Coupled Surge-Heave Motions of a Moored System.
I: Model Calibration and Parametric Study

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Abstract: Model calibration and parametric studies of the coupled, complex surge-heave motions of a medium-scale experimental, nonlinear, submerged, moored structural system are presented here. The experimental system excited by periodic wave fields consists of a spherical buoy and attached multipoint mooring lines. Sources of nonlinearity include complex geometric restoring forces and coupled fluid–structure interaction exciting forces. The sphere moves mainly in a two-dimensional fashion [or two-degree-of-freedoms (2DOF)] of surge and heave, with negligible pitch. Characteristic experimental results include harmonic, subharmonic, and superharmonic responses. As an extension of single-degree-of-freedom, independent-flow-field (IFF) models, a 2DOF, IFF model is derived and employed. Good agreement is shown between the analytical predictions and experimental results. Existence of complex nonlinear responses, including chaos and multiple coexisting steady states, are numerically identified when the coupling is strong and damping is light. Degree of complexity and nonlinearity of responses diminish with decreasing coupling and/or increasing damping.

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Introduction

Complex nonlinear responses of compliant ocean structures are well known and frequently observed in fields, experiments, and numerical simulations (e.g., Virgin and Bishop 1988; Garza-Ríos and Bernitsas 1995; Kim and Bernitsas 2001). This class of systems usually possesses a high degree of nonlinear restoring forces and fluid–structure interactions. These structural responses are often coupled in various degree-of-freedoms (DOFs). The degree of coupling is generally dependent on the direction of incident waves and the nonlinear configurations of structures. However, due to the high degree of complexity of the system and associated analyses involved, most of the studies investigating the response characteristics are usually conducted on simplified, single-degree-of-freedom (SDOF) models, focusing only on the predominant motion (e.g., surge) to keep analyses manageable (e.g., Choi and Lou 1991; Gottlieb and Yim 1993). Results of the analyses of SDOF models often reveal some intricate response characteristics (e.g., underlying bifurcation patterns and chaos) and provide guidelines for model developments of coupled motions and response investigations.

The responses of SDOF, compliant offshore structures subjected to periodic wave excitation, including nonharmonics, resonances, and coexistence had been analytically and numerically studied (e.g., Thompson 1983; Virgin 1987; Gottlieb and Yim 1992). Using a surge-only SDOF model, Gottlieb et al. (1997) identified the existence of an underlying organized transitions structure embedded in bifurcation domains of the response of a moored experimental system and employed the analytical structure to facilitate numerical search of possible routes to quasiperiodic and chaotic responses. Their numerical results indicated that under the same system parameters with different initial conditions, coexisting (and competing) nonlinear ocean structural responses were found near resonances where local bifurcations occur.

Gottlieb and Yim (1997) later presented a preliminary analytical study on a two-DOF (2DOF) coupled surge-and-heave model development and numerical response prediction. Higher degree nonlinear, complex responses were frequently observed in the simulations. The analysis and numerical predictions were to be validated by filed data and/or experimental results.

A medium-scale experimental investigation of the nonlinear response behavior of a submerged, moored ocean structural system was conducted to classify nonlinear responses, and validate and calibrate analytical predictions (Yim et al. 1993). In the study, two experimental models, a SDOF in surge and a 2DOF in coupled surge and heave, were employed to examine the response characteristics and possible correlations. Experimental responses of both SDOF and 2DOF models exhibited highly nonlinear characteristics. Response analyses were designed to first examine the surge motion of the SDOF model to identify possible intricate patterns in response characteristics. Characteristics of coupled surge-and-heave responses of the 2DOF model were later to be examined and compared with their SDOF counterparts. The effects of the surge-and-heave coupling and damping mechanism on the response behavior were to be examined via parametric studies.

To date, the experimental SDOF response classifications, analyses, and comparisons with numerical simulations had been conducted (Lin and Yim 1998). The analytical SDOF model was...
derived based on a standard Morison formulation with an intention to predict the experimental response behavior by a simple, small-body model with a set of constant system parameters. The existence of nonlinear characteristic motions in the experimental results (e.g., superharmonics and subharmonics) verified analytical predictions in a previous study. An underlying bifurcation pattern was identified to exist in the experimental results, and it was presented by a frequency response diagram. Coexistence of distinctively different nonlinear responses, e.g., harmonics and subharmonics, were also experimentally observed near resonances. Despite good agreement in general between numerical simulations and experimental results, the simulations could not capture nonlinear response phenomena near secondary resonances, e.g., jump and coexistence. Moreover, various (inconsistent) system parameter sets needed to be identified in different frequency ranges because of the wave-frequency dependency (Lin and Yim 1998). The model also failed to determine a single set of constant coefficients to closely predict overall response behavior over the frequency range experimentally considered. A more suitable model was suggested to be developed for detailed comparisons.

In identifying a model to improve prediction capability of the experimental results, an investigation of modeling and parameter identification of the experiment had recently been conducted (Narayanan and Yim 2004; Yim and Narayanan 2004). Numerical results indicated that an independent-flow-field (IFF) Morison model with three-term polynomial approximation for the restoring force was the most suitable for the chosen experimental results, especially near subharmonic resonance. A set of “optimal” constant coefficients was identified for the wave frequency range considered. With this set of constant coefficients, simulations of the IFF model were in good agreement with the experimental results under various wave conditions (Lin and Yim 2005; Yim and Lin 2005).

As an extension of the IFF model investigations on SDOF responses, a two-part study on response characteristics of the coupled surge-and-heave (2DOF) model subjected to various wave excitations, including both deterministic and stochastic, is presented here. Part I conducts classifications, analyses, and comparisons of deterministic, coupled surge-heave, experimental responses, and Part II examines the stochastic response characteristics subjected to random wave excitations. Taking the surge-heave coupling into account, a 2DOF, IFF, Morison model governing coupled surge-heave motions is derived and employed for numerical predictions and simulations. The 2DOF model consists of an IFF, Morison hydrodynamic drag, a three-term polynomial approximation of the nonlinear restoring force, and a nonlinear surge-heave coupling term. Based on the identified parameters and preliminary comparisons, an “optimal” set of constant system and hydrodynamic parameters is identified for all examinations and comparisons over the entire wave frequency range considered.

In the deterministic analysis (Part I), comparisons are conducted for all tests, and representative samples are chosen for demonstration purposes. The existence of highly nonlinear responses, including chaos, is assessed based on numerical simulations. Parametric studies by independently and coherently varying the intensities of surge-heave coupling and structural damping are conducted to examine their effects on the response characteristics. Comparisons of responses of the 2DOF and SDOF models are also carried out and discussed.

**System Considered**

A multipoint, moored system is modeled by a hydrodynamically excited, submerged, rigid body moored by elastic mooring cables with geometric nonlinearity. As experimentally observed, the system response predominantly moves in a two-dimensional (2D), surge-and-heave plane. The wave field is governed by kinematics and return flow of prescribed periodic waves. The exciting force components include coupled fluid–structure interaction drag and inertial terms (Gottlieb and Yim 1992).

**Model Configuration**

The experiments were conducted at the O. H. Hinsdale Wave Laboratory at Oregon State Univ. in a 2D wave flume, which had gross dimensions of 104.3 m (length), 3.66 m (width), and 4.57 m (depth) and a hydraulically driven, hinged flap wave board. Data recorded during each test included wave profiles, water particle velocities, sphere displacements, and restoring force on the springs. The sampling rate was 50 Hz for the tests subjected to deterministic waves, or 16 Hz for those subjected to random waves. The experimental model setup and configuration and data acquisition had been reported by Yim et al. (1993). A brief description of the experiment is summarized as follows for convenient reference.

The experimental model considered in this study is a geometrically nonlinear two-point moored system free to move in all 6 DOF, i.e., surge, heave, pitch, yaw, sway, and roll. However, because of the direction of incident waves and symmetry of experimental configuration, it was observed that the movement of the sphere was predominantly two dimensional in surge and heave only. The model consisted of a neutrally buoyant sphere submerged in the closed wave channel (Fig. 1). The sphere, made of polyvinyl chloride with diameter of 0.4572 m, was suspended about 1.83 m above the bottom of the flume. Springs of stiffness of 291.86 N/m were attached to the sphere at a 90° angle to
provide a nonlinear restoring force. The restoring force, which contained geometric nonlinearity, was derived by a Lagrangian formulation (Gottlieb and Yim 1992). The energy dissipation mechanism included a linear system (structural) component (associated with system connections and contact points of instrumentation), and a quadratic hydrodynamic drag component. The initial tension in the mooring cables (springs) varied from 66.72 to 133.44 N. The majority of the tests were performed with relatively low initial tension (111.2 N) to ensure nonlinear motion response (Lin and Yim 1998).

A total of 16 channels of measurements, including six wave gages, two current meters, and eight string pots, were used to measure wave profiles, water particle velocities, and sphere movements, respectively. The data were collected, conditioned, filtered, and stored for later reduction. The six wave gages were located at 3.35, 1.22, and 0.46 m up- and downstream of the quiescent position of the sphere, respectively. Based on a geometric relationship between instrumentation and sphere’s displacements, all the measurements (Channels 9–16) of the sphere’s movements were initially decomposed into surge, heave, and roll components for later analyses and comparisons. Roll motions based on the decomposition were found negligible for all tests as experimentally observed.

Four measurements (Channels 11, 14–16) are used in this study to compute surge and heave motions. The measurements at 0.46 m upstream are chosen to estimate incident wave conditions.

**Equations of Motion**

Based on the experimental observations and results, the sphere moves predominantly in a coupled surge-and-heave fashion. Extending the SDOF, IFF model by taking into account the surge-heave coupling effects (Lin and Yim 2005), the equations of coupled surge-and-heave motion of the cable-moored system subjected to deterministic wave excitations are given by (Yim and Narayanan 2004)

\[
M \ddot{x}_s + C_{s1} \dot{x}_s + C_{d1} \dot{x}_s \dot{x}_h + R_1(x_s) + R_3(x_h) = F_{D1}(u_1) + F_{H1}(u_1)
\]

\[
M \ddot{x}_h + C_{s3} \dot{x}_s + C_{d3} \dot{x}_s \dot{x}_h + R_1(x_s) + R_3(x_h) = F_{D3}(u_3) + F_{H3}(u_3)
\]

where \(M\) is mass of the sphere, and \(x_s\), \(\dot{x}_s\), \(x_h\), and \(\dot{x}_h\) denote the surge displacement, velocity, heave displacement, and velocity, respectively; and \(R_1\) and \(R_3\) are nonlinear restoring forces. Note that indices 1 and 3 represent the components in surge and heave, respectively. \(C_{s1}\) and \(C_{s3}\) denote the effective (linear) system damping coefficients (=\(\xi_{s1} M / C_{s1}\)); \(C_{d1}\) and \(C_{d3}\) = critical damping); \(C_{D1}\) and \(C_{D3}\) = hydrodynamic damping coefficients; \(C_{d1}\) and \(C_{d3}\) = coupling coefficients of surge and heave; and \(F_{D1}\) and \(F_{H1}\) = drag and inertial components of the exciting force, respectively.

The restoring forces \(R_1\) include the forces due to the mooring \(R_g\) and the hydrostatic buoyancy \(R_b\). The sphere used for this experiment is virtually neutrally buoyant when submerged, thus the forcing component caused by \(R_b\) is negligible and not considered here. Restoring forces \(R_3\) can be further approximated by three-term polynomials, which are given as

\[
R_1(x_s) = k_{11}x_s + k_{12}x_s^2 + k_{13}x_s^3
\]

\[
R_3(x_h) = k_{31}x_h + k_{32}x_h^2 + k_{33}x_h^3
\]

The three-term polynomials show a good approximation to the restoring forces (Yim and Narayanan 2004). The exciting force consists of a Morison drag \((F_{D1,3})\) and an inertial component \((F_{H1,3})\) are given by

\[
F_{D1} = \frac{\rho}{2} C_{D1} A_p u_1 |u_1|\]

\[
F_{D3} = \frac{\rho}{2} C_{D3} A_p u_3 |u_3|\]

and

\[
F_{H1} = \rho \forall (1 + C_{A1}) \frac{\partial u_1}{\partial t} - \rho \forall C_{A1} \dot{x}_s
\]

\[
F_{H3} = \rho \forall (1 + C_{A3}) \frac{\partial u_3}{\partial t} - \rho \forall C_{A3} \dot{x}_h
\]

where \(C_{D1,3}\) = hydrodynamic viscous drag coefficients; \(C_{A1,3}\) = added mass coefficients; \(A_p\) = projected drag area; \(\forall\) = displaced volume; \(\rho\) = water density; and \(u_1\) and \(u_3\) = water particle velocities in surge and heave, respectively.

**Parameters Identification**

A set of estimates based on a frequency domain identification technique on noisy subharmonic experimental test cases is employed as an initial guess to identify the IFF model parameters (Narayanan and Yim 2004; Yim and Narayanan 2004). The estimated parameters are later fine tuned by extensive numerical results compared with experimental results in the time domain. An “optimal” set of constant coefficients is thus identified (see Table 1) and employed in the numerical response predictions that are consistently in good agreement with experimental results. Extensive parametric studies in response frequency diagrams will further validate the model with the set of parameters in later sections.

**Experimental Results and Comparisons**

**Observations**

Employing analysis results of the SDOF model as a guideline to search for highly nonlinear responses, a total of nine 2DOF experimental tests (E01-08 and 13) under various wave-forcing conditions have been conducted (Yim et al. 1993). Wave amplitude and period were varied within the ranges of 0.183–0.488 m and 1.25–12 s, respectively. Lengths of the tests were varied from 5 to 30 min to assure steady-state behaviors. Nonlinear

**Table 1. “Optimal” Set of Constant System Parameters Identified**

<table>
<thead>
<tr>
<th>(C_{A1,3})</th>
<th>(C_{D1,3})</th>
<th>(k_{11,3}) (N/m)</th>
<th>(k_{12,3}) (N/m²)</th>
<th>(k_{13,3}) (N/m³)</th>
<th>(C_{D1,3})</th>
<th>(C_{D1,3})</th>
<th>(k_{11,3}) (N/m)</th>
<th>(k_{12,3}) (N/m²)</th>
<th>(k_{13,3}) (N/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.20</td>
<td>135.78</td>
<td>191.60</td>
<td>628.40</td>
<td>0.02</td>
<td>1%</td>
<td>0.311/0.283</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

- \(C_{A1,3}\) = added mass coefficients.
- \(C_{D1,3}\) = hydrodynamic viscous drag coefficients.
- \(k_{11,3}\) = linear stiffness coefficients.
- \(k_{12,3}\) = quadratic stiffness coefficients.
- \(k_{13,3}\) = cubic stiffness coefficients.
- \(C_{D1,3}\) = hydrodynamic damping coefficients.
- \(C_{A1,3}\) = added mass coefficients.
- \(A_p\) = projected drag area.
- \(\forall\) = displaced volume.
- \(\rho\) = water density.
- \(u_1\) and \(u_3\) = water particle velocities in surge and heave, respectively.
model responses, including harmonic, subharmonic, and superharmonic responses were frequently observed. A brief summary of these tests can be found in Table 2.

Wave profile and typical subharmonic surge and heave responses (Test E01) are shown in Figs. 2(a–c), respectively. Figs. 2(d–f), respectively, show the wave profile and typical superharmonic surge and heave responses (Test E04). A profound nonlinearity is noted in the wave profile of Test E04, which is in the superharmonic, lower frequency range. Also, note that similar response characteristics are observed in surge and heave for both cases, and a high degree of correlation between surge and heave is indicated.

Comparisons of Predictions and Experimental Results

Numerical results show that in comparison with the SDOF counterpart, frequency dependency of the parameters of the 2DOF is more prominent, especially in the lower frequency range (<0.15 Hz) (i.e., the superharmonic domain). Nonetheless, variations of the parameters are relatively small, and extensive numerical results show that the simulations are in reasonably good agreement with experimental results by employing an “optimal” set of constant system coefficients. Thus, this set of coefficients is used for further comparisons, predictions, and analyses in this study.

Time History

Comparisons have been performed on all test results, and good agreement is observed. Selected representative test results, which include harmonics, subharmonics, and superharmonics are presented here for demonstration purposes. In addition, response characteristics of each test and the discrepancies between predictions and experimental results are also discussed below.

Comparisons of the numerical predictions and experimental results of a harmonic response (Test E13) in both surge and heave displacements with wave frequency at 0.45 Hz are shown in Figs. 3(a and b). It is observed that the simulations of surge and heave are in very good agreement with experimental results in both response characteristics and amplitude.

Comparisons of the numerical predictions and experimental results of a subharmonic response (Test E06) in both surge and heave displacements with wave frequency at 0.45 Hz are shown in Figs. 4(a and b). Again, it is observed that the simulations of surge and heave are in reasonably good agreement with experimental results in response characteristics and amplitude. Small differences between the simulated and experimental results in the harmonic response component (smaller amplitude component) are nonetheless noted.

Comparisons of the numerical predictions and experimental results of a superharmonic response (Test E05) in both surge and heave displacements with wave frequency at 0.15 Hz are shown in Figs. 5(a and b). Observe that the simulations of surge and heave are in good agreement with experimental results in

<table>
<thead>
<tr>
<th>Test case</th>
<th>$H$ (m)</th>
<th>$T$ (s)</th>
<th>Response type</th>
</tr>
</thead>
<tbody>
<tr>
<td>E01</td>
<td>0.204</td>
<td>2.00</td>
<td>Subharmonic</td>
</tr>
<tr>
<td>E02</td>
<td>0.332</td>
<td>1.43</td>
<td>Harmonic</td>
</tr>
<tr>
<td>E03</td>
<td>0.183</td>
<td>1.25</td>
<td>Harmonic</td>
</tr>
<tr>
<td>E04</td>
<td>0.357</td>
<td>10.00</td>
<td>Superharmonic</td>
</tr>
<tr>
<td>E05</td>
<td>0.405</td>
<td>6.67</td>
<td>Superharmonic</td>
</tr>
<tr>
<td>E06</td>
<td>0.488</td>
<td>2.22</td>
<td>Subharmonic</td>
</tr>
<tr>
<td>E07</td>
<td>0.366</td>
<td>12.00</td>
<td>Superharmonic</td>
</tr>
<tr>
<td>E08</td>
<td>0.262</td>
<td>1.33</td>
<td>Harmonic</td>
</tr>
<tr>
<td>E13</td>
<td>0.335</td>
<td>2.22</td>
<td>Harmonic</td>
</tr>
</tbody>
</table>

Fig. 2. Sample experimental nonlinear structural responses: (a) wave profile, (b) subharmonic surge, (c) subharmonic heave, (d) wave profile, (e) superharmonic surge, and (f) superharmonic heave; wave height of 0.24 m and period of 2 s for test E01, and 0.38 m and 10 s for test E04, respectively.
response amplitude and characteristics. Discrepancies between predictions and experimental results in the details of the response characteristics are noted. The discrepancies may be inherited from the variations between the linear wave model and actual nonlinear waves as indicated by Fig. 2. Small variations in the identified system parameters are also noted, and higher frequency dependency in hydrodynamic parameters in the lower frequency range is indicated.

Frequency Response Diagram
A frequency response diagram is employed here to exhibit the overall system response characteristics, including resonances, coexistence, and possible highly nonlinear responses, which can be depicted (Nayfeh and Mook 1979). We point out here that an alternative means of presentation of the responses as a function of wave number $k$ and wave height $H$, conventional in hydrodynamics, has also been considered. However, in such a presentation format, the underlying bifurcation structure was not as clearly revealed. Therefore, frequency response diagrams are chosen for interpretations and comparisons throughout this study.

With identified constant system parameters, frequency response diagrams of surge and heave are generated by numerical simulations and compared with limited experimental results as shown in Figs. 6(a and b), respectively. Based on the identified parameters, it is indicated that the 2DOF model is lightly damped and strongly coupled in surge and heave motions. The frequency response diagrams are computed by varying wave frequency from 0.01 to 1.00 Hz, and at each frequency, 25 various initial conditions uniformly distributed over specified initial displacement-velocity domain are employed to compute steady-state responses to identify possible multiple coexisting responses. In both figures [Figs. 6(a and b)], experimental results are denoted by circles ($\bigcirc$)
and simulations by crosses (+). The simulation results are shown to be in good agreement with the experimental results for both surge and heave displacements.

The comparison results indicate that the hydrodynamic properties associated with the coupled surge-heave motions are relatively sensitive, nonlinear, and wave frequency dependent, especially in the low frequency range. However, predictions of the 2DOF, IFF Morison model with constant coefficients can capture overall response characteristics and behaviors.

**Numerical Predictions**

The frequency response diagrams obtained by numerical simulations indicate an intricate nature of system response in both surge and heave displacements [Figs. 6(a and b)]. A secondary, superharmonic resonance (tilting to the left) is indicated by a hump in the low frequency range near 0.15 Hz. In the frequency range, superharmonic response is observed experimentally and verified numerically. Another hump is shown near 0.35 Hz, which indicates multiple coexistence of steady-state responses. Figs. 7(a and b) show the coexisting harmonic and subharmonic responses with wave frequency at 0.35 Hz.

The primary resonance (tilting to the right) is found near 0.25 Hz. In the frequency range of 0.5–0.8 Hz, multiple steady-state responses, including chaos, are predicted to coexist. Samples of the coexisting surge responses near frequency 0.69 Hz are shown in Figs. 8(a–d). For the same excitation, four various, distinct characteristic responses, including subharmonics, chaos, small, and large amplitude harmonics are found using different initial conditions. The highly nonlinear chaotic response shows random-like characteristics in its time history [Fig. 8(c)]. Fractal nature of the chaotic attractor is fully revealed in the corresponding Poincaré maps as shown in Figs. 9(a and b). Note that a

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**Fig. 7.** Coexisting harmonic (a) and (b) and subharmonic (c) and (d) responses with wave amplitude of 0.61 m and frequency at 0.35 Hz

**Fig. 8.** Coexisting harmonic, subharmonic, and chaotic responses: (a) small-amplitude harmonic and (b) subharmonic, (c) chaotic, and (d) large-amplitude harmonic; initial conditions—(0,0,0,0), (0.3,−0.3,0,0), (−1.5,1.5,0,0), and (1.5,0,0,0), respectively, and wave amplitude of 0.61 m and frequency at 0.69 Hz
A Poincaré map is a collection of Poincaré points, e.g., \( X_{1p} \), \( X_{2p} \), which are sampled from the corresponding time history at every forcing period. Another secondary, subharmonic resonance tilting to the right is found near 0.5 Hz. Multiple responses are also found to coexist near the subharmonic frequency range. The coexisting harmonic and subharmonic responses are shown in Figs. 10(a) and (b).

As indicated by the frequency response diagrams presented in Figs. 6(a and b), the surge and heave show similar overall response characteristics, suggesting a high degree of correlation. It is also indicated that under a wave frequency of 0.5–0.9 Hz the system response is highly nonlinear, even chaotic, and exhibits multiple coexisting response characteristics [Figs. 6(a and b)]. However, in the experiment, because of quiescent initial settings for most tests and the presence of tank noise, the model response may (and often does) settle and remain in the stronger, smaller amplitude response modes, e.g., small amplitude harmonic or subharmonic. Higher degree and/or larger amplitude nonlinear response, e.g., chaos and large-amplitude harmonics as predicted to exist, however, can only be reached with large initial displacements and velocities.

**Parametric Studies**

Numerical results show that high-order nonlinear responses with intricate characteristics exist when surge-heave couplings \( C_{13} \) and \( C_{31} \) are strong and structural dampings \( \xi_{1} \) and \( \xi_{3} \) are weak. Effects of each of the factors (coupling and damping) on the structural responses are examined in this section. Comparisons of response characteristics via response frequency diagrams of the 2DOF and SDOF \( (C_{13}=C_{31}=0, \text{ and } \xi_{1}=\xi_{3}=6\% \text{, c.f. Lin and Yim 2005}) \) are also shown and discussed here.

**Surge-Heave Coupling \( (C_{13} \text{ and } C_{31}) \)**

Based on comparisons of numerical and experimental results, surge-heave coupling coefficients \( C_{13} \) and \( C_{31} \) are found to be 0.311 and 0.283 N/m³, respectively. A sensitivity study is then conducted on the two coupling coefficients by holding other system parameters constant to independently examine their effects on the response characteristics. The surge-heave coupling coefficients \( C_{13} \) and \( C_{31} \) are, respectively, decreased incrementally from 0.311 and 0.283 to a limiting case of 0 and 0 as two uncoupled SDOF models of surge and heave motions.

Numerical results show that fine, intricate, nonlinear response patterns in the frequency response diagrams diminish with decreasing couplings (Fig. 11). Chaotic motions and multicoexistence disappear when \( C_{13}=C_{31}=0.15 \text{ N/m³} \). When surge and heave motions are completely uncoupled \( (C_{13}=C_{31}=0) \), there are jumps and coexisting harmonics and subharmonics found near 0.25 and 0.5 Hz, and other complex nonlinear phenomena are absent. This indicates that results of a SDOF model may provide information about the general behavior of system responses; however, a 2DOF model is necessary for accurate predictions of sensitive, higher order nonlinear, complex responses.

**Structural Damping \( (\xi_{1} \text{ and } \xi_{3}) \)**

Based on comparisons of numerical simulations and experimental results, structural damping of the 2DOF model is identified to be around 1% of the critical damping. The structural damping...
consists of energy dissipation mechanisms caused by instrumentations and model configurations. As for its SDOF counterpart, because of the presence of a rod model constricting the sphere’s motion in surge, the structural damping was found to be much higher (around 6% of the critical damping). The damping ratio is then varied from 1 to 6% to examine its effects on response characteristics.

Numerical results indicate that the system exhibits a similar higher-order nonlinear, intricate frequency response pattern until the structural damping ratio is increased to 3%. When the damping ratio is increased to 4% and beyond [e.g., 6% in Fig. 12(c)], complex, higher-order nonlinear phenomena, including two multiple coexistences (near 0.7 Hz, including chaos and near 0.8 Hz, respectively), decrease. However, other essential trends, e.g., jump phenomena, primary, and secondary resonances still remain (Fig. 12).

Comparison of 2DOF and SDOF Response
Comparisons of characteristic responses of 2DOF and SDOF models are discussed in this section (Fig. 13). The 2DOF model is identified to have strong coupling ($C_{13}=0.311$ and $C_{31}=0.283$ N/m$^3$) and light damping ($\xi_{51} = 1\%$). Numerical results show that there is multiple coexistence, including chaos. The SDOF model, on the other hand, has zero coupling ($C_{13}=C_{31}=0$) and heavy damping ($\xi_{51} = 6\%$) because of its configuration constraints (Lin and Yim 2005). Numerical results indicate that harmonic and subharmonic responses are the most dominant character responses. Other higher order nonlinear responses have not been identified and observed either experimentally or numerically.

It is observed that strong energy transfer between surge and heave, characterized by coupling terms, leads to more sensitive, complex, nonlinear system response behavior. Numerical results also show that when the energy dissipating mechanism is strong, the sensitive, higher-order nonlinear, complex responses are damped out, and only relatively more stable responses remain. Hence, despite the fact that the SDOF model provides an overall insight into the system, the 2DOF model supplies more accurate and detailed response predictions for the underlying response behavior.

Besides the prominent effects of the surge-and-heave coupling and structural damping terms on the response characteristics, the presence of random perturbations in wave profiles may also play an important role in response characteristic transitions. With

Fig. 11. Effects of surge-heave coupling on response characteristics: (a) $C_{13}=0.311$ and $C_{31}=0.283$; (b) 0.24, (c) 0.15, and (d) 0 (N/m$^3$) with wave amplitude of 0.61 m

Fig. 12. Parametric study on effects of structural damping ratio $\xi_{51,3}$ on surge response characteristics: (a) 1%, (b) 3%, and (c) 6% with wave amplitude of 0.61 m
various perturbation intensities, the response may be led to transitioning and remain in different coexisting attractors, as experimentally observed. The perturbation-induced response transitions are examined in detail in Part II.

Concluding Remarks

As an extension of studies of SDOF responses, this study presents the model calibration and response comparisons of a medium-scale experimental, 2DOF, nonlinear, moored ocean structural system. Some concluding remarks are accordingly drawn as follows:

1. Predictions of a proposed deterministic 2DOF, IFF Morison model are in good agreement with the experimental results using both frequency response diagrams and time histories.
2. Simulations of the frequency response diagram imply the existence of resonances, jump phenomenon, and coexistence of multiple attractors. Not all of these behaviors are exhibited in the experimental results because of limited tests and range of parameters. It is noted that the hydrodynamic parameters of the 2DOF model are relatively more frequency dependent than those of its SDOF counterpart. The dependency is found to be more prominent in the lower-frequency range.
3. Simulations also indicate the existence of very large-amplitude harmonic response which, with the presence of variations in wave amplitude, may lead the sphere to potentially very large-amplitude motions as indicated in the experiment.
4. Predictions show the coexistence of high-order nonlinear responses, including chaos. Chaotic response is identified to exist with large initial displacements and/or velocities, which may provide guidelines in future experimental search for higher-order nonlinear responses.
5. Numerical results show that multiple, complex, nonlinear responses coexist when the surge-heave coupling is strong and structural damping is light (consistent with parameters identified from experimental results). The degree of response coexistence, nonlinearity, and complexity decreases with decreasing coupling and/or increasing damping.
6. Parametric studies indicated that results of both 2DOF and SDOF models may possess similar qualitative response characteristics; however, 2DOF model studies are necessary to identify detailed bifurcation patterns and the existence of sensitive, higher-order nonlinear, intricate responses.

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References


