SECTION 7: FAULT ANALYSIS

ESE 470 – Energy Distribution Systems

Introduction

Power System Faults

- ***Faults*** in three-phase power systems are ***short circuits***
  - Line-to-ground
  - Line-to-line
- Result in the flow of excessive current
  - Damage to equipment
    - Heat – burning/melting
    - Structural damage due to large magnetic forces
- ***Bolted*** short circuits
  - True short circuits – i.e., zero impedance
- In general, fault impedance may be non-zero
- Faults may be ***opens*** as well
  - We'll focus on short circuits

Types of Faults

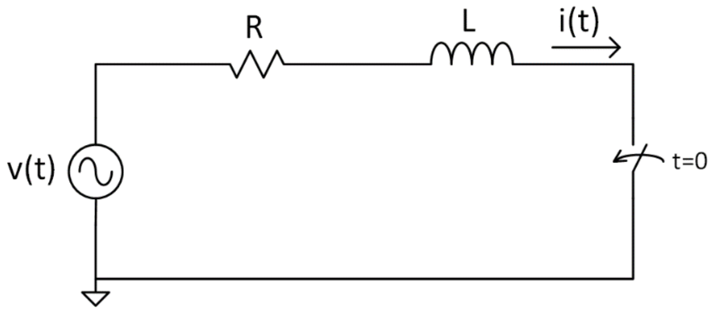
- Type of faults from most to least common:
  - Single line-to-ground faults
  - Line-to-line faults
  - Double line-to-ground faults
  - Balanced three-phase (symmetrical) faults

- We'll look first at the least common type of fault – the symmetrical fault – due to its simplicity

Subtransient Fault Current

Fault Current

- Faults occur nearly instantaneously
  - Lightening, tree fall, arcing over insulation, etc.
- Step change from steady-state behavior
  - Like throwing a switch to create the fault at $t = 0$
- Consider an unloaded synchronous generator
  - Equivalent circuit model:



$v(t) = \sqrt{2} V_G \sin(\omega t + \alpha)$

  - R: generator resistance
  - L: generator inductance
  - $i(t) = 0$ for $t < 0$
  - Source phase, α, determines voltage at $t = 0$
    - Short circuit can occur at any point in a 60 Hz cycle

Fault Current

- The governing differential equation for $t > 0$ is

$$\frac{di}{dt} + i(t)\frac{R}{L} = \frac{\sqrt{2}V_G}{L}\sin(\omega t + \alpha)$$

- The solution gives the fault current

$$i(t) = \frac{\sqrt{2}V_G}{Z}\left[\sin(\omega t + \alpha - \theta) - \sin(\alpha - \theta)\, e^{-t\frac{R}{L}}\right]$$

where $Z = \sqrt{R^2 + (\omega L)^2}$ and $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

- This total fault current is referred to as the ***asymmetrical fault current***
  - It has a ***steady-state*** component

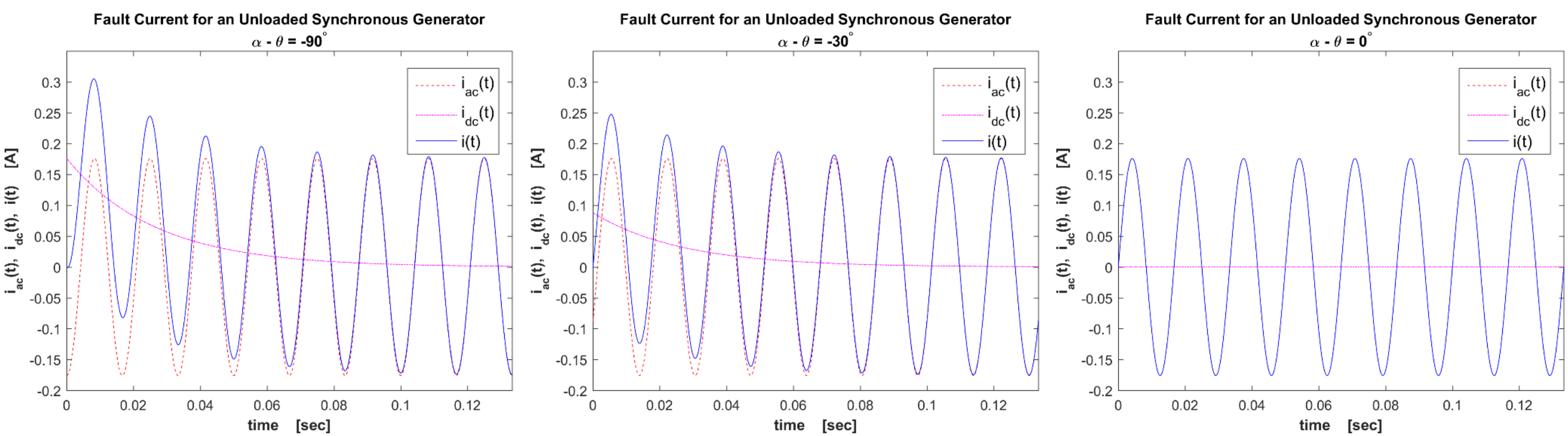
$$i_{ac}(t) = \frac{\sqrt{2}V_G}{Z}\sin(\omega t + \alpha - \theta)$$

  - And a ***transient*** component

$$i_{dc}(t) = -\frac{\sqrt{2}V_G}{Z}\sin(\alpha - \theta)\, e^{-t\frac{R}{L}}$$

Fault Current

- Magnitude of the transient fault current, i_{dc}, depends on α
  - $i_{dc}(0) = 0 \quad \text{for} \quad \alpha = \theta$
  - $i_{dc}(0) = \sqrt{2} I_{ac} \quad \text{for} \quad \alpha = \theta - 90°$
    - $I_{ac} = V_G / Z$ is the rms value of the steady-state fault current



- Worst-case fault current occurs for $\alpha = \theta - 90°$

$$i(t) = \frac{\sqrt{2} V_G}{Z} \left[\sin\left(\omega t - \frac{\pi}{2}\right) + e^{-t\frac{R}{L}} \right]$$

Fault Current

- Important points here:
  - Total fault current has both ***steady-state*** and ***transient*** components – asymmetrical
  - Magnitude of the asymmetry (transient component) depends on the ***phase of the generator voltage*** at the time of the fault
  - In this class, we will use the ***steady-state*** current component, $\mathbf{I}_{ac}$, as our primary fault current metric

Generator Reactance

- The reactance of the generator was assumed constant in the previous example
- Physical characteristics of real generators result in a ***time-varying reactance*** following a fault
  - Time-dependence modeled with ***three reactance values***
    - X_d'': subtransient reactance
    - X_d': transient reactance
    - X_d: synchronous reactance
  - Reactance increases with time, such that

$$X_d'' < X_d' < X_d$$

Sub-Transient Fault Current

- Transition rates between reactance values are dictated by two time constants:
  - τ_d'': short-circuit subtransient time constant
  - τ_d': short-circuit transient time constant

- Neglecting generator resistance, i.e. assuming $\theta = 90°$, the synchronous portion of the fault current is

$$i_{ac}(t) = \sqrt{2}V_G\left[\left(\frac{1}{X_d''} - \frac{1}{X_d'}\right)e^{-\frac{t}{\tau_d''}} + \left(\frac{1}{X_d'} - \frac{1}{X_d}\right)e^{-\frac{t}{\tau_d'}} + \frac{1}{X_d}\right]\sin\left(\omega t + \alpha - \frac{\pi}{2}\right)$$

- At the instant of the fault, $t = 0$, the rms synchronous fault current is

$$I_F'' = \frac{V_G}{X_d''}$$

  - This is the rms ***subtransient fault current***, I_F''
  - This will be our primary metric for assessing fault current

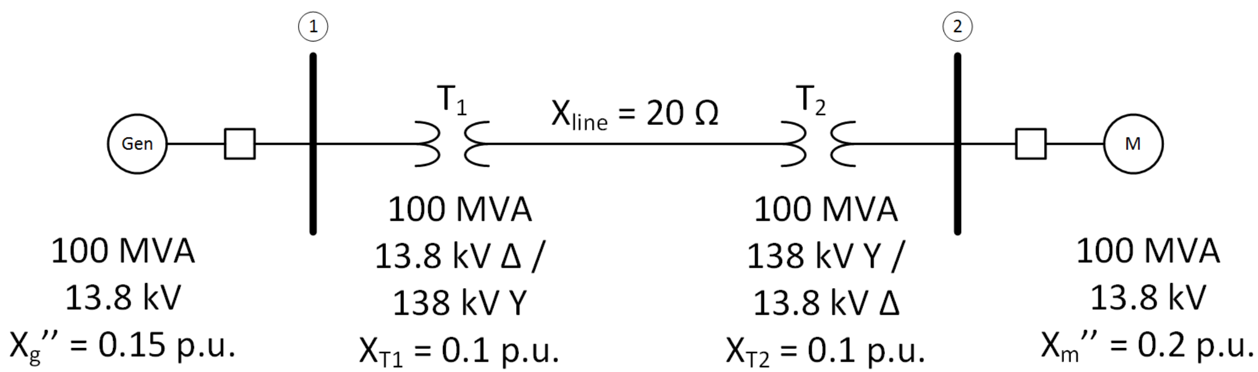
Symmetrical Three-Phase Short Circuits

Symmetrical 3-ϕ Short Circuits

- Next, we'll calculate the subtransient fault current resulting from a balanced three-phase fault
- We'll make the following simplifying assumptions:
  - Transformers modeled with leakage reactance only
    - Neglect winding resistance and shunt admittances
    - Neglect Δ-Y phase shifts
  - Transmission lines modeled with series reactance only
  - Synchronous machines modeled as constant voltage sources in series with subtransient reactances
    - Generators and motors
  - Induction motors are neglected or modeled as synchronous motors
  - Non-rotating loads are neglected

Symmetrical 3-ϕ Short Circuits

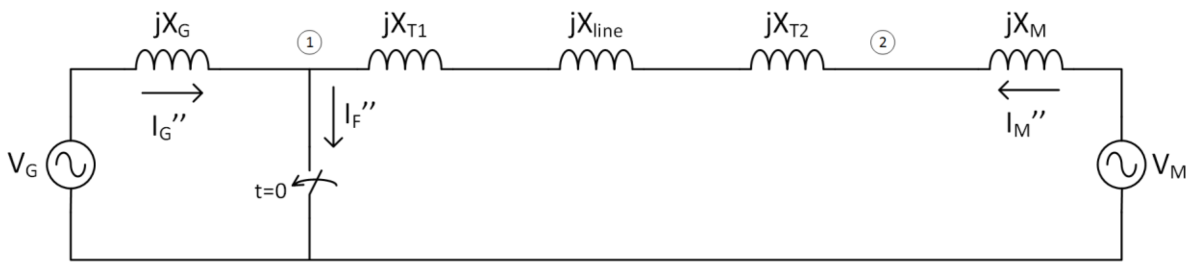
- We'll apply superposition to determine three-phase subtransient fault current
- Consider the following power system:



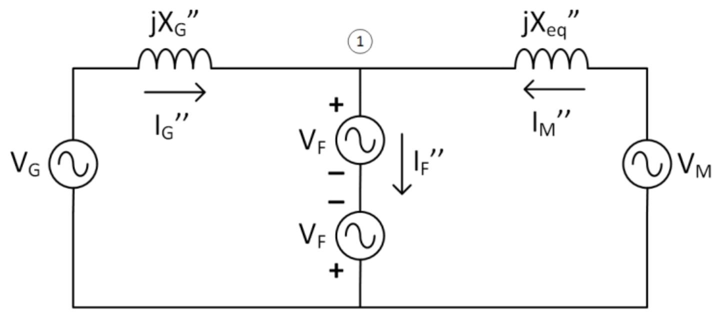
- Assume there is a balanced three-phase short of bus 1 to ground at $t = 0$

Symmetrical 3-ϕ Short Circuits

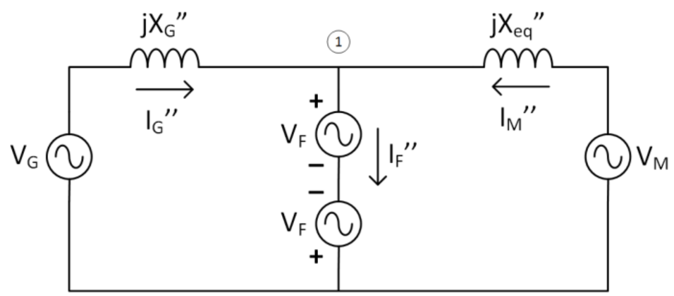
- The instant of the fault can be modeled by the switch closing in the following line-to-neutral schematic



- The short circuit (closed switch) can be represented by two back-to-back voltage sources, each equal to $\boldsymbol{V_F}$

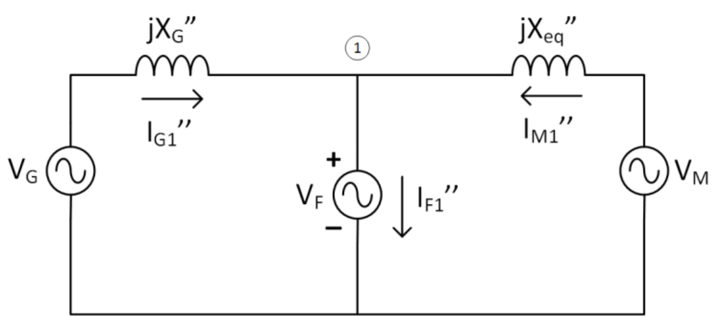


Symmetrical 3-ϕ Short Circuits

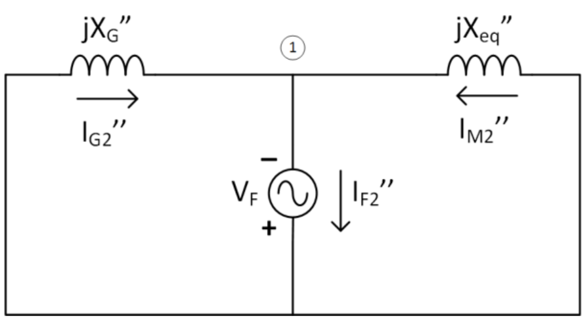


- Applying superposition, we can represent this circuit as the sum of two separate circuits:

Circuit 1



Circuit 2



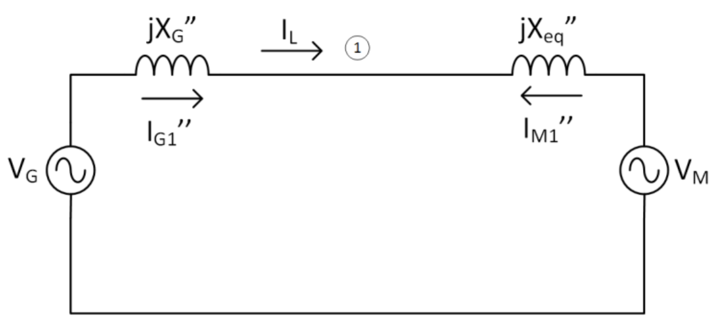
Symmetrical 3-ϕ Short Circuits

- Assume that the value of the fault-location source, $\boldsymbol{V}_F$, is the ***pre-fault voltage*** at that location
  - Circuit 1, then, represents the ***pre-fault circuit***, so

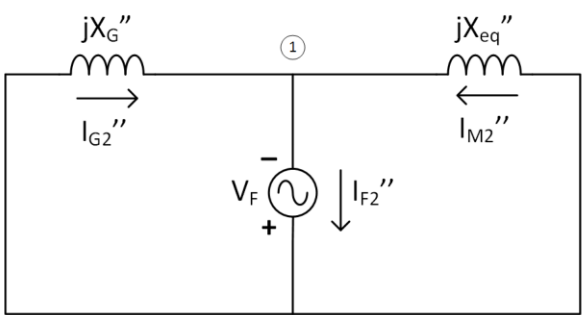
$$\boldsymbol{I}''_{F1} = 0$$

  - The $\boldsymbol{V}_F$ source can therefore be removed from circuit 1

Circuit 1



Circuit 2



Symmetrical 3-ϕ Short Circuits

- The current in circuit 1, $\boldsymbol{I}_L$, is the pre-fault line current
- Superposition gives the fault current

$$\boldsymbol{I}_F'' = \boldsymbol{I}_{F1}'' + \boldsymbol{I}_{F2}'' = \boldsymbol{I}_{F2}''$$

- The generator fault current is

$$\boldsymbol{I}_G'' = \boldsymbol{I}_{G1}'' + \boldsymbol{I}_{G2}''$$

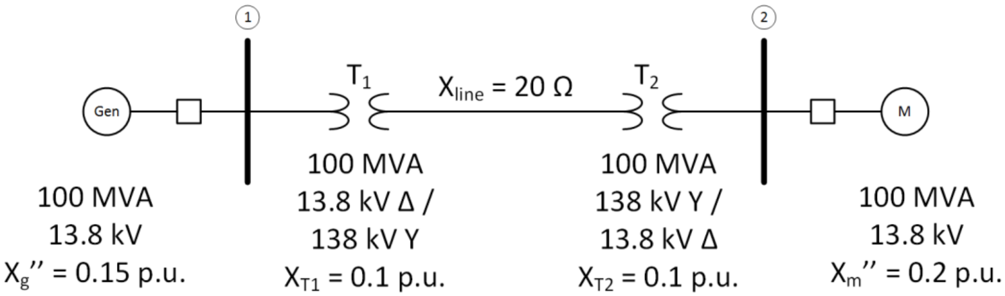
$$\boldsymbol{I}_G'' = \boldsymbol{I}_L + \boldsymbol{I}_{G2}''$$

- The motor fault current is

$$\boldsymbol{I}_{\boldsymbol{M}}'' = \boldsymbol{I}_{M1}'' + \boldsymbol{I}_{M2}''$$

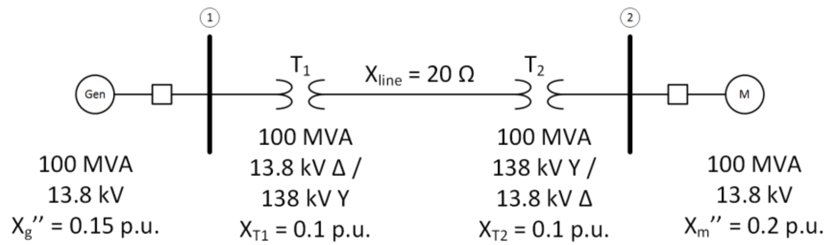
$$\boldsymbol{I}_M'' = -\boldsymbol{I}_L + \boldsymbol{I}_{M2}''$$

Symmetrical 3-ϕ Fault – Example



- For the simple power system above:
  - Generator is supplying rated power
  - Generator voltage is 5% above rated voltage
  - Generator power factor is 0.95 lagging
- A bolted three-phase fault occurs at bus 1
- Determine:
  - Subtransient fault current
  - Subtransient generator current
  - Subtransient motor current

Symmetrical 3-ϕ Fault – Example



- First convert to per-unit
  - Use $S_b = 100\ MVA$
- Base voltage in the transmission line zone is

$$V_{b,tl} = 138\ kV$$

- Base impedance in the transmission line zone is

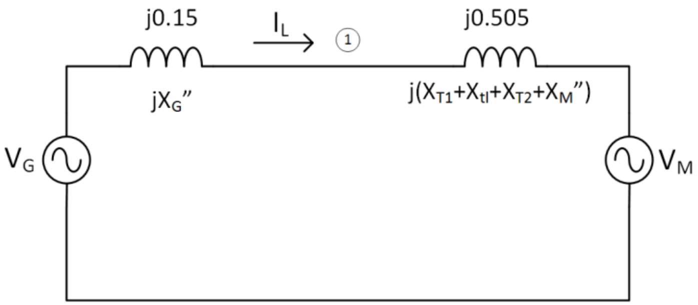
$$Z_{b,tl} = \frac{V_{b,tl}^2}{S_b} = \frac{(138\ kV)^2}{100\ MVA} = 190.4\ \Omega$$

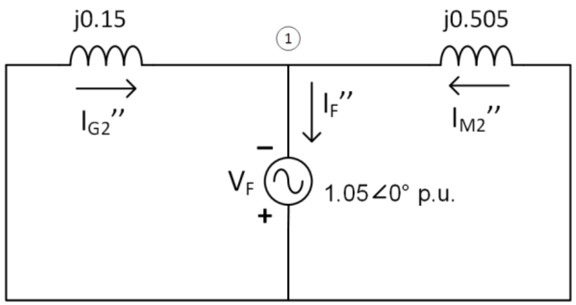
- The per-unit transmission line reactance is

$$X_{tl} = \frac{20\ \Omega}{190.4\ \Omega} = 0.105\ p.u.$$

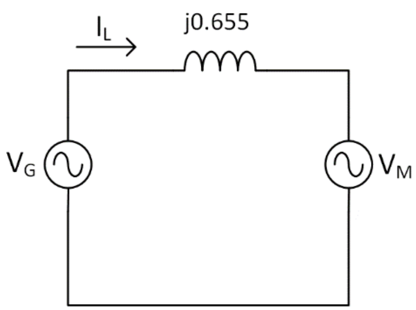
Symmetrical 3-ϕ Fault – Example

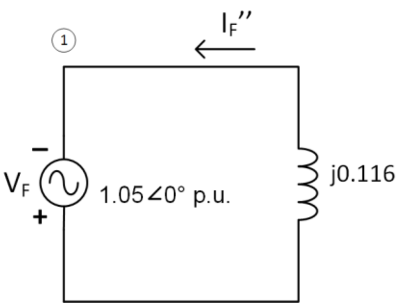
- The two per-unit circuits are





- These can be simplified by combining impedances





Symmetrical 3-ϕ Fault – Example

- Using circuit 2, we can calculate the subtransient fault current

$$\boldsymbol{I}_F'' = \frac{1.05\angle 0°}{j0.116} = 9.079\angle -90°\ p.u.$$

- To convert to kA, first determine the current base in the generator zone

$$I_{b,G} = \frac{S_b}{\sqrt{3}V_{b,G}} = \frac{100\ MVA}{\sqrt{3}\cdot 13.8\ kV} = 4.18\ kA$$

- The ***subtransient fault current*** is

$$\boldsymbol{I}_F'' = (9.079\angle -90°)\cdot 4.18\ kA$$

$$\boxed{\boldsymbol{I}_F'' = 37.98\angle -90°\ kA}$$

Symmetrical 3-ϕ Fault – Example

- The ***pre-fault line current*** can be calculated from the pre-fault generator voltage and power

$$\boldsymbol{I}_L = \left(\frac{\boldsymbol{S}_G/3}{\boldsymbol{V}_G''/\sqrt{3}} \right)^* = \left(\frac{\boldsymbol{S}_G}{\sqrt{3}\boldsymbol{V}_G''} \right)^* = \frac{(100\angle\cos^{-1}(0.95)\ MVA)^*}{\left(\sqrt{3}\cdot 1.05\cdot 13.8\angle 0°\ kV\right)^*}$$

$$\boldsymbol{I}_L = \frac{100\angle -18.19°\ MVA}{\sqrt{3}\cdot 1.05\cdot 13.8\angle 0°\ kV}$$

$$\boldsymbol{I}_L = 3.98\angle -18.19°\ kA$$

  - Or, in per-unit:

$$\boldsymbol{I}_L = \frac{3.98\angle -18.19°\ kA}{4.18\ \text{kA}} = 0.952\angle -18.19°\ p.u.$$

  - This will be used to find the generator and motor fault currents

Symmetrical 3-ϕ Fault – Example

- The generator's contribution to the fault current is found by applying current division

$$\boldsymbol{I}''_{G2} = \boldsymbol{I}''_{F}\frac{0.505}{0.505 + 0.15} = 7.0\angle - 90°\ p.u.$$

- Adding the pre-fault line current, we have the ***subtransient generator fault current***

$$\boldsymbol{I}''_{G} = \boldsymbol{I}_{L} + \boldsymbol{I}''_{G2}$$

$$\boldsymbol{I}''_{G} = 0.952\angle - 18.19° \ + 7.0\angle - 90°$$

$$\boldsymbol{I}''_{G} = 7.35\angle - 82.9°\ p.u.$$

- Converting to kA

$$\boldsymbol{I}''_{G} = (7.35\angle - 82.9°) \cdot 4.18\ kA$$

$$\boxed{\boldsymbol{I}''_{G} = 30.74\angle - 82.9°\ kA}$$

Symmetrical 3-ϕ Fault – Example

- Similarly, for the motor

$$\boldsymbol{I}''_{M2} = \boldsymbol{I}''_F \frac{0.15}{0.505 + 0.15} = 2.08\angle -90^\circ\ p.u.$$

- Subtracting the pre-fault line current gives the ***subtransient motor fault current***

$$\boldsymbol{I}''_M = -\boldsymbol{I}_L + \boldsymbol{I}''_{M2}$$

$$\boldsymbol{I}''_M = -0.952\angle -18.19^\circ + 2.08\angle -90^\circ$$

$$\boldsymbol{I}''_M = 2.0\angle -116.9^\circ$$

- Converting to kA

$$\boldsymbol{I}''_M = (2.0\angle -116.9^\circ) \cdot 4.18\ kA$$

$$\boxed{\boldsymbol{I}''_M = 8.36\angle -116.9^\circ\ kA}$$

Symmetrical Components

Symmetrical Components

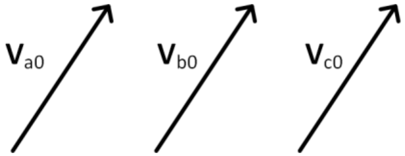
- In the previous section, we saw how to calculate subtransient fault current for balanced three-phase faults
- Unsymmetrical faults are much more common
  - Analysis is more complicated
- We'll now learn a tool that will simplify the analysis of unsymmetrical faults
  - The ***method of symmetrical components***

Symmetrical Components

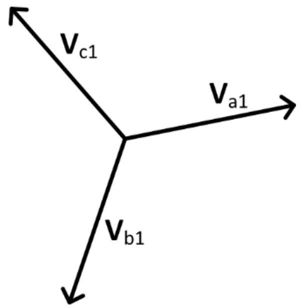
- The ***method of symmetrical components***:
  - Represent an asymmetrical set of N phasors as a sum of N sets of symmetrical component phasors
  - These N sets of phasors are called ***sequence components***
- Analogous to:
  - Decomposition of electrical signals into differential and common-mode components
  - Decomposition of forces into orthogonal components
- For a three-phase system ($N = 3$), sequence components are:
  - Zero sequence components
  - Positive sequence components
  - Negative sequence components

Sequence Components

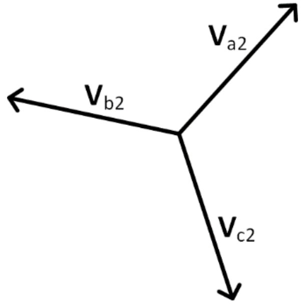
- ***Zero sequence components***
  - Three phasors with equal magnitude and equal phase
  - $\boldsymbol{V}_{a0}, \boldsymbol{V}_{b0}, \boldsymbol{V}_{c0}$



- ***Positive sequence components***
  - Three phasors with equal magnitude and $\pm 120°$, positive-sequence phase
  - $\boldsymbol{V}_{a1}, \boldsymbol{V}_{b1}, \boldsymbol{V}_{c1}$



- ***Negative sequence components***
  - Three phasors with equal magnitude and $\pm 120°$, negative-sequence phase
  - $\boldsymbol{V}_{a2}, \boldsymbol{V}_{b2}, \boldsymbol{V}_{c2}$



Sequence Components

- Note that the absolute phase and the magnitudes of the sequence components is not specified
  - Magnitude and phase define a unique set of sequence components

- ***Any set of phasors – balanced or unbalanced – can be represented as a sum of sequence components***

$$\begin{bmatrix} \boldsymbol{V}_a \\ \boldsymbol{V}_b \\ \boldsymbol{V}_c \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}_{a0} \\ \boldsymbol{V}_{b0} \\ \boldsymbol{V}_{c0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{V}_{a1} \\ \boldsymbol{V}_{b1} \\ \boldsymbol{V}_{c1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{V}_{a2} \\ \boldsymbol{V}_{b2} \\ \boldsymbol{V}_{c2} \end{bmatrix} \tag{1}$$

Sequence Components

- The phasors of each sequence component have a fixed phase relationship
  - If we know one, we know the other two
  - Assume we know phase a – use that as the reference
- For the zero sequence components, we have

$$\boldsymbol{V}_0 = \boldsymbol{V}_{a0} = \boldsymbol{V}_{b0} = \boldsymbol{V}_{c0} \tag{2}$$

- For the positive sequence components,

$$\boldsymbol{V}_1 = \boldsymbol{V}_{a1} = (1\angle 120°) \cdot \boldsymbol{V}_{b1} = (1\angle 240°) \cdot \boldsymbol{V}_{c1} \tag{3}$$

- And, for the negative sequence components,

$$\boldsymbol{V}_2 = \boldsymbol{V}_{a2} = (1\angle 240°) \cdot \boldsymbol{V}_{b2} = (1\angle 120°) \cdot \boldsymbol{V}_{c2} \tag{4}$$

- Note that we're using phase a as our reference, so

$$\boldsymbol{V}_0 = \boldsymbol{V}_{a0}, \quad \boldsymbol{V}_1 = \boldsymbol{V}_{a1}, \quad \boldsymbol{V}_2 = \boldsymbol{V}_{a2}$$

Sequence Components

- Next, we define a complex number, a, that has unit magnitude and phase of $120°$

$$a = 1\angle 120° \tag{5}$$

  - Multiplication by a results in a rotation (a phase shift) of $120°$
  - Multiplication by a^2 yields a rotation of $240° = -120°$

- Using (5) to rewrite (3) and (4)

$$\boldsymbol{V}_1 = \boldsymbol{V}_{a1} = a\boldsymbol{V}_{b1} = a^2\boldsymbol{V}_{c1} \tag{6}$$

$$\boldsymbol{V}_2 = \boldsymbol{V}_{a2} = a^2\boldsymbol{V}_{b2} = a\boldsymbol{V}_{c2} \tag{7}$$

Sequence Components

- Using (2), (6), and (7), we can rewrite (1) in a simplified form

$$\begin{bmatrix} \boldsymbol{V}_a \\ \boldsymbol{V}_b \\ \boldsymbol{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_0 \\ \boldsymbol{V}_1 \\ \boldsymbol{V}_2 \end{bmatrix} \tag{8}$$

  - The vector on the left is the vector of phase voltages, $\boldsymbol{V}_p$
  - The vector on the right is the vector of (phase a) sequence components, $\boldsymbol{V}_s$
  - We'll call the 3×3 transformation matrix $\boldsymbol{A}$

- We can rewrite (8) as

$$\boldsymbol{V}_p = \boldsymbol{A}\boldsymbol{V}_s \tag{9}$$

Sequence Components

- We can express the sequence voltages as a function of the phase voltages by inverting the transformation matrix

$$\boldsymbol{V}_s = \boldsymbol{A}^{-1}\boldsymbol{V}_p \tag{10}$$

where

$$\boldsymbol{A}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{11}$$

So

$$\begin{bmatrix} \boldsymbol{V}_0 \\ \boldsymbol{V}_1 \\ \boldsymbol{V}_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \boldsymbol{V}_a \\ \boldsymbol{V}_b \\ \boldsymbol{V}_c \end{bmatrix} \tag{12}$$

Sequence Components

- The same relationships hold for three-phase currents
- The phase currents are

$$\boldsymbol{I}_p = \begin{bmatrix} \boldsymbol{I}_a \\ \boldsymbol{I}_b \\ \boldsymbol{I}_c \end{bmatrix}$$

- And, the sequence currents are

$$\boldsymbol{I}_s = \begin{bmatrix} \boldsymbol{I}_0 \\ \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix}$$

Sequence Components

- The transformation matrix, $\boldsymbol{A}$, relates the phase currents to the sequence currents

$$\boldsymbol{I}_p = \boldsymbol{A}\boldsymbol{I}_s$$

$$\begin{bmatrix} \boldsymbol{I}_a \\ \boldsymbol{I}_b \\ \boldsymbol{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_0 \\ \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} \tag{13}$$

- And vice versa

$$\boldsymbol{I}_s = \boldsymbol{A}^{-1}\boldsymbol{I}_p$$

$$\begin{bmatrix} \boldsymbol{I}_0 \\ \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_a \\ \boldsymbol{I}_b \\ \boldsymbol{I}_c \end{bmatrix} \tag{14}$$

Sequence Components – Balanced System

- Before applying sequence components to unbalanced systems, let's first look at the ***sequence components for a balanced, positive-sequence, three-phase system***
- For a ***balanced system***, we have

$$\boldsymbol{V}_b = \boldsymbol{V}_a \cdot 1\angle -120° = a^2\boldsymbol{V}_a$$

$$\boldsymbol{V}_c = \boldsymbol{V}_a \cdot 1\angle 120° = a\boldsymbol{V}_a$$

- The sequence voltages are given by (12)
  - The ***zero sequence voltage*** is

$$\boldsymbol{V}_0 = \frac{1}{3}[\boldsymbol{V}_a + \boldsymbol{V}_b + \boldsymbol{V}_c] = \frac{1}{3}[\boldsymbol{V}_a + a^2\boldsymbol{V}_a + a\boldsymbol{V}_a]$$

$$\boldsymbol{V}_0 = \frac{1}{3}\boldsymbol{V}_a[1 + a^2 + a]$$

- Applying the identity $1 + a^2 + a = 0$, we have

$$\boldsymbol{V}_0 = 0$$

Sequence Components – Balanced System

- The ***positive sequence component*** is given by

$$\boldsymbol{V}_1 = \frac{1}{3}[\boldsymbol{V}_a + a\boldsymbol{V}_b + a^2\boldsymbol{V}_c]$$

$$\boldsymbol{V}_1 = \frac{1}{3}[\boldsymbol{V}_a + a \cdot a^2\boldsymbol{V}_a + a^2 \cdot a\boldsymbol{V}_a]$$

$$\boldsymbol{V}_1 = \frac{1}{3}[\boldsymbol{V}_a + a^3\boldsymbol{V}_a + a^3\boldsymbol{V}_a]$$

  - Since $a^3 = 1\angle 0°$, we have

$$\boldsymbol{V}_1 = \frac{1}{3}[3\boldsymbol{V}_a]$$

$$\boldsymbol{V}_1 = \boldsymbol{V}_a$$

Sequence Components – Balanced System

- The ***negative sequence component*** is given by

$$\boldsymbol{V}_2 = \frac{1}{3}[\boldsymbol{V}_a + a^2\boldsymbol{V}_b + a\boldsymbol{V}_c]$$

$$\boldsymbol{V}_2 = \frac{1}{3}[\boldsymbol{V}_a + a^2 \cdot a^2\boldsymbol{V}_a + a \cdot a\boldsymbol{V}_a]$$

$$\boldsymbol{V}_2 = \frac{1}{3}[\boldsymbol{V}_a + a^4\boldsymbol{V}_a + a^2\boldsymbol{V}_a]$$

- Again, using the identity $1 + a^2 + a = 0$, along with the fact that $a^4 = a$, we have

$$\boldsymbol{V}_2 = 0$$

Sequence Components – Balanced System

- So, for a ***positive-sequence, balanced***, three-phase system, the ***sequence voltages*** are

$$\boldsymbol{V}_0 = 0, \quad \boldsymbol{V}_1 = \boldsymbol{V}_a, \quad \boldsymbol{V}_2 = 0$$

- Similarly, the ***sequence currents*** are

$$\boldsymbol{I}_0 = 0, \quad \boldsymbol{I}_1 = \boldsymbol{I}_a, \quad \boldsymbol{I}_2 = 0$$

- This is as we would expect
  - No zero- or negative-sequence components for a positive-sequence balanced system
  - Zero- and negative-sequence components are only used to account for imbalance

Sequence Components

- We have just introduced the concept of ***symmetric components***
  - Allows for decomposition of, possibly unbalanced, three-phase phasors into ***sequence components***

- We'll now apply this concept to power system networks to develop ***sequence networks***
  - ***Decoupled networks*** for each of the sequence components
  - Sequence networks become ***coupled only at the point of imbalance***
  - Simplifies the analysis of unbalanced systems

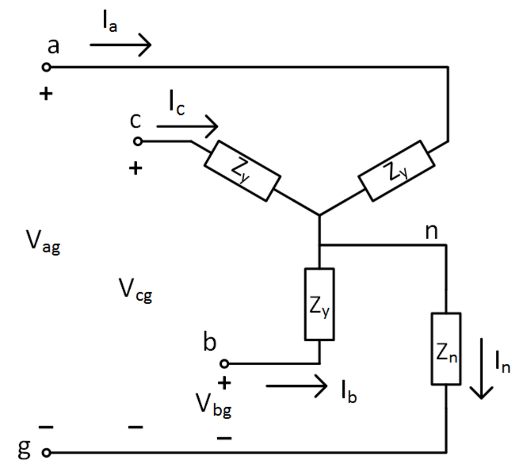
Sequence Networks

Sequence Networks

- ***Power system components each have their own set of sequence networks***
  - Non-rotating loads
  - Transmission lines
  - Rotating machines – generators and motors
  - Transformers
- Sequence networks for overall systems are interconnections of the individual sequence network
- Sequence networks become coupled in a particular way at the fault location depending on type of fault
  - Line-to-line
  - Single line-to-ground
  - Double line-to-ground
- Fault current can be determined through simple analysis of the coupled sequence networks

Sequence Networks – Non-Rotating Loads

Sequence Networks – Non-Rotating Loads



- Consider a balanced Y-load with the neutral grounded through some non-zero impedance
- Applying KVL gives the phase-a-to-ground voltage

$$\boldsymbol{V}_{ag} = Z_y\boldsymbol{I}_a + Z_n\boldsymbol{I}_n$$

$$\boldsymbol{V}_{ag} = Z_y\boldsymbol{I}_a + Z_n(\boldsymbol{I}_a + \boldsymbol{I}_b + \boldsymbol{I}_c)$$

$$\boldsymbol{V}_{ag} = (Z_y + Z_n)\boldsymbol{I}_a + Z_n\boldsymbol{I}_b + Z_n\boldsymbol{I}_c \qquad (15)$$

- For phase b:

$$\boldsymbol{V}_{bg} = Z_y\boldsymbol{I}_b + Z_n\boldsymbol{I}_n = Z_y\boldsymbol{I}_b + Z_n(\boldsymbol{I}_a + \boldsymbol{I}_b + \boldsymbol{I}_c)$$

$$\boldsymbol{V}_{bg} = Z_n\boldsymbol{I}_a + (Z_y + Z_n)\boldsymbol{I}_b + Z_n\boldsymbol{I}_c \qquad (16)$$

- Similarly, for phase c:

$$\boldsymbol{V}_{ag} = Z_n\boldsymbol{I}_a + Z_n\boldsymbol{I}_b + (Z_y + Z_n)\boldsymbol{I}_c \qquad (17)$$

Sequence Networks – Non-Rotating Loads

- Putting (15) – (17) in matrix form

$$\begin{bmatrix} \boldsymbol{V}_{ag} \\ \boldsymbol{V}_{bg} \\ \boldsymbol{V}_{cg} \end{bmatrix} = \begin{bmatrix} (Z_y + Z_n) & Z_n & Z_n \\ Z_n & (Z_y + Z_n) & Z_n \\ Z_n & Z_n & (Z_y + Z_n) \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_a \\ \boldsymbol{I}_b \\ \boldsymbol{I}_c \end{bmatrix}$$

or

$$\boldsymbol{V}_p = \boldsymbol{Z}_p \boldsymbol{I}_p \tag{18}$$

where $\boldsymbol{V}_p$ and $\boldsymbol{I}_p$ are the phase voltages and currents, respectively, and $\boldsymbol{Z}_p$ is the ***phase impedance matrix***

- We can use (9) and (13) to rewrite (18) as

$$\boldsymbol{A}\boldsymbol{V}_s = \boldsymbol{Z}_p \boldsymbol{A} \boldsymbol{I}_s$$

- Solving for $\boldsymbol{V}_s$

$$\boldsymbol{V}_s = \boldsymbol{A}^{-1} \boldsymbol{Z}_p \boldsymbol{A} \boldsymbol{I}_s$$

or

$$\boldsymbol{V}_s = \boldsymbol{Z}_s \boldsymbol{I}_s \tag{19}$$

Sequence Networks – Non-Rotating Loads

$$\boldsymbol{V}_s = \boldsymbol{Z}_s \boldsymbol{I}_s \tag{19}$$

where Z_s is the ***sequence impedance matrix***

$$\boldsymbol{Z}_s = \boldsymbol{A}^{-1}\boldsymbol{Z}_p\boldsymbol{A} = \begin{bmatrix} \left(Z_y + 3Z_n\right) & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix} \tag{20}$$

- Equation (19) then becomes a set of three uncoupled equations

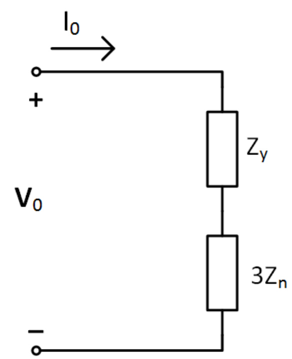
$$\boldsymbol{V}_0 = \left(Z_y + 3Z_n\right)\boldsymbol{I}_0 = Z_0\boldsymbol{I}_0 \tag{21}$$

$$\boldsymbol{V}_1 = Z_y\boldsymbol{I}_1 = Z_1\boldsymbol{I}_1 \tag{22}$$

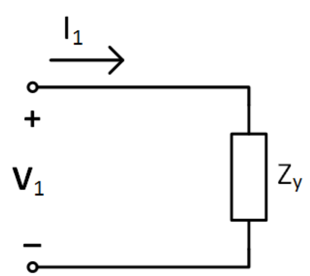
$$\boldsymbol{V}_2 = Z_y\boldsymbol{I}_2 = Z_2\boldsymbol{I}_2 \tag{23}$$

Sequence Networks – Non-Rotating Loads

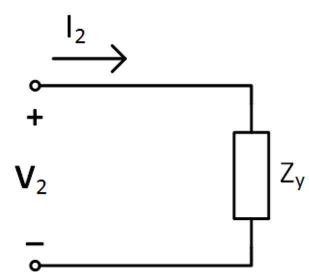
- Equations (21) – (23) describe the uncoupled ***sequence networks***
  - Zero-sequence network:



  - Positive-sequence network:

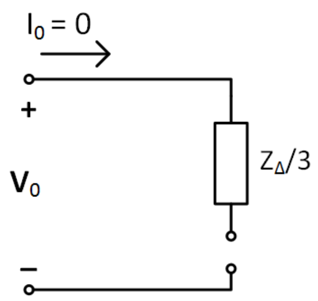


  - Negative-sequence network:

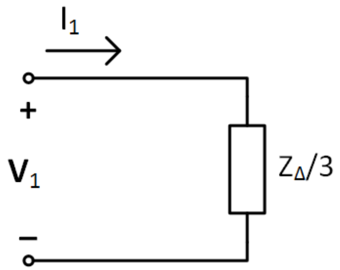


Sequence Networks – Non-Rotating Loads

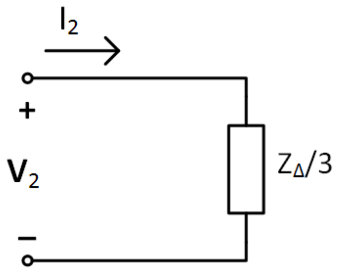
- We can develop similar sequence networks for a balanced Δ-connected load
  - $Z_y = Z_\Delta/3$
  - There is no neutral point for the Δ-network, so $Z_n = \infty$ - an open circuit
  - Zero-sequence network:



  - Positive-sequence network:



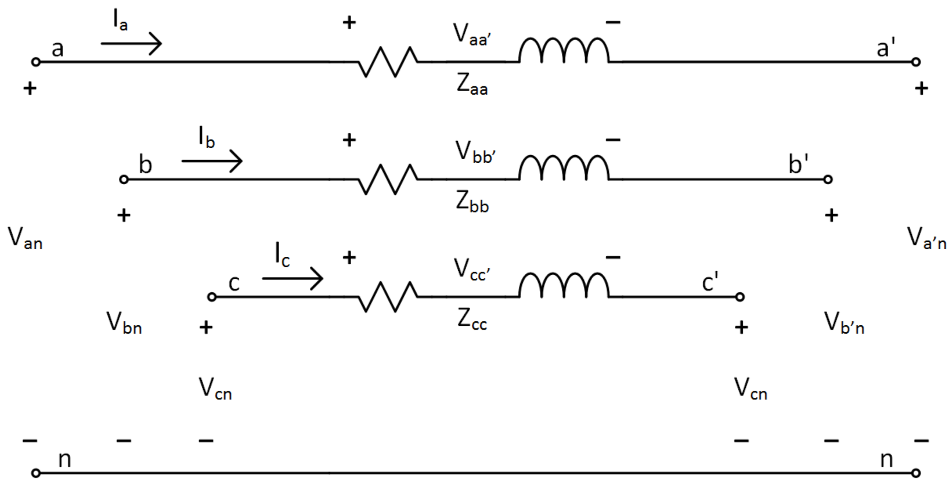
  - Negative-sequence network:



Sequence Networks – 3-ϕ Lines

Sequence Networks – 3-ϕ Lines

- Balanced, three-phase lines can be modeled as



- The voltage drops across the lines are given by the following system of equations

$$\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} V_{an} - V_{a'n} \\ V_{bn} - V_{b'n} \\ V_{cn} - V_{c'n} \end{bmatrix} \qquad (24)$$

Sequence Networks – 3-ϕ Lines

- Writing (24) in compact form

$$\boldsymbol{V}_p - \boldsymbol{V}_{p'} = \boldsymbol{Z}_p \boldsymbol{I}_p \tag{25}$$

- $\boldsymbol{Z}_p$ is the ***phase impedance matrix***
  - Self impedances along the diagonal
  - Mutual impedances elsewhere
  - Symmetric
  - Diagonal, if we neglect mutual impedances

Sequence Networks – 3-ϕ Lines

- We can rewrite (25) in terms of sequence components

$$\boldsymbol{AV}_s - \boldsymbol{AV}_{s'} = \boldsymbol{Z}_p\boldsymbol{AI}_s$$

$$\boldsymbol{V}_s - \boldsymbol{V}_{s'} = \boldsymbol{A}^{-1}\boldsymbol{Z}_p\boldsymbol{AI}_s$$

$$\boldsymbol{V}_s - \boldsymbol{V}_{s'} = \boldsymbol{Z}_s\boldsymbol{I}_s \qquad (26)$$

where $\boldsymbol{Z}_s$ is the sequence impedance matrix

$$\boldsymbol{Z}_s = \boldsymbol{A}^{-1}\boldsymbol{Z}_p\boldsymbol{A} \qquad (27)$$

- $\boldsymbol{Z}_s$ is diagonal so long as the system impedances are balanced, i.e.
  - Self impedances are equal: $Z_{aa} = Z_{bb} = Z_{cc}$
  - Mutual impedances are equal: $Z_{ab} = Z_{ac} = Z_{bc}$

Sequence Networks – 3-ϕ Lines

- For balanced lines, $\boldsymbol{Z}_s$ is diagonal

$$\boldsymbol{Z}_s = \begin{bmatrix} Z_{aa} + 2Z_{ab} & 0 & 0 \\ 0 & Z_{aa} - Z_{ab} & 0 \\ 0 & 0 & Z_{aa} - Z_{ab} \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}$$

- Because Z_s is diagonal, (26) represents three uncoupled equations

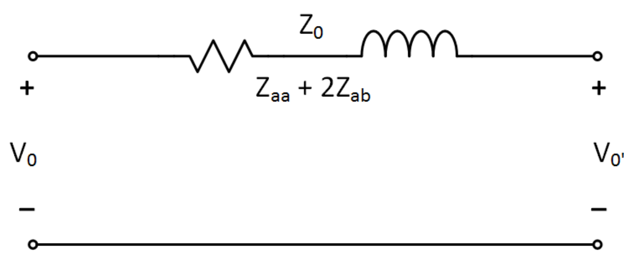
$$\boldsymbol{V}_0 - \boldsymbol{V}_{0'} = Z_0 \boldsymbol{I}_0 \tag{28}$$

$$\boldsymbol{V}_1 - \boldsymbol{V}_{1'} = Z_1 \boldsymbol{I}_1 \tag{29}$$

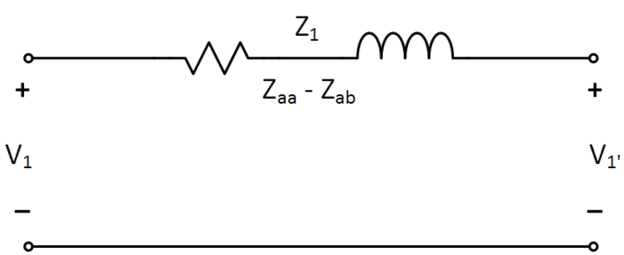
$$\boldsymbol{V}_2 - \boldsymbol{V}_{2'} = Z_2 \boldsymbol{I}_2 \tag{30}$$

Sequence Networks – 3-ϕ Lines

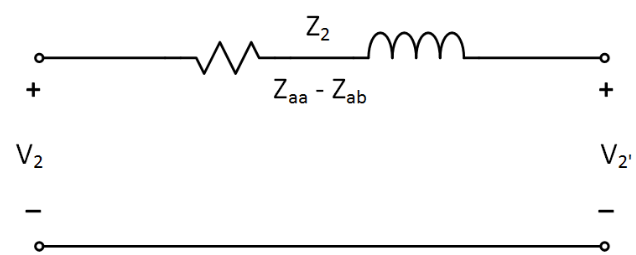
- Equations (28) – (30) describe the voltage drop across three uncoupled ***sequence networks***
  - Zero-sequence network:



  - Positive-sequence network:



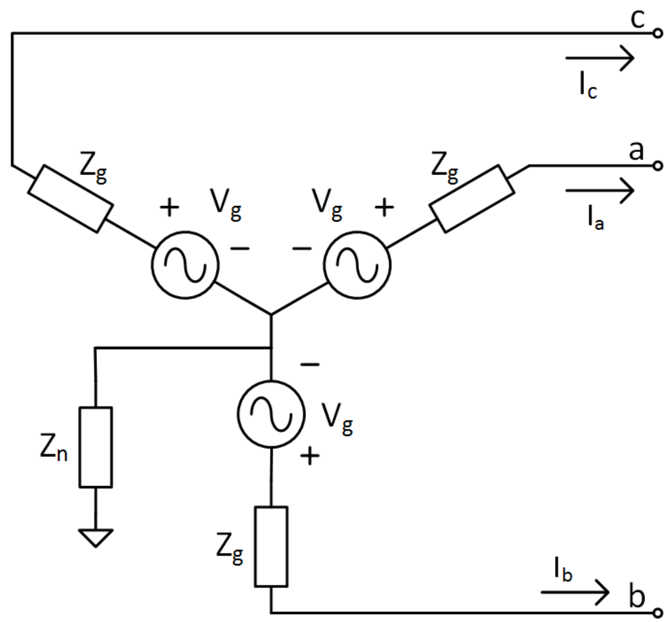
  - Negative-sequence network:



Sequence Networks –Rotating Machines

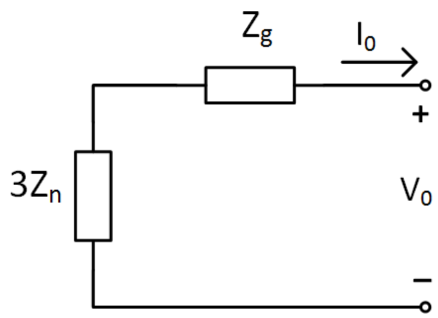
Sequence Networks – Rotating Machines

- Consider the following model for a ***synchronous generator***
- Similar to the Y-connected load
  - Generator includes ***voltage sources*** on each phase
- Voltage sources are ***positive sequence***
  - Sources will appear ***only in the positive-sequence network***

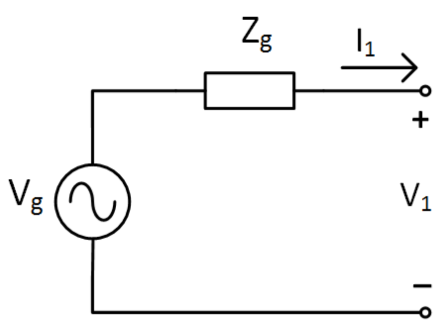


Sequence Networks – Synchronous Generator

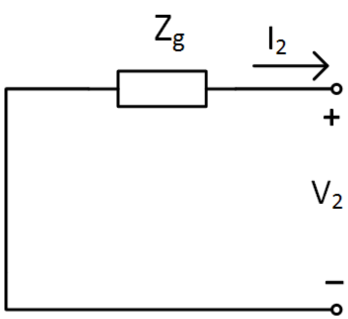
- Sequence networks for Y-connected synchronous generator
  - Zero-sequence network:



  - Positive-sequence network:



  - Negative-sequence network:



Sequence Networks – Motors

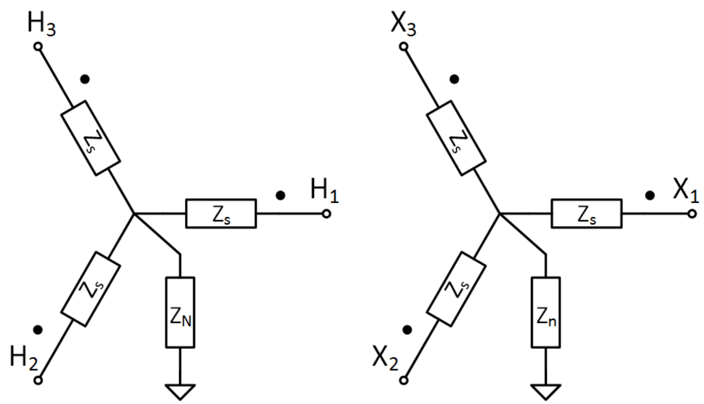
- **Synchronous motors**
  - Sequence networks identical to those for synchronous generators
  - Reference current directions are reversed

- **Induction motors**
  - Similar sequence networks to synchronous motors, except source in the positive sequence network set to zero

Sequence Networks –Transformers

Sequence Networks - Y-Y Transformers

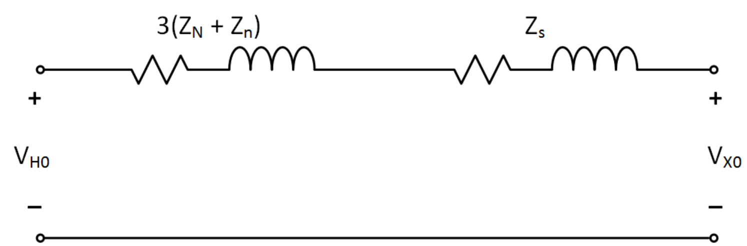
- Per-unit sequence networks for transformers
  - Simplify by neglecting transformer shunt admittances
- Consider a Y-Y transformer



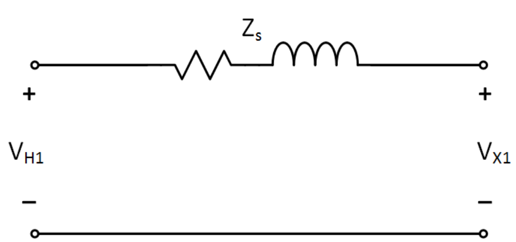
- Similar to the Y-connected load, the voltage drops across the neutral impedances are $3I_0Z_N$ and $3I_0Z_n$
  - $3Z_N$ and $3Z_n$ each appear in the zero-sequence network
  - Can be combined in the per-unit circuit as long as shunt impedances are neglected

Sequence Networks – Y-Y Transformers

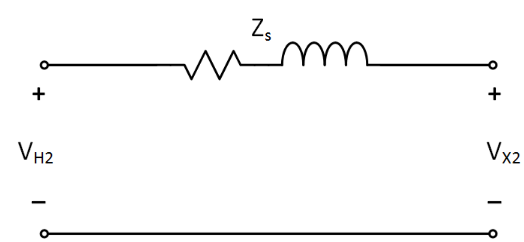
- Impedance accounting for leakage flux and winding resistance for each winding can be referred to the primary
  - Add together into a single impedance, Z_s, in the per-unit model
- Y-Y transformer sequence networks
  - Zero-sequence network:



  - Positive-sequence network:

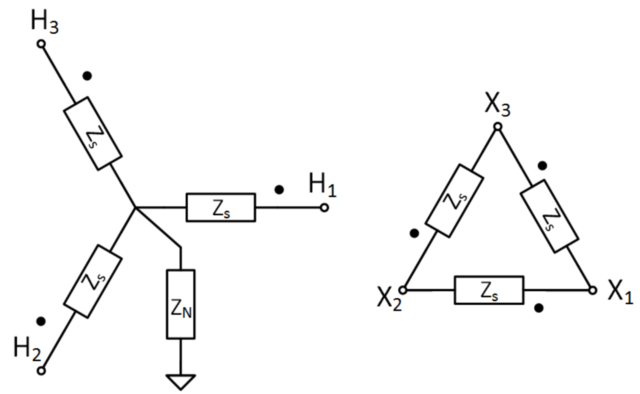


  - Negative-sequence network:



Sequence Networks – Y-Δ Transformers

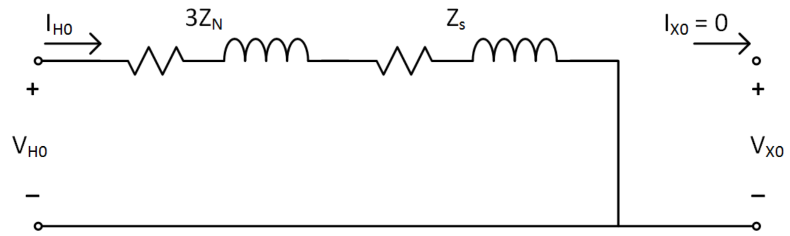
- Y-Δ transformers differ in a couple of ways



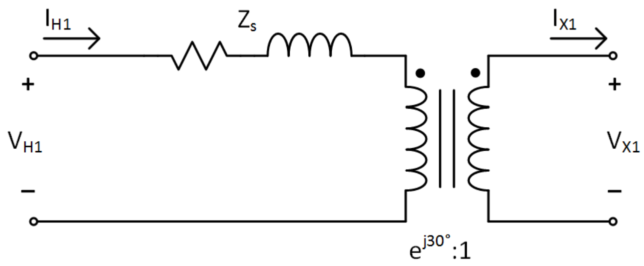
- Must account for phase shift from primary to secondary
  - For positive-sequence network, Y-side voltage and current lead Δ-side voltage and current
  - For negative-sequence network, Y-side voltage and current lag Δ-side voltage and current
- No neutral connection on the Δ side
  - Zero-sequence current cannot enter or leave the Δ winding

Sequence Networks – Y-ΔTransformers

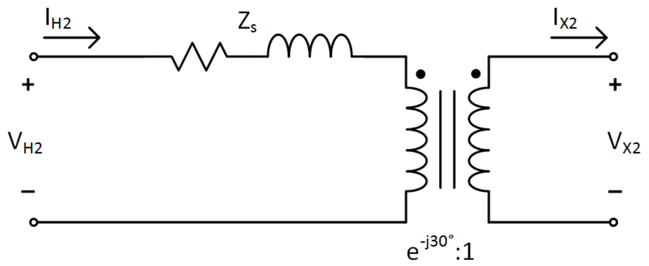
- Sequence networks for Y-ΔTransformers
  - Zero-sequence network:



  - Positive-sequence network:

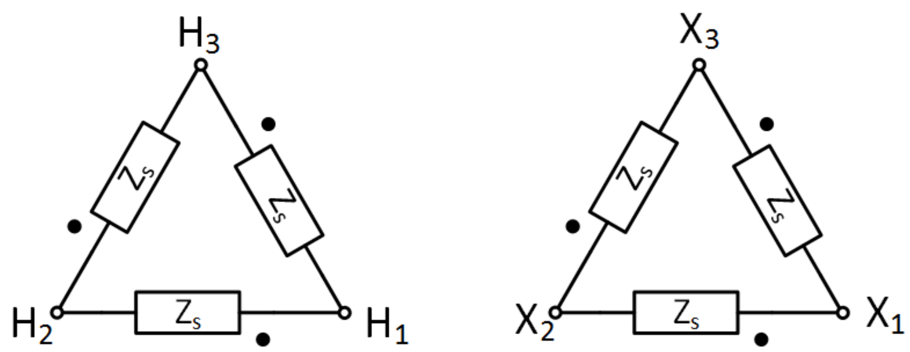


  - Negative-sequence network:



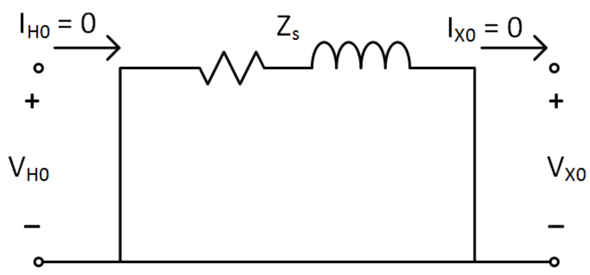
Sequence Networks – Δ-ΔTransformers

- Δ-Δ transformers
  - Like Y-Y transformers, no phase shift
  - No neutral connections
    - Zero-sequence current cannot flow into or out of either winding

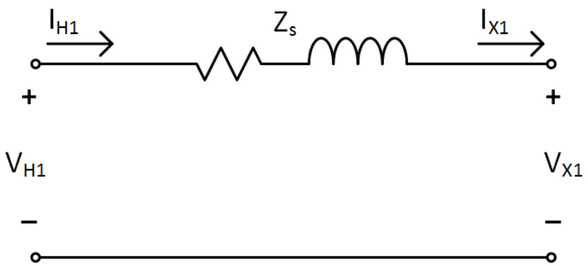


Sequence Networks – Δ-ΔTransformers

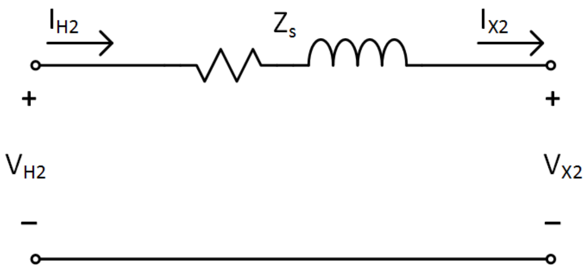
- Sequence networks for Δ-ΔTransformers
  - <u>Zero-sequence network</u>:



  - <u>Positive-sequence network</u>:



  - <u>Negative-sequence network</u>:



Power in Sequence Networks

Power in Sequence Networks

- We can relate the power delivered to a system's sequence networks to the three-phase power delivered to that system
- We know that the complex power delivered to a three-phase system is the sum of the power at each phase

$$S_p = \boldsymbol{V}_{an}\boldsymbol{I}_a^* + \boldsymbol{V}_{bn}\boldsymbol{I}_b^* + \boldsymbol{V}_{cn}\boldsymbol{I}_c^*$$

- In matrix form, this looks like

$$S_p = [\boldsymbol{V}_{an} \quad \boldsymbol{V}_{bn} \quad \boldsymbol{V}_{cn}]\begin{bmatrix}\boldsymbol{I}_a^* \\ \boldsymbol{I}_b^* \\ \boldsymbol{I}_c^*\end{bmatrix}$$

$$S_p = \boldsymbol{V}_p^T\boldsymbol{I}_p^* \qquad (28)$$

Power in Sequence Networks

- Recall the following relationships

$$\boldsymbol{V}_p = \boldsymbol{A}\boldsymbol{V}_s \tag{9}$$

$$\boldsymbol{I}_p = \boldsymbol{A}\boldsymbol{I}_s \tag{13}$$

- Using (9) and (13) in (28), we have

$$\boldsymbol{S}_p = (\boldsymbol{A}\boldsymbol{V}_s)^T(\boldsymbol{A}\boldsymbol{I}_s)^*$$

$$\boldsymbol{S}_p = \boldsymbol{V}_s^T\boldsymbol{A}^T\boldsymbol{A}^*\boldsymbol{I}_s^* \tag{29}$$

- Computing the product in the middle of the right-hand side of (29), we find

$$\boldsymbol{A}^T\boldsymbol{A}^* = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3\boldsymbol{I}_3$$

where $\boldsymbol{I}_3$ is the 3×3 identity matrix

Power in Sequence Networks

- Equation (29) then becomes

$$\boldsymbol{S}_p = \boldsymbol{V}_s^T 3\boldsymbol{I}_3 \boldsymbol{I}_s^*$$

$$\boldsymbol{S}_p = 3\boldsymbol{V}_s^T \boldsymbol{I}_s^*$$

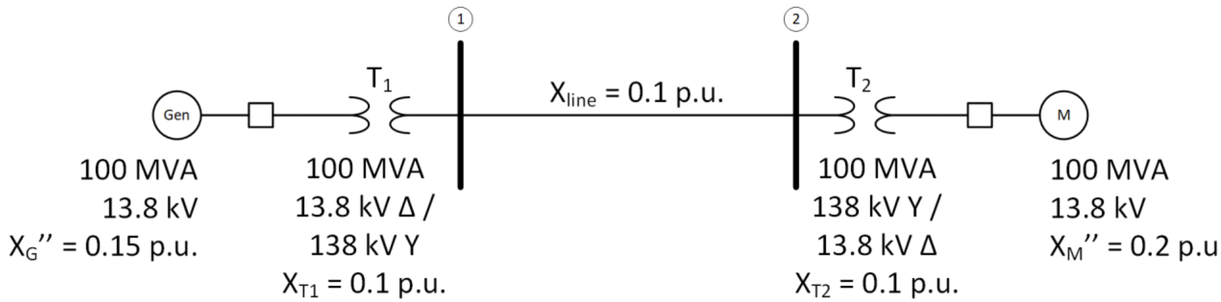
$$\boldsymbol{S}_p = 3[\boldsymbol{V}_0 \quad \boldsymbol{V}_1 \quad \boldsymbol{V}_2]\begin{bmatrix}\boldsymbol{I}_0^* \\ \boldsymbol{I}_1^* \\ \boldsymbol{I}_2^*\end{bmatrix}$$

$$\boldsymbol{S}_p = 3(\boldsymbol{V}_0\boldsymbol{I}_0^* + \boldsymbol{V}_1\boldsymbol{I}_1^* + \boldsymbol{V}_2\boldsymbol{I}_2^*)$$

- The total power delivered to a three-phase network is three times the sum of the power delivered to the three sequence networks
  - The three sequence networks represent only one of the three phases – recall, we chose to consider only phase a

Example Problems

A bolted, symmetric, three-phase fault occurs 60% of the way from bus 1 to bus 2. Determine the subtransient fault current in per-unit and in amperes. The load is consuming rated power at rated voltage and unity power factor.



Determine the sequence components for the following unbalanced set of three-phase voltage phasors:

$$\mathbf{V}_a = 1\angle 0^\circ\ p.u.$$
$$\mathbf{V}_b = 0.5\angle -60^\circ\ p.u.$$
$$\mathbf{V}_c = 2\angle 200^\circ\ p.u.$$

Determine the phase components for the following set of sequence components:

$$\mathbf{V}_0 = 1\angle 60°\ p.u.$$
$$\mathbf{V}_1 = 1\angle 0°\ p.u.$$
$$\mathbf{V}_2 = 0\ p.u.$$

Unsymmetrical Faults

Unsymmetrical Faults

- The majority of faults that occur in three-phase power systems are unsymmetrical
  - Not balanced
  - Fault current and voltage differ for each phase
- The method of symmetrical components and sequence networks provide us with a tool to analyze these unsymmetrical faults
- We'll examine three types of unsymmetrical faults
  - Single line-to-ground (SLG) faults
  - Line-to-line (LL) faults
  - Double line-to-ground (DLG) faults

Unsymmetrical Fault Analysis - Procedure

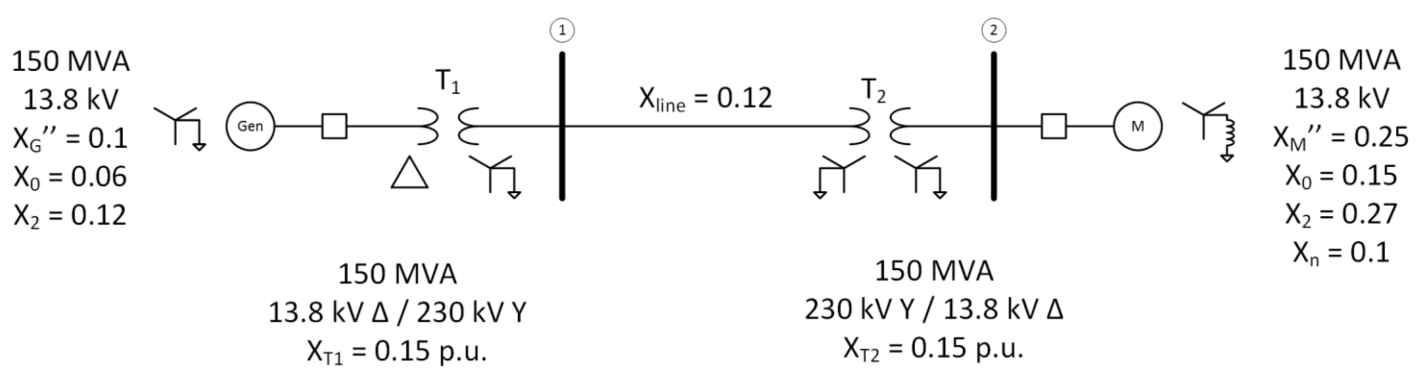
- Basic procedure for fault analysis:
  1. Generate sequence networks for the system
  2. Interconnect sequence networks appropriately at the fault location
  3. Perform circuit analysis on the interconnected sequence networks

Unsymmetrical Fault Analysis

- To simplify our analysis, we'll make the following assumptions
  1. System is balanced before the instant of the fault
  2. Neglect pre-fault load current
     - All pre-fault machine terminal voltages and bus voltages are equal to V_F
  3. Transmission lines are modeled as series reactances only
  4. Transformers are modeled with leakage reactances only
  5. Non-rotating loads are neglected
  6. Induction motors are either neglected or modeled as synchronous motors

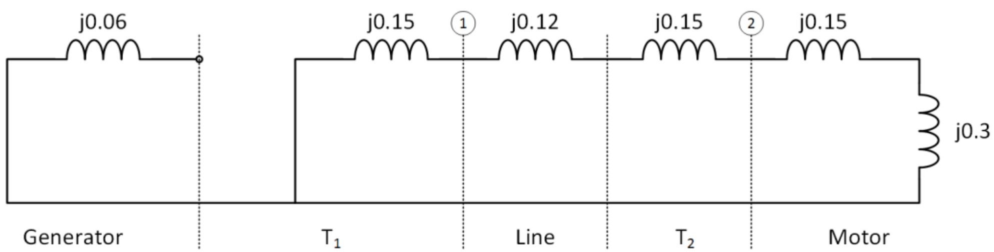
Unsymmetrical Fault Analysis

- Each sequence network includes all interconnected power-system components
  - Generators, motors, lines, and transformers
- Analysis will be simplified if we represent each sequence network as its Thévenin equivalent
  - From the perspective of the fault location
- For example, consider the following power system:

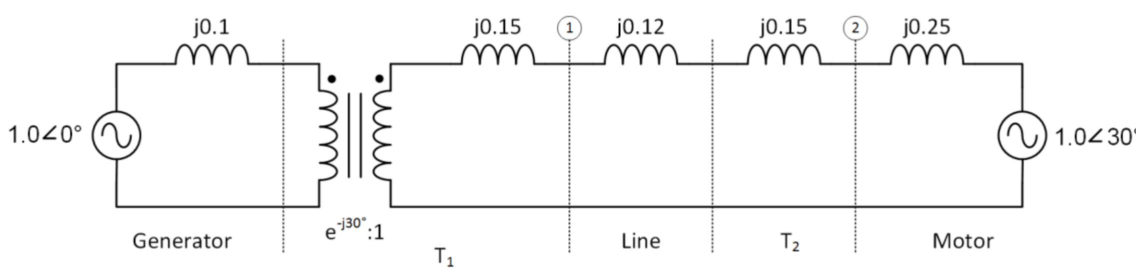


Unsymmetrical Fault Analysis – Sequence Networks

- The sequence networks for the system are generated by interconnecting the sequence networks for each of the components
- The zero-sequence network:

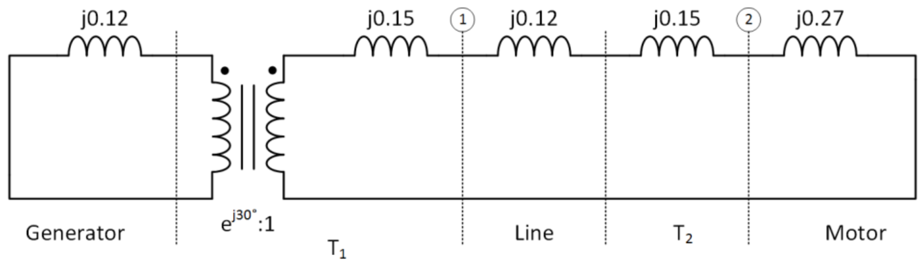


- The positive-sequence network:
  - Assuming the generator is operating at the rated voltage at the time of the fault

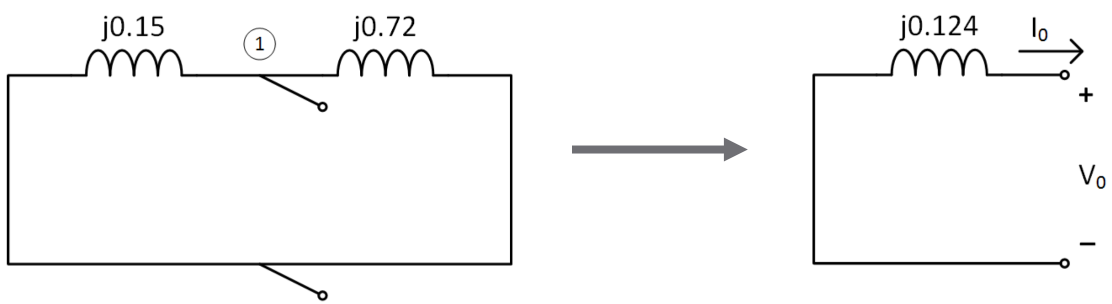


Unsymmetrical Fault Analysis – Sequence Networks

- The negative sequence network:

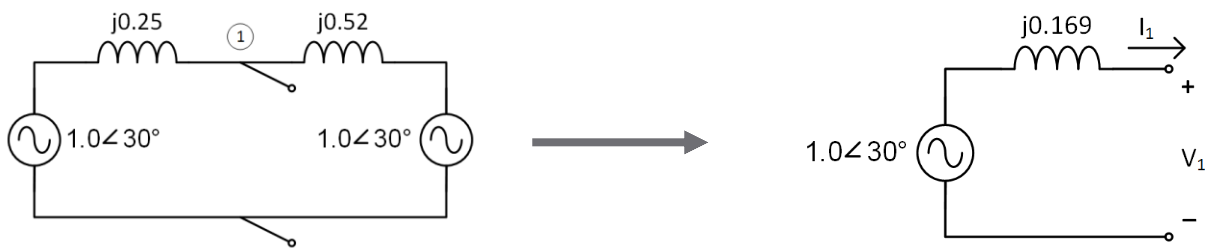


- Now, let's assume there is some sort of fault at bus 1
  - Determine the Thévenin equivalent for each sequence network from the perspective of bus 1
- Simplifying the zero-sequence network to its Thévenin equivalent

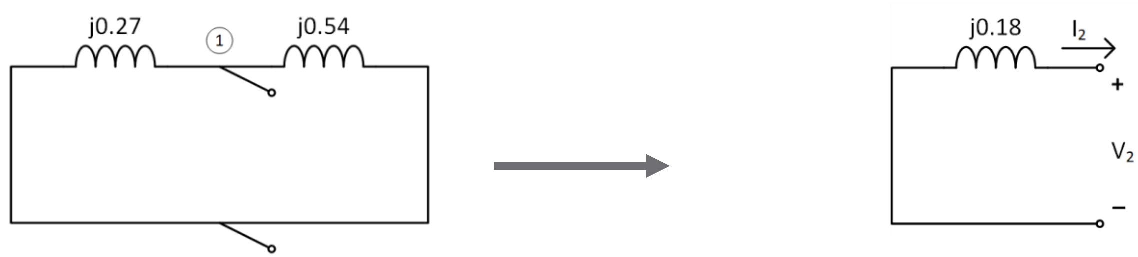


Unsymmetrical Fault Analysis – Sequence Networks

- The positive-sequence network simplifies to the following circuit with the following Thévenin equivalent



- Similarly, for the negative-sequence network, we have

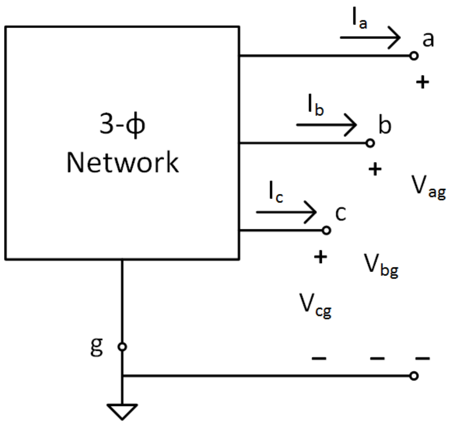


- Next, we'll see how to interconnect these networks to analyze different types of faults

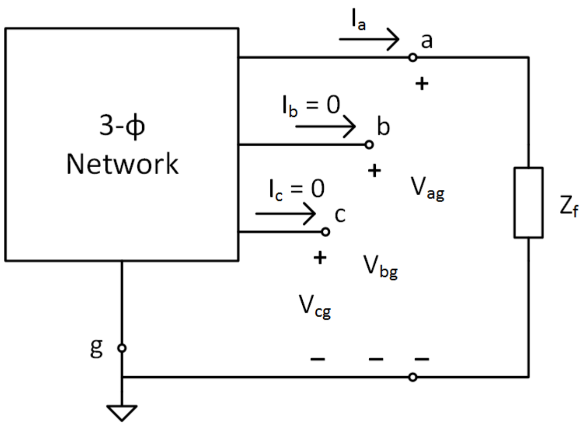
Single-Line-to-Ground Fault

Unsymmetrical Fault Analysis – SLG Fault

- The following represents a generic three-phase network with terminals at the fault location:

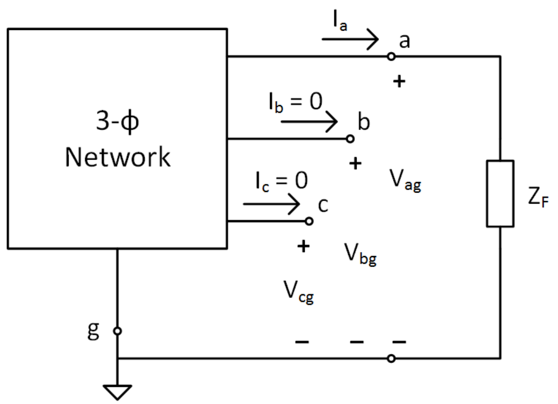


- If we have a single-line-to-ground fault, where phase a is shorted through Z_f to ground, the model becomes:



Unsymmetrical Fault Analysis – SLG Fault

- The ***phase-domain fault conditions***:



$$\boldsymbol{I}_a = \frac{\boldsymbol{V}_{ag}}{Z_f} \quad (1)$$

$$\boldsymbol{I}_b = \boldsymbol{I}_c = 0 \quad (2)$$

- Transforming these phase-domain currents to the sequence domain

$$\begin{bmatrix} \boldsymbol{I}_0 \\ \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \boldsymbol{V}_{ag}/Z_f \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} \boldsymbol{V}_{ag}/Z_f \\ \boldsymbol{V}_{ag}/Z_f \\ \boldsymbol{V}_{ag}/Z_f \end{bmatrix} \quad (3)$$

- This gives one of our ***sequence-domain fault conditions***

$$\boldsymbol{I}_0 = \boldsymbol{I}_1 = \boldsymbol{I}_2 \quad (4)$$

Unsymmetrical Fault Analysis – SLG Fault

- We know that

$$\boldsymbol{I}_a = \frac{\boldsymbol{V}_{ag}}{Z_f} = \boldsymbol{I}_0 + \boldsymbol{I}_1 + \boldsymbol{I}_2 \qquad (5)$$

and

$$\boldsymbol{V}_{ag} = \boldsymbol{V}_0 + \boldsymbol{V}_1 + \boldsymbol{V}_2 \qquad (6)$$

- Using (5) and (6) in (1), we get

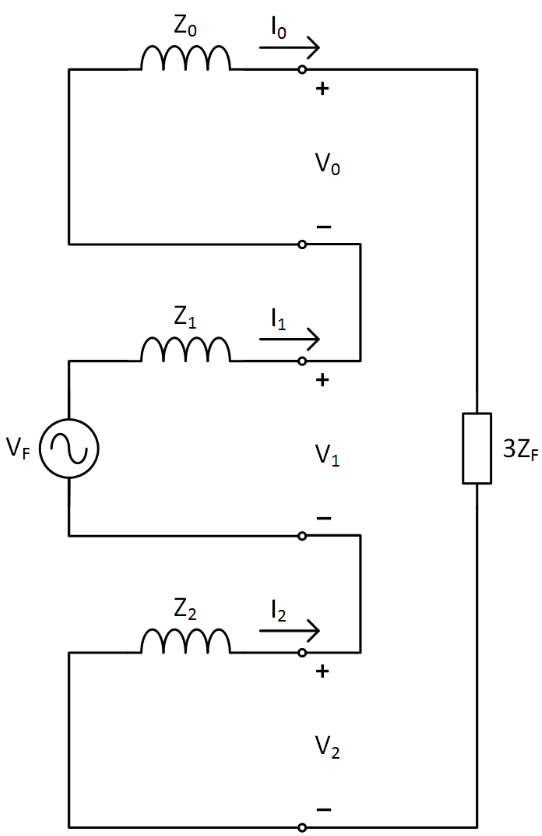
$$\boldsymbol{I}_0 + \boldsymbol{I}_1 + \boldsymbol{I}_2 = \frac{1}{Z_f}(\boldsymbol{V}_0 + \boldsymbol{V}_1 + \boldsymbol{V}_2)$$

- Using (4), this gives our second ***sequence-domain fault condition***

$$\boldsymbol{I}_0 = \boldsymbol{I}_1 = \boldsymbol{I}_2 = \frac{1}{3Z_f}(\boldsymbol{V}_0 + \boldsymbol{V}_1 + \boldsymbol{V}_2) \qquad (7)$$

Unsymmetrical Fault Analysis – SLG Fault

- The sequence-domain fault conditions are satisfied by connecting the sequence networks in series along with three times the fault impedance
- We want to find the phase domain fault current, $\boldsymbol{I}_F$



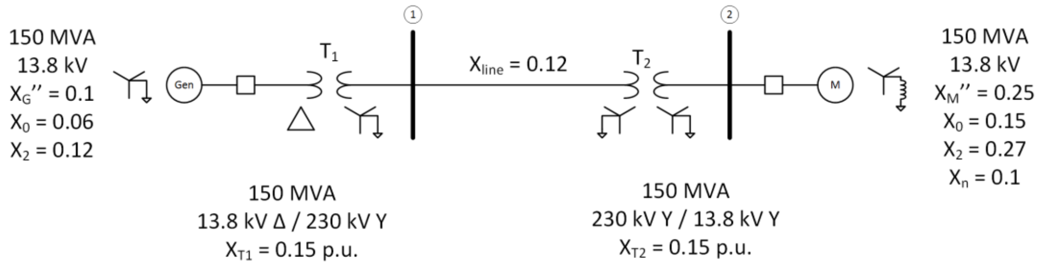
$$\boldsymbol{I}_F = \boldsymbol{I}_a = \boldsymbol{I}_0 + \boldsymbol{I}_1 + \boldsymbol{I}_2 = 3\boldsymbol{I}_1$$

$$\boldsymbol{I}_1 = \frac{\boldsymbol{V}_F}{Z_0+Z_1+Z_2+3Z_F}$$

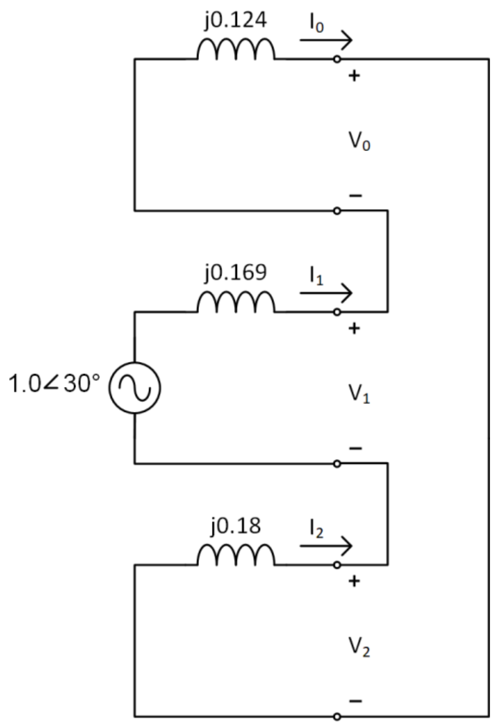
$$\boxed{\boldsymbol{I}_F = \frac{3\boldsymbol{V}_F}{Z_0+Z_1+Z_2+3Z_F}} \tag{8}$$

SLG Fault - Example

- Returning to our example power system



- The interconnected sequence networks for a ***bolted fault*** at bus 1:



SLG Fault - Example

- The fault current is

$$\boldsymbol{I}_F = \frac{3\boldsymbol{V}_F}{Z_0+Z_1+Z_2+3Z_F}$$

$$\boldsymbol{I}_F = \frac{3.0\angle 30°}{j0.473} = 6.34\angle -60°\ p.u.$$

- The current base at bus 1 is

$$\boldsymbol{I}_b = \frac{S_b}{\sqrt{3}V_{b1}} = \frac{150\ MVA}{\sqrt{3}\ 230\ kV} = 376.5\ A$$

- So the fault current in kA is

$$\boldsymbol{I}_F = (6.34\angle -60°)(376.5\ A)$$

$$\boxed{\boldsymbol{I}_F = 2.39\angle -60°\ kA}$$

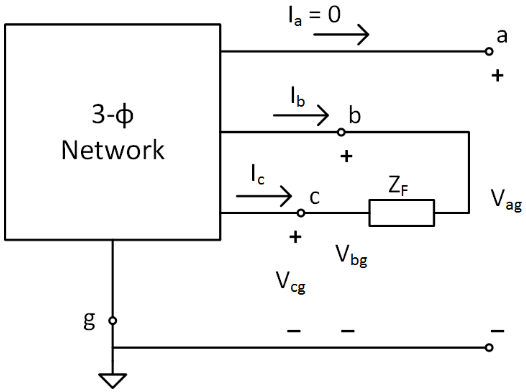
Line-to-Line Fault

Unsymmetrical Fault Analysis – LL Fault

- Now consider a ***line-to-line fault*** between phase b and phase c through impedance Z_F
- Phase-domain fault conditions:

$$\boldsymbol{I}_a = 0 \qquad (9)$$

$$\boldsymbol{I}_b = -\boldsymbol{I}_c = \frac{\boldsymbol{V}_{bg} - \boldsymbol{V}_{cg}}{Z_F} \qquad (10)$$



- Transforming to the sequence domain

$$\begin{bmatrix} \boldsymbol{I}_0 \\ \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ \boldsymbol{I}_b \\ -\boldsymbol{I}_b \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ (a - a^2)\boldsymbol{I}_b \\ (a^2 - a)\boldsymbol{I}_b \end{bmatrix} \qquad (11)$$

- So, the first two sequence-domain fault conditions are

$$\boldsymbol{I}_0 = 0 \qquad (12)$$

$$\boldsymbol{I}_2 = -\boldsymbol{I}_1 \qquad (13)$$

Unsymmetrical Fault Analysis – LL Fault

- To derive the remaining sequence-domain fault condition, rearrange (10) and transform to the sequence domain

$$\boldsymbol{V}_{bg} - \boldsymbol{V}_{cg} = \boldsymbol{I}_b Z_F$$

$$(\boldsymbol{V}_0 + a^2\boldsymbol{V}_1 + a\boldsymbol{V}_2) - (\boldsymbol{V}_0 + a\boldsymbol{V}_1 + a^2\boldsymbol{V}_2)$$

$$= (\boldsymbol{I}_0 + a^2\boldsymbol{I}_1 + a\boldsymbol{I}_2) Z_F$$

$$a^2\boldsymbol{V}_1 + a\boldsymbol{V}_2 - a\boldsymbol{V}_1 - a^2\boldsymbol{V}_2 = (a^2 - a)\boldsymbol{I}_1 Z_F$$

$$(a^2 - a)\boldsymbol{V}_1 - (a^2 - a)\boldsymbol{V}_2 = (a^2 - a)\boldsymbol{I}_1 Z_F$$

- The last sequence-domain fault condition is

$$\boldsymbol{V}_1 - \boldsymbol{V}_2 = \boldsymbol{I}_1 Z_F \qquad (14)$$

Unsymmetrical Fault Analysis – LL Fault

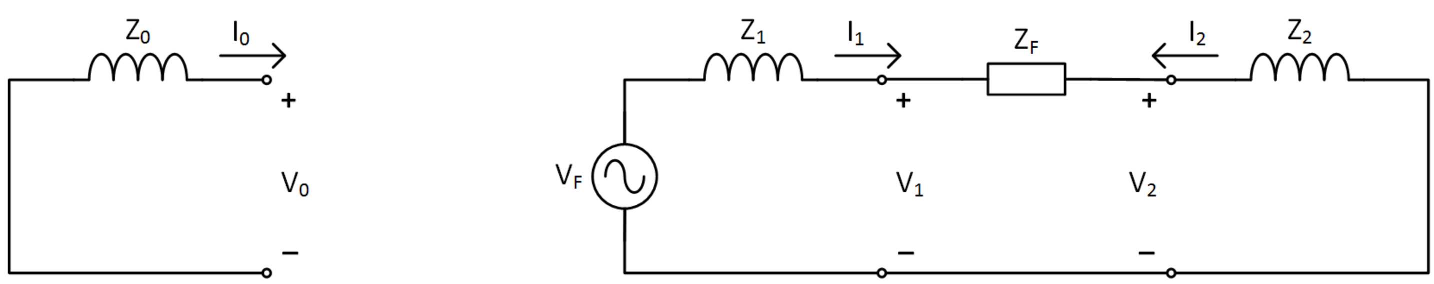
- Sequence-domain fault conditions

$$I_0 = 0 \quad (12)$$

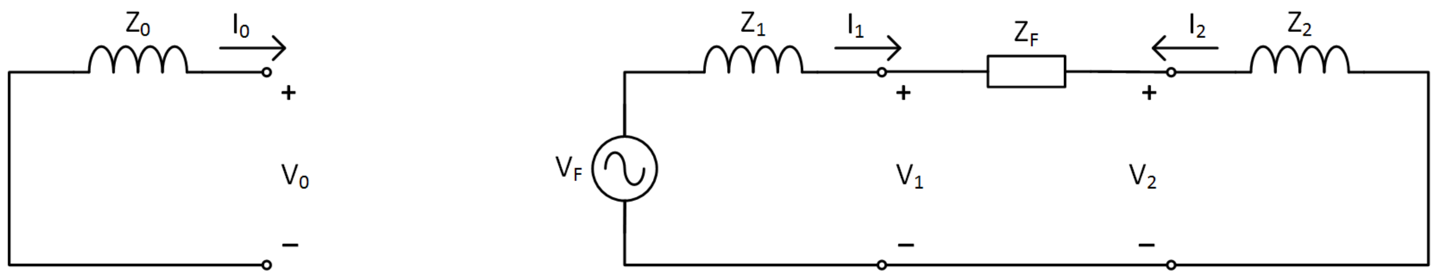
$$I_2 = -I_1 \quad (13)$$

$$V_1 - V_2 = I_1 Z_F \quad (14)$$

- These can be satisfied by:
  - Leaving the zero-sequence network open
  - Connecting the terminals of the positive- and negative-sequence networks together through Z_F



Unsymmetrical Fault Analysis – LL Fault



- The fault current is the phase b current, which is given by

$$\boldsymbol{I}_F = \boldsymbol{I}_b = \boldsymbol{I}_0 + a^2\boldsymbol{I}_1 + a\boldsymbol{I}_2$$

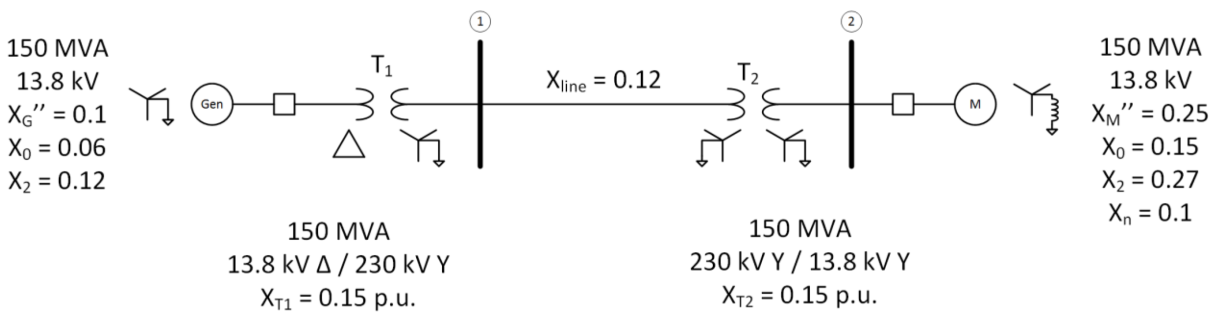
$$\boldsymbol{I}_F = a^2\boldsymbol{I}_1 - a\boldsymbol{I}_1$$

$$\boldsymbol{I}_F = -j\sqrt{3}\,\boldsymbol{I}_1 = \frac{-j\sqrt{3}\,\boldsymbol{V}_F}{Z_1+Z_2+Z_F}$$

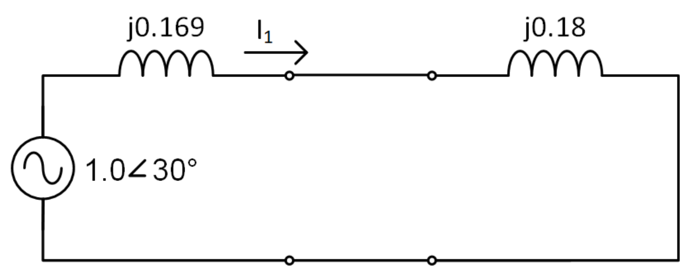
$$\boxed{\boldsymbol{I}_F = \frac{\sqrt{3}\,\boldsymbol{V}_F\angle -90°}{Z_1+Z_2+Z_F}} \qquad (15)$$

LL Fault - Example

- Now consider the same system with a bolted line-to-line fault at bus 1



- The sequence network:



LL Fault - Example

$$\boldsymbol{I}_1 = \frac{1\angle 30°}{j0.349} = 2.87\angle - 60°$$

- The subtransient fault current is given by (15) as

$$\boldsymbol{I}_F = \left(\sqrt{3}\angle - 90°\right)(2.87\angle - 60°)$$

$$\boldsymbol{I}_F = 4.96\angle - 150°\ p.u.$$

- Using the previously-determined current base, we can convert the fault current to kA

$$\boldsymbol{I}_F = I_{b1} \cdot 4.96\angle - 150°$$

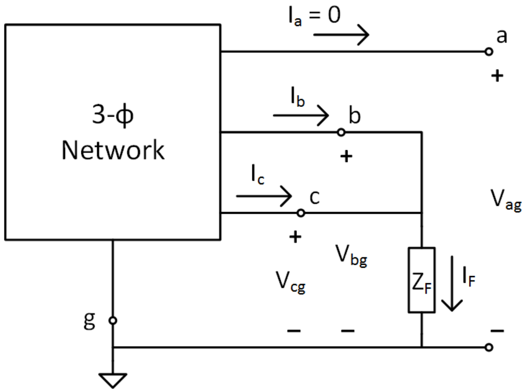
$$\boldsymbol{I}_F = (4.96\angle - 150°)(376.5A)$$

$$\boxed{\boldsymbol{I}_F = 1.87\angle - 150°\ kA}$$

Double-Line-to-Ground Fault

Unsymmetrical Fault Analysis – DLG Fault

- Now consider a ***double line-to-ground fault***
  - Assume phases b and c are shorted to ground through Z_F
- Phase-domain fault conditions:



$$I_a = 0 \quad (16)$$

$$I_b + I_c = \frac{V_{bg}}{Z_F} = \frac{V_{cg}}{Z_F} \quad (17)$$

- It can be shown that (16) and (17) transform to the following sequence-domain fault conditions (analysis skipped here)

$$I_0 + I_1 + I_2 = 0 \quad (18)$$

$$V_1 = V_2 \quad (19)$$

$$I_0 = \frac{1}{3Z_F}(V_0 - V_1) \quad (20)$$

Unsymmetrical Fault Analysis – DLG Fault

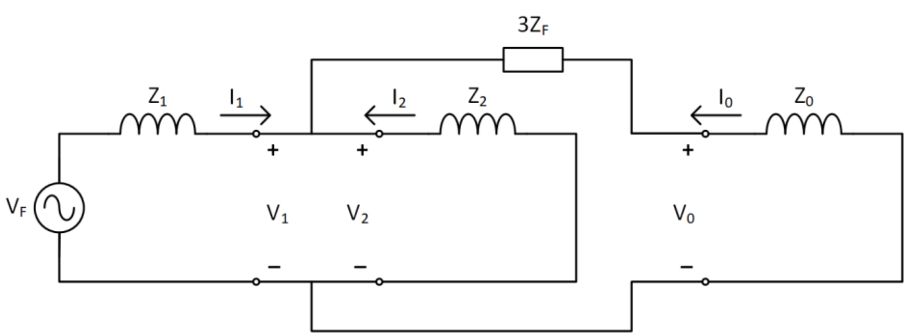
- Sequence-domain fault conditions

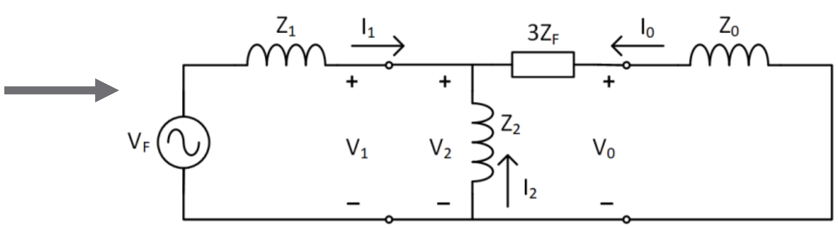
$$I_0 + I_1 + I_2 = 0 \quad (18)$$

$$V_1 = V_2 \quad (19)$$

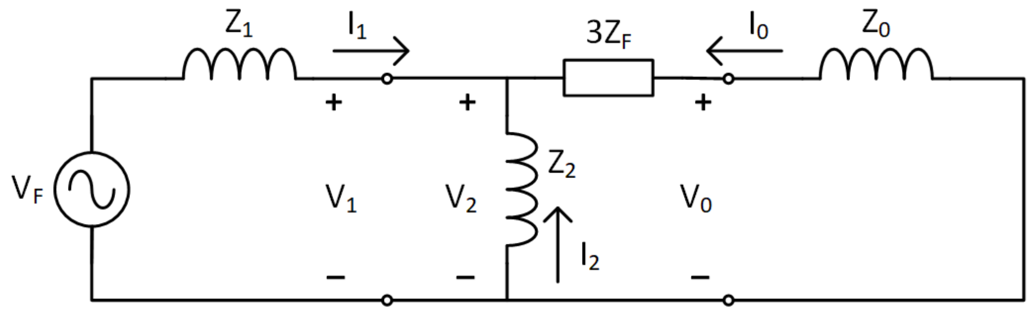
$$I_0 = \frac{1}{3Z_F}(V_0 - V_1) \quad (20)$$

- To satisfy these fault conditions
  - Connect the positive- and negative-sequence networks together directly
  - Connect the zero- and positive-sequence networks together through $3Z_F$





Unsymmetrical Fault Analysis – DLG Fault



- The fault current is the sum of the phase b and phase c currents, as given by (17)
  - In the sequence domain the fault current is

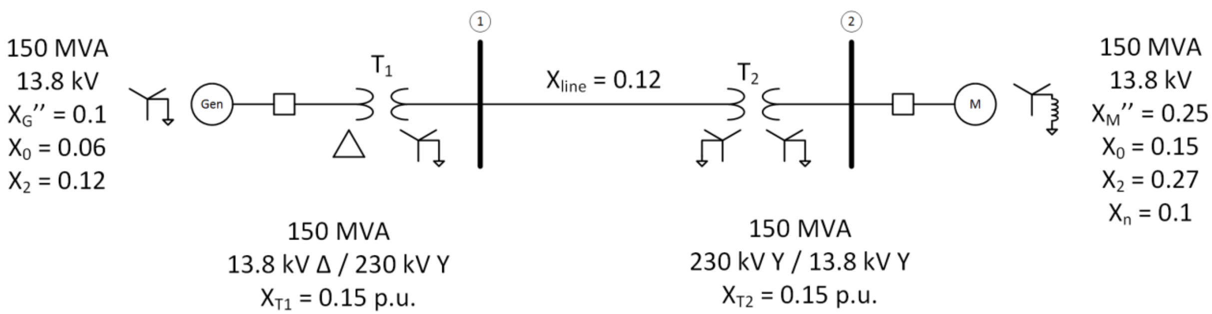
$$\boldsymbol{I}_F = \boldsymbol{I}_b + \boldsymbol{I}_c = 3\boldsymbol{I}_0$$

$$\boxed{\boldsymbol{I}_F = 3\boldsymbol{I}_0} \qquad (21)$$

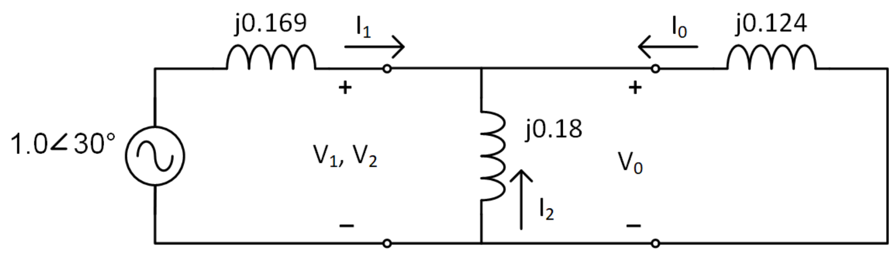
- I_0 can be determined by a simple analysis (e.g. nodal) of the interconnected sequence networks

DLG Fault - Example

- Now determine the subtransient fault current for a bolted double line-to-ground fault at bus 1

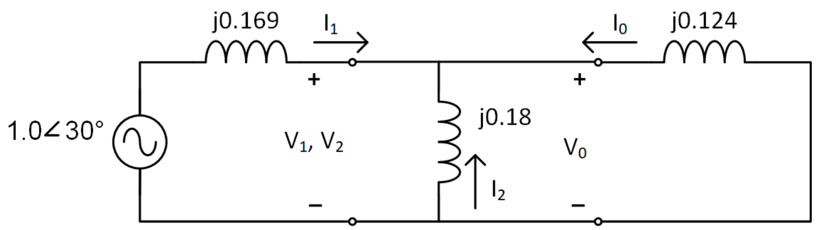


- The sequence network:



- Here, because $Z_F = 0$, $\boldsymbol{V}_0 = \boldsymbol{V}_1 = \boldsymbol{V}_2$

DLG Fault - Example



- To find $\boldsymbol{I}_F$, we must determine $\boldsymbol{I}_0$
- We can first find V_0 by applying voltage division

$$\boldsymbol{V}_0 = \boldsymbol{V}_F \frac{Z_2||Z_0}{Z_1 + Z_2||Z_0}$$

$$\boldsymbol{V}_0 = 1.0\angle 30° \frac{j0.18||j0.124}{j0.169 + j0.18||j0.124}$$

$$\boldsymbol{V}_0 = 0.303\angle 30°$$

DLG Fault - Example

- Next, calculate $\boldsymbol{I}_0$

$$\boldsymbol{I}_0 = \frac{-\boldsymbol{V}_0}{Z_0} = \frac{-0.303\angle 30°}{j0.124} = 2.44\angle 120°\, p.u.$$

- The per-unit fault current is

$$\boldsymbol{I}_F = 3\boldsymbol{I}_0 = 7.33\angle 120°\, p.u.$$

- Using the current base to convert to kA, gives the subtransient DLG fault current

$$\boldsymbol{I}_F = (7.33\angle 120°)(376.5\ A)$$

$$\boxed{\boldsymbol{I}_F = 2.76\angle 120°\ kA}$$

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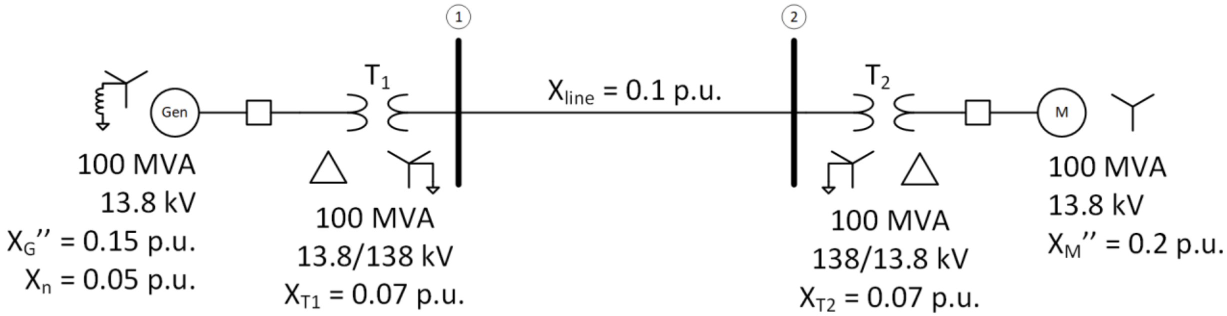
Example Problems

Draw the sequence networks for the following power system. Assume the generator is operating at rated voltage.



Reduce the sequence networks to their Thévenin equivalents for a fault occurring half of the way along the transmission line.

Determine the subtransient fault current resulting from a DLG fault, half way along the transmission line, through an impedance of j0.2 p.u.