

# CONTROL OF NOISY NONLINEAR RESPONSE IN OCEAN SYSTEMS

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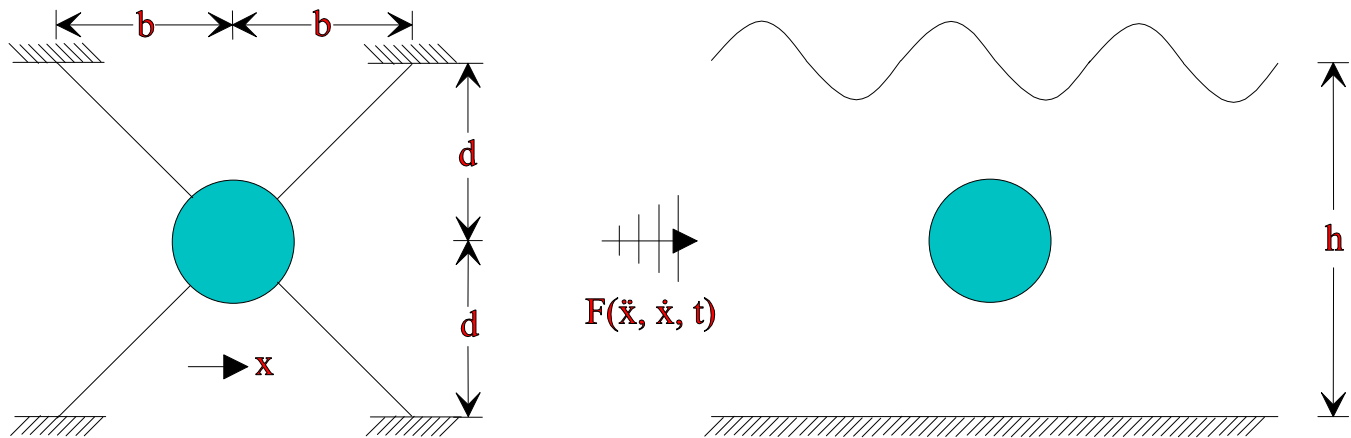
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# System Description



- An ocean structural system subject to wave and current excitation
- Analysis and simulation indicates presence of highly nonlinear motions
- Experimental data verify complex oscillations



# Analytical and Numerical Predictions

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{y} \\ \dot{\mathbf{y}} &= -\mathbf{R}(\mathbf{x}) + (\mathbf{y} + \mathbf{F}(\mathbf{y}, \mathbf{x}, \mathbf{2})) \\ \dot{\mathbf{2}} &= \mathbf{T}\end{aligned}$$

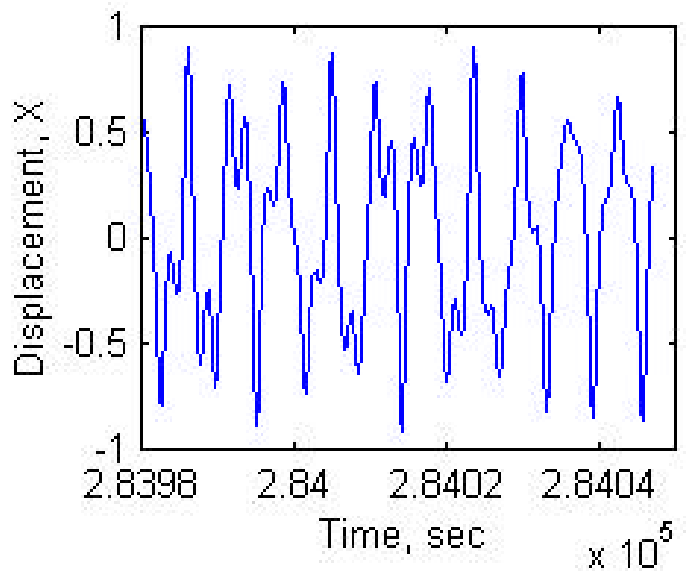
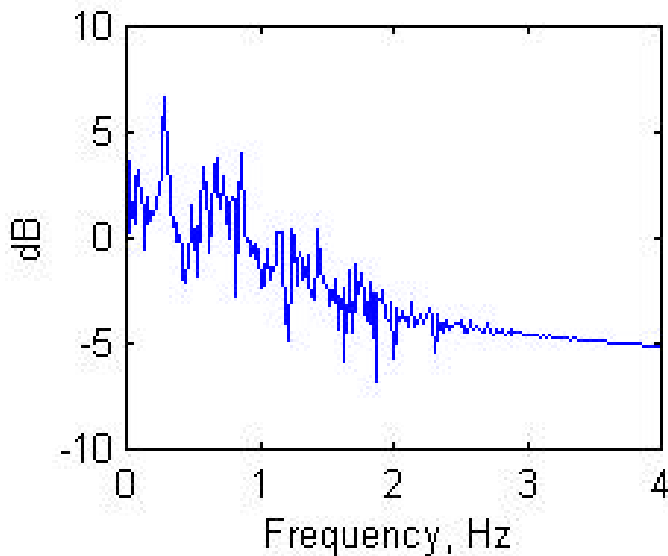
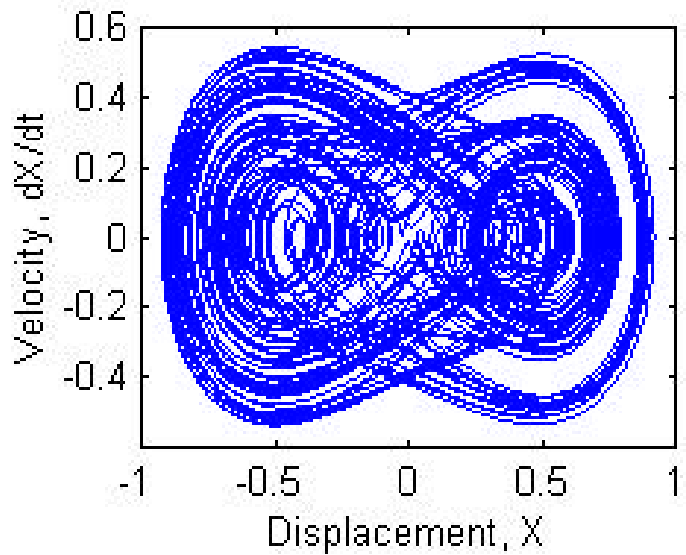
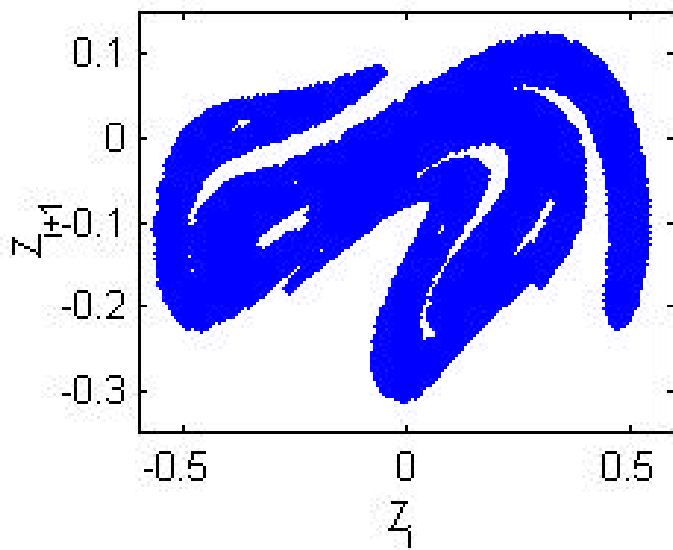
$$\mathbf{R}(\mathbf{x}) = \mathbf{R}(\mathbf{x} + \text{sgn}(\mathbf{x})) \left( \frac{1}{\sqrt{1 + \mathbf{S}^2}} - \frac{1}{\sqrt{1 + (\mathbf{x} + \mathbf{S}\text{sgn}(\mathbf{x}))^2}} \right)$$

$$\mathbf{F}(\mathbf{y}, \mathbf{x}, \mathbf{2}) = f_0 - f_1 \sin(\mathbf{2})$$

- Fluid loads modelled as added inertia and coupled drag
- Nonlinear coupling of the Morrison Form
- Small body theory
- Restraints on vertical and rotational motion



# Chaotic Response



- Analysis and simulation indicates presence of highly nonlinear motions
- Experimental data verify complex oscillations



# Control Algorithm for Nonlinear Response

- Determine all points such that

$$|Z_i - Z_{i+1}| < \epsilon,$$

$$|Z_i - Z_n| < \epsilon$$

- Form error vectors

$$d = |Z_i - Z_p|$$

$$e = |Z_{i+1} - Z_p|$$

- Calculate linear mapping of errors

$$A = (d^T d)^{-1} d^T e$$

- Define feedback control input

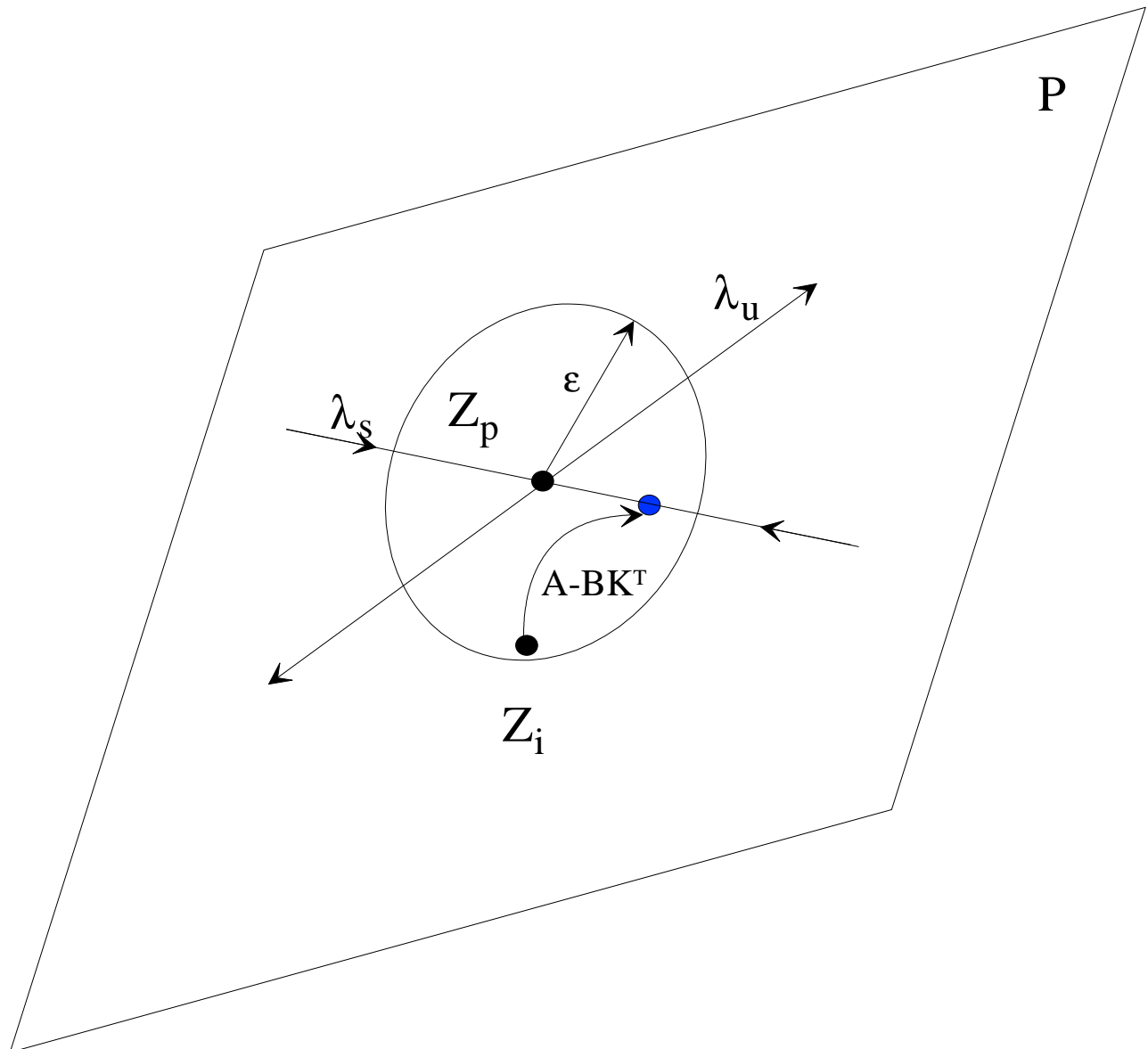
$$u = -K^T(Z_i - Z_p)$$

- Feedback control rule

$$Z_{i+1} - Z_p = (A - BK^T)(Z_i - Z_p)$$



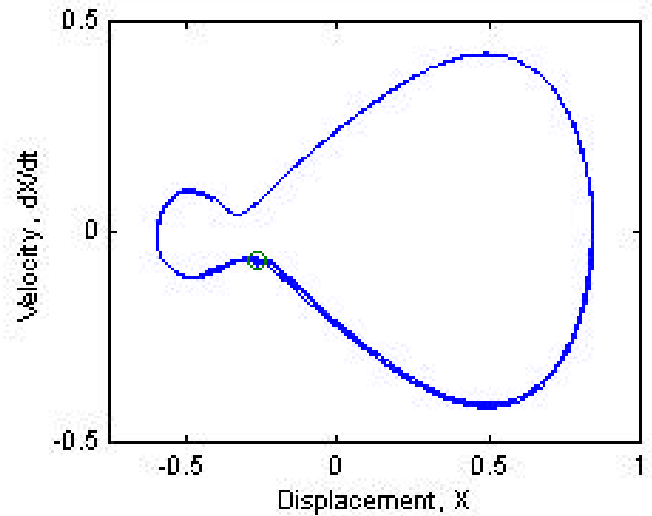
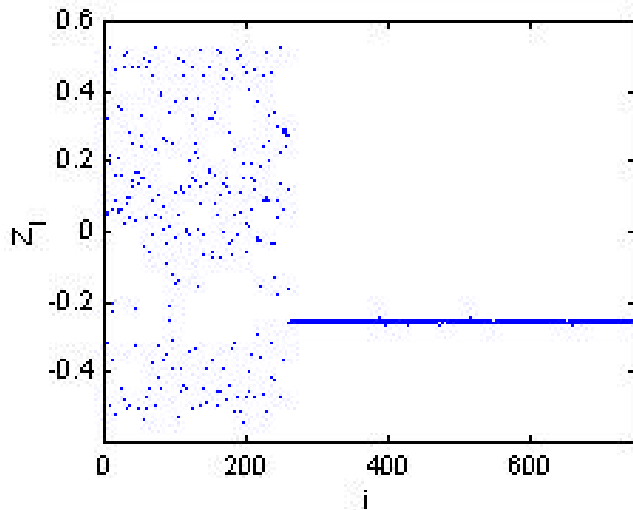
# Control Action



- The controller moves the trajectory towards the stable eigenvector of the unstable first return map



# Controlled Nonlinear Response



- Primary resonance cycle identified at  $x = -0.2623, \dot{x} = -0.0677$
- Choose  $K^T = [\mathbf{8}_u, -\mathbf{8}_u \mathbf{8}_s]$  (OGY)



## Relative Power to Maintain Control

- Define

$$P_{\text{wr}_{\text{Controller}}} = \left| \frac{\int_0^T \mathbf{x}^T \mathbf{u}(t) dt}{T} \right| + \left| \frac{\int_0^T \dot{\mathbf{x}}^2 dt}{T} \right|$$

$$P_{\text{wr}_{\text{System}}} = \frac{1}{T} \int_0^T \mathbf{f}^T \mathbf{F}(t) \frac{d\mathbf{x}}{dt} dt$$

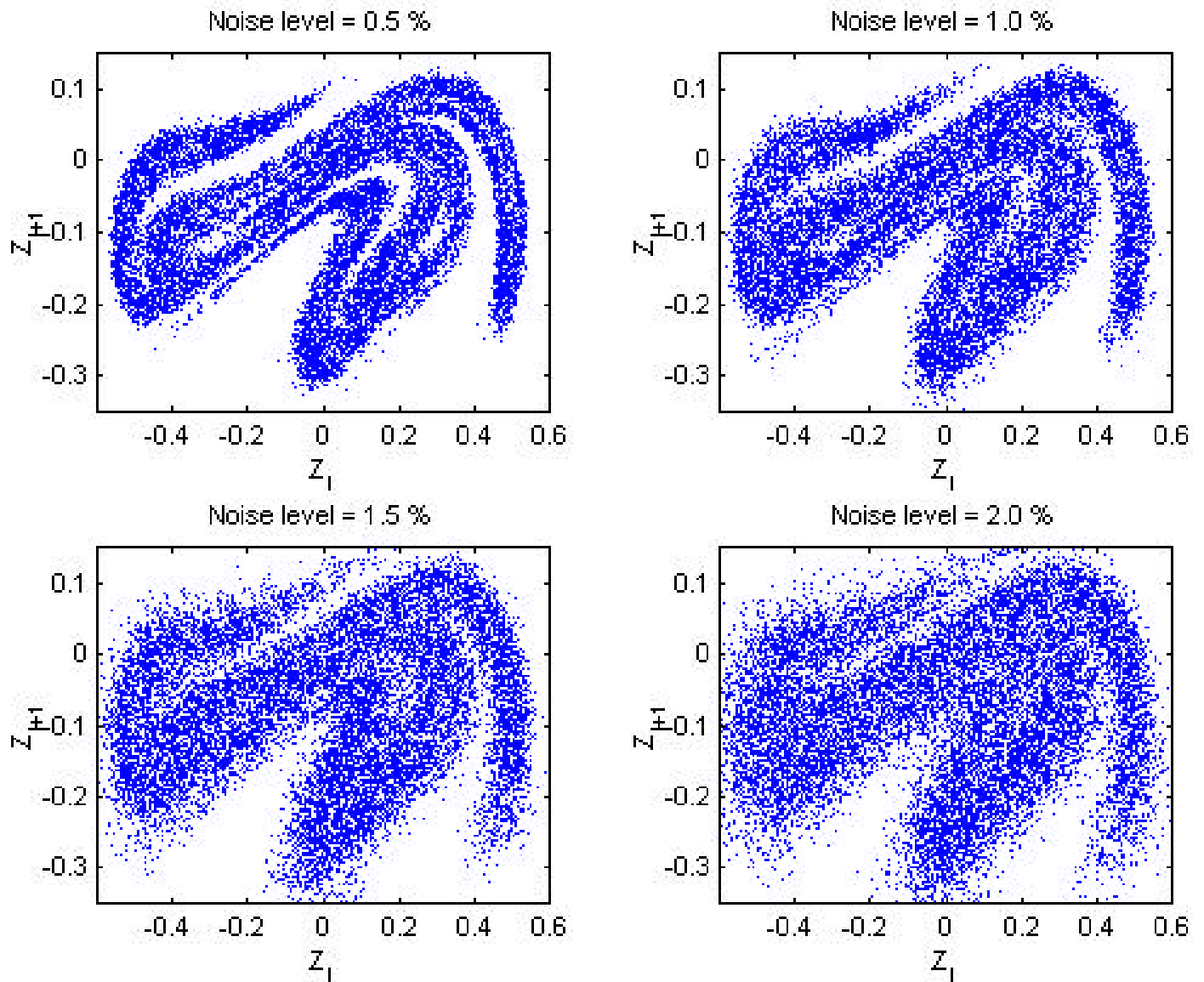
- The relative power required to maintain control of the nonlinear oscillations is expressed as the ratio of the power required to initiate the incremental shift over the input power to the system.

$$\frac{P_{\text{wr}_{\text{Controller}}}}{P_{\text{wr}_{\text{System}}}} 100 = 0.23$$





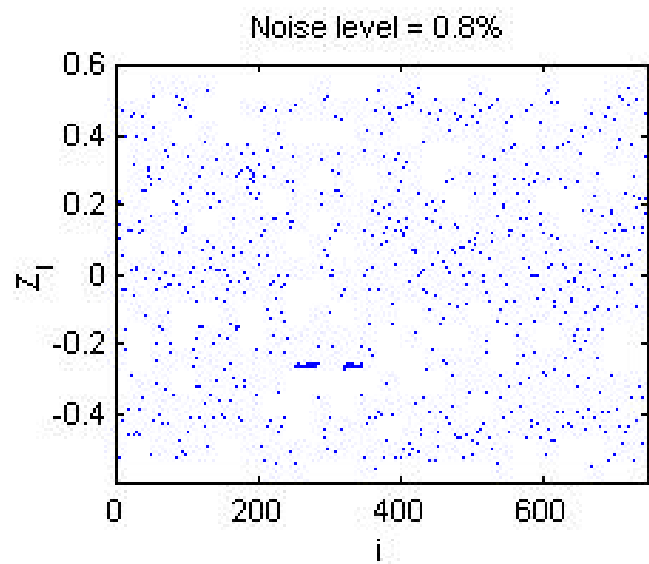
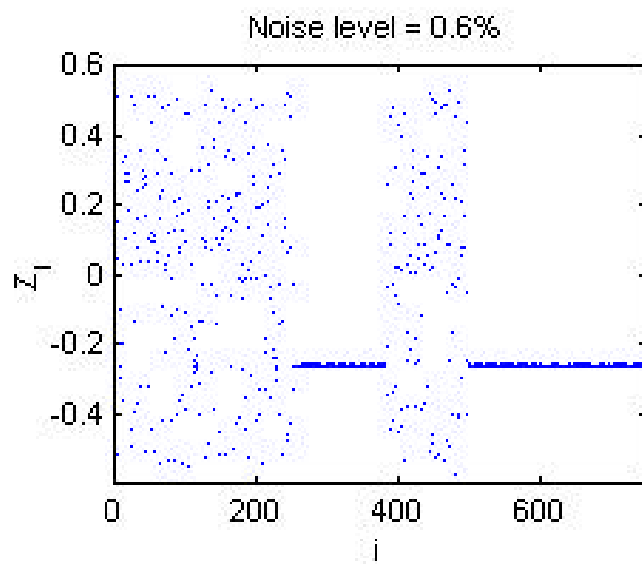
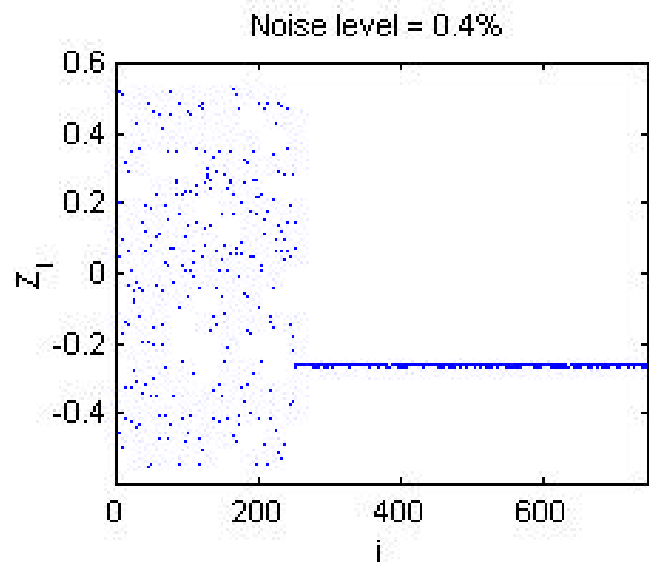
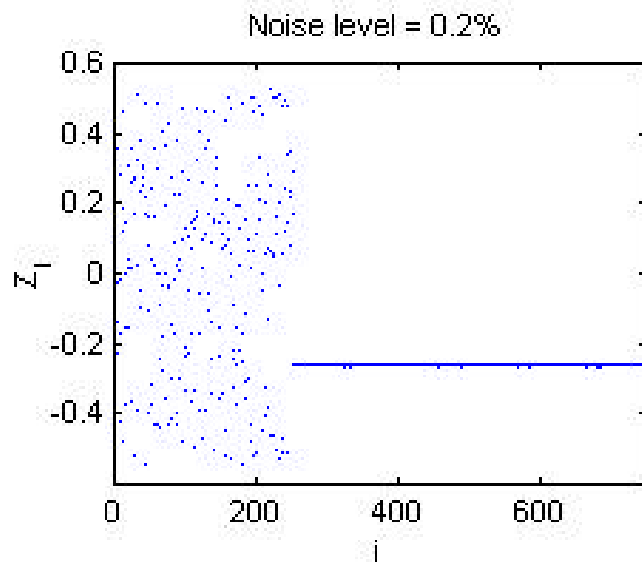
# Effects of Random Noise



- Noise destabilizes the chaotic response



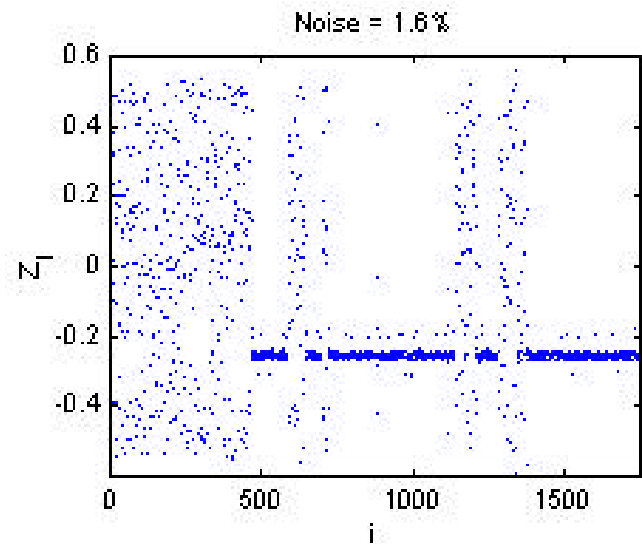
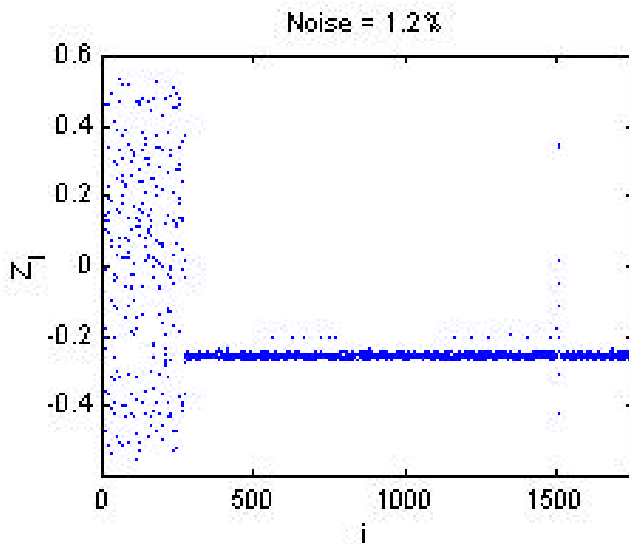
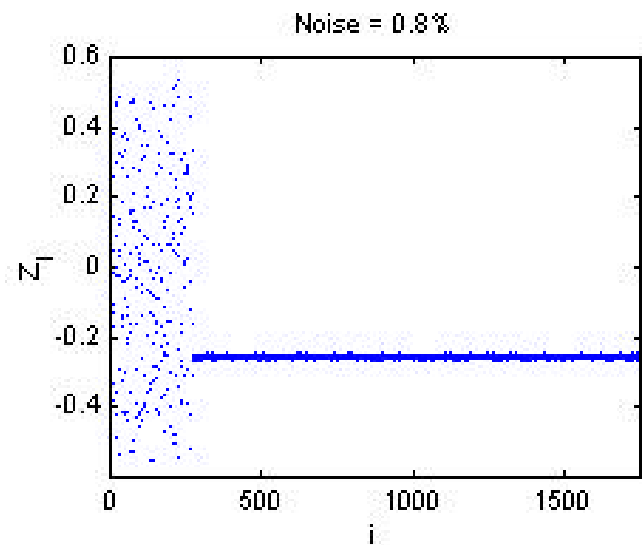
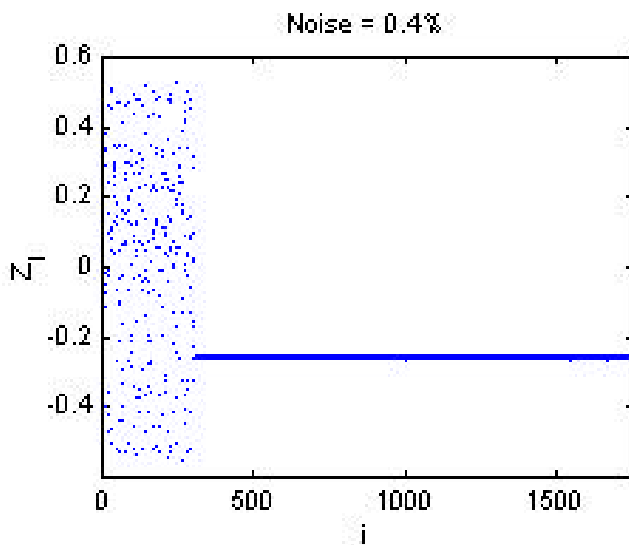
# Noise Destabilizes the Controller



- A loss in controller robustness results



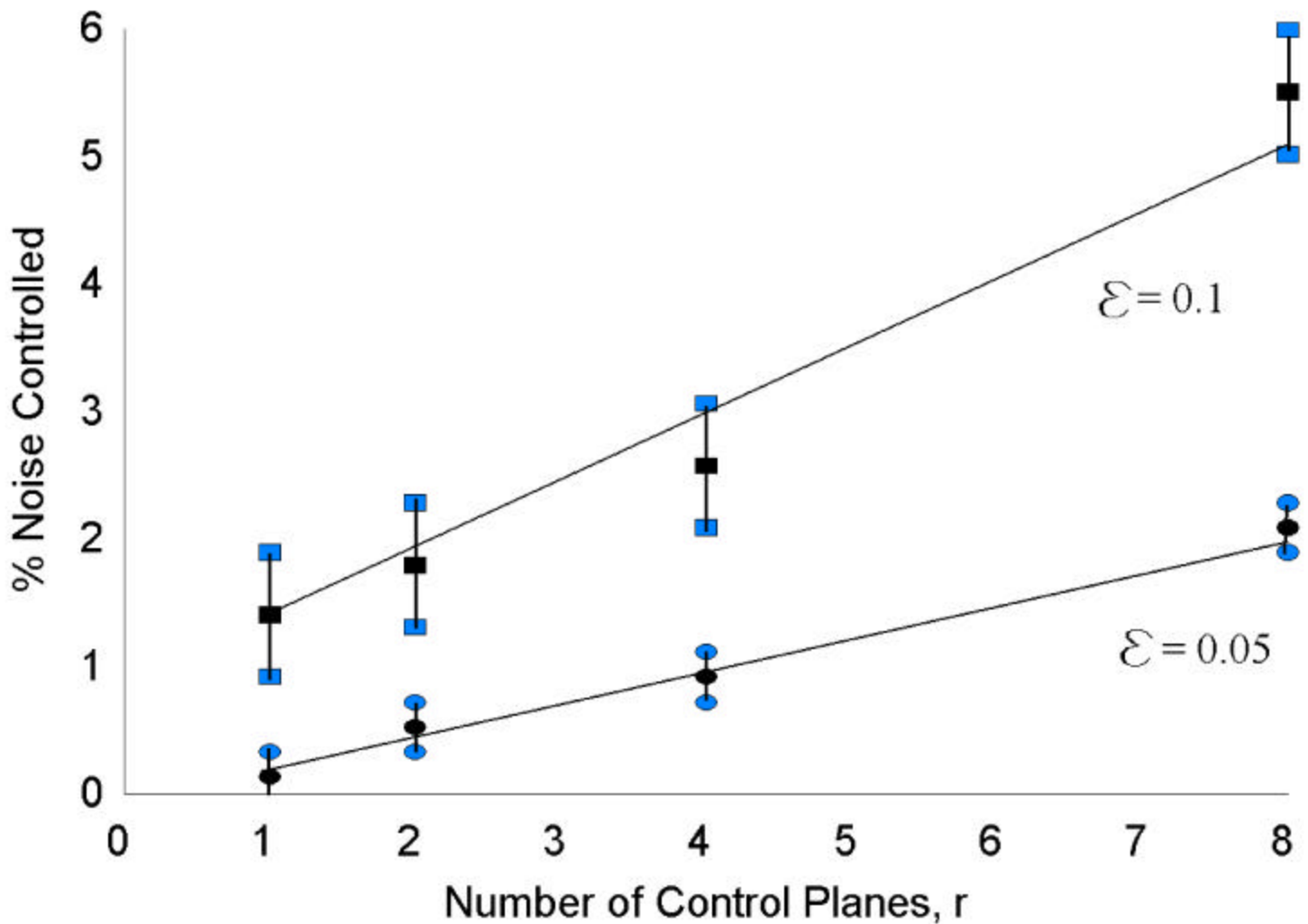
# Multi-plane Control



- Sample time series at  $2\mathbf{B} / r'$  intervals
- Create  $r$  separate control planes and build feedback controller on each plane



# Controllable Noise Level with Multi-Plane Control



- A net increase in robustness to small amounts of additive random noise results by increasing then number of controllers per cycle



# CONCLUDING REMARKS

- Control of chaotic oscillations of moored system
- Low energy perturbations implemented
- Effects of additive noise investigated
- In extreme cases, state estimation may be required

