Problem 1: We have the following pixel values with the corresponding frequency of occurrence. (6pts)

<table>
<thead>
<tr>
<th>Pixel values</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>100</td>
<td>20</td>
<td>300</td>
<td>120</td>
<td>1000</td>
<td>600</td>
<td>900</td>
<td>100</td>
<td>400</td>
<td>123</td>
</tr>
</tbody>
</table>

a) Suppose you are using a codebook of size 3, and the initial codewords are C(0) = 1, C(2) = 2, and C(3) = 3. Show steps by steps the final codeword after running the Loyd-Max Algorithm. What is the distortion value?

b) Suppose you are using a codebook of size 3, and the initial codewords are C(0) = 8, C(2) = 9, and C(3) = 10. Repeat question (a).

c) Do you think the final codewords will always be the same? Explain your answer.

Problem 2: Show that distortion value in the Loyd-Max quantizer monotonically decreases with the number of iterations. (4pts)

Problem 3: In this problem, we will use MatLab to perform vector quantization on image. We will use the image Lena512.pgm (10pts).

1) Download the lena512.pgm from the class website
2) Start Matlab, and go into the directory that contains the downloaded image lena512.pgm.
3) At the Matlab prompt, load the image lena512.pgm using
   “data = imread(‘lena512.pgm’);”
4) Display image lena512 to make sure we load it in properly using
   “imshow(data);” You should see the lena image.
5) Now the data is a 512x512 matrix contains the pixel value of the lena image.
6) Your job is to design a codebook consisting of 16 codewords, each codeword is a 2x2 block, using the GLA algorithm. (How many iterations do you need to run before the relative distortion error does not change much from iteration to iteration?)
7) Use your codebook to reconstruct the lena image.

Note: to access entry (i,j) in the matrix data, use data(i,j).
For example, if you want to add the entry (i,j) to the entry (i+1, j+1), and store the result in the entry (i,j), type “data(i,j) = data(i,j) + data(i+1,j+1);”
For tutorial on MatLab, see the link http://www.math.ufl.edu/help/matlab-tutorial/
Turn in your matlab source code and a hard copy of the quantized lena image.
What is the compression ratio?

**Extra credits:**
Implement k-d tree using the codebook in problem 3. Encode the image Barbara using the k-d tree. Email me the source code.