Reinforcement learning: Markov Decision Processes

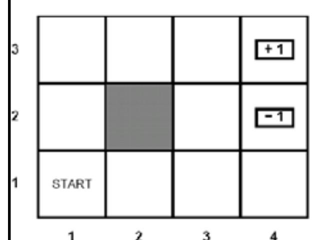
Reinforcement Learning

- You can think of supervised learning as the teacher providing answers (the class labels)
- In reinforcement learning, the agent learns based on a punishment/reward scheme
- Before we can talk about reinforcement learning, we need to introduce Markov Decision Processes

Decision Processes: General Description

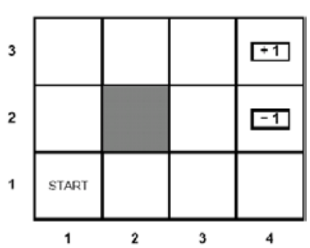
- Decide what action to take next given that your action will affect what happens in the future
- Real world examples:
  - Robot path planning
  - Elevator scheduling
  - Travel route planning
  - Aircraft navigation
  - Manufacturing processes
  - Network switching and routing

Sequential Decisions

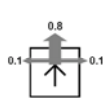


- Assume a fully observable, deterministic environment
- Each grid cell is a state
- The goal state is marked +1
- At each time step, agent must move Up, Right, Down, or Left
- How do you get from start to the goal state?

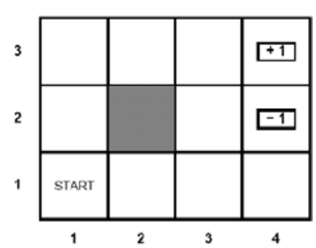
Sequential Decisions



- Suppose the environment is now stochastic
- With 0.8 probability you go in the direction you intend
- With 0.2 probability you move at right angles to the intended direction (0.1 in either direction – if you hit the wall you stay put)
- What is the optimal solution now?



Sequential Decisions



- Up, Up, Right, Right, Right reaches the goal state with probability 0.8^5=032768
- But in this stochastic world, going Up, Up, Right, Right, Right might end up with you actually going Right, Right, Up, Up, Right with probability $(0.1^4)(0.8)=0.00008$
- Even worse, you might end up in the -1 state accidentally

Transition Model

- Transition model: a specification of the outcome probabilities for each action in each possible state
- T(*s*, *a*, *s'*) = probability of going to state *s'* if you are in state *s* and do action *a*
- The transitions are **Markovian** ie. the probability of reaching state s' from s depends only on s and not on the history of earlier states (aka The Markov Property)
- Mathematically:

  Suppose you visited the following states in chronological order: $s_0, \ldots, s_t$

  $P(s_{t+1} \mid a, s_0, \ldots, s_t) = P(s_{t+1} \mid a, s_t)$

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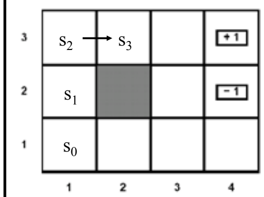
Markov Property Example



Suppose

$s_0 = (1,1)$, $s_1 = (1,2)$, $s_2 = (1,3)$

If I go right from state s_2, the probability of going to s_3 only depends on the fact that I am at state s_2 and not the entire state history $\{s_0, s_1, s_2\}$

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The Reward Function

- Depends on the sequence of states (known as the environment history)
- At each state *s*, the agent receives a reward R(s) which may be positive or negative (but must be bounded)
- For now, we'll define the utility of an environment history as the sum of the rewards received

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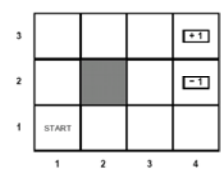
Utility Function Example

R(4,3) = +1 (Agent wants to get here)

R(4,2) = -1 (Agent wants to avoid this

R(s) = -0.04 (for all other states)

$U(s_1, \ldots, s_n) = R(s_1) + \ldots + R(s_n)$



If the states an agent goes through are Up, Up, Right, Right, Right, the utility of this environment history is: -0.04-0.04-0.04-0.04-0.04+1

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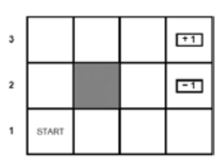
Utility Function Example

If there's no uncertainty, then the agent would find the sequence of actions that maximizes the sum of the rewards of the visited states



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Markov Decision Process

The specification of a sequential decision problem for a fully observable environment with a *Markovian transition model* and *additive rewards* is called a Markov Decision Process (MDP)

An MDP has the following components:

1. A finite set of states S along with an initial state S_0
2. A finite set of actions A
3. Transition Model: T(s, a, s') = P(s' | a, s)
4. Reward Function: R(s)

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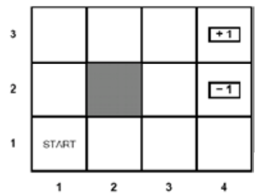
Solutions to an MDP

- Why is the following not a satisfactory solution to the MDP?

[1,1]-Up
[1,2]-Up
[1,3]-Right
[2,3]-Right
[3,3]-Right



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A Policy

- Policy: mapping from a state to an action
- Need this to be defined for all states so that the agent will always know what to do
- Notation:
  - π denotes a policy
  - π(s) denotes the action recommended by the policy π for state s

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Optimal Policy

- There are obviously many different policies for an MDP
- Some are better than others. The "best" one is called the optimal policy π* (we will define best more precisely in later slides)
- Note: every time we start at the initial state and execute a policy, we get a different environment history (due to the stochastic nature of the environment)
- This means we get a different utility every time we execute a policy
- Need to measure expected utility ie. the average of the utilities of the possible environment histories generated by the policy

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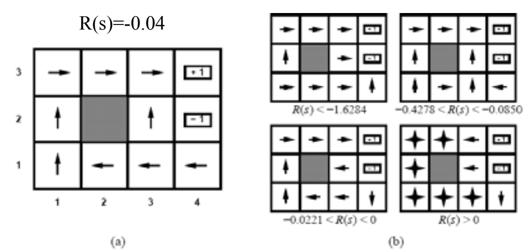
Optimal Policy Example

R(s)=-0.04



Notice the tradeoff between risk and reward!

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Roadmap for the Next Few Slides

We need to describe how to compute optimal policies

1. Before we can do that, we need to define the utility of a state
2. Before we can do (1), we need to explain stationarity assumption
3. Before we can do (2), we need to explain finite/infinite horizons

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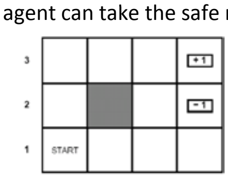
Finite/Infinite Horizons

- Finite horizon: fixed time N after which nothing matters (think of this as a deadline)
- Suppose our agent starts at (3,1), R(s)=-0.04, and N=3. Then to get to the +1 state, agent must go up.
- If N=100, agent can take the safe route around



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Nonstationary Policies

- Nonstationary policy: the optimal action in a given state changes over time
- With a finite horizon, the optimal policy is nonstationary
- With an infinite horizon, there is no incentive to behave differently in the same state at different times
- With an infinite horizon, the optimal policy is stationary
- We will assume infinite horizons

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Utility of a State Sequence

Under stationarity, there are two ways to assign utilities to sequences:

1. Additive rewards: The utility of a state sequence is:

   $U(s_0, s_1, s_2, \ldots) = R(s_0) + R(s_1) + R(s_2) + \ldots$

2. Discounted rewards: The utility of a state sequence is:

   $U(s_0, s_1, s_2, \ldots) = R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + \ldots$

   Where $0 \leq \gamma \leq 1$ is the discount factor

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The Discount Factor

- Describes preference for current rewards over future rewards
- Compensates for uncertainty in available time (models mortality)
- Eg. Being promised $10000 next year is only worth 90% of being promised $10000 now
- γ near 0 means future rewards don't mean anything
- $\gamma = 1$ makes discounted rewards equivalent to additive rewards

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Utilities

We assume infinite horizons. This means that if the agent doesn't get to a terminal state, then environmental histories are infinite, and utilities with additive rewards are infinite. How do we deal with this?

Discounted rewards makes utility finite.

Assuming largest possible reward is R_{max} and $\gamma < 1$,

$$U(s_0, s_1, s_2, \ldots) = \sum_{t=0}^{\infty} \gamma^t R(s_t)$$

$$\leq \sum_{t=0}^{\infty} \gamma^t R_{\max} = \frac{R_{\max}}{(1-\gamma)}$$

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Computing Optimal Policies

- A policy π generates a sequence of states
- But the world is stochastic, so a policy π has a range of possible state sequences, each of which has some probability of occurring
- The value of a policy is the expected sum of discounted rewards obtained

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The Optimal Policy

- Given a policy π, we write the expected sum of discounted rewards obtained as:

$$E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \mid \pi\right]$$

- An optimal policy π^* is the policy that maximizes the expected sum above

$$\pi^* = \arg\max_{\pi} E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \mid \pi\right]$$

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The Optimal Policy

- For every MDP, there exists an optimal policy
- There is no better option (in terms of expected sum of rewards) than to follow this policy
- How do you calculate this optimal policy? Can't evaluate all policies…too many of them
- First, need to calculate the utility of each state
- Then use the state utilities to select an optimal action in each state

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Rewards vs Utilities

- What's the difference between R(s) the reward for a state and U(s) the utility of a state?
  - R(s) – the short term reward for being in s
  - U(s) – The long-term total expected reward for the sequence of states starting at s (not just the reward for state s)

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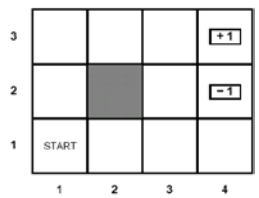
Utilities in the Maze Example



Start at state (1,1). Let's suppose we choose the action Up.

U(1,1) = R(1,1) + ...

Reward for current state

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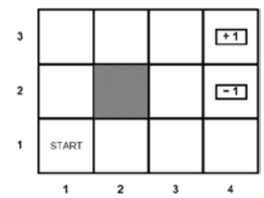
Utilities in the Maze Example



Start at state (1,1). Let's choose the action Up.

Prob of moving right

U(1,1) = R(1,1) + 0.8*U(1,2) + 0.1*U(2,1) + 0.1*U(1,1)

Prob of moving up

Prob of moving left (into the wall) and staying put

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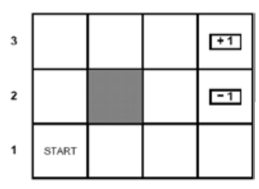
Utilities in the Maze Example



Now let's throw in the discounting factor

U(1,1) = R(1,1) + γ*0.8*U(1,2) + γ*0.1*U(2,1) + γ*0.1*U(1,1)

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The Utility of a State

If we choose action a at state s, expected future rewards (discounted) are:

$$U(s) = R(s) + \gamma \sum_{s'} T(s,a,s')U(s')$$

Reward at current state s

Probability of moving from state s to state s' by doing action a

Expected sum of future discounted rewards starting at state s

Expected sum of future discounted rewards starting at state s'

The Utility of a State

- In the previous example, we chose the action a then determined the utility of the state.
- The utility is really defined in terms of the optimal action.
- We modify the previous formula slightly by adding a max term over actions.

$$U(s) = R(s) + \gamma \max_a \sum_{s'} T(s,a,s')U(s')$$

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The Utility of a State

The utility of a state is the immediate reward for that state plus the expected discounted utility of the next state, assuming the agent chooses the optimal action

$$U(s) = R(s) + \gamma \max_a \sum_{s'} T(s,a,s')U(s')$$

This is the famous Bellman Equation

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The Optimal Policy

- Selection of the action π*(s) = a which maximizes the expected utility U(s)
- Intuitively, π* gives us the best action we can take from any state to maximize our future discounted rewards

$$\pi^*(s) = \arg\max_a \sum_{s'} T(s,a,s')U(s')$$

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What You Should Know

- How to formulate a problem as an MDP
- What the Markov property is
- How to calculate the utility for a state in an MDP
- What the Bellman equation is

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