SECTION 4: ULTRACAPACITORS

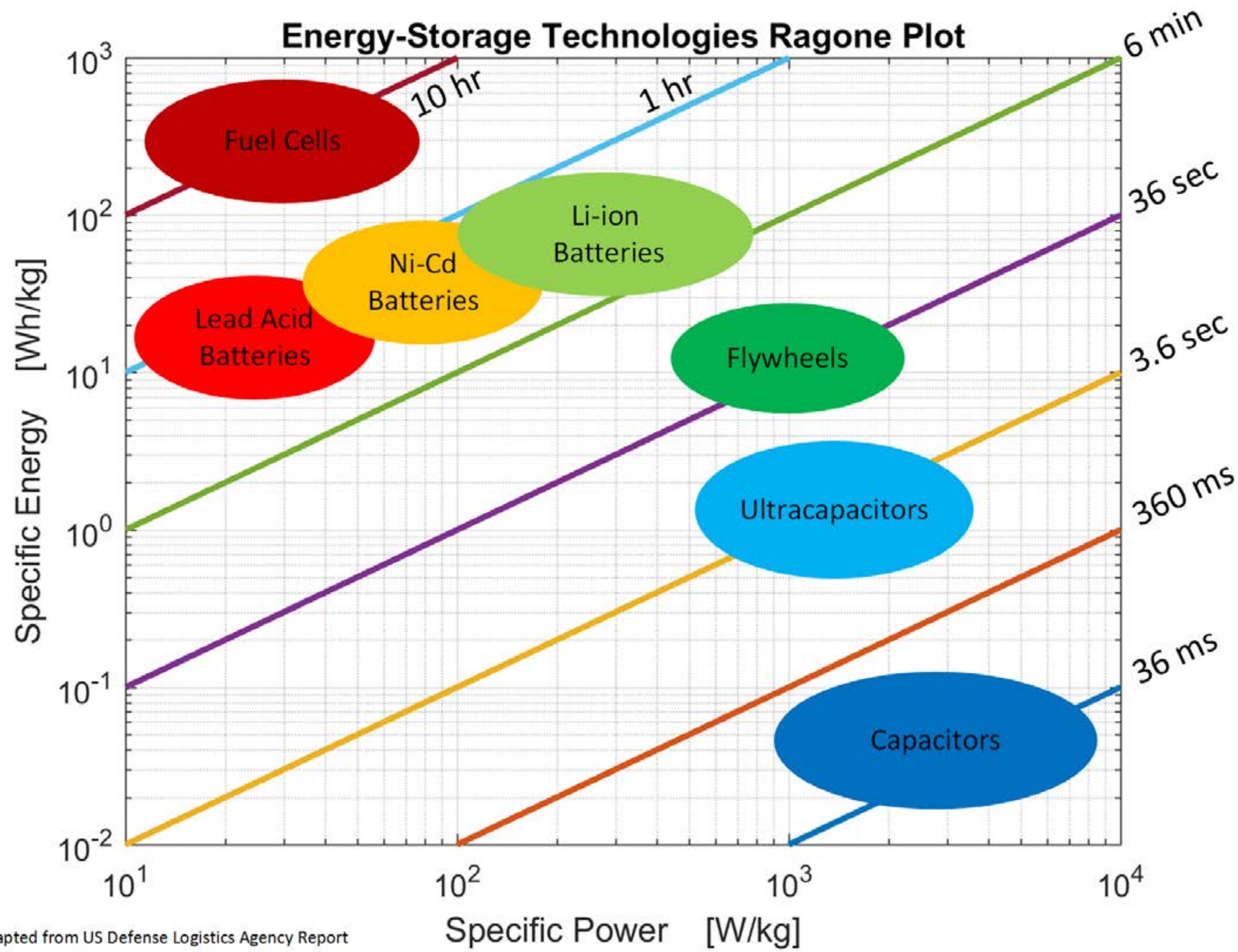
ESE 471 – Energy Storage Systems

Introduction

Ultracapacitors

- Capitors are ***electrical energy*** storage devices
  - Energy is stored in an ***electric field***
- ***Advantages*** of capacitors for energy storage
  - High specific power
  - High efficiency
  - Equal charge and discharge rates
  - Long lifetime
- ***Disadvantages*** of capacitors for energy storage
  - Low specific energy
- ***Ultracapacitors*** (or ***supercapacitors***) are variations of traditional capacitors with significantly improved specific energy
  - Useful in high-power energy-storage applications

Ultracapacitors – Ragone Plot



Adapted from US Defense Logistics Agency Report

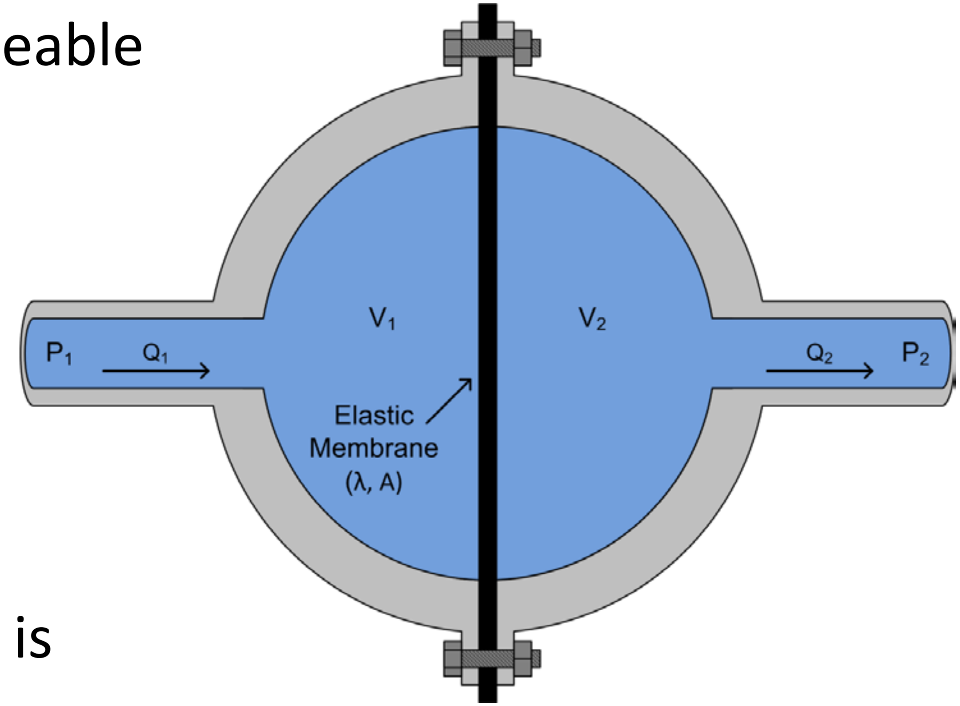
Ultracapacitors - Applications

- Ultracapacitors are useful in relatively ***high-power***, ***low-energy*** applications
  - They occupy a similar region in the Ragone plane as flywheels
- ***Energy recovery*** and ***regenerative braking*** applications
  - Cars
    - EV, HEV, ICE (e.g. Mazda 6 i-ELOOP)
  - Buses
  - Trains
  - Cranes
  - Elevators
- ***Uninterruptible power supply*** (***UPS***) applications
  - Fast-responding, short-term power until generators take over
- ***Wind turbine pitch control***
  - Put turbine blades in safe position during loss of power

Capacitor Fundamentals

Fluid Capacitor

- Consider the following device:
  - Two rigid hemispherical shells
  - Separated by an impermeable elastic membrane
    - Modulus of elasticity, λ
    - Area, A
  - Incompressible fluid
  - External pumps set pressure or flow rate at each port
  - Total volume inside shell is constant
  - Volume on either side of the membrane may vary



Fluid Capacitor – Equilibrium

- Equal pressures

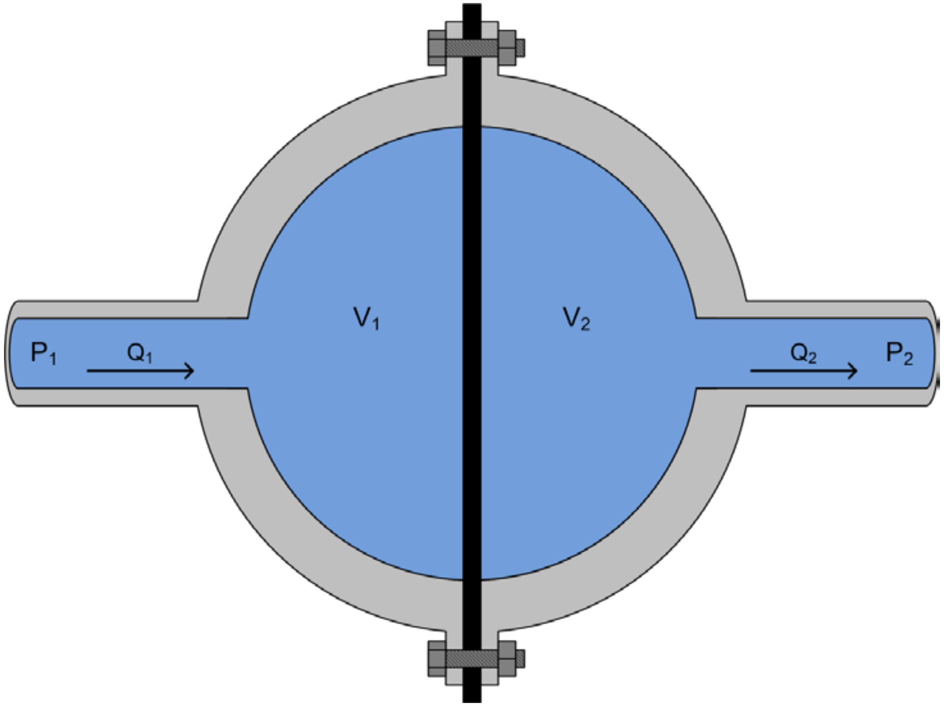
$$\Delta P = P_1 - P_2 = 0$$

- No fluid flow

$$Q_1 = Q_2 = 0$$

- Membrane does not deform
- Equal volume on each side

$$V_1 = V_2 = \frac{V}{2}$$



Fluid Capacitor – $P_1 > P_2$

- Pressure differential

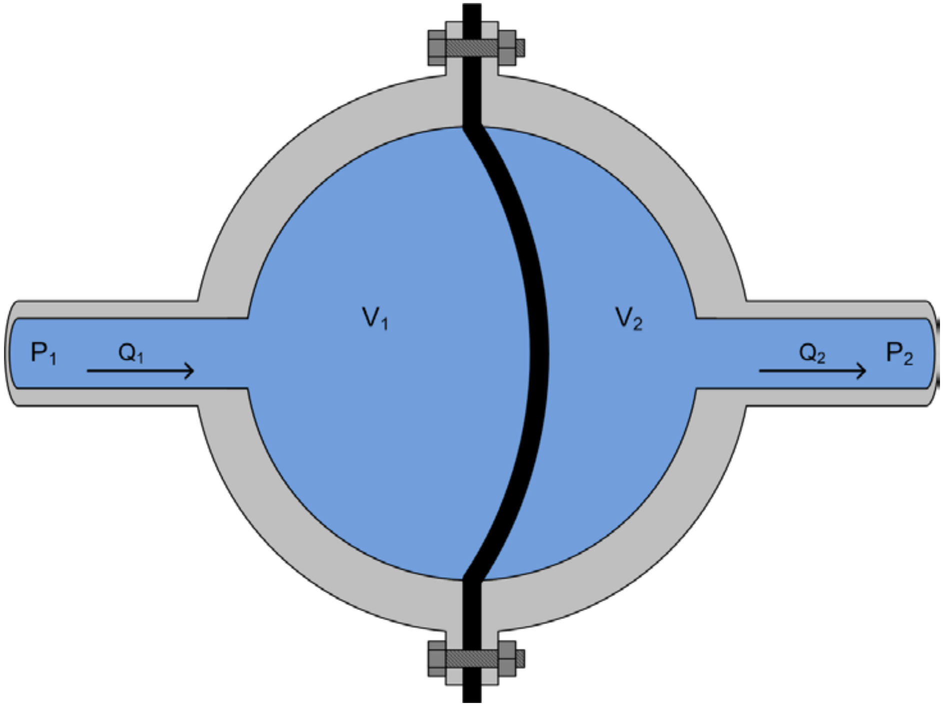
$$\Delta P = P_1 - P_2 > 0$$

- Membrane deforms
- Volume differential

$$\Delta V = V_1 - V_2 > 0$$

- Transient flow as membrane stretches, but...
- No steady-state flow
  - As $t \to \infty$

$$Q_1 = Q_2 = 0$$



Fluid Capacitor – $P_1 < P_2$

- Pressure differential

$$\Delta P = P_1 - P_2 < 0$$

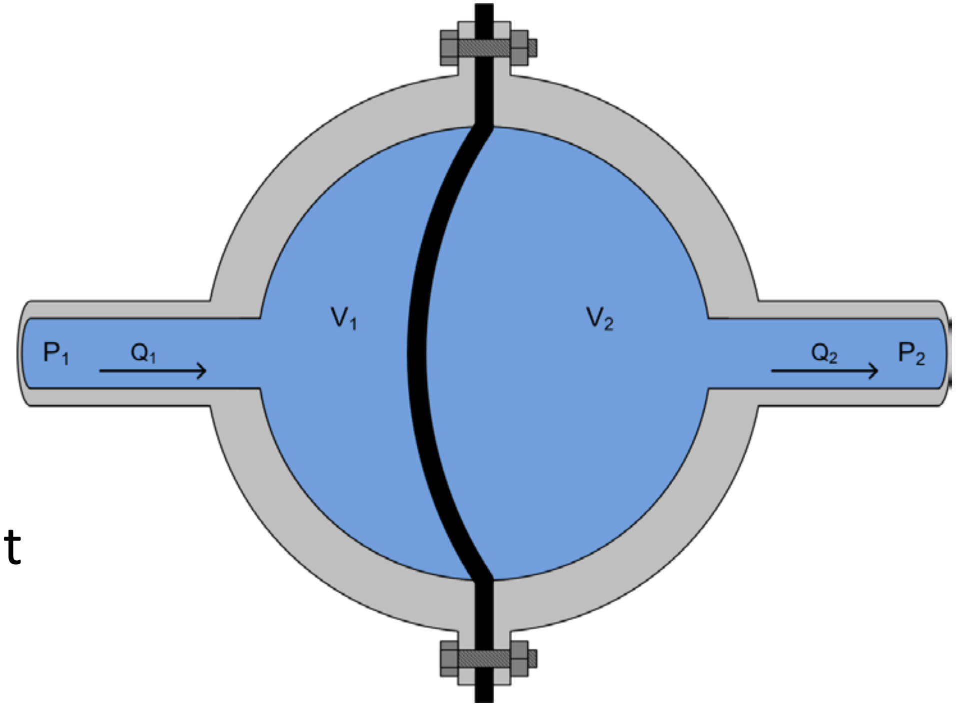
- Volume differential

$$\Delta V = V_1 - V_2 < 0$$

- ΔV proportional to:
  - Pressure differential
  - Physical properties, λ, A
- Total volume remains constant

$$V_1 + V_2 = V$$

- Again, no steady-state flow



Fluid Capacitor – Constant Flow Rate

- Constant flow rate forced into port 1

$$Q_1 \neq 0$$

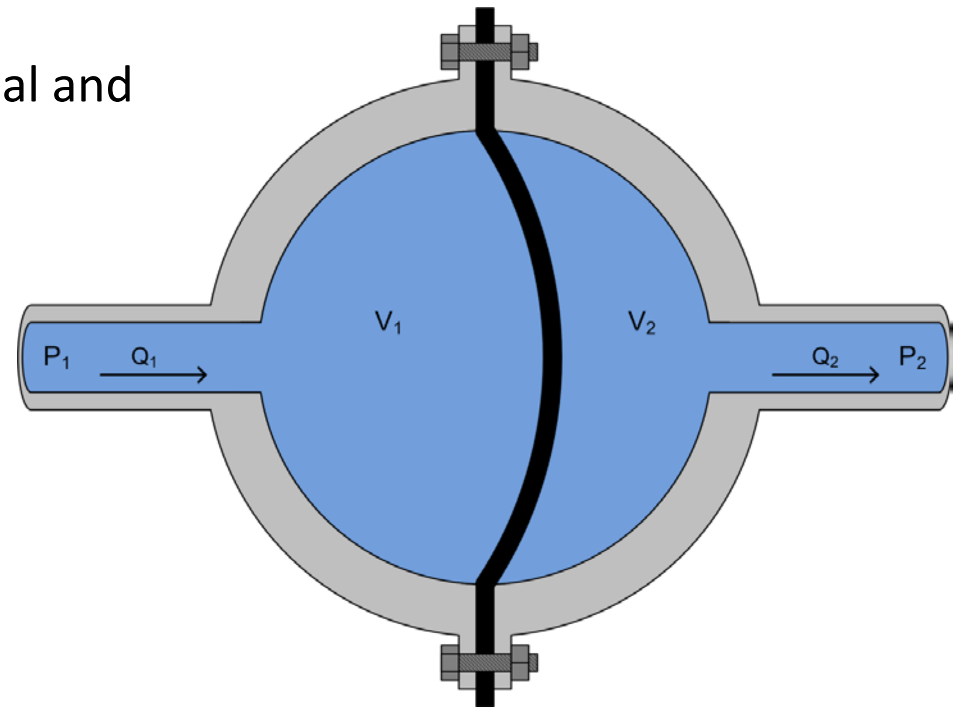
- Incompressible, so flows are equal and opposite

$$Q_1 = Q_2$$

- Volume on each side proportional to time

$$V_1 = \frac{V}{2} + Q_1 \cdot t$$

$$V_2 = \frac{V}{2} - Q_2 \cdot t = \frac{V}{2} - Q_1 \cdot t$$



- Volume differential proportional to time

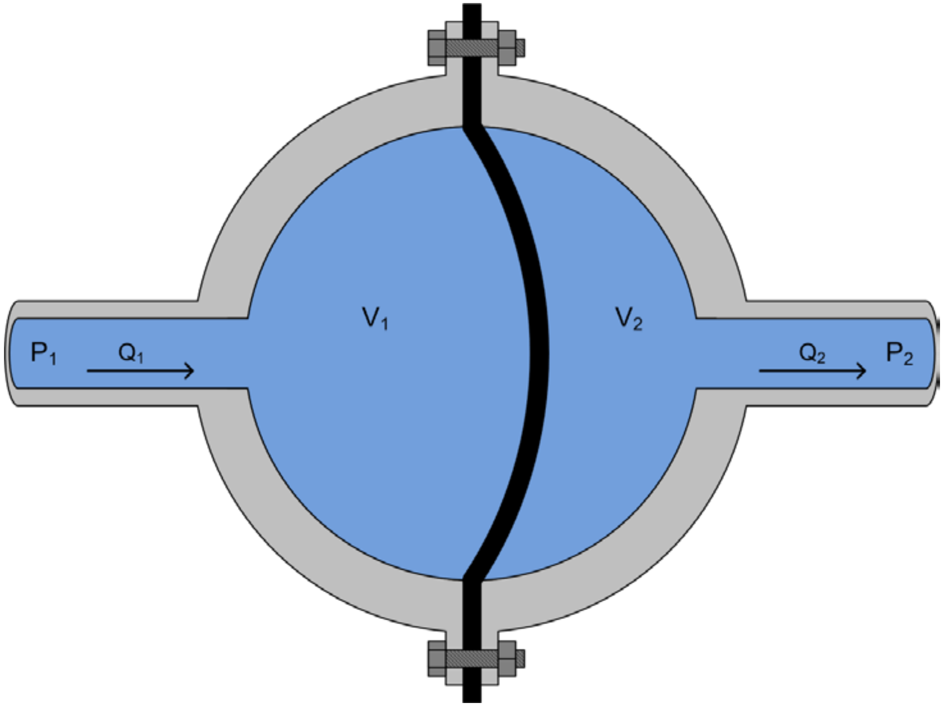
$$\Delta V = V_1 - V_2 = 2Q_1 \cdot t$$

Fluid Capacitor – Capacitance

- Define a relationship between differential volume and pressure
- ***Capacitance***

$$C = \frac{\Delta V}{\Delta P}$$

- Intrinsic device property
- Determined by physical parameters:
  - Membrane area, A
  - Modulus of elasticity, λ



Fluid Capacitor – DC vs. AC

- In steady-state (DC), no fluid flows

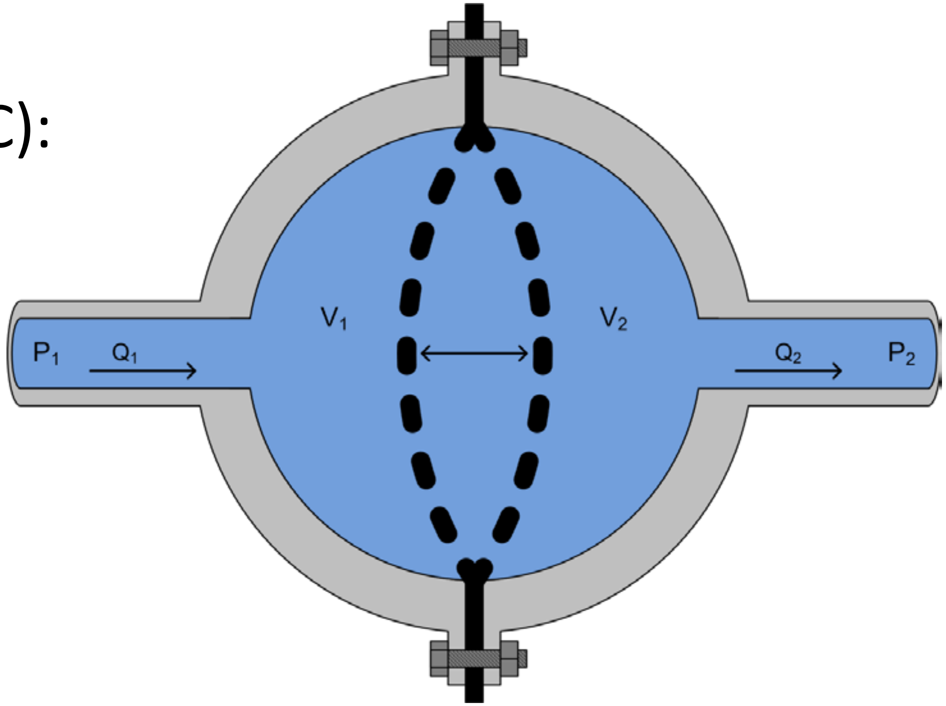
$$Q_1 = Q_2 = 0$$

- Consider sinusoidal ΔP (AC):

$$\Delta P = P\sin(\omega t)$$

- Resulting flow rate is proportional to:
  - Rate of change of differential pressure
  - Capacitance

$$Q_1 = Q_2 = C\frac{dP}{dt} = \omega CP\cos(\omega t)$$

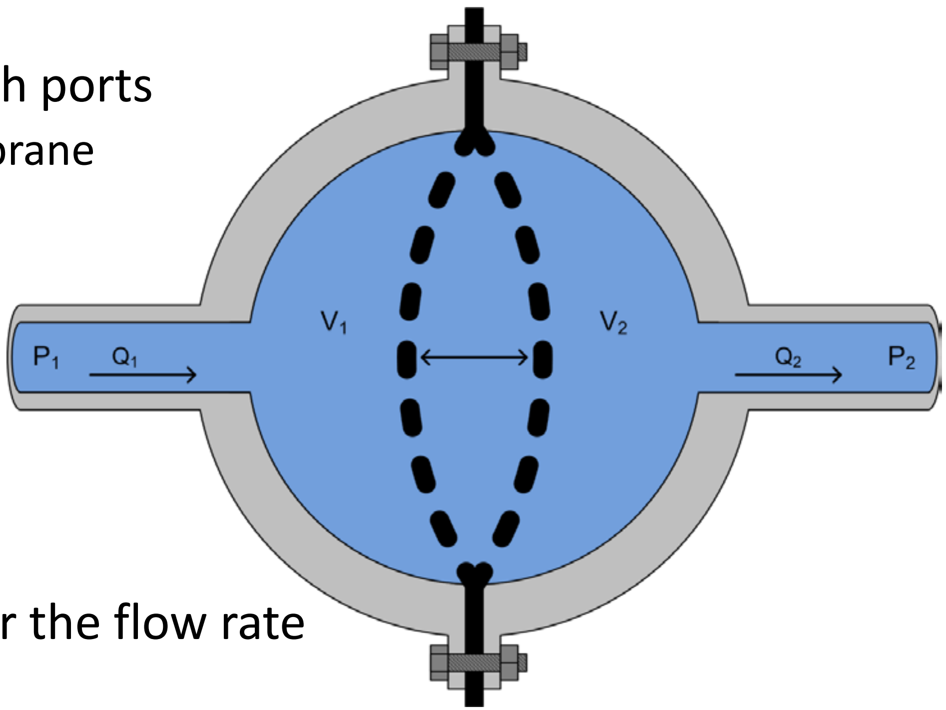


Fluid Capacitor – Time-Varying ΔP

- Equal and opposite flow at both ports

$$Q_1 = Q_2$$

- Not the same fluid flowing at both ports
  - Fluid cannot permeate the membrane



- ***Fluid appears to flow through the device***
  - Due to the displacement of the membrane
  - A ***displacement flow***
- The faster ΔP changes, the higher the flow rate

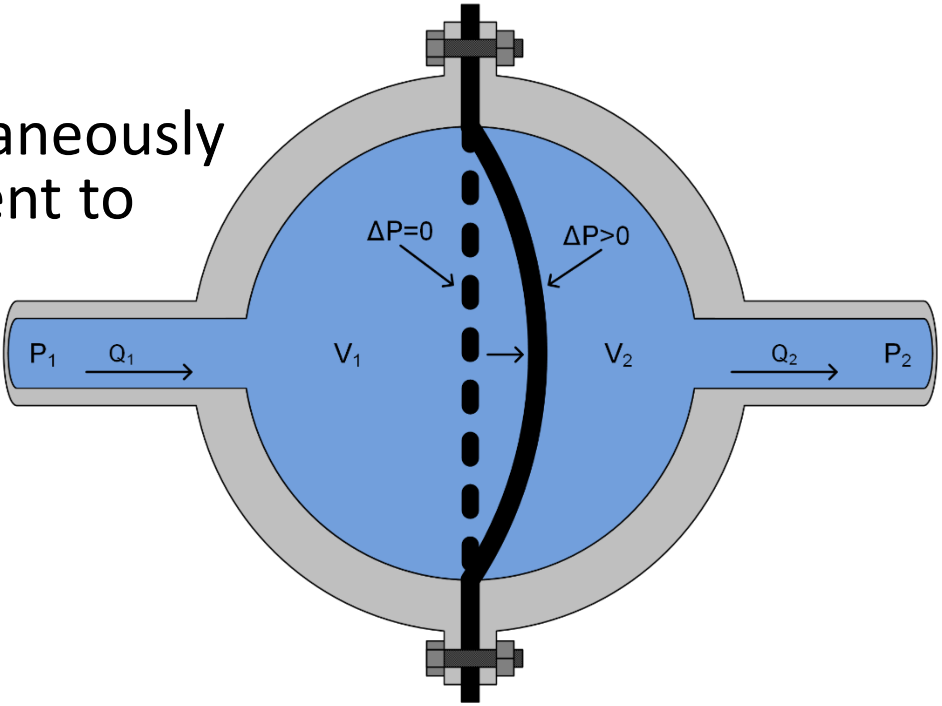
$$Q \propto \omega$$

- The larger the capacitance, the higher the flow rate

$$Q \propto C$$

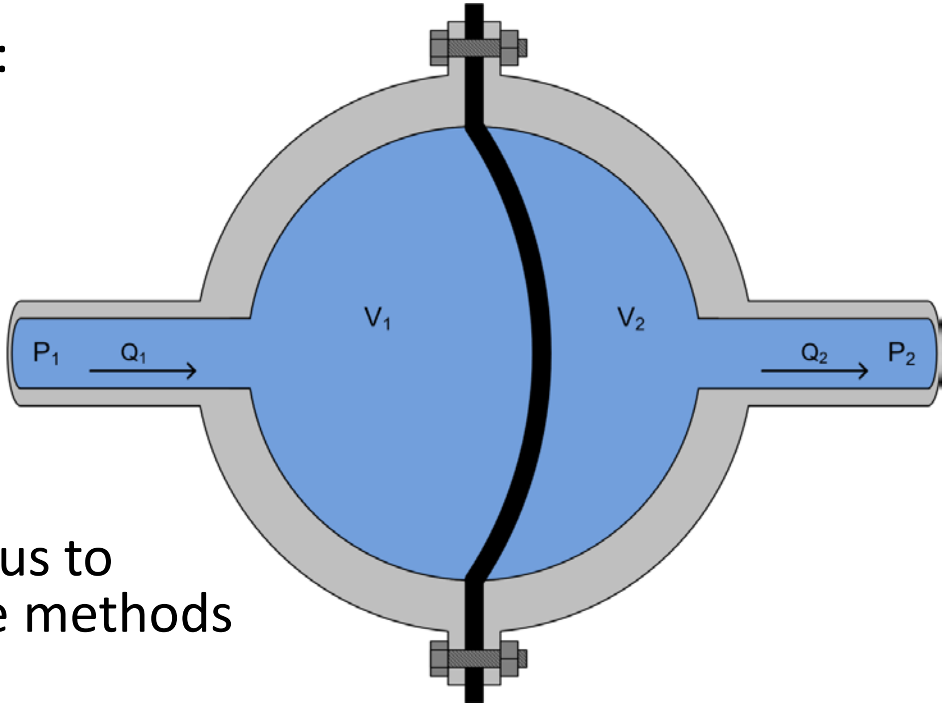
Fluid Capacitor – Changing ΔP

- A given ΔP corresponds to a particular membrane displacement
  - Forces must balance
- Membrane cannot instantaneously jump from one displacement to another
- Step change in displacement/pressure is impossible
  - Would require an infinite flow rate
- ***Pressure across a fluid capacitor cannot change instantaneously***



Fluid Capacitor – Energy Storage

- Stretched membrane ***stores energy***
  - ***Potential energy***
- Stored energy proportional to:
  - ΔP
  - ΔV
- Energy released as membrane returns
  - P and Q are supplied
- Not a real device, but analogous to other potential energy storage methods
  - PHES
  - CAES
  - ***Electrical capacitors***



Electrical Capacitor

- In the electrical domain, our “working fluid” is ***positive electrical charge***
- In either domain, we have a ***potential-driven flow***

Fluid Domain	Electrical Domain
Pressure – P	Voltage – V
Volumetric flow rate – Q	Current – I
Volume – V	Charge – Q

Electrical Capacitor

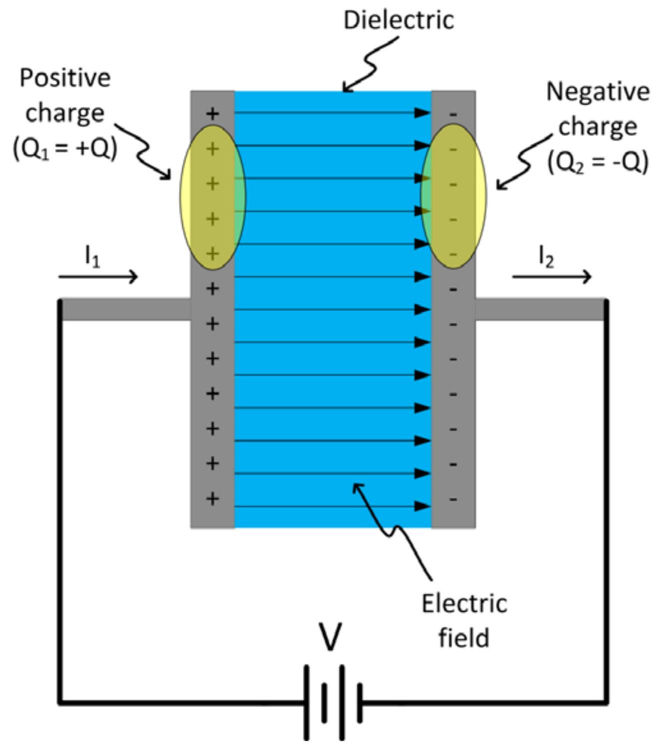
- Parallel-plate capacitor
  - Parallel metal plates
  - Separated by an insulator
- ***Applied voltage creates charge differential***
  - Equal and opposite charge

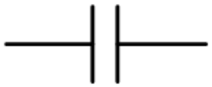
$$Q_1 = -Q_2$$

  - Zero net charge
- Equal current

$$I_1 = I_2$$

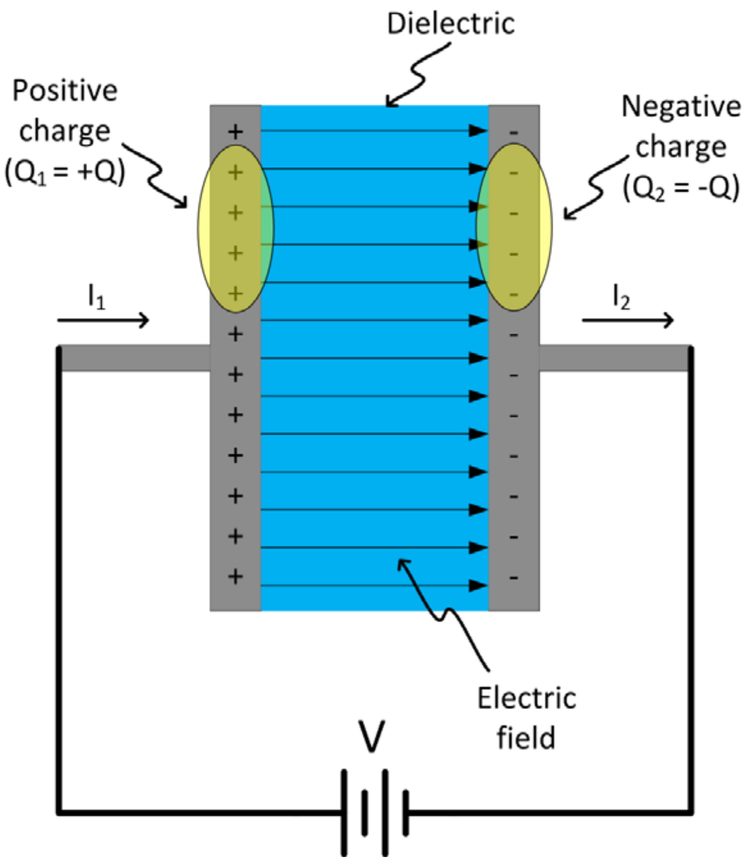
  - What flows in one side flows out the other



- Schematic symbol: 
- Units: Farads (F)

Electrical Capacitor – Electric Field

- Charge differential results in an ***electric field***, $\boldsymbol{E}$, in the dielectric
  - Units: V/m
- $|\boldsymbol{E}|$ is inversely proportional to dielectric thickness, d
- Above some maximum electric field strength, dielectric will ***break down***
  - Conducts electrical current
  - Maximum capacitor voltage rating

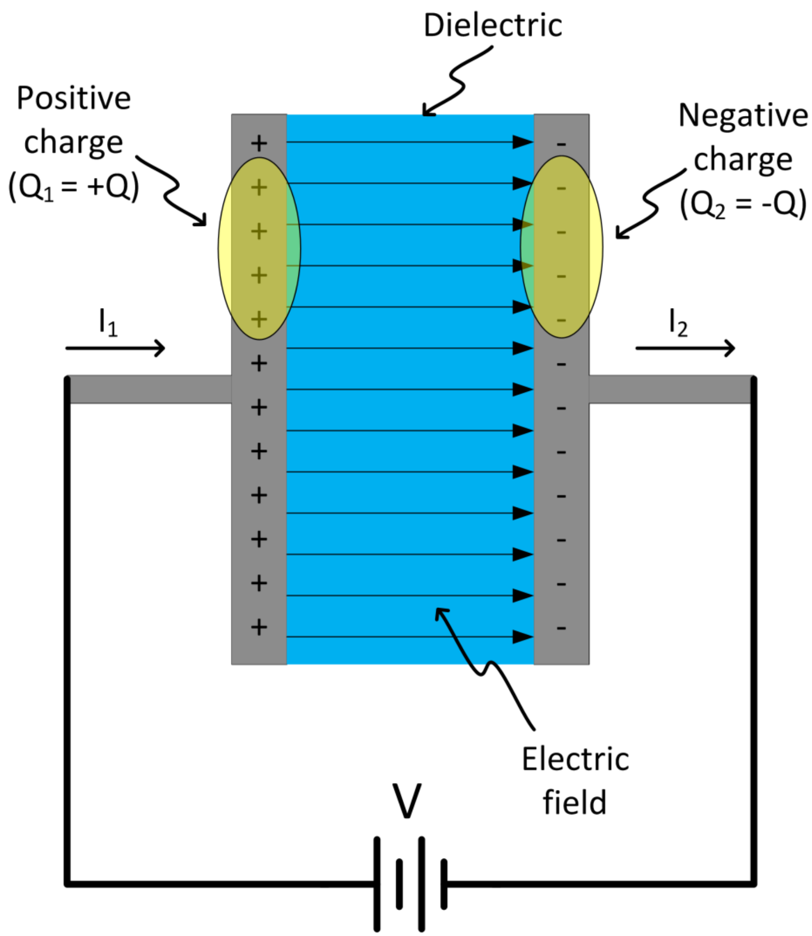


Electrical Capacitor - Capacitance

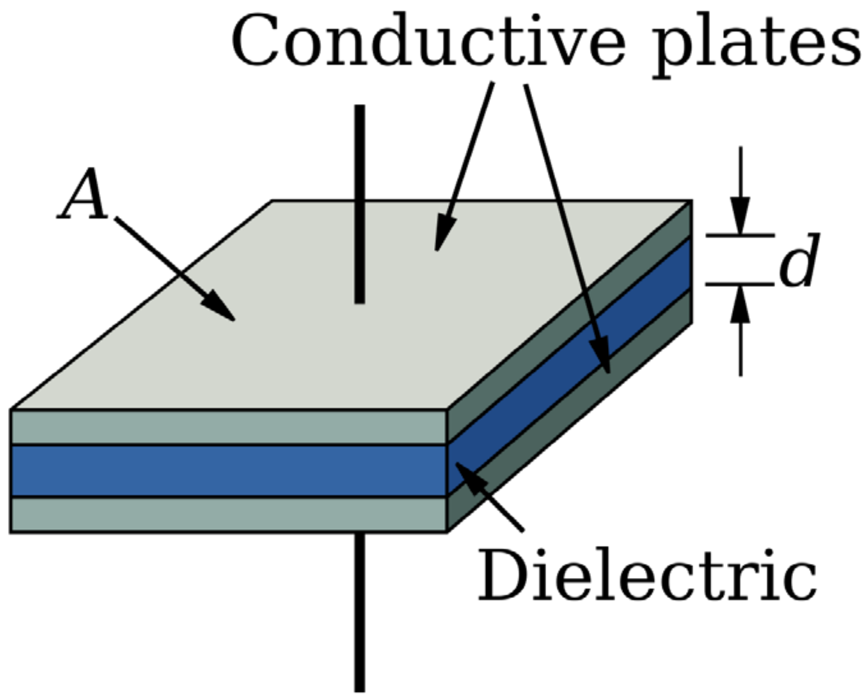
- ***Capacitance***
  - Ratio of charge to voltage

$$C = \frac{Q}{V}$$

- Intrinsic device property
- Proportional to physical parameters:
  - Dielectric thickness, d
  - Dielectric constant, ε
  - Area of electrodes, A



Parallel-Plate Capacitor



- Capacitance

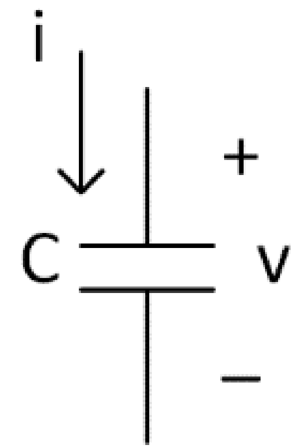
$$C = \frac{\varepsilon A}{d}$$

  - ε: dielectric permittivity
  - A: area of the plates
  - d: dielectric thickness

- Capacitance is maximized by using:
  - High-dielectric-constant materials
  - Thin dielectric
  - Large-surface-area plates

Capacitors – Voltage and Current

- Current through a capacitor is proportional to
  - Capacitance
  - Rate of change of the voltage



$$i(t) = C\frac{dv}{dt}$$

- Voltage across capacitor results from an accumulation of charge differential
  - Capacitor integrates current

$$v(t) = \frac{1}{C}\int i(t)dt$$

Voltage Change Across a Capacitor

- For a step change in voltage,

$$\frac{dv}{dt} = \infty$$

  - The corresponding current would be ***infinite***

- ***Voltage across a capacitor cannot change instantaneously***

- Current can change instantaneously, but voltage is the integral of current

$$\lim_{\Delta t \to 0} \Delta V = \lim_{\Delta t \to 0} \int_{t_0}^{t_0+\Delta t} i(t)dt = 0$$

Capacitors – Open Circuits at DC

- Current through a capacitor is proportional to the time rate of change of the voltage across the capacitor

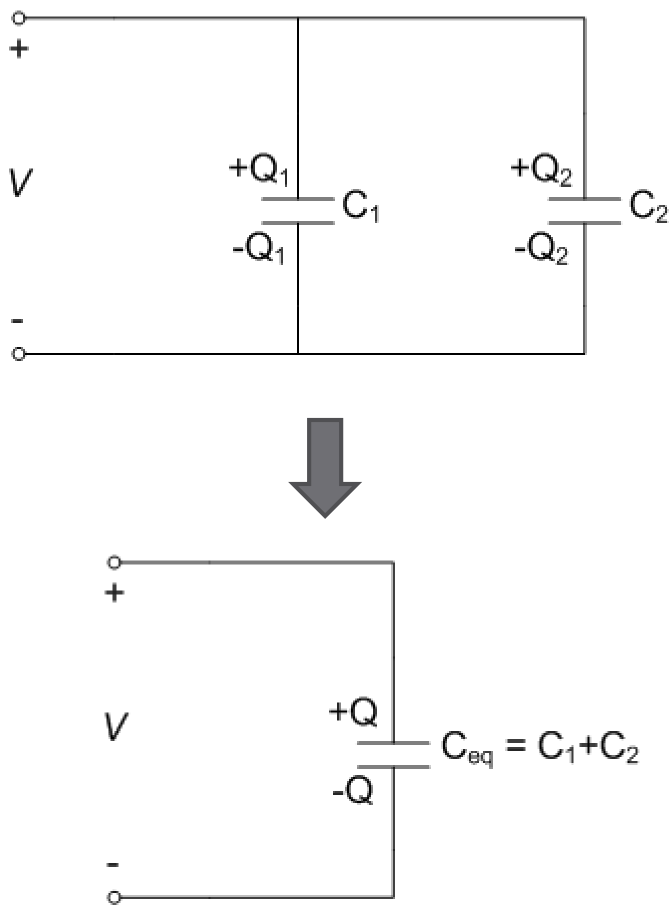
$$i(t) = C\frac{dv}{dt}$$

- A DC voltage does not change with time, so

$$\frac{dv}{dt} = 0 \quad \text{and} \quad i(t) = 0$$

- ***A capacitor is an open circuit at DC***

Capacitors in Parallel



- Total charge on two parallel capacitors is

$$Q = Q_1 + Q_2$$

$$Q = C_1V + C_2V$$

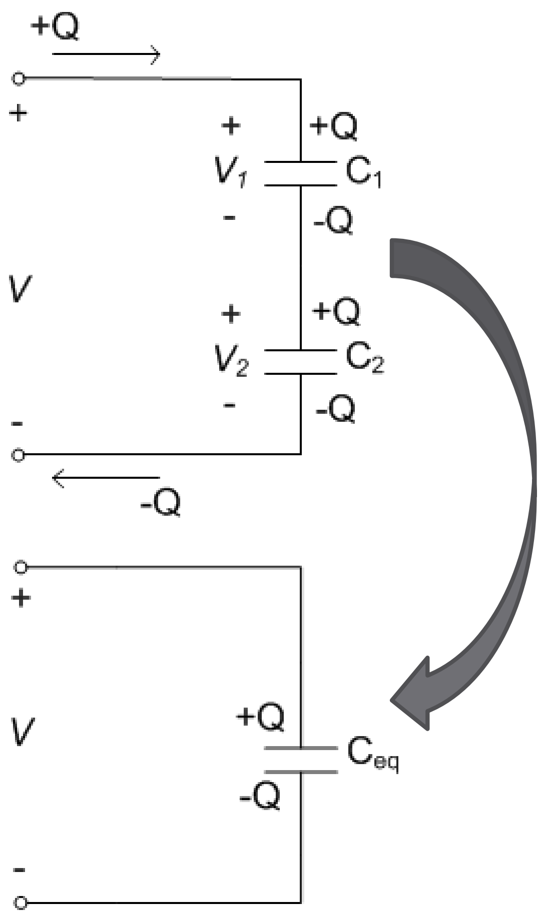
$$Q = (C_1 + C_2)V$$

$$Q = C_{eq}V$$

- ***Capacitances in parallel add***

$$\boxed{C_{eq} = C_1 + C_2}$$

Capacitors in Series



- Total voltage across the series combination is

$$V = V_1 + V_2$$

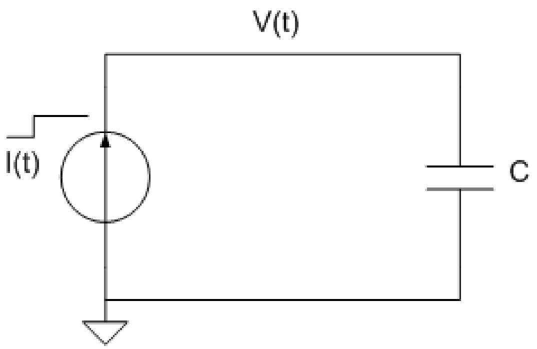
$$V = \frac{Q}{C_1} + \frac{Q}{C_2}$$

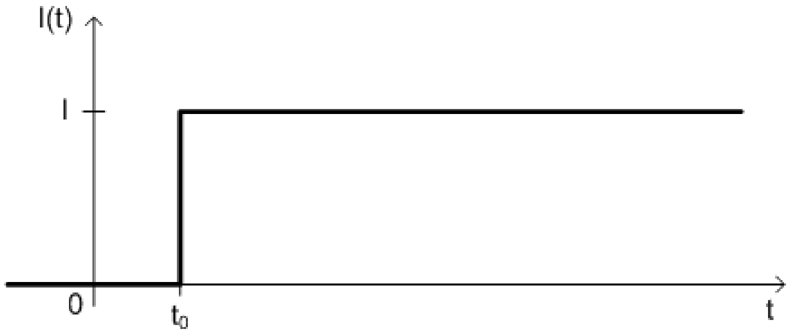
$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{Q}{C_{eq}}$$

- ***The inverses of capacitors in series add***

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \frac{C_1 C_2}{C_1 + C_2}$$

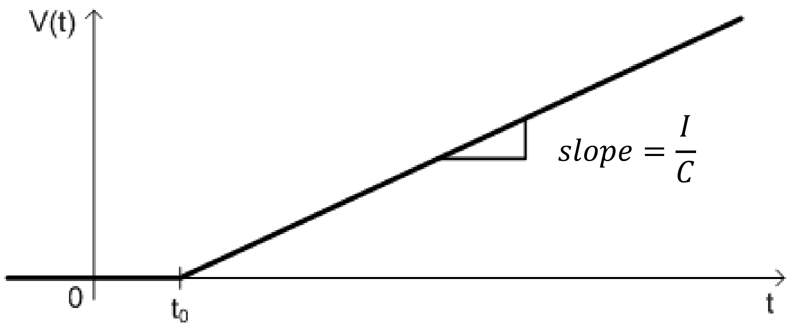
Constant Current Onto a Capacitor





- Capacitor voltage increases linearly for constant current

$$v(t) = \frac{I(t-t_0)}{C}, \quad t \geq t_0$$

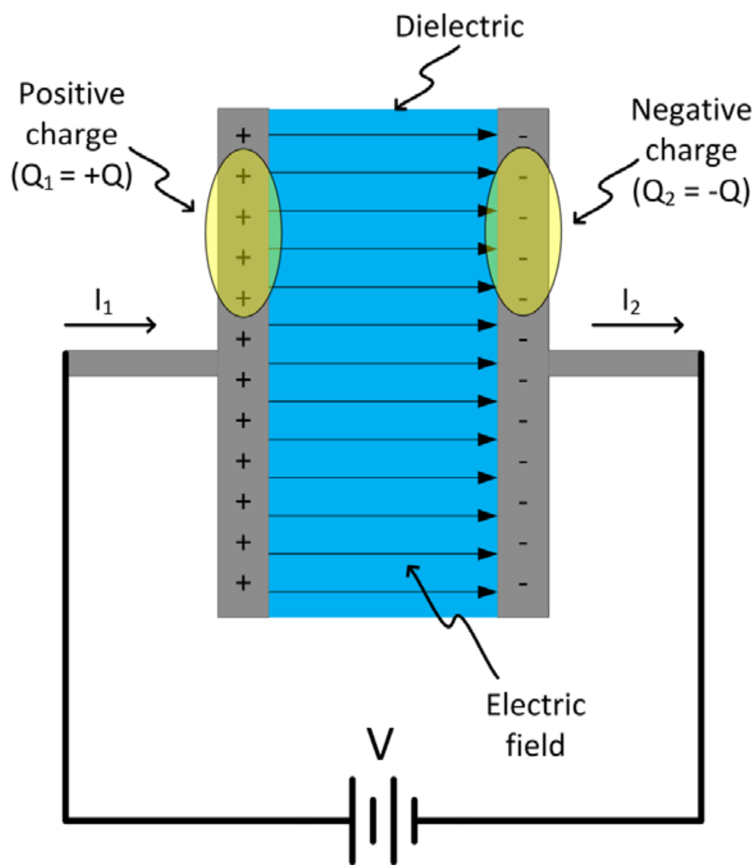


Electrical Capacitor – Energy Storage

- Capacitors store ***electrical energy***
  - Energy stored in the ***electric field***
- Stored energy is proportional to:
  - Voltage
  - Charge differential

$$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

- Energy released as E-field collapses
  - V and I supplied



Energy Storage – Example

- A capacitor is charged to 100 V
  - The stored energy will be used to lift a 1000 kg elevator car 10 stories (35 m)
  - Determine the required capacitance

- The required energy is

$$E = mgh = 1000\ kg \cdot 9.81 \frac{m}{s^2} \cdot 35\ m$$

$$E = 343.4\ kJ$$

- Energy stored on the capacitor is

$$E = \frac{1}{2} C (100\ V)^2$$

- The required capacitance is

$$C = \frac{2 \cdot 343.4\ kJ}{(100\ V)^2} = \underline{68.7\ F}$$

Ultracapacitors

Ultracapacitors - Introduction

- Energy stored by a capacitor

$$E = \frac{1}{2} C V^2$$

- Would like to maximize capacitance in order to maximize energy storage
- Recall the capacitance of a parallel-plate capacitor

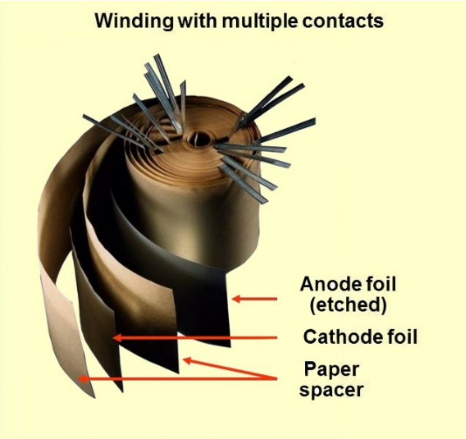
$$C = \frac{\varepsilon A}{d}$$

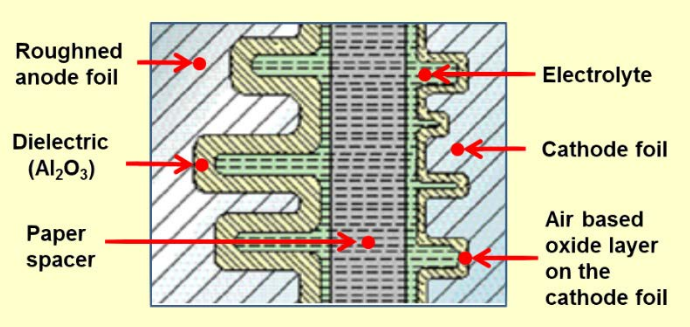
- To increase capacitance:
  - Use a higher-permittivity dielectric
  - Increase surface area of the plates
  - Decrease dielectric thickness
- Traditional capacitors do all of these things
  - ε limited by available materials and dielectric strength
  - A limited by practical overall device size
  - d limited by dielectric breakdown field strength

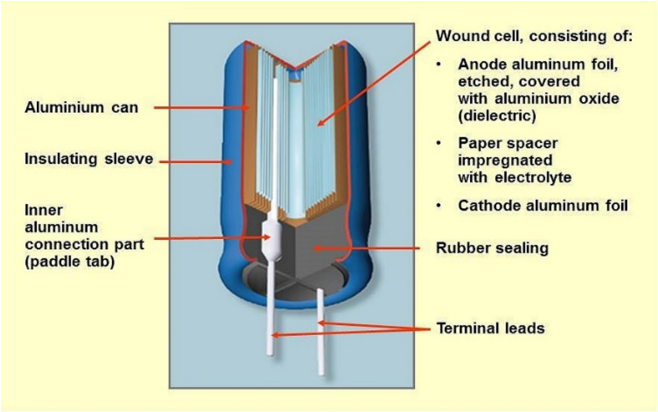
Traditional Capacitors – Construction

- Let's take a look at the construction of two high-capacitance traditional capacitors
  - Aluminum electrolytic
  - Tantalum electrolytic

- ***Aluminum electrolytic capacitor:***

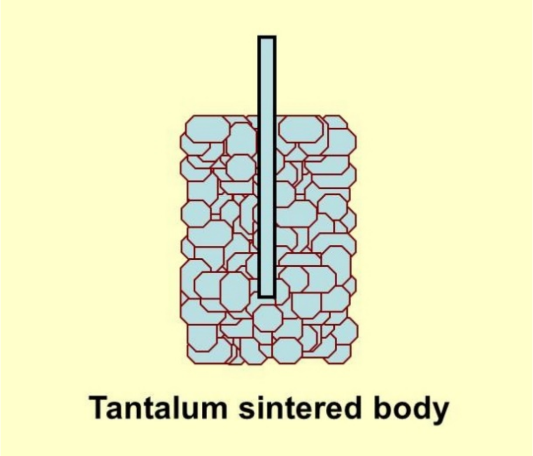


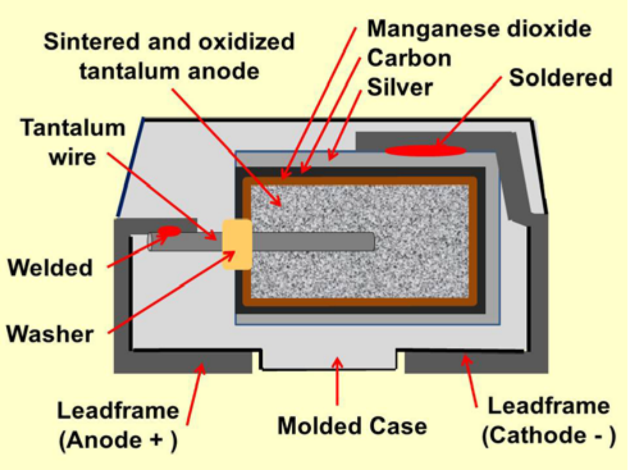


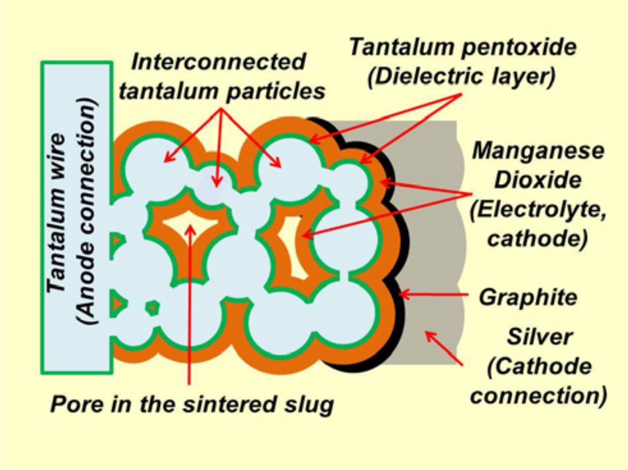


Traditional Capacitors – Construction

- ***Tantalum electrolytic capacitor:***







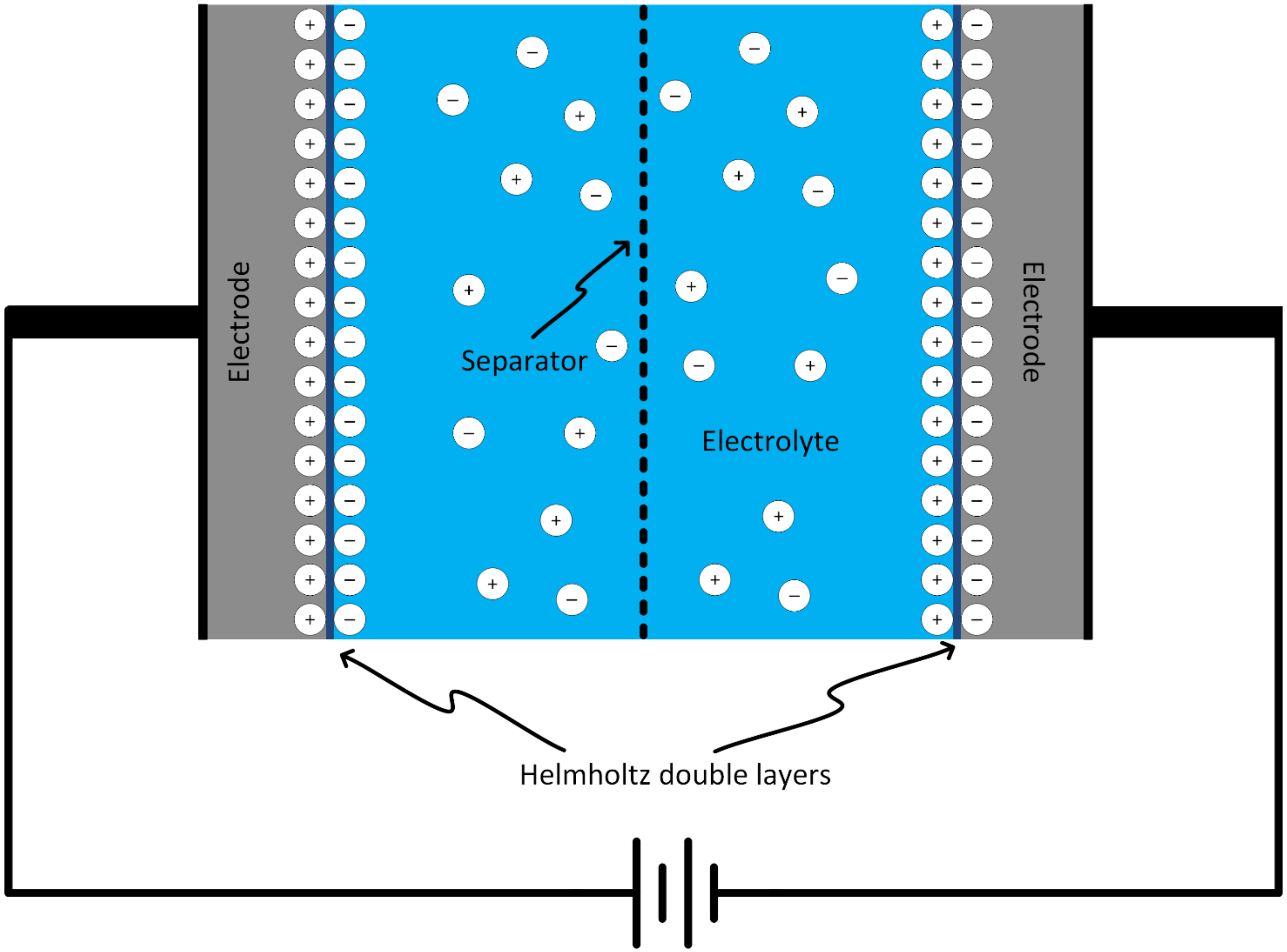
- In both of these types of capacitors, efforts are made to maximize A and minimize d
- But, a physical dielectric layer of non-zero thickness is used

Ultracapacitors

- In a previous example, we found we needed a capacitance of 68.7 F
  - Impractically large for a traditional capacitor
  - Not so for an ***ultracapacitor***
- ***Ultracapacitors*** or ***supercapacitors*** achieve very high capacitance values by eliminating the solid dielectric layer of traditional capacitors
- Energy is stored in an E-field
  - Not in a dielectric layer
  - In an ***electric double layer*** (***Helmholtz double layer***)
  - ***Electric double-layer capacitors*** (***EDLC***)

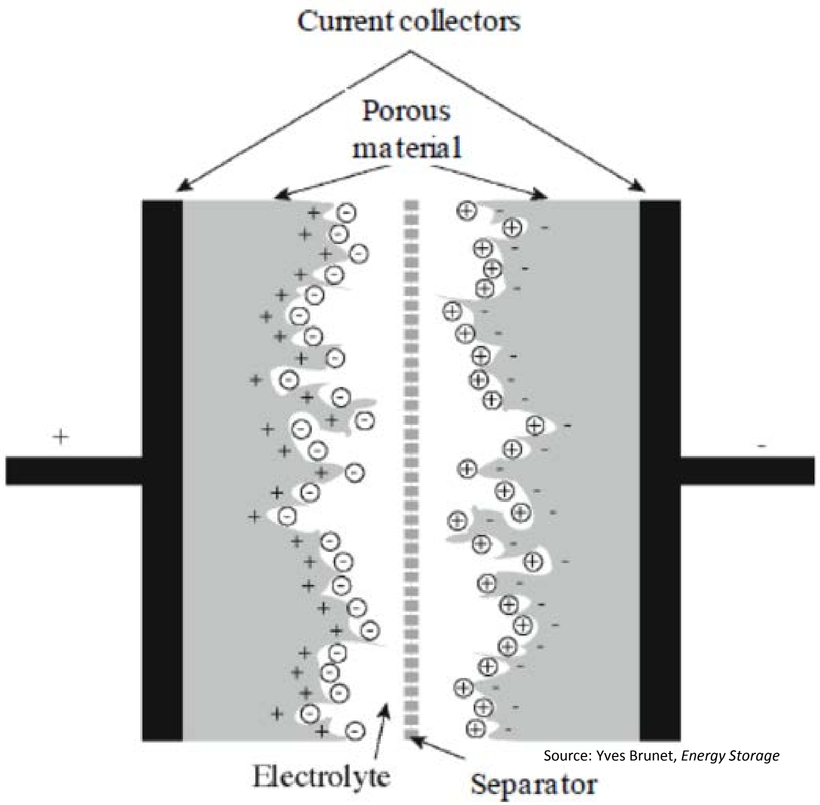
Ultracapacitors

- Electric double-layer capacitor



Ultracapacitors

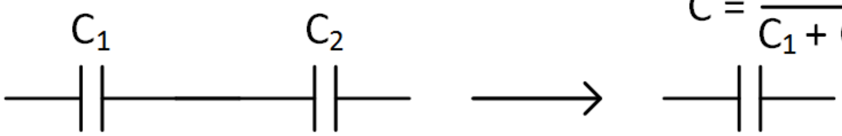
- Electrodes are rough and porous
  - Surface area is increased
  - Activated charcoal
  - Aerogel
- No charge transfer between the electrolyte and the electrode
- Separator is permeable
  - Mechanical separation preventing contact between electrodes
- Thickness of double layers is on the molecular scale



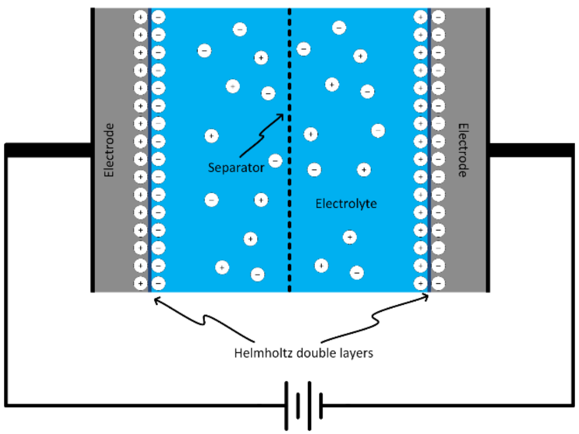
Source: Yves Brunet, *Energy Storage*

Ultracapacitors

- Two double layers
  - Two capacitors in series



$$C = \frac{C_1 C_2}{C_1 + C_2}$$

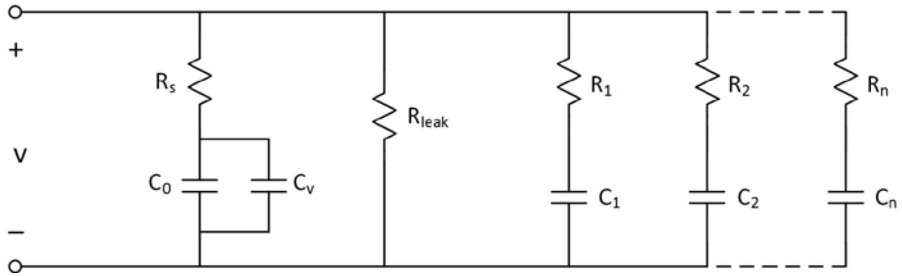


- Capacitance values in the range of 1 … 1000s of farads are common
- Ultracapacitors are ***polarized***
  - Positive electrode must be kept at a higher potential
- ***Maximum voltage*** determined by the electrolyte dissociation voltage
  - Typically ~2.5 V
  - For higher-voltage operation, multiple ultracapacitors are connected in series

Equivalent Circuit Model

Equivalent Circuit Model

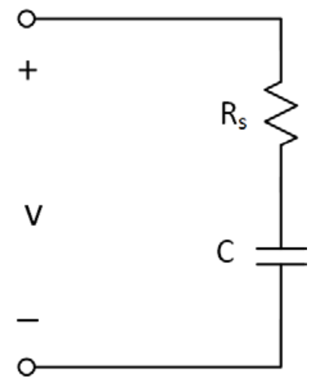
- ***Ultracapacitor equivalent circuit model***



- R_s: equivalent series resistance (ESR)
  - Primarily due to ionic conduction in the electrolyte
- C_0: primary capacitance of the ultracapacitor
- C_v: voltage-dependent capacitance
  - Associated with diffusion layers near the double layers
  - $C_v = k \cdot v$
- R_{leak}: leakage resistance
  - Typically specified as a leakage current at V_{max}
- R_1, C_1, … R_n, C_n: distributed resistance and capacitance of the porous electrodes
  - Models multiple time constants

Equivalent Circuit Model

- We will typically simplify this model significantly
  - Account for only capacitance and ESR
  - Typical ESR values: 0.5 mΩ … 500 mΩ



- Account for leakage resistance, R_{leak}, when appropriate
  - Self-discharge
  - Typical leakage resistance: 100 Ω … 100 kΩ
  - Typical leakage currents: 10μA … 10 mA

Charging and Discharging

Charging and Discharging

- The voltage seen across a capacitor is proportional to the stored charge differential

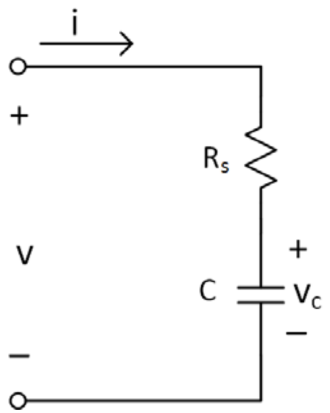
$$V = \frac{Q}{C}$$

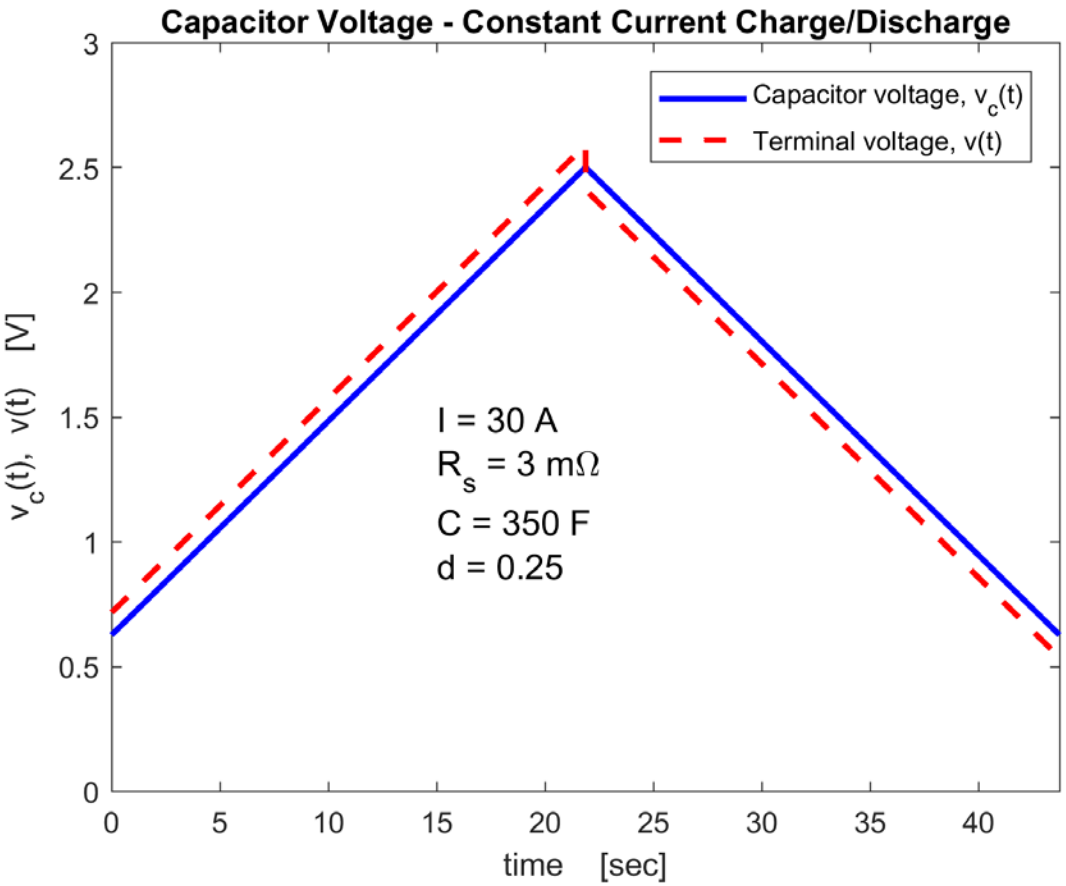
- So, unlike batteries, capacitor voltage does not remain constant as a capacitor discharges
- Power electronic circuitry generally required to interface between ultracapacitors and load
  - DC-DC converters
  - Inverters – DC-AC and AC-DC converters
- Interface circuitry also provides charge/discharge control
  - Current/power control

Charging and Discharging

- Two primary modes of charging/discharging
  - ***Constant current***
  - ***Constant power***
- Unlike batteries, capacitors can be ***charged and discharged at the same rates***
- Constant-current charging is simple
  - Both in terms of circuitry and analysis/design
- Constant-power charging useful in many applications, such as ***regenerative braking***
  - Charging while drawing constant power from the vehicle
  - Discharging while supplying constant power to the vehicle

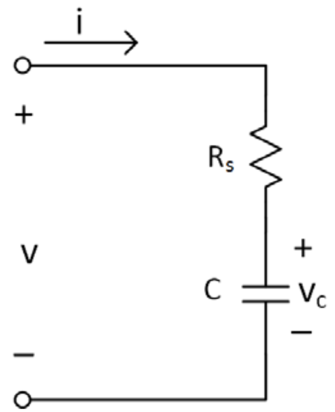
Constant-Current Charging

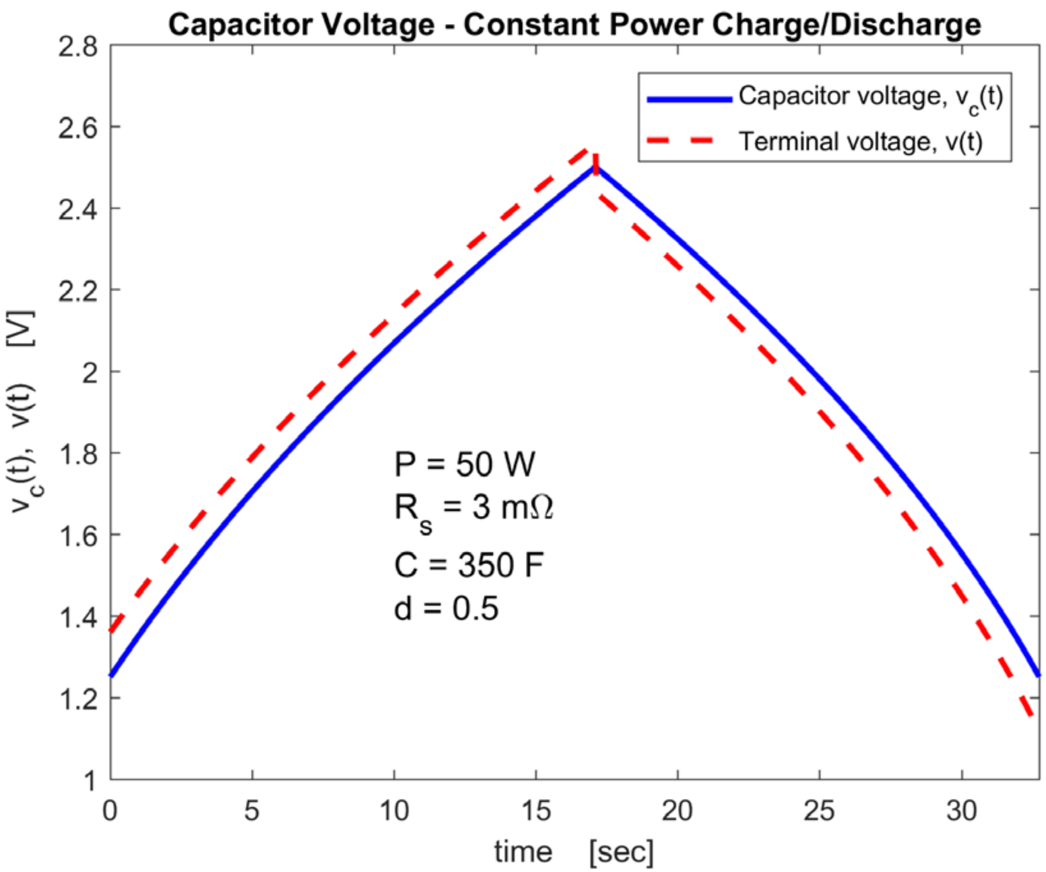




- Voltage drop across R_s during charge/discharge
- Constant rate of voltage change
- Power varies

Constant-Power Charging





- Varying rate of voltage change
- Current varies depending on state of charge
  - Higher current at lower state of charge
  - Lower current near full charge

Cell Balancing

Cell Balancing

- Typically, $V_{max} = 2.5\ V \ldots 3.0\ V$
  - Series-connected cells provide higher voltages
- Consider a series connection of four cells
- Equal charge differential, $\pm Q$, on each cell
- The voltage across each capacitor is

$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2},\ V_3 = \frac{Q}{C_3},\ V_4 = \frac{Q}{C_4}$$

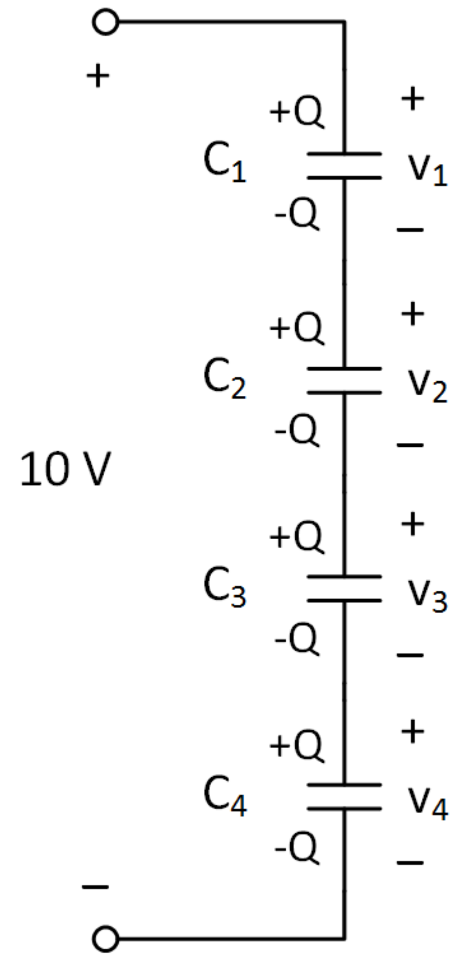
- Nominally, all capacitors are equal

$$C_1 = C_2 = C_3 = C_4 = C$$

- Nominally, all voltages are equal

$$V_1 = V_2 = V_3 = V_4 = \frac{Q}{C} = 2.5\text{V}$$

- But, capacitances may vary by as much as $\pm 20\%$



Cell Balancing

- Consider the following scenario:
- Total equivalent capacitance

$$C_{eq} = 0.24C$$

- Stored charge

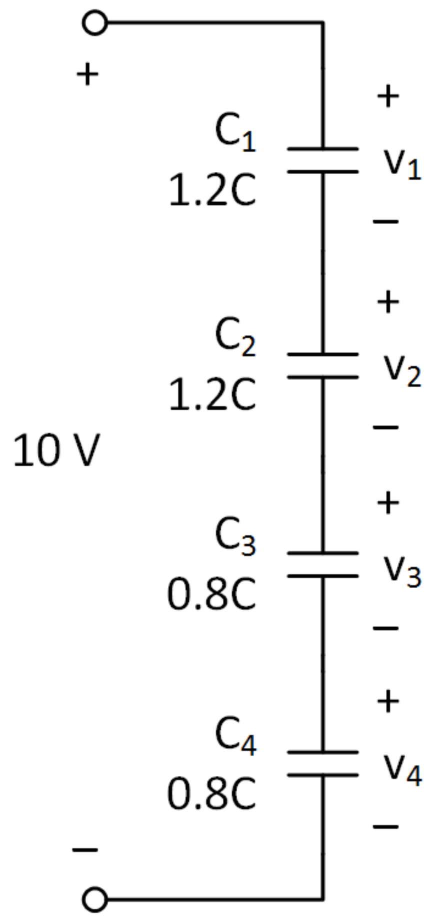
$$Q = 10\ V \cdot 0.24C$$

- Now, ***cell voltages are imbalanced***

$$V_1 = V_2 = \frac{Q}{1.2C} = \frac{10\ V \cdot 0.24C}{1.2C} = 2\ V$$

$$V_3 = V_4 = \frac{Q}{0.8C} = \frac{10\ V \cdot 0.24C}{0.8C} = 3\ V$$

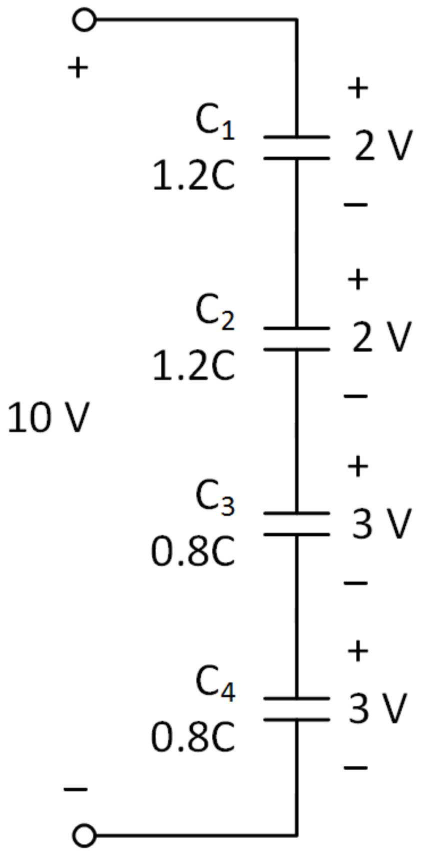
- If $V_{max} = 2.5\ V$, then
  - C_1 and C_2 are ***underutilized***
  - C_3 and C_4 are ***overstressed***



Cell Balancing

- Cell balancing circuitry
  - Safely utilize each cell's storage capacity

- Two balancing approaches:
  - ***Resistive balancing***
    - Resistors placed in parallel with the cells
    - Slow – not for high-duty-cycle applications
  - ***Active balancing***
    - Cell voltages monitored and electronic switches balance voltages
    - Fast – good for high-duty-cycle applications



Efficiency

Ultracapacitors – Efficiency

- Ultracapacitors have small, but non-zero, ESR
  - They are lossy devices
  - Not all input energy is available for use
  - Efficiency is less than 100%
- We will define ***round-trip efficiency*** as the efficiency through an entire charge/discharge cycle
  - Ratio of output energy to input energy

$$\eta_{rt} = \frac{E_{out}}{E_{in}} \cdot 100\% \qquad (1)$$

- Efficiency depends on how a capacitor is used
  - Rate of charge/discharge
  - Depth of discharge

Ultracapacitors – Efficiency

- Energy stored by a capacitor is proportional to the capacitor voltage squared

$$E_c = \frac{1}{2} C V^2 \qquad (2)$$

- Capacitor's effectiveness at storing energy depends on its state of charge (SOC)
  - Energy stored more quickly at high SOC
  - Energy stored more slowly at low SOC
- Loss in ESR depends on the current
- Therefore, ***instantaneous efficiency***, $\eta(t)$, varies with SOC
  - Total round-trip efficiency depends on depth of discharge
  - Ultracapacitors are typically not discharged completely

Discharge Factor

- ***Discharge factor***

$$d = \frac{V_{min}}{V_{max}} \qquad (3)$$

  - V_{min}: voltage at the lowest allowable SOC
  - V_{max}: maximum allowable (fully-charged) capacitor voltage

- We'll now examine the round-trip efficiency for capacitors operated at constant current and at constant power

Efficiency – Constant Current

- For a capacitor operating at a non-zero discharge factor, only some of the stored energy is usable
- ***Usable energy***

$$E_u = \frac{1}{2}CV_{max}^2 - \frac{1}{2}CV_{min}^2$$

$$E_u = \frac{1}{2}CV_{max}^2 - \frac{1}{2}C(dV_{max})^2$$

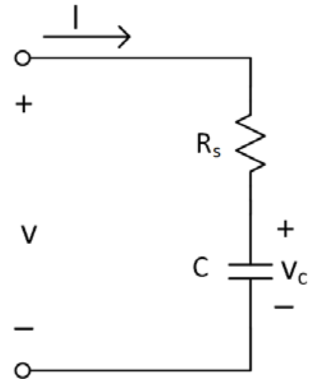
$$\boxed{E_u = \frac{1}{2}CV_{max}^2(1 - d^2)} \qquad (4)$$

- Power dissipated in the ESR at ***constant current***, I, is

$$P_R = I^2 R_s$$

- Energy lost in the ESR is

$$E_R = I^2 R_s \cdot t$$



Charging efficiency – Constant Current

- During the charging cycle, the efficiency is

$$\eta_1 = \frac{E_u}{E_u + E_R} = \frac{\frac{1}{2}CV_{max}^2\left(1-d^2\right)}{\frac{1}{2}CV_{max}^2(1-d^2) + I^2R_s \cdot t_c}$$

where t_c is the duration of the charging cycle

- We can solve for t_c as follows

$$V_{max} - V_{min} = \frac{I \cdot t_c}{C} \quad \rightarrow \quad t_c = \frac{CV_{max}(1-d)}{I}$$

- The efficiency then becomes

$$\eta_1 = \frac{\frac{1}{2}CV_{max}^2\left(1-d^2\right)}{\frac{1}{2}CV_{max}^2(1-d^2) + IR_sCV_{max}(1-d)}$$

$$\boxed{\eta_1 = \frac{\frac{1}{2}V_{max}\left(1-d^2\right)}{\frac{1}{2}V_{max}(1-d^2) + IR_s(1-d)}} \qquad (5)$$

Round-Trip Efficiency – Constant Current

- Similar loss is incurred in the ESR during discharge
  - Energy output is the stored energy minus resistive loss
- ***Round-trip efficiency*** is

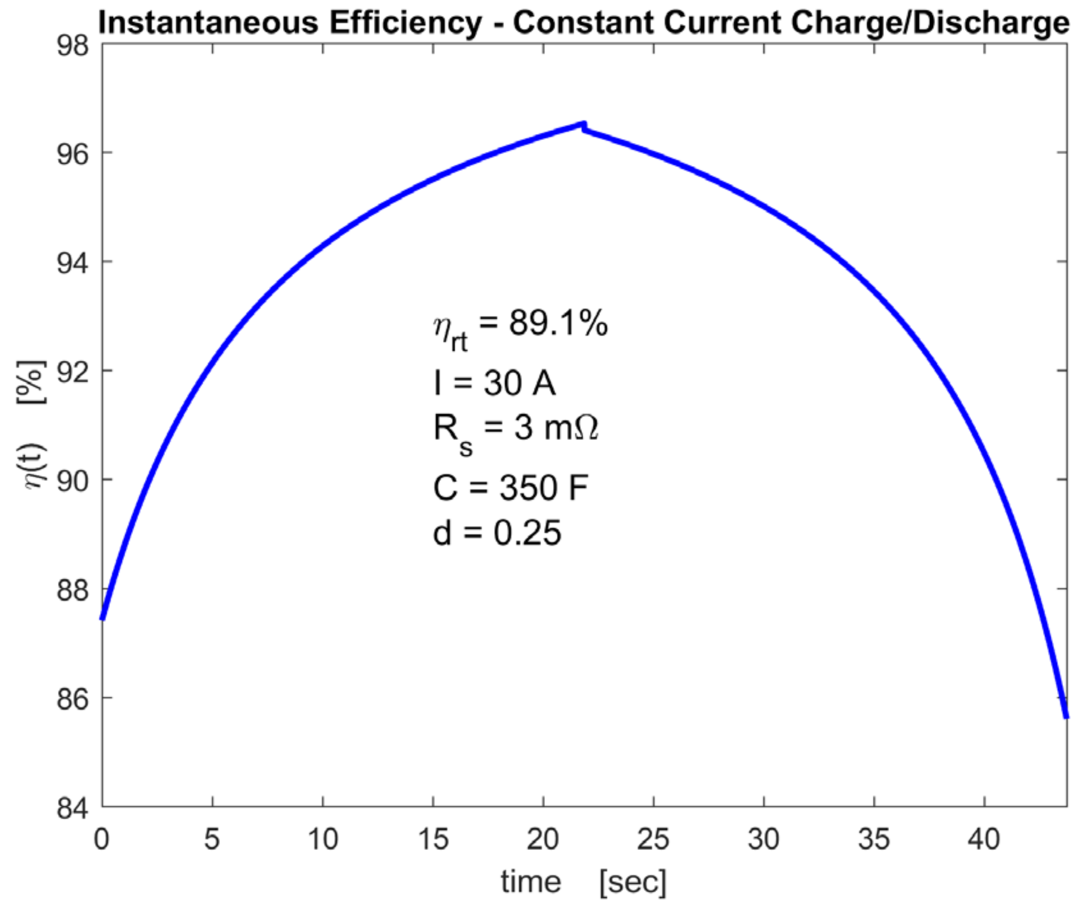
$$\eta_{rt} = \frac{E_{out}}{E_{in}} = \frac{E_u - E_R}{E_u + E_R}$$

$$\eta_{rt} = \frac{\frac{1}{2}CV_{max}^2(1-d^2) - IR_sCV_{max}(1-d)}{\frac{1}{2}CV_{max}^2(1-d^2) + IR_sCV_{max}(1-d)}$$

$$\boxed{\eta_{rt} = \frac{\frac{1}{2}V_{max}(1-d^2) - IR_s(1-d)}{\frac{1}{2}V_{max}(1-d^2) + IR_s(1-d)}} \qquad (6)$$

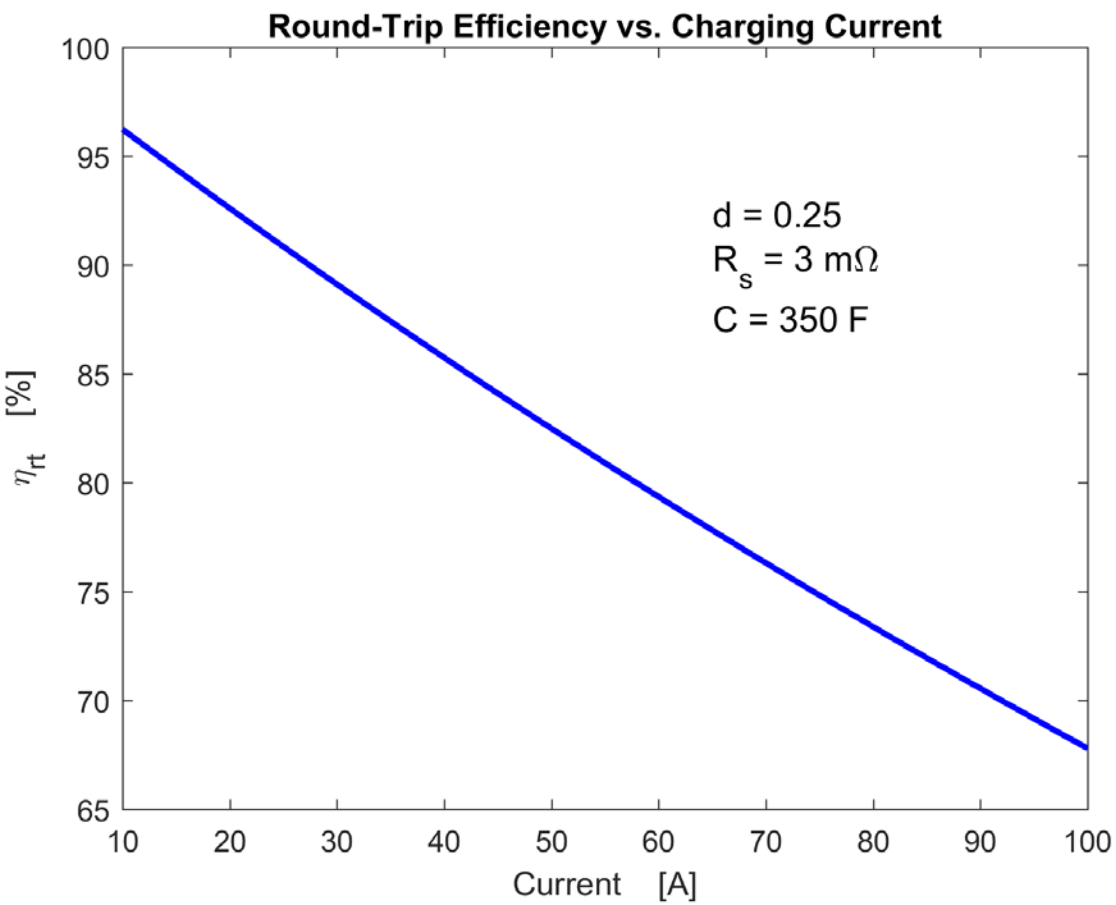
Instantaneous Efficiency – Constant Current

- At lower SOC:
  - Low rate of energy storage
  - Low efficiency
- At higher SOC:
  - Higher rate of energy storage
  - Higher efficiency



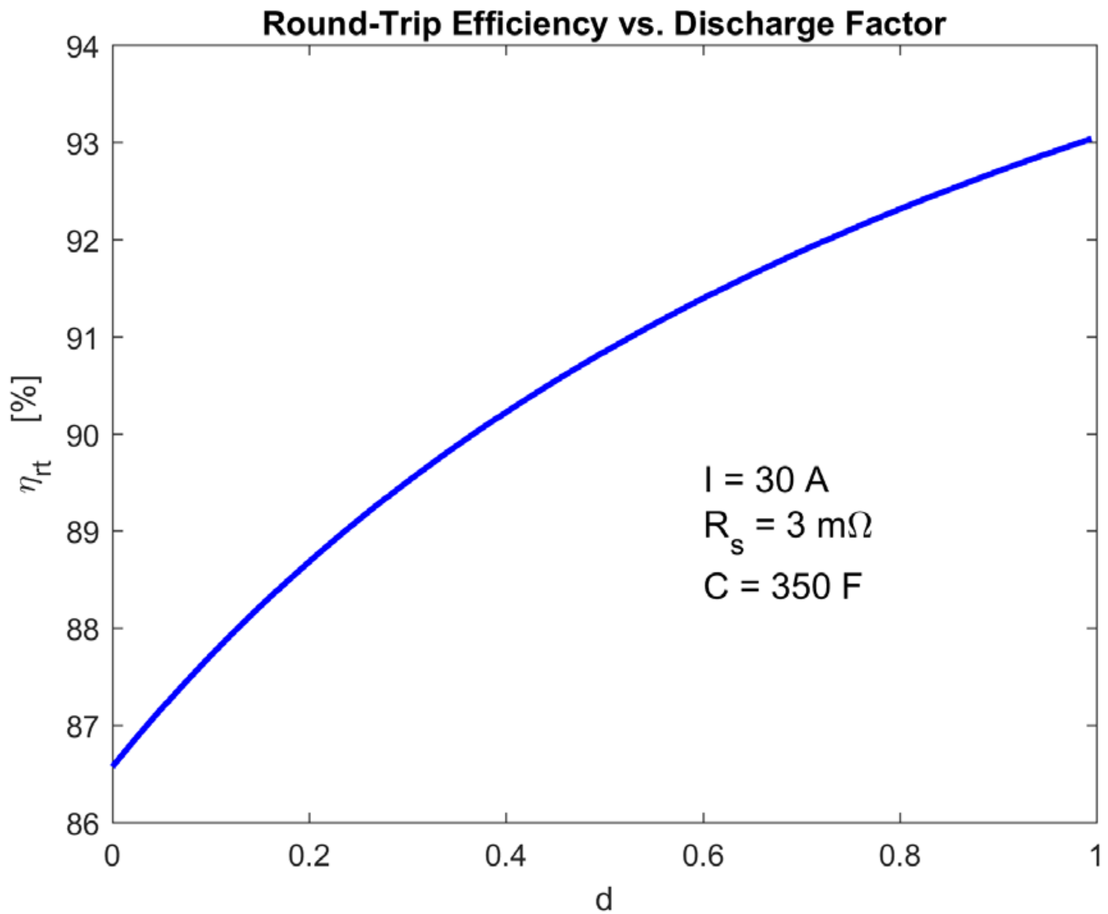
Efficiency vs. Current – Constant Current

- As current increases, loss in ESR increases



Efficiency vs. Discharge Factor – Constant Current

- Greater depth-of-discharge corresponds to lower efficiency



Efficiency – Constant Power

- For constant-power charging/discharging, we start with the same expression for efficiency

$$\eta_{rt} = \frac{E_u - E_R}{Eu + E_R}$$

- Usable energy is the same

$$E_u = \frac{1}{2} C V_{max}^2 (1 - d^2)$$

- But, since current is now time varying, the energy lost in the resistance (in one direction) is

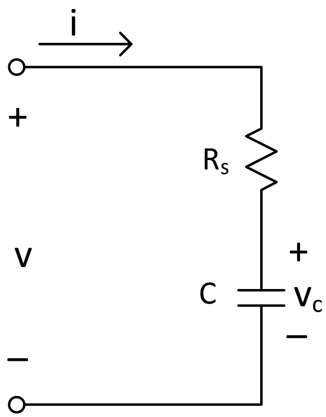
$$E_R = \int_0^{t_c} i^2(t) R_S \, dt$$

- Things get a bit more complicated, as we now need to solve a differential equation to determine the capacitor voltage

Efficiency – Constant Power

- The input to the capacitor is now constant power
  - Write a power-balance equation

$$P - i^2(t) \cdot R_s - i(t)v_c(t) = 0$$



- Current is given by

$$i(t) = C\frac{dv_c}{dt}$$

- The power balance becomes

$$R_s C^2 \left(\frac{dv_c}{dt}\right)^2 + C\frac{dv_c}{dt} v_c(t) - P = 0 \qquad (7)$$

- An ordinary differential equation in ***quadratic*** form

Efficiency – Constant Power

- Applying the quadratic formula to (7), we get

$$\frac{dv_C}{dt} = \frac{-Cv_C(t) \pm \sqrt{C^2 v_C^2(t) + 4R_S C^2 P}}{2R_S C^2}$$

- Simplifying, and keeping only the valid '+' solution

$$\frac{dv_C}{dt} = \frac{-v_C(t) + \sqrt{v_C^2(t) + 4R_S P}}{2R_S C} \qquad (8)$$

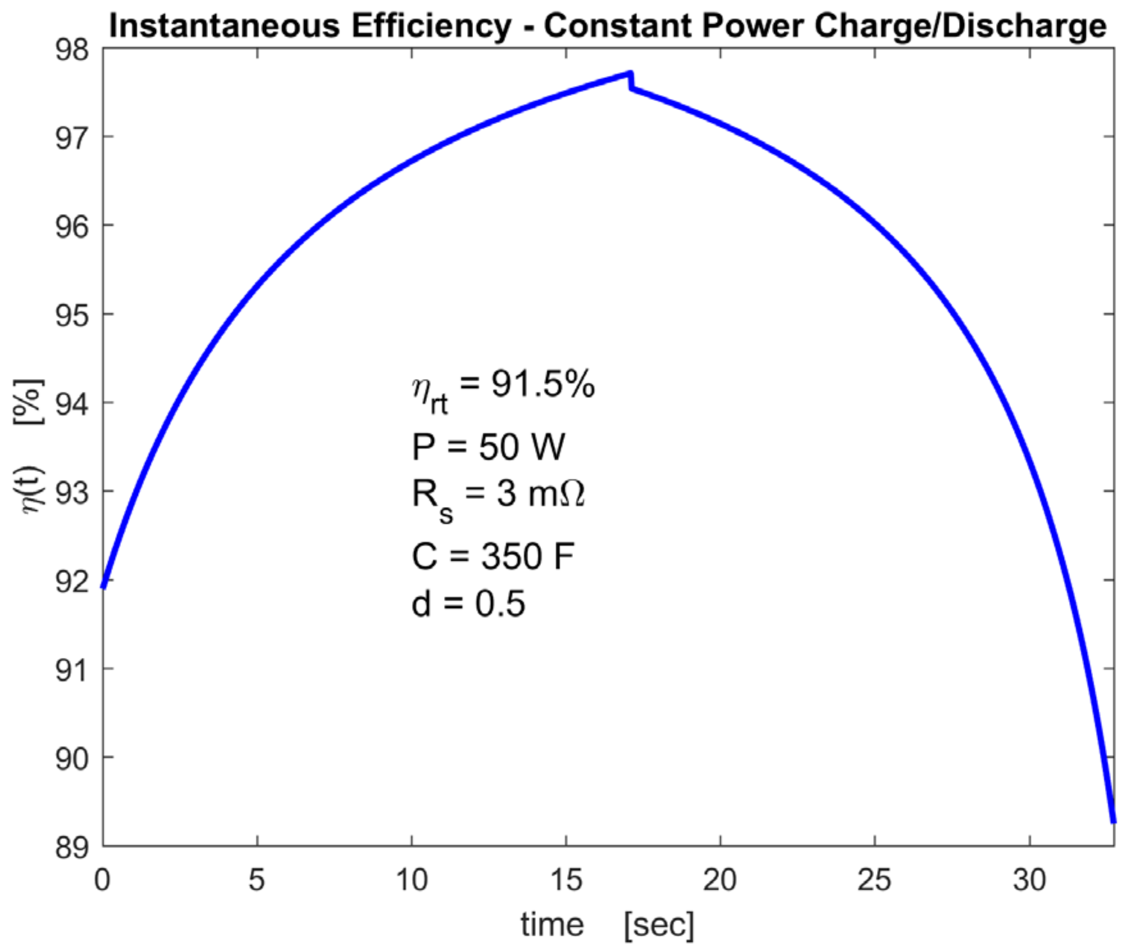
- For ***discharging*** this becomes

$$\frac{dv_C}{dt} = \frac{-v_C(t) + \sqrt{v_C^2(t) - 4R_S P}}{2R_S C} \qquad (9)$$

- We don't have a nice closed-form solutions to (8) or (9), but we can solve them numerically

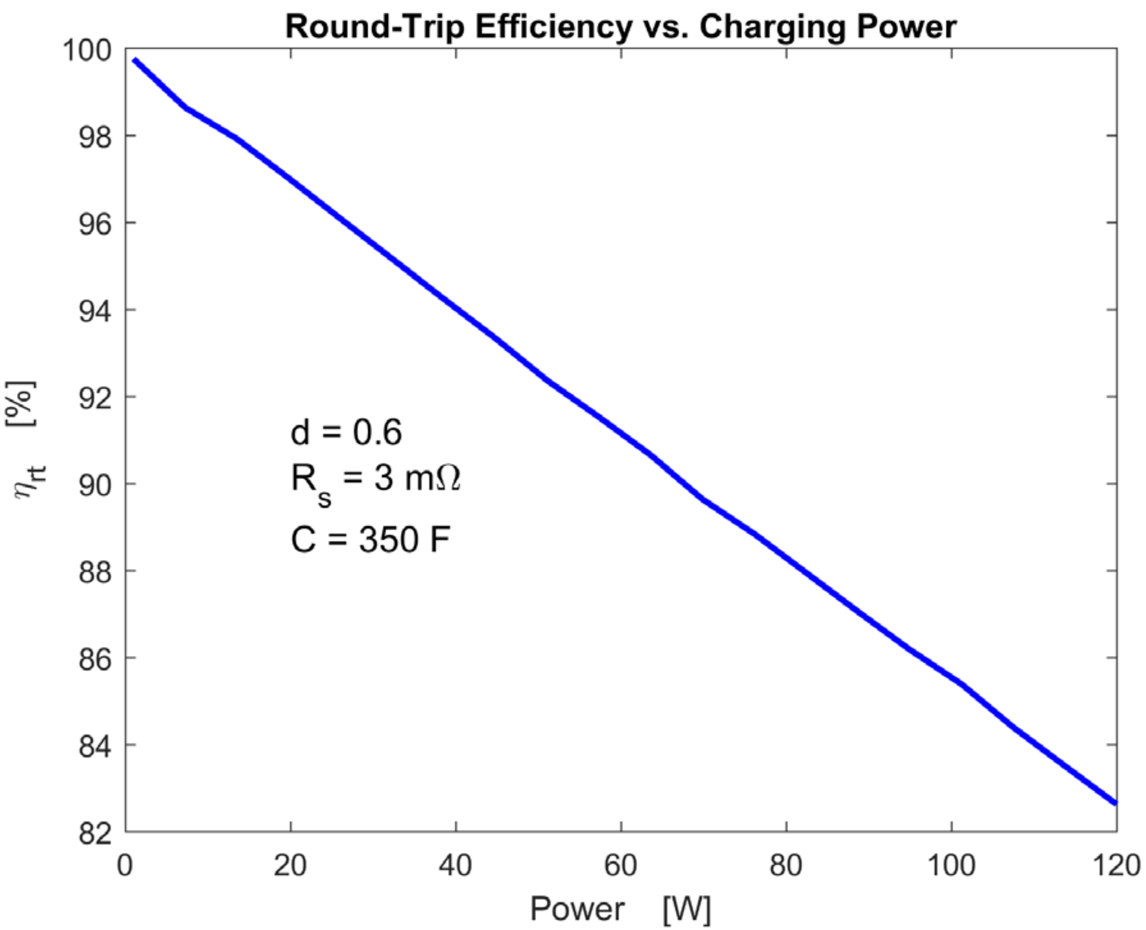
Instantaneous Efficiency – Constant Power

- At lower SOC:
  - High current
  - High I^2R loss
  - Low efficiency
- At higher SOC:
  - Lower current
  - Lower I^2R loss
  - Higher efficiency



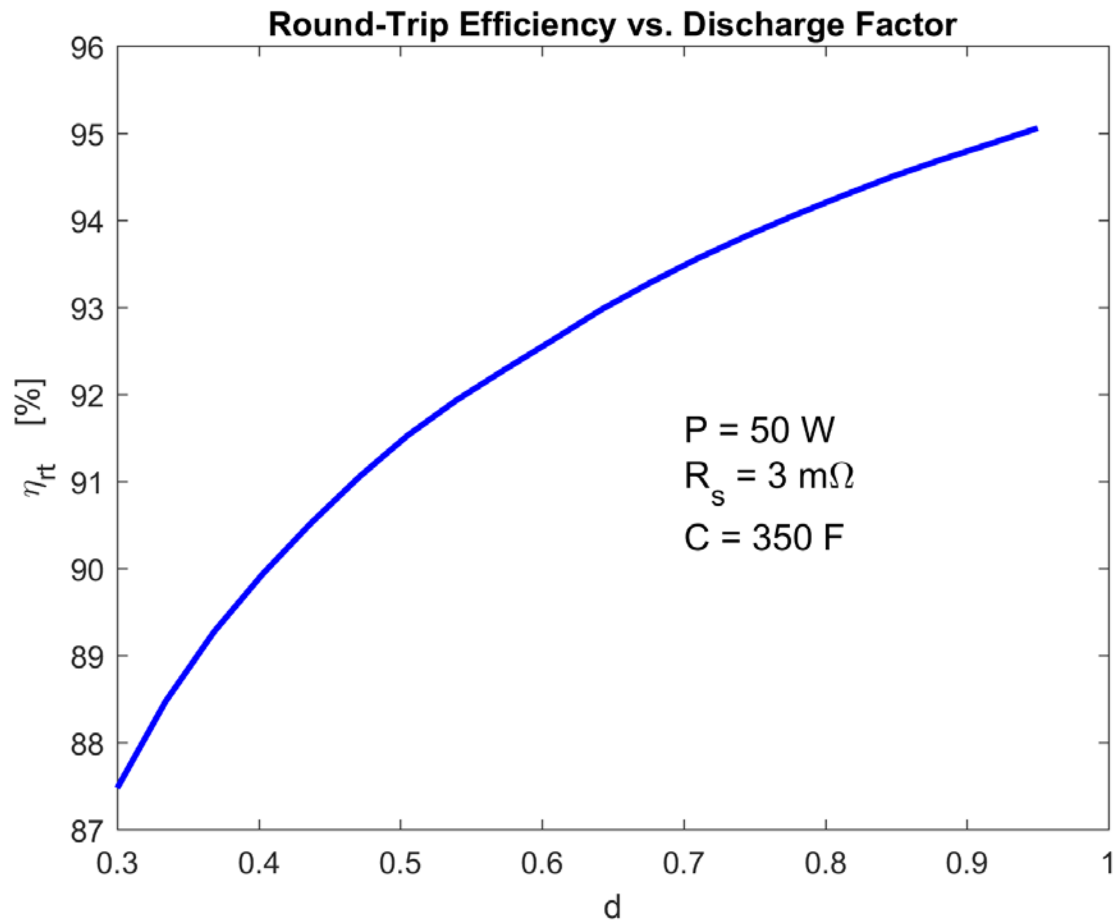
Efficiency vs. Current – Constant Power

- As power increases, current increases and resistive loss increases



Efficiency vs. Discharge Factor – Constant Power

- Greater depth-of-discharge corresponds to lower efficiency



Approximate Efficiency – Constant Power

- Sometimes we may want a quick way to approximate efficiency for constant-power charging/discharging
  - Calculate an average current
  - Calculate efficiency as you would for constant-current charging/discharging
- Approximate average current as the average of the maximum and minimum currents

$$I_{avg} \approx \frac{I_{max} + I_{min}}{2}$$

where

$$I_{max} = \frac{P}{V_{min}} \quad \text{and} \quad I_{min} = \frac{P}{V_{max}}$$

- Then, efficiency is approximately given by

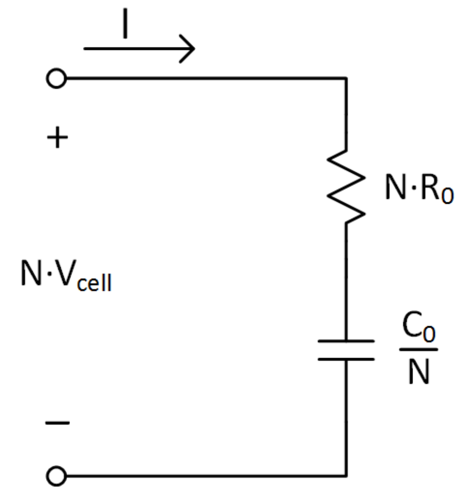
$$\eta_{rt} \approx \frac{\frac{1}{2}V_{max}(1-d^2)-I_{avg}R_s(1-d)}{\frac{1}{2}V_{max}(1-d^2)+I_{avg}R_s(1-d)} \qquad (10)$$

Series-Connected Capacitor Cells

Series-Connected Capacitor Cells

- To achieve higher working voltages, multiple capacitor cells are connected in series
- How does this effect
  - Energy storage?
  - Efficiency?

- Consider a series connection of N cells, each with a capacitance of C_0 and a maximum voltage of V_{cell}
  - Equivalent circuit model:



Series-Connected Capacitor Cells

- Energy stored:

$$E = \frac{1}{2}\frac{C_0}{N}(N \cdot V_{cell})^2$$

$$E = N \cdot \frac{1}{2} C_0 V_{cell}^2$$

- As expected, this is N times the energy stored in a single cell
- For a discharge factor of d, the usable stored energy is

$$E_u = N\frac{1}{2}C_0V_{cell}^2 - N\frac{1}{2}C_0(dV_{cell})^2$$

$$E_u = N\frac{1}{2}C_0V_{cell}^2(1-d^2) \quad (11)$$

Cells in Series – Constant Current

- Energy lost in the resistance during charging (or discharging):

$$E_R = I^2 N R_0 t_c$$

- For ***constant-current*** operation

$$V_{max} - V_{min} = V_{max}(1-d) = NV_{cell}(1-d)$$

$$NV_{cell}(1-d) = \frac{I\,t_c}{C} = \frac{I\,t_c}{C_0/N}$$

- The charging (or discharging) time is

$$t_c = \frac{C_0 V_{cell}(1-d)}{I}$$

- So, losses during charging (or discharging) are

$$E_R = I^2 N R_0 \frac{C_0 V_{cell}(1-d)}{I}$$

$$E_R = NIR_0 C_0 V_{cell}(1-d) \qquad (12)$$

Cells in Series – Constant Current

- Round-trip efficiency is

$$\eta_{rt} = \frac{E_u - E_R}{E_u + E_R}$$

- Using (11) and (12), we get

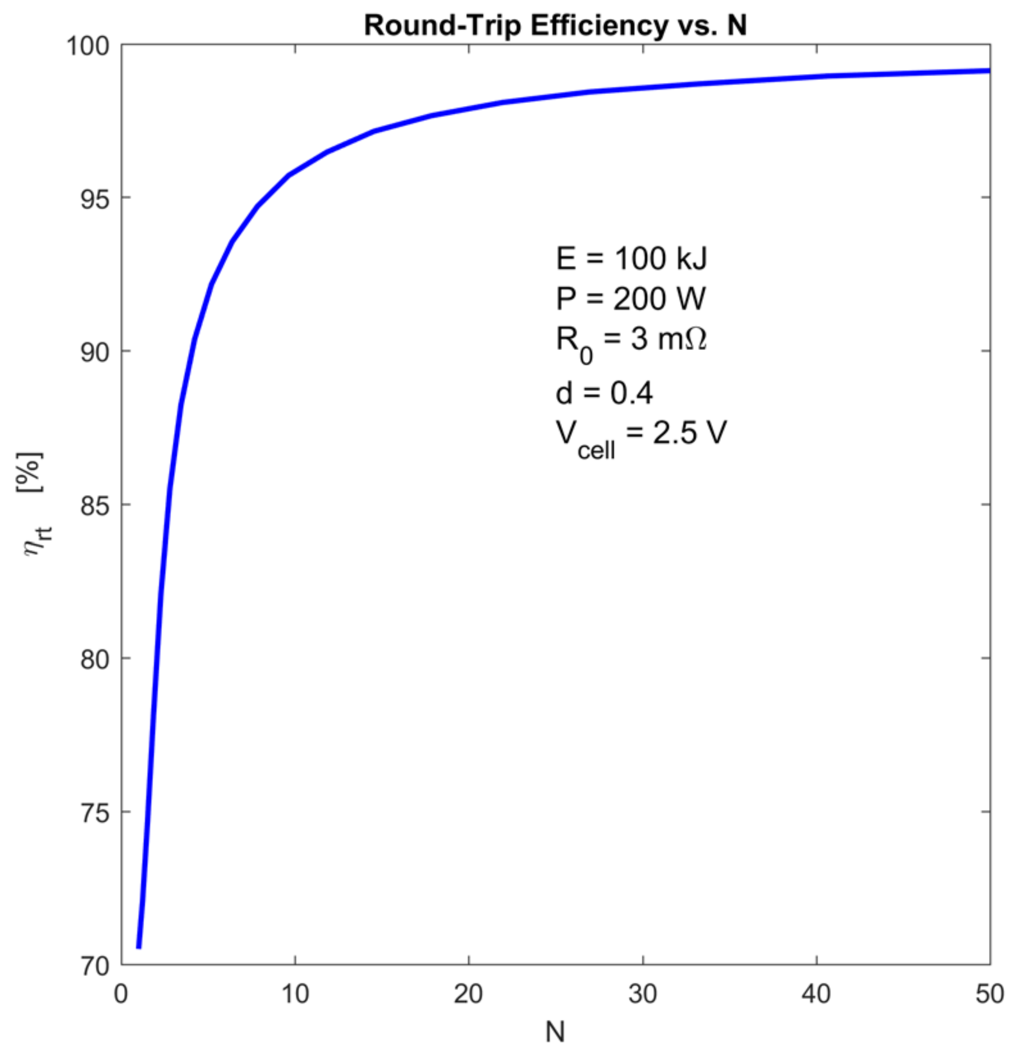
$$\eta_{rt} = \frac{N\frac{1}{2}C_0V_{cell}^2(1-d^2) - NIR_0C_0V_{cell}(1-d)}{N\frac{1}{2}C_0V_{cell}^2(1-d^2) + NIR_0C_0V_{cell}(1-d)}$$

$$\eta_{rt} = \frac{\frac{1}{2}V_{cell}(1-d^2) - IR_0(1-d)}{\frac{1}{2}V_{cell}(1-d^2) + IR_0(1-d)}$$

- This is, of course, the same as for a single cell, *but*
  - ***Current can be reduced at a higher maximum voltage***
    - Efficiency will improve

Cells in Series – Constant Power

- For ***constant-power*** operation, we'll again investigate numerically
- Round-trip efficiency vs. # of series-connected cells:
  - Assuming:
    - $E = 100\ kJ$
    - $P = 200\ W$
    - $\mathrm{R_0 = 3\ m\Omega}$
    - $d = 0.25$
    - $V_{cell} = 2.5\ V$



Capacitor Bank Sizing

Ultracapacitor Sizing

- Sizing a capacitor bank involves determining the following parameters
  - Stored energy: E
  - Available power: P
  - Maximum voltage: V_{max}
  - Minimum voltage: V_{min}
  - Discharge factor: d
  - # of cells in series and/or parallel: N_s, N_p
  - Total capacitance: C
  - Cell capacitance: C_0
  - Efficiency: η
- Sizing procedure depends on which of these parameters are specified
- We'll outline a procedure assuming the specified requirements are:
  - Energy storage
  - Power
  - Voltage range

Ultracapacitor Sizing Procedure

1. Discharge factor

$$d = \frac{V_{min}}{V_{max}}$$

2. Number of series-connected cells

$$N_s = \frac{V_{max}}{V_{cell}}$$

3. Total required capacitance

$$E = \frac{1}{2} C V_{max}^2 (1 - d^2) \quad \rightarrow \quad C = \frac{2E}{V_{max}^2 (1 - d^2)}$$

4. Cell capacitance

$$C_0 = N_s \cdot C$$

5. Determine the resulting efficiency using the required power
   - Evaluate numerically or approximate
6. Iterate if necessary
   - Adjust N as needed

Ultracapacitor Sizing – Example

- Size a capacitor bank for an energy recovery system for a tower crane with the following specifications
  - Height: $h = 50\ m$
  - Capacity: $m = 5{,}000\ kg$
  - Time to lift max load: $t_d = 30\ s$
- Let's assume we have a power conversion system that can operate over the range of $60\ V_{DC}\ \ldots 150\ V_{DC}$ at an efficiency of $\eta_{pcs} = 97\%$

- The required energy to lift the maximum load is

$$E = mgh \cdot \frac{1}{\eta_{pcs}} = 5000\ kg \cdot 9.81 \frac{m}{s^2} \cdot 50\ m \cdot \frac{1}{0.97}$$

$$E = 2.53\ MJ$$

- Performing that lift in $30\ sec$ corresponds to a required power of

$$P = \frac{E}{t_d} = \frac{2.53\ MJ}{30\ s} = 84.3\ kW$$

Ultracapacitor Sizing – Example

- The discharge factor is

$$d = \frac{V_{min}}{V_{max}} = \frac{60\ V}{150\ V} = 0.4$$

- The required number of cells in series is

$$N_s = \frac{V_{max}}{V_{cell}} = \frac{150\ V}{2.5\ V} = 60$$

- The total required capacitance is

$$C = \frac{2E}{V_{max}^2(1 - d^2)}$$

$$C = \frac{2 \cdot 2.53\ MJ}{(150\ V)^2(1 - 0.4^2)}$$

$$C = 267.6\ F$$

Ultracapacitor Sizing – Example

- The capacitance of each individual cell is

$$C_0 = N_s \cdot C = 60 \cdot 267.6\ F$$

$$C_0 = 16.1\ kF$$

- So, the capacitor bank would consist of sixty $16.1\ kF$ capacitors connected in series
  - $16.1\ kF$ is a large capacitance – likely unavailable
  - Connect multiple capacitors in parallel
- Let's say we have access to individual capacitor cells with the following specifications
  - $C_0 = 3400\ F$
  - $R_0 = 0.28\ m\Omega$
- Five capacitors in parallel will give

$$C_{0p} = N_p C_0 = 5C_0 = 17\ kF$$

$$R_{0p} = \frac{R_0}{N_p} = \frac{R_0}{5} = \frac{0.28\ m\Omega}{5} = 56\ \mu\Omega$$

Ultracapacitor Sizing – Example

- Placing 60 of the 5-capacitor groups in series, the total series resistance is

$$R_s = N_s \cdot R_{0p} = 60 \cdot 56\ \mu\Omega = 3.36\ m\Omega$$

- For constant-power charge/discharge, we can ***approximate efficiency***
- The maximum current is

$$I_{max} = \frac{P}{V_{min}} = \frac{84.3\ kW}{60\ V} = 1.4\ kA$$

- The minimum current is

$$I_{min} = \frac{P}{V_{max}} = \frac{84.3\ kW}{150\ V} = 562\ A$$

- The approximate average current is

$$I_{avg} \approx \frac{I_{max} + I_{min}}{2} = 981\ A$$

Ultracapacitor Sizing – Example

- Using the average current, we can approximate the round trip efficiency for the capacitor bank as

$$\eta_{rt} \approx \frac{\frac{1}{2}V_{cell}(1-d^2) - I_{avg}R_{0p}(1-d)}{\frac{1}{2}V_{cell}(1-d^2) + I_{avg}R_{0p}(1-d)}$$

$$\eta_{rt} \approx \frac{1.25\,V(1-0.4^2) - 981\,A \cdot 56\,\mu\Omega(1-0.4)}{1.25\,V(1-0.4^2) + 981\,A \cdot 56\,\mu\Omega(1-0.4)}$$

$$\eta_{rt} \approx 0.92 \quad \rightarrow \quad \eta_{rt} \approx 92\%$$

  - Note that R_{0p} is used, because that is the resistance of each of the 60 series-connected parallel combinations

- Total round-trip efficiency must include the power conversion system

$$\eta_{rt} = 0.97 \cdot 0.92 \cdot 0.97 = 0.87 \quad \rightarrow \quad \eta_{rt} = 87\%$$

Ultracapacitor Sizing – Example

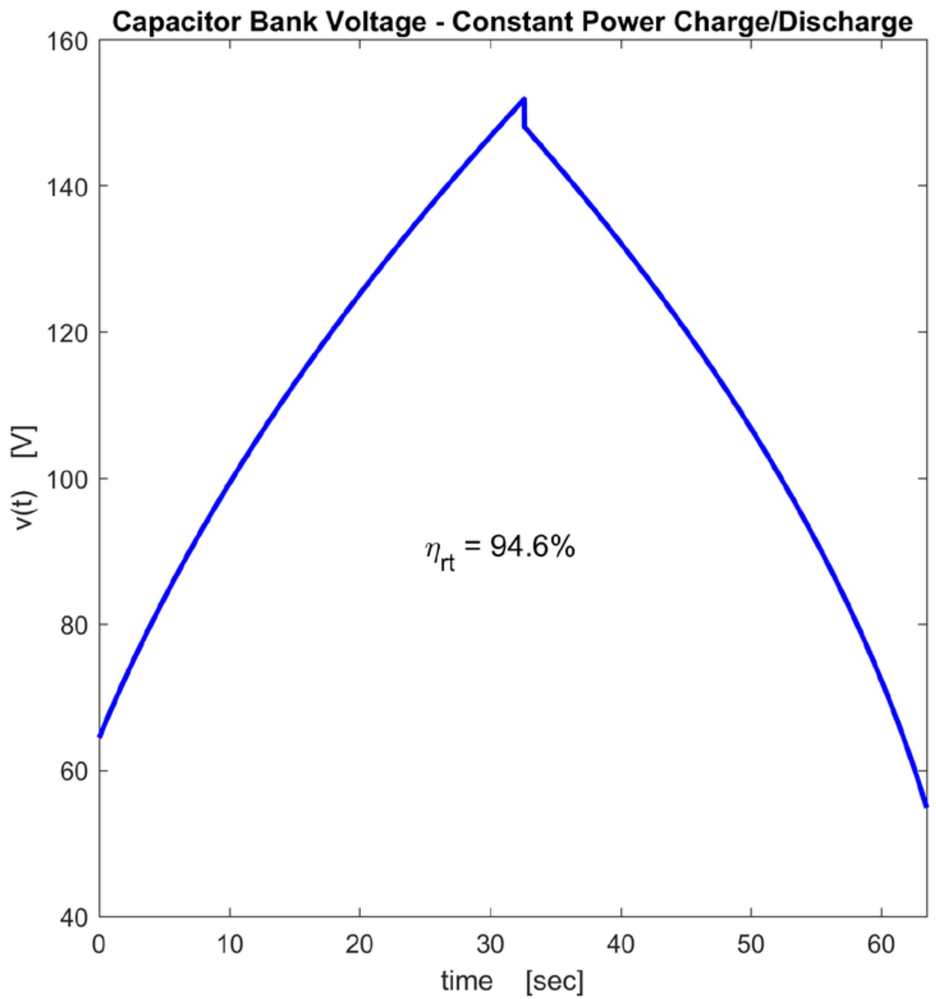
- Solving numerically, we find that the efficiency of the capacitor bank is a bit higher

$$\eta_{rt} = 94.6\%$$

  - I_{avg} overestimates the time-average current

- Accounting for conversion losses, we have

$$\eta_{rt} = 89\%$$



Ultracapacitor Efficiency Summary

Efficiency Summary

Configuration	Round-trip efficiency
Single capacitor	$$\eta_{rt} = \frac{\frac{1}{2}V_{cell}(1-d^2) - IR_0(1-d)}{\frac{1}{2}V_{cell}(1-d^2) + IR_0(1-d)}$$
N_s in series	$$\eta_{rt} = \frac{\frac{1}{2}N_sV_{cell}(1-d^2) - IN_sR_0(1-d)}{\frac{1}{2}N_sV_{cell}(1-d^2) + IN_sR_0(1-d)}$$ $$\eta_{rt} = \frac{\frac{1}{2}V_{max}(1-d^2) - IR_s(1-d)}{\frac{1}{2}V_{max}(1-d^2) + IR_s(1-d)}$$ $$\eta_{rt} = \frac{\frac{1}{2}V_{cell}(1-d^2) - IR_0(1-d)}{\frac{1}{2}V_{cell}(1-d^2) + IR_0(1-d)}$$

Efficiency Summary

Configuration	Round-trip efficiency
N_p in parallel	$$\eta_{rt} = \frac{\frac{1}{2}V_{cell}(1-d^2) - I\frac{R_0}{N_p}(1-d)}{\frac{1}{2}V_{cell}(1-d^2) + I\frac{R_0}{N_p}(1-d)}$$ $$\eta_{rt} = \frac{\frac{1}{2}V_{cell}(1-d^2) - IR_{0p}(1-d)}{\frac{1}{2}V_{cell}(1-d^2) + IR_{0p}(1-d)}$$

Efficiency Summary

Configuration	Round-trip efficiency
N_s in series, N_p in parallel	$\eta_{rt} = \frac{\frac{1}{2}N_s V_{cell}(1-d^2) - I\frac{N_s}{N_p}R_0(1-d)}{\frac{1}{2}N_s V_{cell}(1-d^2) + I\frac{N_s}{N_p}R_0(1-d)}$ $\eta_{rt} = \frac{\frac{1}{2}V_{cell}(1-d^2) - IR_{0p}(1-d)}{\frac{1}{2}V_{cell}(1-d^2) + IR_{0p}(1-d)}$ $\eta_{rt} = \frac{\frac{1}{2}V_{max}(1-d^2) - IR_{eq}(1-d)}{\frac{1}{2}V_{max}(1-d^2) + IR_{eq}(1-d)}$

- All of the expressions on this and the previous two pages are for ***constant-current*** charging/discharging
  - For ***constant power***, use an approximate ***average current***