Global and Local Nonlinear System Responses under Narrowband Random Excitations. I: Semianalytical Method

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Abstract: A single-degree-of-freedom nonlinear structural system under narrowband random excitation can exhibit very complex global and local response behaviors. In order to develop a stochastic method to analyze the nonlinear responses, the system under deterministic excitation is first modeled and examined in the primary and subharmonic resonance regions. Typical response behaviors including coexistence of attractors and (global) jump phenomena are observed. Governing equations of the probability for the response–amplitude perturbations (a local transition) within an attraction domain and a jump between different attraction domains (a global transition) are derived under the assumption of a stationary Markov condition. The overall response–amplitude probability distribution is obtained by applying the Bayes formula to the two types of response transition probability distributions. In this study, we focus on understanding the physics of the transitions using the proposed probability method.

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Introduction

Dynamic system response behaviors under narrowband stochastic excitations have been studied for decades (Rice 1954; Lyon et al. 2005; Bense et al. 2006; Dimenberg 1971; Richard and Anand 1983; Davies and Liu 1990; Koliopulos and Bishop 1993). To date, the stochastic behavior of linear systems is well understood by using the spectral analysis technique (Crandall and Mark 1963; Crandall et al. 1967; Nigam 1983; Roberts and Spanos 1990; Newland 1993; Soong and Grigoriu 1993; Lutes and Sarkani 1997). However, relatively little understanding on the stochastic behavior of nonlinear systems subjected to narrowband excitations has been achieved because of the complexity of the system response characteristics.

The complex response behavior of a nonlinear mechanical or structural system under deterministic excitations includes the jump phenomenon (a global bifurcation behavior), subharmonic and superharmonic resonance, and even chaotic response (coexisting attraction domains) (Nayfeh and Mook 1979; Guckenheimer and Holmes 1986; Thompson and Stewart 1986; Jordan and Smith 1999). To investigate these complex responses, semianalytical methods and numerical techniques (Gottlieb and Yim 1992; Yim and Lin 1992) are required in general.

In recent years, Roberts and Spanos (1986) and Davies and Liu (1990) approximated the excitation and response as Markov processes and pointed out a general rule of applying the stochastic averaging method in analyzing the stochastic system response under narrowband excitations. By solving the associated Fokker–Planck equation relating the excitation envelope and the response envelope, an approximate probability density function of the response process was obtained. A direct numerical solution of the Fokker–Planck equation using the path integral procedure was developed by Naess and Johnsen (1993) (see also Naess 1994, 1997). The method was successfully applied to the analysis and predictions of nonlinear system responses by Naess and Moe (2000) and Yim et al. (2005a and b). Probabilities of exceedence were calculated by integrating the four-dimensional state space process formed by the response displacement and velocity and the second order filter approximating the force excitation spectrum. Although the path integral solution method provides accurate numerical data in the form of joint probability densities, the results have to be presented in graphical form and are not convenient for understanding the physics of the response behaviors.

Alternatively, a quasi-harmonic method was introduced by Koliopulos and Bishop (1993). Under the assumption that both the excitation and response processes are narrowband, the quasi-harmonic analysis leads to the formulation of a probability density function of the response envelope. However, the excitation bandwidth effect on the response behavior is not taken into account by this method, although the validity of this method can be determined by an extra parameter that indicates the occurrence and persistence of the response amplitude jump phenomenon.

In previous studies, the analytical methods developed were based on deterministic techniques. However, due to the random nature of stochastic excitation processes (e.g., waves in the ocean), the system response should be considered as a transient state, and nearly steady-state behavior should be a special case.
under small randomness of the excitation. Moreover, current technology in predicting nonlinear stochastic response behavior is still limited to the case when the system is in a primary resonance region.

In this study, a semianalytical method capable of more accurately characterizing and predicting stochastic nonlinear response behavior under broadband excitations is developed and extended to analyze the system behavior in the primary and subharmonic resonance regions. We focus on understanding the physics and response–amplitude transitions (local perturbations) using the proposed methodology. The information obtained is more useful for long-term fatigue calculations.

### System Considered

The governing equation of motion of a nonlinear single-degree-of-freedom dynamic system such as a moored offshore semisubmersible platform can be represented by the general normalized Duffing oscillator (Gottlieb and Yim 1992; Lin and Yim 1997) and is expressed as

\[ \ddot{x} + c_1 \dot{x} + a_1 x + a_2 x^3 = f(t) \]  

(1)

where \( c_1 \) is (normalized) damping coefficient; and \( a_1 \) and \( a_2 \) are linear and nonlinear stiffness coefficients, respectively. The system is subjected to a normalized external excitation \( f(t) \). The elastic restoring force represented by the cubic polynomial is the only source of nonlinearity in the system. For the excitation, both deterministic and stochastic excitation models are used. When the excitation is deterministic, the excitation parameters are all time-independent and \( f(t) \) can be expressed as in Eq. (2)

\[ f(t) = A \cos(\omega t + \phi) \]  

(2)

If the excitation is a stochastic process, the parameters \((A, \omega, \phi)\) then becomes time dependent random variables instead of constants and their behavior may vary significantly depending on the spectral bandwidth of the process. The narrowband process is modeled as the output process of a lightly damped linear system with a white noise process as the excitation (Stratonovich 1963). This linear system can be expressed as

\[ \dot{f} + \gamma f + \omega_f f = \gamma^{1/2} \omega_f W_0(t) \]  

(3)

where \( \gamma \) serves as a bandwidth parameter; \( \omega_f \) is peak frequency of the output process \( f(t) \); and \( W_0(t) \) is stationary Gaussian white noise process with zero mean and spectral intensity \( S_\omega \).

The envelope and phase processes associated with a given random process are useful concepts in the theory of random vibration. A narrowband stochastic process \( f(t) \) can be represented as a cosine function having time varying amplitude governed by the envelope process, and time-varying frequency governed by the phase process (Langley 1986). That is

\[ f(t) = A(t) \cos[\omega_f t + \phi(t)] \]  

(4)

where \( A(t) \) and \( \phi(t) \) are envelope and the phase processes, respectively.

By assuming the narrowband process \( f(t) \) to be Gaussian with zero mean and variance \( \sigma_f^2 \), the joint probability density function of the envelope \( A(t) \) and the phase \( \phi(t) \) processes is obtained as (Ochi 1990)

### Deterministic Nonlinear System Response Behavior

**Coexisting Attraction Domains and Initial Condition Dependency**

A major difference in the response characteristics between a linear system and a nonlinear system with multiple attraction domains is the dependence of the steady-state response on the system initial conditions. To demonstrate the system response initial condition dependency in the subharmonic resonance region, the four coexisting attraction domains corresponding to a deterministic excitation amplitude \( A = 9 \) and time series of four different responses are shown in Figs. 1 and 2. Fig. 1 shows that the response behavior may be totally different depending on the initial condition even though the system parameters and deterministic excitations are identical. Notice that the system response behavior with certain set of parameters is sensitive to initial conditions around the boundaries of coexisting attraction domains. As shown in Fig. 2, coexisting attraction domains have such complex patterns that a probabilistic description is needed to describe sensitive system response behavior. For convenience of notations, the large primary resonance, small primary resonance, 1/2 subharmonic and 1/3 subharmonic attraction domains are denoted as \( D_1, D_2, D_3, \) and \( D_4 \), respectively.

### Response–Amplitude Domain Overlapping

Previous investigations showed that when the excitation frequency \( \omega \) is close to three times the natural frequency of the corresponding linearized system five attraction domains coexist (Thompson and Stewart 1986). These attraction domains include...
two primary resonances (large and small amplitude), two 1/2 subharmonic, and one 1/3 subharmonic responses. It is found that the two coexisting 1/2 subharmonic response attractors are of the same steady-state amplitude but with different biases in time series. Thus, in this study they are considered as being parts of the same attraction domain (i.e., they belong to the same attractor).

Amplitude–response curves of the system in the primary and subharmonic resonance regions are shown in Fig. 2. Due to the complexity of solving approximate steady-state response amplitudes in these attraction domains, these curves are obtained by direct integration of Eq. (1). In the figure, the vertical dashed lines indicate intervals of excitation amplitudes where different types of system response, or attraction domain $D_d$ exist and those excitation amplitude domains are denoted as $D_d^a$ ($d=1,2,3,4$). Note that, in each domain $D_d$ ($d=1,2,3,4$), the steady-state response amplitudes also form a response amplitude domain $D_d^R$ ($d=1,2,3,4$), as shown in Fig. 3. In addition, unlike the case in the primary resonance regions, response amplitude domains overlapping are observed among $D_d^R$ ($d=2,3,4$). That is, a single response amplitude may belong to more than one attraction domain and thus may correspond to different deterministic excitation amplitudes.

Throughout this study, the excitation amplitude domain specified explicitly according to Fig. 3 will be used to clearly demonstrate the sensitive global and local characteristics of the nonlinear system responses.

**Stochastic System Analysis Methodology**

**Assumptions**

For the narrowband excitation process, $f(t)$, the frequency is assumed constant and equal to the peak frequency, $\omega_p$, of the spectrum of the system output in Eq. (3). The effect of excitation frequency variations on the response behavior is taken into account through consideration of the excitation phase-angle variations (Rice 1954; Stratonovich 1963; Langley 1986). For the...
201 Stochastic Behavior of Excitation Parameters

To investigate the stochastic behavior of the amplitude and the phase processes, a four-dimensional joint probability density function (PDF) of the random variables representing the excitation amplitudes and phase angles corresponding to consecutive excitation cycles, i.e., \( A^{(1)} \), \( A^{(2)} \), \( \phi^{(1)} \) and \( \phi^{(2)} \), can be obtained as

\[
\begin{align*}
\mathbf{p}(A^{(1)}, A^{(2)}, \phi^{(1)}, \phi^{(2)}) &= 
\frac{A^{(1)}A^{(2)}}{4\pi^2\sqrt{\Sigma}} \exp\left( -\frac{1}{2\sqrt{\Sigma}} \left( \sigma_0^2 (A^{(1)})^2 + (A^{(2)})^2 \right) 
- 2A^{(1)}A^{(2)}[\rho \cos(\phi^{(2)} - \phi^{(1)}) + \lambda \sin(\phi^{(2)} - \phi^{(1)})] \right) 
\end{align*}
\]

where

\[
0 \leq A^{(1)}, \quad A^{(2)} < \infty, \quad 0 \leq \phi^{(1)}, \quad \phi^{(2)} \leq 2\pi \tag{6}
\]

and superscripts \((1)\) and \((2)\) indicate that the quantities are in the current and the next excitation cycles, respectively; \( \Sigma \) is the joint PDF of \( A^{(1)}, A^{(2)} \), and \( \Phi \) can be obtained from Eq. (6) by the transformation of the random variables (Ochi 1990)

\[
p(A^{(1)}, A^{(2)}, \Phi) = 
\frac{A^{(1)}A^{(2)}}{2\pi\sqrt{\Sigma}} \exp\left( -\frac{1}{2\sqrt{\Sigma}} \left( \sigma_0^2 (A^{(1)})^2 + (A^{(2)})^2 \right) 
- 2A^{(1)}A^{(2)}[\rho \cos(\Phi) + \lambda \sin(\Phi)] \right) - 2\pi \leq \Phi \leq 2\pi \tag{8}
\]

In addition, the joint PDF of \( A^{(1)} \) and \( A^{(2)} \) can be obtained by integrating Eq. (6) with respect to \( \phi^{(1)} \) and \( \phi^{(2)} \) or integrating Eq. (8) with respect to \( \Phi \)

\[
p(A^{(1)}, A^{(2)}) = 
\frac{A^{(1)}A^{(2)}}{\sqrt{\Sigma}} \times \exp\left( -\frac{1}{2\sqrt{\Sigma}} \sigma_0^2 (A^{(1)})^2 + (A^{(2)})^2 \right) 
\times I_0\left( \frac{A^{(1)}A^{(2)}}{\sqrt{\Sigma}} \sqrt{\rho^2 + \lambda^2} \right) \tag{9}
\]

where \( I_0 \) is modified Bessel function of order zero. The excitation bandwidth dependency of Eq. (6) is transferred to Eqs. (8) and (9).

Transition Types and Probabilities

Two types of transitions from the current state to a future state in the next excitation cycle under the stationary Markov process assumption and their associated probabilities are examined in this study. The first, called attraction–domain transition, is a global bifurcation or "jump phenomenon" pertaining to the response motion transitioning from one attractor to another. The second, called response–amplitude perturbation, is a local transition pertaining to the response amplitude change between the current state and the next cycle (with or without an associated attraction–domain transition). Expressions of the probabilities governing these transitions are derived in the following subsections.

Attraction–Domain Transition

When the excitation amplitude varies outside of the attraction domain boundaries defined in the response amplitude curves, the system response may be attracted to a competing attraction domain, a global bifurcation or "jump" will occur and this bifurcation is defined as an attraction–domain transition. For a stationary Markov response process, the response attraction–domain transition among finite number of domains \( D^o_n \) can be modeled as a stationary Markov process with discrete states (Gillespie 1992) or a stationary Markov chain (Ochi 1990; Bouleau and Lepingle 1994).

The stochastic behavior of the attraction–domain transition is characterized by an attraction–domain transition probability matrix \( K \). The governing equation of the attraction–domain transition probability can be expressed as

\[
\hat{p}(D^{(2)}) = K\hat{p}(D^{(1)}) \tag{10}
\]

where \( \hat{p}(D^{(1)}) \) and \( \hat{p}(D^{(2)}) \) are probability vectors of the system response being in each individual attraction domain in the current and the next excitation cycles, respectively. Dimensions of \( \hat{p} \) and
\[ p(D^{(2)}) = p_{ij} \] 

where, \( p_{ij} \) is the conditional probability that the system response is going to the \( j \)th attraction domain given that it is currently in the \( i \)th attraction domain. The conditional probabilities \( p_{ij} \) can be evaluated from \( E_{ij} \) and \( E_{ji} \), and the probability that an attraction domain becomes the destination domain is

\[ p(E_{ij}) = 1 - \int D^{(2)} dA_{ij} \]

Accordingly, the probability, \( p(E_{ij}) \), that the system response exits from the attraction domain \( D_d \) at the domain lower limit \( A_{dl} \) can be obtained by

\[ p(E_{dl}) = 1 - \int D^{(2)} dA_{dl} \]

Note also that, after the system response exits from an attraction domain, there may exist multiple possible destination domains of the attraction-domain transition. In this case, transient-state system mean energy, or equivalently, the transient-state system response amplitude is employed to determine the attraction domain that the system will settle to during the attraction-domain transition. The system energy level can be represented by the system mean energy that is defined as the averaged system total energy over one excitation cycle. Fig. 4 shows the relationship between the system total energy and the system mean energy. When the system response has higher total energy, the system response will also have higher mean energy, the response also has larger amplitudes as shown in Fig. 4.

For example, consider the 1/2 subharmonic domain lower boundary \( A_{2L} \). At the 1/2 subharmonic domain lower bound \( A_{2L} \), the possible destination domains are the large and the small amplitude harmonic domains, and the 1/3 subharmonic domain (see Fig. 3). However, a jump from a 1/2 subharmonic response domain to the large amplitude resonance response domain is highly unlikely due to the decreasing excitation amplitude (hence input energy). Thus, during the attraction-domain transition, the system response may transition to either the small amplitude harmonic or the 1/3 subharmonic domain (see Fig. 3) when the excitation amplitude varies from \( A > A_{2L} \) (A = 7 in this case) to \( A < A_{2L} \) (A = 6). Fig. 5 shows that the system mean energy of a typical response in the 1/2 subharmonic domain is higher than those corresponding to the 1/3 subharmonic domain, which is turn is higher than those in the small amplitude harmonic domain. Therefore, it is assumed that, after the response exits from the 1/2 subharmonic domain, it will first visit the 1/3 subharmonic domain before it can visit the small amplitude harmonic domain.

**Response-Amplitude Perturbation**

The response-amplitude perturbation is defined as the local behavior of the response-amplitude variation (whether it is within an attraction domain or accompanied by an attraction-domain transition). Due to variations in the excitation amplitude during each excitation cycle, variations of the response-amplitude will occur within its current response-amplitude domain, \( D_d^R \), corresponding to different excitation amplitude in \( D_d^A \). The governing equation of the response-amplitude perturbation from \( D_d^R \) at the current excitation cycle to \( D_d^R \) at the next excitation cycle reads

\[ p(R^{(2)}|A^{(1)}) = \int D^{(2)} dA^{(2)} \]

Note that \( p(R^{(2)}|A^{(1)}) \) is a diagonal element of \( K \); \( p_{ij} \), in Eqs. (10) and (11). The probability, \( p(E_{ji}) \), that the system response exits from the attraction domain \( D_d \) at the domain upper limit \( A_{ul} \) is equivalent to the probability that the excitation amplitude \( A^{(2)} \) is greater than \( A_{dl} \). Thus, from Eq. (13), \( p(E_{dl}) \) can be obtained by

\[ p(E_{dl}) = 1 - \int D^{(2)} dA^{(2)} \]

\[ p(R^{(2)}|A^{(1)}) = \int D^{(2)} dA^{(2)} \]

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\[ p(R^{(2)}|A^{(1)}) = \int D^{(2)} dA^{(2)} \]
where \( p(R(2)|r^{(1)},A^{(2)},D_d) \) is domain-dependent response-amplitude perturbation probability density function.

Note that the probability distribution of the response amplitude \( p(R(2)|D_d^{(2)}) \) may be perturbed from all \( p(R(1)|D_d^{(1)},A^{(1)}); D_d^{(2)} \) and is the union of all the possible perturbations including both cases of with and without an associated attraction-domain transition. The events of these events are mutually exclusive and, thus, according to the Bayes formula (Ochi 1990), \( p(R(2)|D_d^{(2)}) \) can be expressed as

\[
p(R(2)|D_d^{(2)}) = \int_{D_d^{(2)}} p(R(2)|A^{(1)},A^{(2)},D_d^{(5)},p(A^{(1)}|D_d^{(1)}))dA^{(1)}
\]

On the other hand, occurrence of a response amplitude \( R \) being in the domains, \( D_d^{(2)} \), corresponding to different attraction domains, \( D_d \), are mutually exclusive events. Therefore, the response-amplitude probability distribution can be expressed as

\[
p(R(2)|D_d^{(2)}) = \sum_{d=1}^{n} p(R(2)|D_d^{(2)},p(D_d^{(6)}) \quad z = 1,2
\]

Within an attraction domain, the response–amplitude probability distribution can also be expressed as

\[
p(R(2)|D_d^{(2)}) = \int_{D_d^{(2)}} p(R(2)|D_d^{(2)},p(A|D_d^{(4)}))dA, \quad z = 1,2
\]

Note that \( \{p(D_d^{(0)})dA\} \) is equivalent to \( \{p(A|D_d^{(0)})\} \) which stands for the probability of the excitation amplitude being equivalent to \( A \) given that \( A \) belongs to \( D_d^{(0)} \). The integration of Eq. (20) is carried out over the entire domain \( D_d^{(1)} \). Thus, Eq. (20) can be rewritten as

\[
p(R(2)|D_d^{(2)}) = \int_{D_d^{(2)}} p(R(2)|D_d^{(2)},p(A|D_d^{(4)}))dA, \quad z = 1,2
\]
By substituting Eqs. (17) and (18) into Eq. (20), the governing equation of the response–amplitude perturbation probability yields

\[ p(R(2)|D_d^2) = \int \left[ \int \left[ \int \left[ \int p(R(2)|R(1)^{(1)}, A(2)^{(1)}, D_d^2)p(R(1)^{(1)}|d_d^1)dA(1)^{(1)} \right] \right] \right] p(A(2)^{(1)}|D_d^2)p(D_d^1) \]

Finally, the overall stationary response–amplitude probability distribution can be approximated as

\[ \tilde{p}(R(1)) = \tilde{p}(R(2)) = \sum_{d=1}^{n} \tilde{p}(R(1)|D_d^2)p(D_d) \]

### 399 Evaluation of Response–Amplitude Perturbation Probability

From a deterministic point of view, the variation in the response amplitude is a function of: (1) the excitation amplitude and the response amplitude in the current excitation cycle; (2) variation in the excitation parameters (amplitude and phase angle); and (3) the system phase status \((x, dx/dt)\) at the time when the excitation parameter variation takes place, which is also considered as the initial condition of the following transient-state response.

That is

\[ R(2) = g(R(1), A(1), A(2), \Phi = \phi(2) - \phi(1), X^0) \]

where \(R(1)\) and \(R(2)\) are response amplitudes in the domains \(D_d^{(1)}\) and \(D_d^{(2)}\), respectively; \(\Phi\) is the variation in the excitation phase angle; and \(X^0\) stands for the system initial condition. Note that the function \(g\) is domain dependent. If the response amplitude \(R(1)\), the excitation amplitude \(A(1)\) and \(A(2)\) are fixed, then \(R(2)\) can be considered as a function of \(\Phi\) and \(X^0\) only, i.e., \(R(2) = g(\Phi, X^0)\) where \(g\) = domain dependent function. As a result, the probability distribution of \(R(2)\) can be derived from the joint probability distribution of \(\Phi\) and \(X^0\) through the functional relationship

\[ p(R(2)|R(1)^{(1)}, A(1)^{(1)}, A(2)^{(1)}, D_d^{(2)}) = p(R(2)|R(1)^{(1)}, A(1)^{(1)}, A(2)^{(1)}, D_d^{(2)}) \]

To facilitate numerical evaluation of \(p(R(2)|R(1)^{(1)}, A(1)^{(1)}, A(2)^{(1)}, D_d^{(2)})\), the response amplitude domains \(D_d^{(2)}\), \(X^0\) domain, the excitation phase angle \(\Phi\) domain, and the excitation amplitude domain \(D_d^{(2)}\) are discretized. The value of \(g(\Phi, X^0)\) is \(R(2)^{(1)}\), given \(R(1)^{(1)}, A(1)^{(1)}\), and \(A(2)^{(1)}\) are obtained by direct numerical integration of Eq. (1), where the subscripts indicate sample points of their corresponding discretized random variables, \(\Phi\), \(X^0, R(1)^{(1)}, R(2)^{(1)}\), and \(D_d^{(2)}\), respectively. Thus, \(p(R(2)|R(1)^{(1)}, A(1)^{(1)}, A(2)^{(1)}, D_d^{(2)})\)

The probability distribution of the phase difference \(\Phi\) is characterized by Eq. (8) and depends on \(A(1)^{(1)}\) and \(A(2)^{(1)}\) only. The initial condition, \(X^0\), is assumed to be uniformly distributed over the domain that is the phase trajectory of the current response cycle.

In addition, \(\Phi\) and \(X^0\) can be assumed as statistically independent because the excitation properties \(\Phi\) are not affected by the system response and the uniformly distributed system initial condition is affected by neither the variation in the excitation parameters nor the system response. Thus

\[ p(R(2)^{(2)}|R(1)^{(1)}, A(1)^{(1)}, A(2)^{(1)}, D_d^{(2)}) = p(R(2)^{(2)}|R(1)^{(1)}, A(1)^{(1)}, A(2)^{(1)}, D_d^{(2)}) = \frac{1}{m} \int_{\Phi_{\min}}^{\Phi_{\max}} \frac{p(A(1)^{(1)}, A(2)^{(1)}|\Phi)}{p(A(1)^{(1)}, A(2)^{(1)}|\Phi)} d\Phi \]

where \(m\) = total number of intervals in discretized \(X^0\) domain; \(p(A(1)^{(1)}, A(2)^{(1)}|\Phi)\) and \(p(A(1)^{(1)}, A(2)^{(1)}|\Phi)\) can be obtained by Eqs. (8) and (9). By varying \(\Phi_{\min}\) and \(\Phi_{\max}\) over all respective domains and lumping all computed \(p(R(2)^{(2)}|R(1)^{(1)}, A(1)^{(1)}, A(2)^{(1)}, D_d^{(2)})\), a probability vector of the response amplitude \(p(R(2)|R(1)^{(1)}, A(1)^{(1)}, A(2)^{(1)}, D_d^{(2)})\) can be obtained.

### Concluding Remarks

A semianalytical method proposed in this study has been successfully extended to the primary and subharmonic resonances. In previous studies, analytical methods can only predict the response behavior in the primary resonance region where only two attraction domains coexist.

The proposed semianalytical method is capable of accurately characterizing the stochastic response behavior of the nonlinear system subject to narrowband excitations by predicting the response–amplitude probability distribution and capturing the trends of variations in the response–amplitude statistical properties.

Up to now, the transient–state response behavior can only be characterized qualitatively, in general. Numerical integrations of the system equation of motion need to be employed to determine the transient–state response amplitudes and thus, the response–amplitude perturbation probability in this study. This procedure of numerical evaluations can be quite time consuming. For example, by using a personal computer, run time of the FORTRAN program to obtain the approximation of \(p(R(1)|D_d^{(1)})\) with 15 intervals of the discretized excitation amplitude domain for all the coexisting attraction domains may take up to 48 h to complete the single case. Thus, the availability of an analytical expression of function \(g\) in Eq. (24) will significantly improve the efficiency of the semianalytical procedure in analyzing the stochastic response behavior.

Finite variations in the excitation frequency may induce significant shifts of the excitation–amplitude domain boundaries, which in turn affect the response attraction–domain transition (global bifurcation) behavior. The excitation–amplitude domain boundaries defined in this study correspond to the constant excitation peak frequency. Thus, the excitation–amplitude domain boundaries are also constant. However, the oscillating excitation frequency is actually varying from cycle to cycle and its variation is governed by the excitation phase process. When the excitation bandwidth is small, the variations in the excitation frequency may be negligible and the excitation–amplitude boundaries may be reasonably considered as constant. However, when the excitation bandwidth increases, the variations in the excitation frequency may be significant enough to shift the excitation–amplitude domain boundaries obviously and affect the response attraction–domain transition behavior. Therefore, to improve the accuracy of the method, the behavior of the excitation–amplitude domain boundary shifts due to variations in the excitation frequency
should be investigated. In addition, the influence of domain boundary shifts on the response attraction–domain transition (global bifurcation) behavior should be incorporated in the semianalytical procedure.

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504 References


