Planning as Satisfiability

Alan Fern *

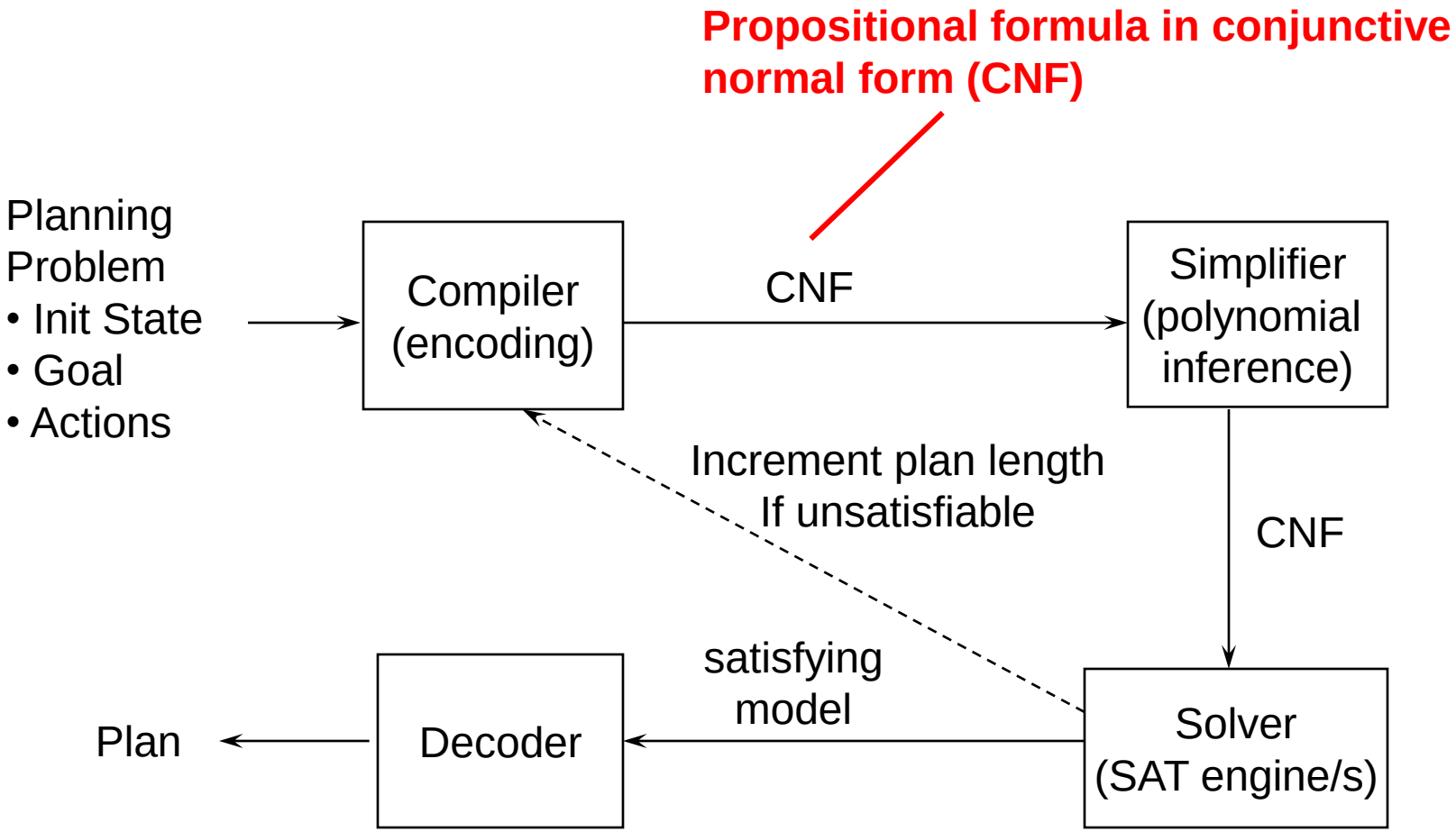
- Review of propositional logic (see chapter 7)
- Planning as propositional satisfiability
- Satisfiability techniques (see chapter 7)
- Combining satisfiability techniques with planning graphs

* Based in part on slides by Stuart Russell and Dana Nau

Planning as Satisfiability

- The “planning as satisfiability” framework lead to large improvements when it was first introduced
  - Especially in the area of optimal planning
- At that time there was much recent progress in satisfiability solvers
  - This work was an attempt to leverage that work by reducing planning to satisfiability
- Planners in this framework are still very competitive in terms of optimal planning

Architecture of a SAT-based Planner



Propositional Logic

- We use logic as a formal representation for knowledge
- Propositional logic is the simplest logic – illustrates basic ideas

Syntax:

- We are given a set of primitive propositions $\{P_1, \ldots, P_n\}$
  - These are the basic statements we can make about the "world"
- From basic propositions we can construct formulas
  - A primitive proposition is a formula
  - If F is a formula, $\neg F$ is a formula (negation)
  - If F_1 and F_2 are formulas, $F_1 \wedge F_2$ is a formula (conjunction)
  - If F_1 and F_2 are formulas, $F_1 \vee F_2$ is a formula (disjunction)
  - If F_1 and F_2 are formula s, $F_1 \Rightarrow F_2$ is a formula (implication)
  - If F_1 and F_2 are formulas, $F_1 \Leftrightarrow F_2$ is a formula (biconditional)

- Really all we need is negation and disjunction or conjunction, but the other connectives are useful shorthand
  - E.g. $F_1 \Rightarrow F_2$ is equivalent to $\neg F_1 \vee F_2$

Propositional Logic: CNF

- A ***literal*** is either a proposition or the negation of a proposition
- A ***clause*** is a disjunction of literals
- A formula is in ***conjunctive normal form (CNF)*** if it is the conjunction of clauses
  - $(\neg R \vee P \vee Q) \wedge (\neg P \vee Q) \wedge (\neg P \vee R)$
- Any formula can be represented in conjunctive normal form (CNF)
  - Though sometimes there may be an exponential blowup in size of CNF encoding compared to original formula
- CNF is used as a canonical representation of formulas in many algorithms

Propositional logic: Semantics

- A **truth assignment** is an assignment of true or false to each proposition: e.g. p1 = false, p2 = true, p3 = false
- A formula is either true or false wrt a truth assignment
- The truth of a formula is evaluated recursively as given below: (P and Q are formulas)

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

The base case is when the formulas are single propositions where the truth is given by the truth assignment.

Propositional Satisfiability

- A formula is ***satisfiable*** if it is true for some truth assignment
  - e.g. $A \vee B$, C
- A formula is unsatisfiable if it is never true for any truth assignment
  - e.g. $A \wedge \neg A$
- Testing satisfiability of CNF formulas is a famous NP-complete problem

Propositional Satisfiability

- Many problems (such as planning) can be naturally encoded as instances of satisfiability
- Thus there has been much work on developing powerful satisfiability solvers
  - these solvers work amazingly well in practice

Complexity of Planning Revisited

- Recall that PlanSAT for propositional STRIPS is PSPACE-complete
  - (Chapman 1987; Bylander 1991; Backstrom 1993, Erol et al. 1994)
- How then it is possible to encode STRIPS planning as SAT, which is only an NP-complete problem?

Bounded PlanSAT

Given: a STRIPS planning problem, and positive integer *n*

Output: "yes" if problem is solvable in *n* steps or less, otherwise "no"

- Bounded PlanSAT is NP-complete
  - (Chenoweth 1991; Gupta and Nau 1992)
- So bounded PlanSAT can be encoded as propositional satisfiability

Encoding Planning as Satisfiability: Basic Idea

- Bounded planning problem (*P*,*n*):
  - *P* is a planning problem; *n* is a positive integer
  - Find a solution for *P* of length *n*
- Create a propositional formula that represents:
  - Initial state
  - Goal
  - Action Dynamics

  for *n* time steps
- We will define the formula for (*P*,*n*) such that:
  1) ***any*** satisfying truth assignment of the formula represent a solution to (*P*,*n*)
  2) if (*P*,*n*) has a solution then the formula is satisfiable

Overall Approach

- Do iterative deepening like we did with Graphplan:
  - for n = 0, 1, 2, …,
    - encode (P,n) as a satisfiability problem Φ
    - if Φ is satisfiable, then
      - From the set of truth values that satisfies Φ, a solution plan can be constructed, so return it and exit
- With a complete satisfiability tester, this approach will produce optimal layered plans for solvable problems
- We can use a GraphPlan analysis to determine an upper bound on n, giving a way to detect unsolvability

Example of Complete Formula for (*P*,1)

$[\ at(r1,l1,0) \wedge \neg at(r1,l2,0)\ ] \wedge$
$at(r1,l2,1) \wedge$

$[\ move(r1,l1,l2,0) \Rightarrow at(r1,l1,0)\ ] \wedge$

$[\ move(r1,l1,l2,0) \Rightarrow at(r1,l2,1)\ ] \wedge$

$[\ move(r1,l1,l2,0) \Rightarrow \neg at(r1,l1,1)\ ] \wedge$

$[\ move(r1,l2,l1,0) \Rightarrow at(r1,l2,0)\ ] \wedge$

$[\ move(r1,l2,l1,0) \Rightarrow at(r1,l1,1)\ ] \wedge$

$[\ move(r1,l2,l1,0) \Rightarrow \neg at(r1,l2,1)\ ] \wedge$
$[\ \neg move(r1,l1,l2,0) \vee \neg move(r1,l2,l1,0)\ ] \wedge$

$[\ \neg at(r1,l1,0) \wedge at(r1,l1,1) \Rightarrow move(r1,l2,l1,0)\ ] \wedge$

$[\ \neg at(r1,l2,0) \wedge at(r1,l2,1) \Rightarrow move(r1,l1,l2,0)\ ] \wedge$

$[\ at(r1,l1,0) \wedge \neg at(r1,l1,1) \Rightarrow move(r1,l1,l2,0)\ ] \wedge$

$[\ at(r1,l2,0) \wedge \neg at(r1,l2,1) \Rightarrow move(r1,l2,l1,0)\ ]$

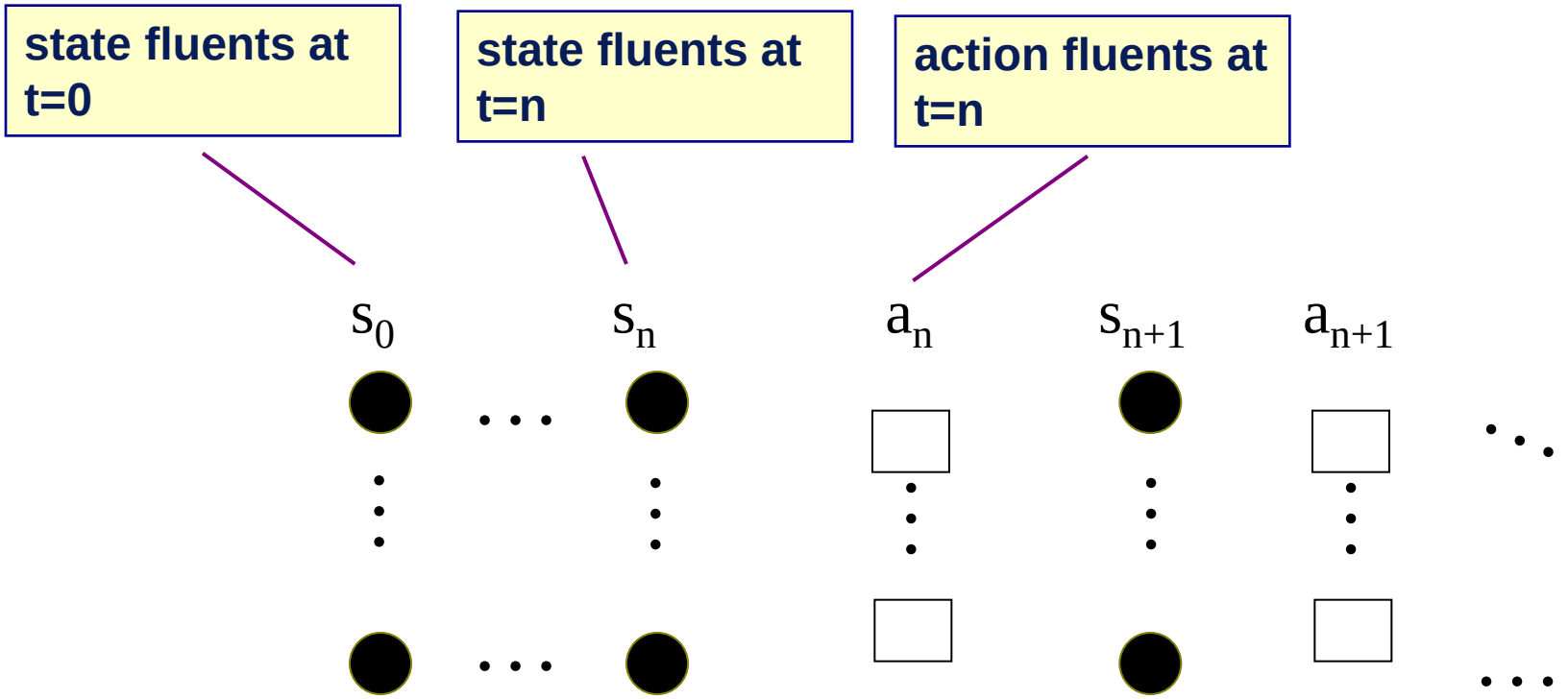
Formula has propositions for actions and states variables at each possible timestep

Lets see how to construct such a formula

Fluents (will be used as propositons)

- If plan $\langle a_0, a_1, \ldots, a_{n-1} \rangle$ is a solution to (P,n), then it generates a sequence of states $\langle s_0, s_1, \ldots, s_{n-1} \rangle$
- A ***fluent*** is a proposition used to describe what's true in each s_i
  - at(r1,loc1,i) is a fluent that's true iff at(r1,loc1) is in s_i
  - We'll use e_i to denote the fluent for a fact e in state s_i
    - e.g. if e = at(r1,loc1)
      then e_i = at(r1,loc1,i)
  - a_i is a fluent saying that a is an action taken at step i
    - e.g., if a = move(r1,loc2,loc1)
      then a_i = move(r1,loc2,loc1,i)
- The set of all possible action and state fluents for (P,n) form the set of primitive propositions used to construct our formula for (P,n)

Pictorial View of Fluents



- A truth assignment selects a subset of these nodes (the ones assigned the value true)
- We want to write down propositional formulas that only allow assignments that correspond to valid plans

Encoding Planning Problems

- We can encode (P,n) so that we consider either layered plans or totally ordered plans
  - an advantage of considering layered plans is that fewer time steps are necessary (i.e. smaller n translates into smaller formulas)
  - for simplicity we first consider totally-ordered plans
- Encode (P,n) as a formula Φ such that

  $\langle a_0, a_1, \ldots, a_{n-1} \rangle$ is a solution for (P,n)

  if and only if

  Φ can be satisfied in a way that makes the fluents $a_0, \ldots, a_{n-1}$ true
- Φ will be conjunction of many formulas
  - We will now consider the types of formulas in

Formulas in Φ

- Formula describing the **initial state**: (let *E* be the set of possible facts in the planning problem)

  $$\bigwedge\{e_0 \mid e \in s_0\} \wedge \bigwedge\{\neg e_0 \mid e \in E - s_0\}$$

  Describes the complete initial state (both positive and negative fact)

  - E.g. $on(A,B,0) \wedge \neg on(B,A,0)$

- Formula describing the **goal**: (*G* is set of goal facts)

  $$\bigwedge\{e_n \mid e \in G\}$$

  says that the goal facts must be true in the final state at timestep *n*

  - E.g. $on(B,A,n)$

- Is this enough?
  - Of course not. The formulas say nothing about actions.

Formulas in Φ

- For every action *a* and timestep *i,* formula describing what fluents must be true if *a* were the *i*'th step of the plan:
  - $a_i \Rightarrow \bigwedge \{e_i \mid e \in \text{Precond}(a)\}$, *a*'s preconditions must be true
  - $a_i \Rightarrow \bigwedge \{e_{i+1} \mid e \in \text{ADD}(a)\}$, a's ADD effects must be true in i+1
  - $a_i \Rightarrow \bigwedge \{\neg e_{i+1} \mid e \in \text{DEL}(a)\}$, a's DEL effects must be false in i+1
- *Complete exclusion* axiom:
  - For all actions *a* and *b* and timesteps *i*, formulas saying *a* and *b* can't occur at the same time

    $$\neg a_i \vee \neg b_i$$
  - this guarantees there can be only one action at a time
- Is this enough?
  - The formulas say nothing about what happens to facts if they are not effected by an action
  - This is known as the ***frame problem***

Frame Axioms

- *Frame axioms*:
  - Formulas describing what *doesn't* change between steps *i* and *i*+1
- Several are many ways to write these
  - Here I show a way that is good in practice
- **explanatory frame axioms**
  - One axiom for every possible fact *e* *at every* timestep *i*
  - Says that if *e* changes truth value between s_i and s_{i+1}, then the action at step *i* must be responsible:

$$\neg e_i \wedge e_{i+1} \Rightarrow \mathrm{V}\{a_i \mid \mathrm{e}\ \textit{in}\ \mathrm{ADD}(a)\}$$

If *e* became true then some action must have added it

$$e_i \wedge \neg e_{i+1} \Rightarrow \mathrm{V}\{a_i \mid \mathrm{e}\ \textit{in}\ \mathrm{DEL}(a)\}$$

If *e* became false then some action must have deleted it

Example

- Planning domain:
  - one robot r1
  - two adjacent locations l1, l2
  - one operator (move the robot)
- Encode (*P*,*n*) where *n* = 1
  - Initial state: {at(r1,l1)}
    Encoding: at(r1,l1,0) $\wedge$ $\neg$at(r1,l2,0)
  - Goal: {at(r1,l2)}
    Encoding: at(r1,l2,1)
  - Action Schema: see next slide

Example (continued)

- Schema: move(r, l, l’)

  PRE: at(r,l)

  ADD: at(r,l’)

  DEL: at(r,l)

Encoding: (for actions move(r1,l1,l2) and move(r1,l2,l1) at time step 0)

$move(r1,l1,l2,0) \Rightarrow at(r1,l1,0)$

$move(r1,l1,l2,0) \Rightarrow at(r1,l2,1)$

$move(r1,l1,l2,0) \Rightarrow \neg at(r1,l1,1)$

$move(r1,l2,l1,0) \Rightarrow at(r1,l2,0)$

$move(r1,l2,l1,0) \Rightarrow at(r1,l1,1)$

$move(r1,l2,l1,0) \Rightarrow \neg at(r1,l2,1)$

Example (continued)

- Schema: move(r, l, l')
  PRE: at(r,l)
  ADD: at(r,l')
  DEL: at(r,l)

- Complete-exclusion axiom:
  $\neg$move(r1,l1,l2,0) $\vee$ $\neg$move(r1,l2,l1,0)

- Explanatory frame axioms:
  $\neg$at(r1,l1,0) $\wedge$ at(r1,l1,1) $\Rightarrow$ move(r1,l2,l1,0)
  $\neg$at(r1,l2,0) $\wedge$ at(r1,l2,1) $\Rightarrow$ move(r1,l1,l2,0)
  at(r1,l1,0) $\wedge$ $\neg$at(r1,l1,1) $\Rightarrow$ move(r1,l1,l2,0)
  at(r1,l2,0) $\wedge$ $\neg$at(r1,l2,1) $\Rightarrow$ move(r1,l2,l1,0)

Complete Formula for (*P*,1)

$$
\begin{aligned}
&[\ at(r1,l1,0) \wedge \neg at(r1,l2,0)\ ] \wedge \\
&at(r1,l2,1) \wedge \\
&[\ move(r1,l1,l2,0) \Rightarrow at(r1,l1,0)\ ] \wedge \\
&[\ move(r1,l1,l2,0) \Rightarrow at(r1,l2,1)\ ] \wedge \\
&[\ move(r1,l1,l2,0) \Rightarrow \neg at(r1,l1,1)\ ] \wedge \\
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&[\ \neg at(r1,l1,0) \wedge at(r1,l1,1) \Rightarrow move(r1,l2,l1,0)\ ] \wedge \\
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&[\ at(r1,l2,0) \wedge \neg at(r1,l2,1) \Rightarrow move(r1,l2,l1,0)\ ]
\end{aligned}
$$

Convert to CNF and give to SAT solver.

Extracting a Plan

- Suppose we find an assignment of truth values that satisfies Φ.
  - This means *P* has a solution of length *n*
- For *i*=0,…,*n-1*, there will be exactly one action *a* such that a_i = *true*
  - This is the *i*'th action of the plan.
- Example (from the previous slides):
  - Φ can be satisfied with move(r1,l1,l2,0) = *true*
  - Thus ⟨move(r1,l1,l2,0)⟩ is a solution for (*P*,0)
    - It's the only solution - no other way to satisfy Φ

Supporting Layered Plans

- *Complete exclusion* axiom:
  - For **all** actions *a* and *b* and time steps *i* include the formula $\neg a_i \vee \neg b_i$
  - this guaranteed that there could be only one action at a time
- *Partial exclusion axiom*:
  - For any pair of incompatible actions (recall from Graphplan) *a* and *b* and each time step *i* include the formula $\neg a_i \vee \neg b_i$
  - This encoding will allowed for more than one action to be taken at a time step resulting in layered plans
  - This is advantageous because fewer time steps are required (i.e. shorter formulas)

SAT Algorithms

- Systematic Search (Optional Material)
  - DPLL (Davis Putnam Logemann Loveland) backtrack search + unit propagation
  - I won't cover in class
- Local Search
  - Walksat (Selman, Kautz & Cohen) greedy local search + noise to escape minima

DPLL [Davis, Putnam, Loveland & Logemann 1962]

- How to find an assignment of truth values that satisfies Φ?
  - Use a satisfiability algorithm
- DPLL is a complete satisfiability solver
  - First need to put Φ into conjunctive normal form

    e.g., $\Phi = D \wedge (\neg D \vee A \vee \neg B) \wedge (\neg D \vee \neg A \vee \neg B) \wedge (\neg D \vee \neg A \vee B) \wedge A$
  - Write Φ as a set of *clauses*: (where each clause is a set of literals)

    $\Phi = \{(D),\ (\neg D, A, \neg B),\ (\neg D, \neg A, \neg B),\ (\neg D, \neg A, B),\ (A)\}$
  - Two special cases:
    - $\Phi = \{\}$ (i.e. no clauses) is a formula that's always *true*
    - $\Phi = \{\ldots, (), \ldots\}$ (i.e. an empty clause) is a formula that's always *false*
  - DPLL simply searches the space of truth assignments, assigning one proposition a value at each step of the search tree

Basic Observations

If literal L_1 is true, then clause $(L_1 \vee L_2 \vee ...)$ is true

If clause C_1 is true, then $C_1 \wedge C_2 \wedge C_3 \wedge ...$ has the same value as $C_2 \wedge C_3 \wedge ...$

Therefore: Okay to delete clauses containing true literals!

If literal L_1 is false, then clause $(L_1 \vee L_2 \vee L_3 \vee ...)$ has the same value as $(L_2 \vee L_3 \vee ...)$

Therefore: Okay to delete shorten containing false literals!

If literal L_1 is false, then clause (L_1) is false

Therefore: the empty clause means false!

DPLL

Davis – Putnam – Loveland – Logemann

```
dpll(F, literal){
   remove clauses containing literal
   shorten clauses containing ¬literal
   if (F contains no clauses) return true;
   if (F contains empty clause)
       return false;
   ; if F has a clause with single literal then force it
   ; to be true since it must be the case
   if (F contains a unit or pure L)
       return dpll(F, L);
   choose V in F;
   if (dpll(F, ¬V))return true;
   return dpll(F, V);
}
```

Can initialize by calling dpll(F,bogus) where bogus is a literal that is not in F

WalkSat

- Local search over space of ***complete*** truth assignments

- Start with random truth assignment
- Repeat until stopping condition
  - With probability P: flip any variable in any unsatisfied clause
  - With probability (1-P): flip best variable in any unsat clause
    - The best variable is the one that when flipped causes the most clauses to be satisfied
  - P, the “temperature” controls the randomness of search

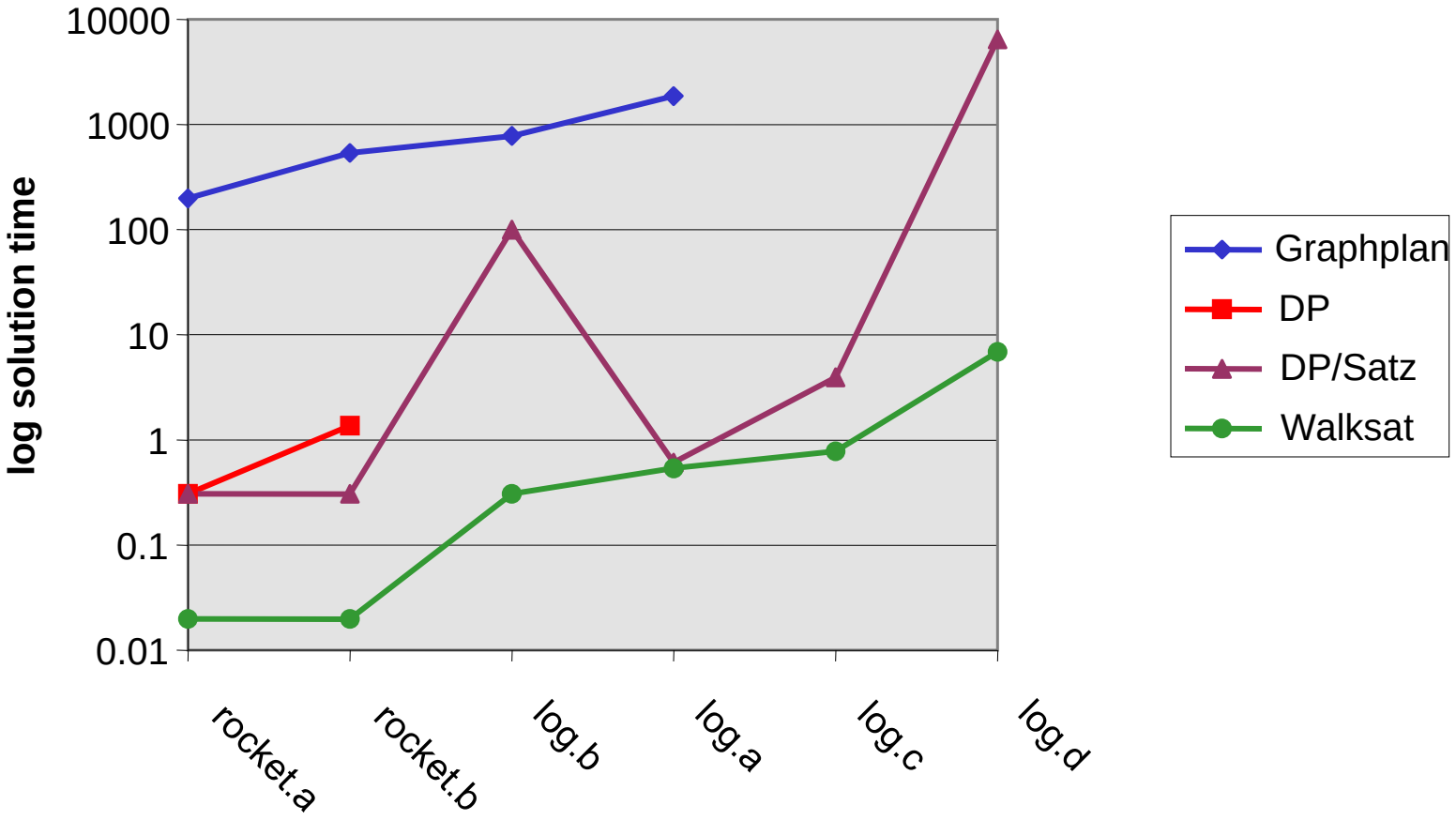
- Randomness can help avoid local minima
- If it finds a solution it typically finds it very quickly
- Not complete: does not test for unsatisfiability

Planning Benchmark Test Set

- Extension of Graphplan test set
- blocks world - up to 18 blocks, 10^{19} states
- logistics - complex, highly-parallel transportation domain.

  Logistics.d:
  - 2,165 possible actions per time slot
  - 10^{16} legal configurations (2^{2000} states)
  - optimal solution contains 74 distinct actions over 14 time slots
- *Problems of this size never previously handled by general-purpose planning systems*

Scaling Up Logistics Planning



What SATPLAN Shows

- General propositional reasoning can compete with state of the art specialized planning systems
  - New, highly tuned variations of DPLL surprising powerful
  - Radically new stochastic approaches to SAT can provide very low exponential scaling
- Why does it work?
  - More flexible than forward or backward chaining
  - Randomized algorithms less likely to get trapped along bad paths

Discussion

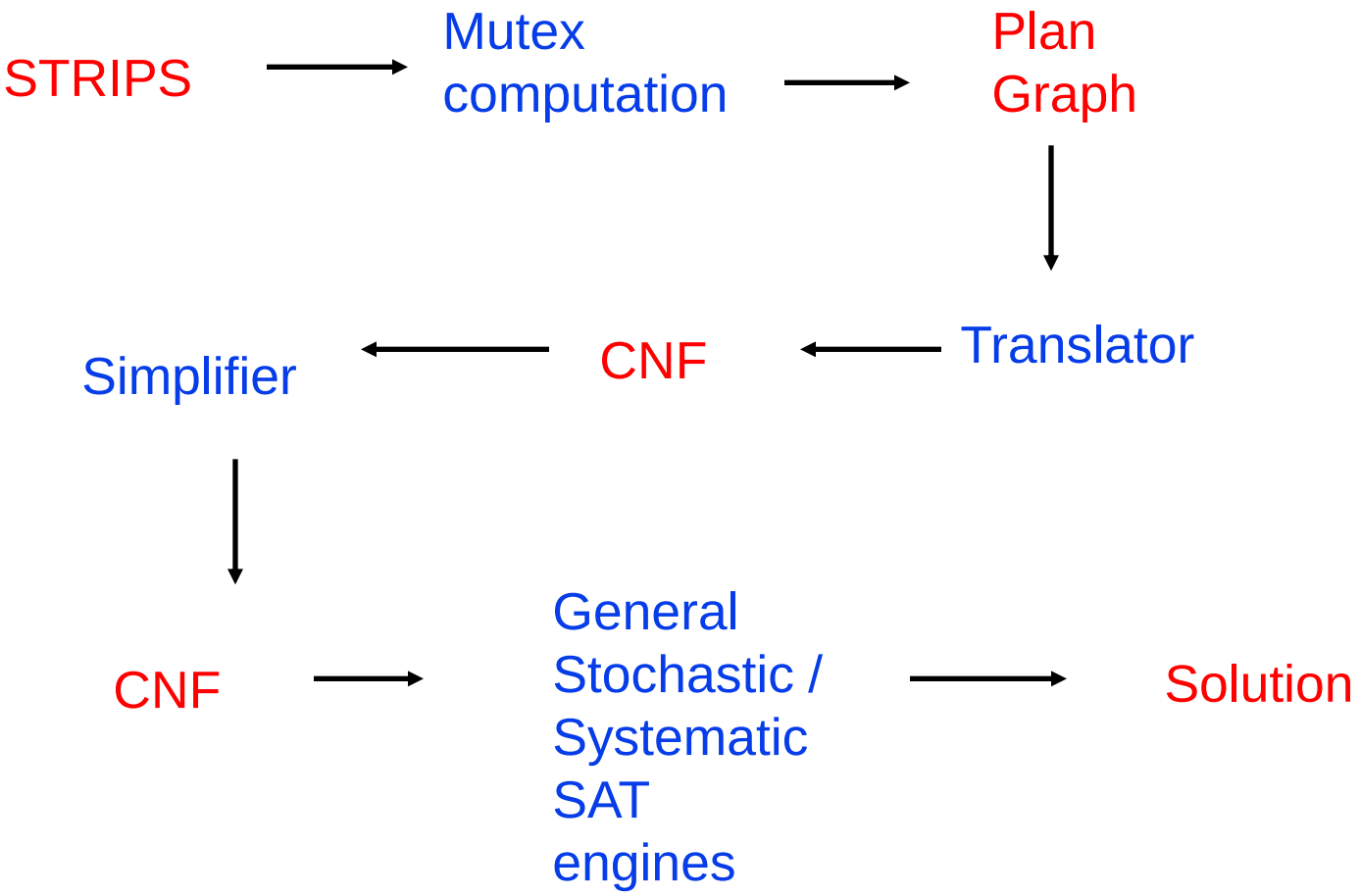
- How well does this work?
  - Created an initial splash but performance can depend strongly on the particular SAT encoding used
- The basic SAT encoding we described does not involve any preprocessing or reasoning about the problem
  - GraphPlan's power is primarily due to such preprocessing (i.e. the graph construction)
- **Idea:** Combine SAT with some GraphPlan-like pre-processing to get more effective encodings

BlackBox (GraphPlan + SatPlan)

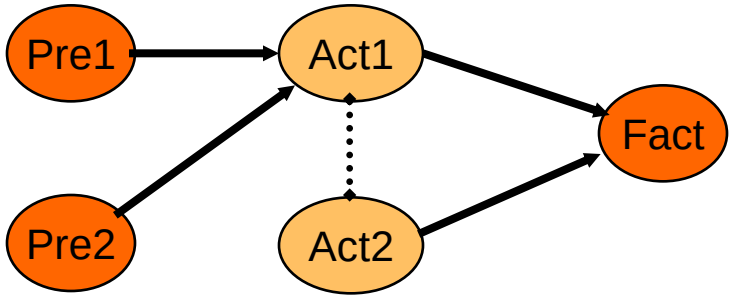
- The BlackBox procedure combines planning-graph expansion and satisfiability checking
  - It is roughly as follows:
- for n = 0, 1, 2, …
  - *Graph expansion:*
    - create a “planning graph” that contains n “levels”
  - Check whether the planning graph satisfies a necessary (but insufficient) condition for plan existence
  - If it does, then
    - Encode (P,n) as a satisfiability problem Φ by translating the planning graph to SAT
    - If Φ is satisfiable then return the solution

Blackbox

Can be thought of as an implementation of GraphPlan that uses an alternative plan extraction technique than the backward chaining of GraphPlan.



Translation of Plan Graph



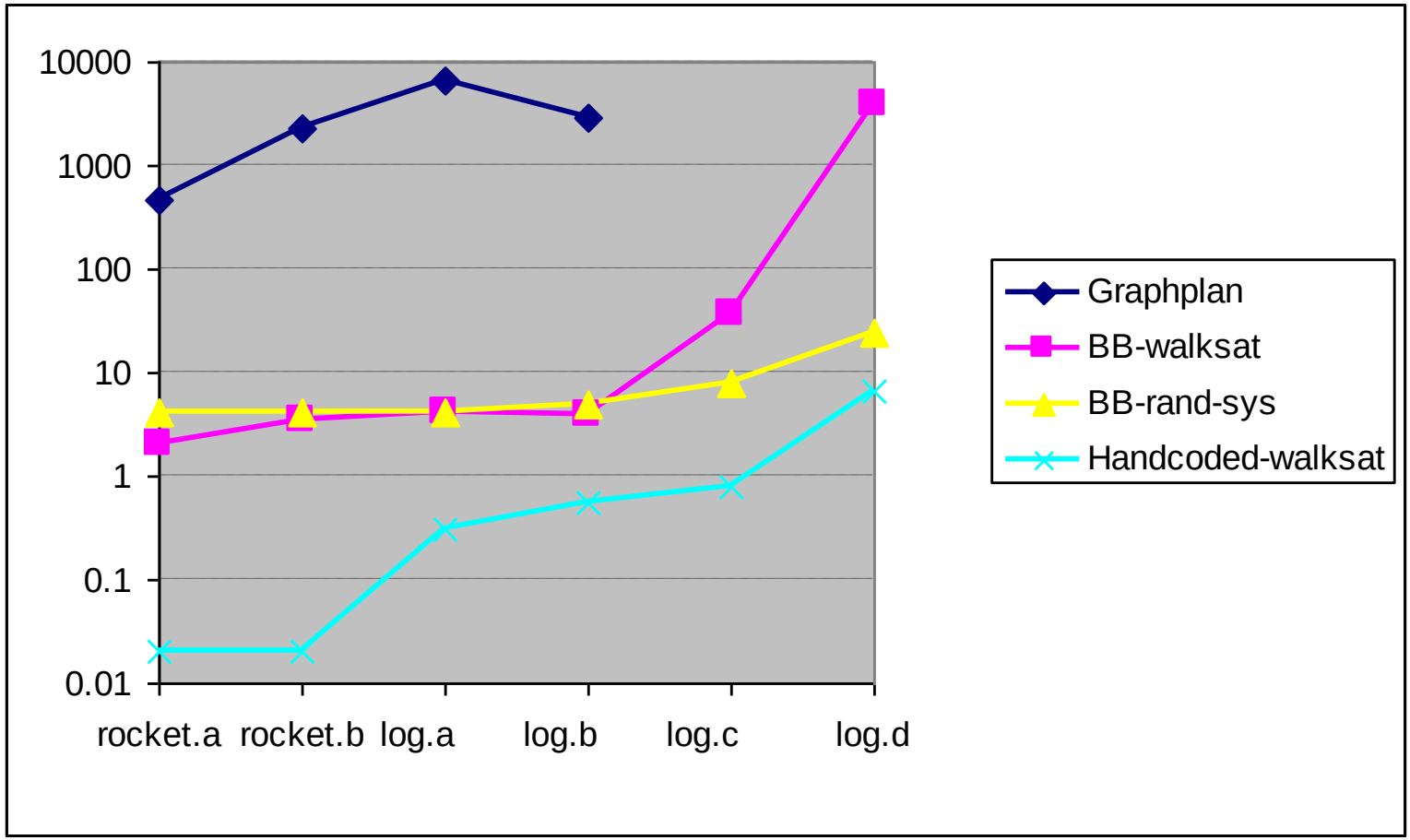
$$Fact \Rightarrow Act1 \vee Act2$$

$$Act1 \Rightarrow Pre1 \wedge Pre2$$

$$\neg Act1 \vee \neg Act2$$

- Can create such constraints for every node in the planning graph.
- Only involves facts and actions in the graph.

Blackbox Results



Applicability

- When is the BlackBox approach *not* a good idea?
  - when domain too large for propositional planning approaches
  - when long *sequential* plans are needed
  - when solution time dominated by reachability analysis (plan-graph generation), not extraction

- Can download a copy of it