1 - V_2 = I_1 Z_F \]

- These can be satisfied by:
  - Leaving the zero-sequence network open
  - Connecting the terminals of the positive- and negative-sequence networks together through \( Z_F \)
The fault current is the phase $b$ current, which is given by

$$I_F = I_b = I_0 + a^2 I_1 + aI_2$$

$$I_F = a^2 I_1 - aI_1$$

$$I_F = -j\sqrt{3} I_1 = \frac{-j\sqrt{3} V_F}{Z_1 + Z_2 + Z_F}$$

$$I_F = \frac{\sqrt{3} V_F \angle -90^\circ}{Z_1 + Z_2 + Z_F}$$

(15)
Now consider the same system with a bolted line-to-line fault at bus 1

The sequence network:
LL Fault - Example

\[ I_1 = \frac{1 \angle 30^\circ}{j0.363} = 2.75 \angle - 60^\circ \]

- The subtransient fault current is given by (15) as
  \[ I_F = \sqrt{3}(2.75 \angle - 60^\circ) \angle - 90^\circ \]
  \[ I_F = 4.76 \angle - 150^\circ \text{ p.u.} \]

- Using the previously-determined current base, we can convert the fault current to kA
  \[ I_F = I_{b1} \cdot 4.76 \angle - 150^\circ \]
  \[ I_F = 4.76 \angle - 150^\circ (376.5A) \]
  \[ I_F = 1.79 \angle - 150^\circ \text{ kA} \]
Double-Line-to-Ground Fault
Now consider a **double line-to-ground fault**

- Assume phases $b$ and $c$ are shorted to ground through $Z_F$

Phase-domain fault conditions:

\[ I_a = 0 \]  

(16)

\[ I_b + I_c = \frac{V_{bg}}{Z_F} = \frac{V_{cg}}{Z_F} \]  

(17)

It can be shown that (16) and (17) transform to the following sequence-domain fault conditions (analysis skipped here)

\[ I_0 + I_1 + I_2 = 0 \]  

(18)

\[ V_1 = V_2 \]  

(19)

\[ I_0 = \frac{1}{3Z_F}(V_0 - V_1) \]  

(20)
Unsymmetrical Fault Analysis – DLG Fault

- Sequence-domain fault conditions

\[ I_0 + I_1 + I_2 = 0 \]  \hspace{1cm} (18)

\[ V_1 = V_2 \]  \hspace{1cm} (19)

\[ I_0 = \frac{1}{3Z_F} (V_0 - V_1) \]  \hspace{1cm} (20)

- To satisfy these fault conditions
  - Connect the positive- and negative-sequence networks together directly
  - Connect the zero- and positive-sequence networks together through \( 3Z_F \)
Unsymmetrical Fault Analysis – DLG Fault

- The fault current is the sum of the phase \( b \) and phase \( c \) currents, as given by (17)
  - In the sequence domain the fault current is
    \[
    I_F = I_b + I_c = 3I_0
    \]
    \[
    I_F = 3I_0
    \] (21)

- \( I_0 \) can be determined by a simple analysis (e.g. nodal) of the interconnected sequence networks
Now determine the subtransient fault current for a bolted double line-to-ground fault at bus 1

The sequence network:

Here, because $Z_F = 0$, $V_0 = V_1 = V_2$
DLG Fault - Example

- To find $I_F$, we must determine $I_0$
- We can first find $V_0$ by applying voltage division

\[ V_0 = V_F \frac{Z_2 || Z_0}{Z_1 + Z_2 || Z_0} \]

\[ V_0 = 1.0 \angle 30^\circ \frac{j0.194 || j0.124}{j0.169 + j0.194 || j0.124} \]

\[ V_0 = 0.309 \angle 30^\circ \]
Next, calculate \( I_0 \)

\[
I_0 = \frac{-V_0}{Z_0} = \frac{-0.309 \angle 30^\circ}{j0.124} = 2.49 \angle 120^\circ \text{ p.u.}
\]

The per-unit fault current is

\[
I_F = 3I_0 = 7.48 \angle 120^\circ \text{ p.u.}
\]

Using the current base to convert to kA, gives the subtransient DLG fault current

\[
I_F = 7.48 \angle 120^\circ (376.5 \text{ A})
\]

\[
I_F = 2.82 \angle 120^\circ \text{ kA}
\]
Example Problems
Draw the sequence networks for the following power system. Assume the generator is operating at rated voltage.
Reduce the sequence networks to their Thévenin equivalents for a fault occurring half of the way along the transmission line.
Determine the subtransient fault current resulting from a DLG fault, half way along the transmission line, through an impedance of j0.2 p.u.