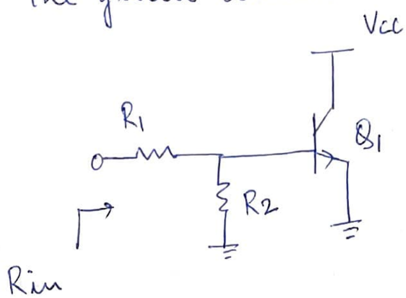
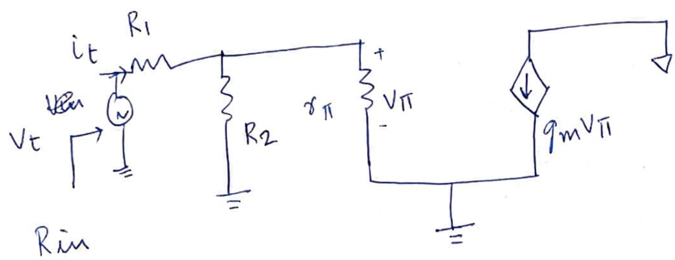
ECE 322 Fall 2021 Exam 2 Practice Question Solutions

Q1) The given circuit:



The small signal model will be given by,



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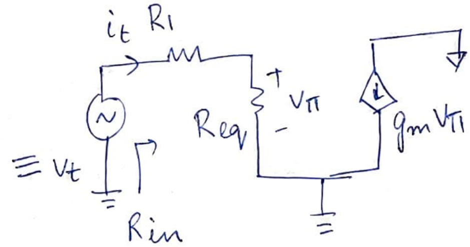
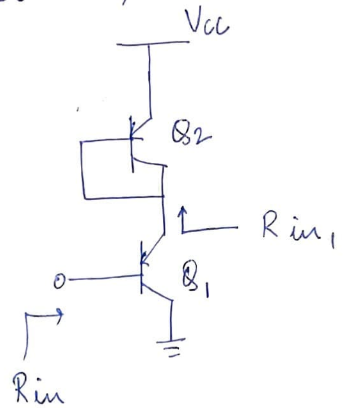


$$R_{in} = \frac{V_t}{i_t} = R_1 + R_{eq}$$

where $R_{eq} = (r_\pi \parallel R_2)$

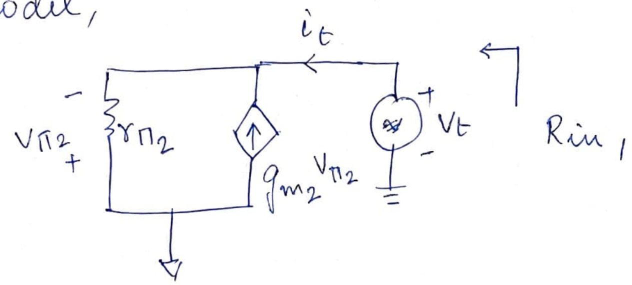
$$\Rightarrow \boxed{R_{in} = R_1 + (R_2 \parallel r_\pi)}$$

Q2) The given circuit,



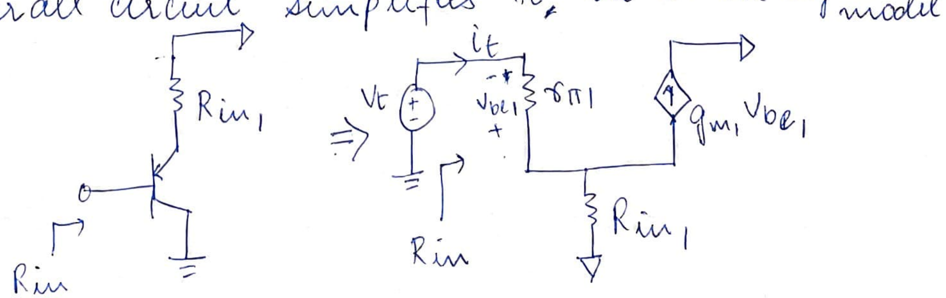
Looking up from the emitter of Q_1, we have an equivalent impedance $R_{in_1} = \frac{1}{g_{m2}} \parallel r_{\pi_2}$

To calculate R_{in_1}, we can use the following small signal model,

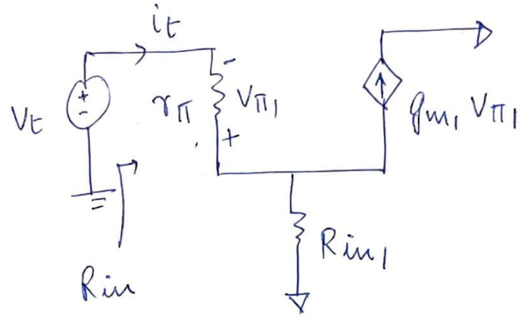


$$R_{in_1} = \frac{V_t}{i_t} = \frac{r_{\pi_2}}{1 + g_{m_2} r_{\pi_2}} = \left(\frac{1}{g_{m_2}} \parallel r_{\pi_2}\right)$$

Hence the overall circuit simplifies to, the small signal model



Q2) The simplified small signal model,



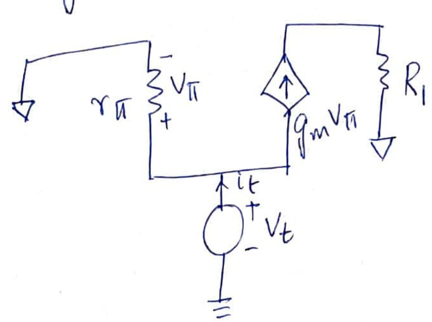
Thus, $\frac{v_t}{i_t} = R_{in}$, $v_{\pi_1} = -r_{\pi_1} i_t$

$$v_t = r_{\pi_1} i_t + i_{R_{in_1}} R_{in_1}$$
$$= r_{\pi_1} i_t + (i_t + g_{m_1} i_t r_{\pi_1}) R_{in_1}$$
$$= i_t \{ r_{\pi_1} + (1+\beta_1) \cancel{i_t} R_{in_1} \}$$

$$\Rightarrow R_{in} = \frac{v_t}{i_t} = r_{\pi_1} + (1+\beta_1) R_{in_1}$$

$$\Rightarrow \boxed{R_{in} = \frac{v_t}{i_t} = r_{\pi_1} + (1+\beta_1)\left(\frac{1}{g_{m_2}} \parallel r_{\pi_2}\right)}$$

Q3) The small signal model to calculate R_{in},



$$R_{in} = \frac{V_t}{i_t}$$

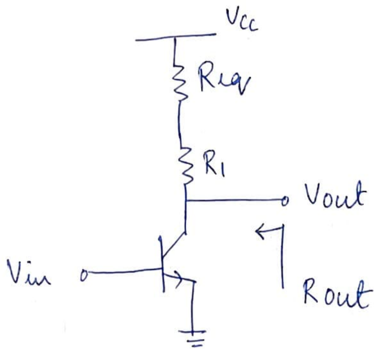
Using KCL,

$$i_t = g_m V_\pi + \frac{V_\pi}{r_\pi}$$

$$V_\pi = V_t$$

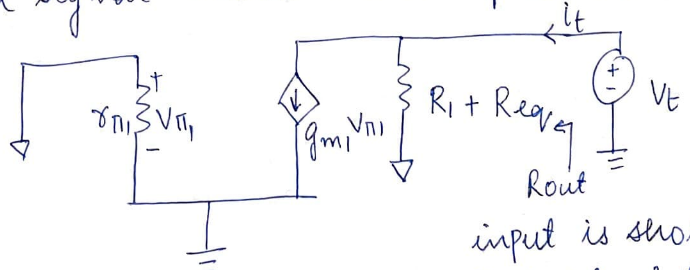
$$\Rightarrow \boxed{R_{in} = \frac{V_t}{i_t} = \frac{V_\pi}{i_t} = \frac{1}{g_m + \frac{1}{r_\pi}} = \frac{1}{g_m} \parallel r_\pi}$$

Q4) The given circuit can be simplified to,



where $R_{eq} = \left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$ (refer to Q2 solution)

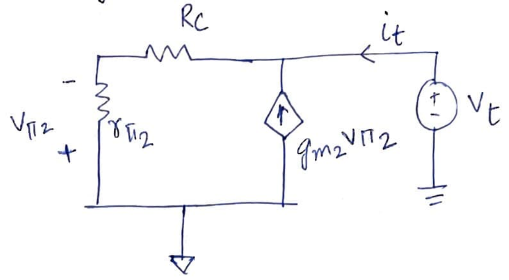
Hence the small signal model can be represented as,



input is shorted for Rout calculation

Hence $$\boxed{R_{out} = \frac{V_t}{i_t} = R_1 + R_{eq} = R_1 + \left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

Q5) The small signal model to calculate Rout,



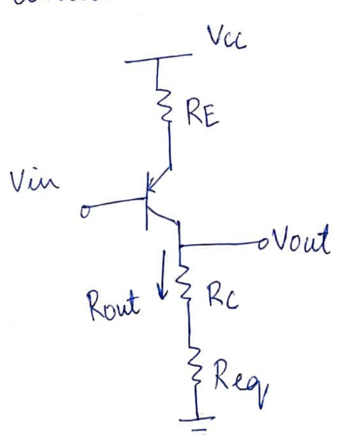
$$i_t = -g_{m2}V_{\pi 2} - \frac{V_{\pi 2}}{r_{\pi 2}}$$

$$V_t = -V_{\pi 2}$$

$$\Rightarrow \quad R_{out} = \frac{V_t}{i_t} = \frac{-V_{\pi 2}}{i_t} = \frac{1}{\left(g_{m2} + \frac{1}{r_{\pi 2}}\right)}$$

$$\Rightarrow \quad \boxed{R_{out} = \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

Q6) The circuit can be simplified to,



where $R_{eq} = \left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$

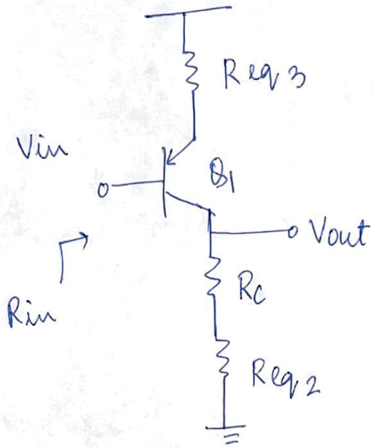
as solved in Q2.

Hence, output resistance

$$R_{out} = R_C + R_{eq}$$

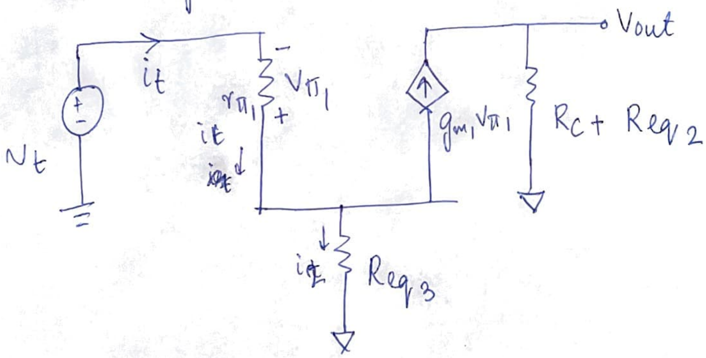
$$\boxed{R_{out} = \left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right) + R_C}$$

87) The given circuit can be simplified to,



where $Req_2 = \left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$, $Req_3 = \left(\frac{1}{g_{m3}} \parallel r_{\pi 3}\right)$

The small signal model to calculate Rin,



$i_t =$

$$v_t = -v_\pi + i_2 \, Req_3 \quad \cdots (1)$$

$$v_\pi = -r_{\pi_1} i_t$$

$$i_2 = i_t - g_{m1} v_{\pi_1} = i_t (1 + g_{m1} r_{\pi_1})$$

$$\Rightarrow i_2 = i_t (1 + \beta_1) \quad \cdots (2)$$

From eq (1) and eq (2), $v_t = -v_\pi + i_t(1+\beta_1) Req_3$

$$v_t = i_t \left[r_{\pi_1} + (1+\beta_1) Req_3 \right]$$

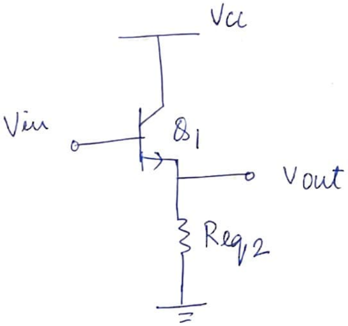
Q7)

$$\therefore\ v_t = i_t\left[r_{\pi 1} + (1+\beta_1)\, R_{eq,3}\right]$$

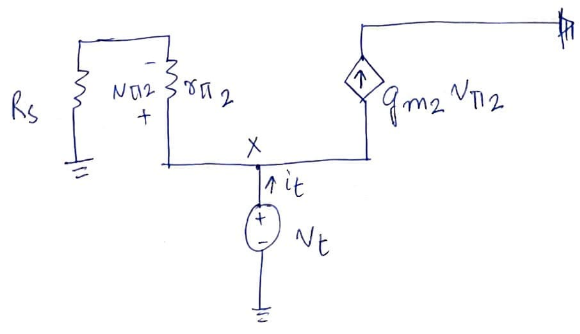
$$\Rightarrow R_{in} = \frac{v_t}{i_t} = r_{\pi 1} + (1+\beta_1)\, R_{eq,3}$$

$$\Rightarrow \boxed{R_{in} = r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m3}} \parallel r_{\pi 3}\right)}$$

Q8) The given circuit can be simplified to,



R_{eq2} can be calculated using the small signal model of Q_2.



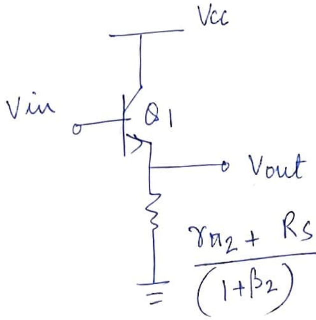
$$i_t - g_{m2}\, v_{\pi 2} = \frac{v_t}{r_{\pi 2} + R_s} \quad \cdots\cdots (1)\ \left[\text{KCL at node } X\right]$$

$$v_{\pi 2} = \frac{+v_t}{r_{\pi 2} + R_s}\, r_{\pi 2} \quad \cdots\cdots (2)$$

From eq (1) and eq (2)

$$R_{eq\,2} = \frac{v_t}{i_t} = \frac{r_{\pi 2} + R_S}{(1+\beta_2)} \quad , \quad \beta_2 = g_{m2}\, r_{\pi 2}$$

Thus the overall circuit reduces to,



$$\frac{r_{\pi 2} + R_S}{(1+\beta_2)} = R_{E_1}$$

The small signal model to calculate Rout can hence be constructed for the Emitter follower configuration above. The Rout derivation can be observed in the lecture notes,

$$R_{out} = \frac{r_{\pi 1}}{\beta_1 + 1} \parallel R_{E_1}$$

$$\Rightarrow \quad R_{out} = \left(\frac{r_{\pi 1}}{1 + g_{m1} r_{\pi 1}}\right) \parallel \left(\frac{r_{\pi 2} + R_S}{1+\beta_2}\right)$$

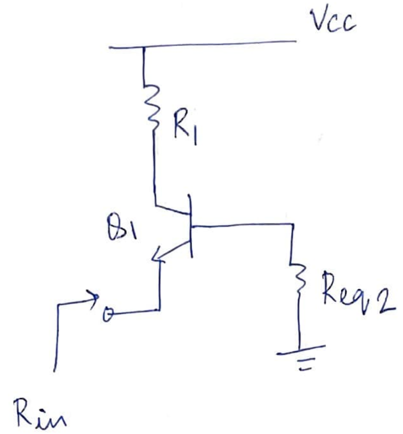
$$\Rightarrow \quad \boxed{R_{out} = \frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel \left(\frac{r_{\pi 2} + R_S}{1+\beta_2}\right)}$$

Referring to the lecture notes,

$$R_{in} = r_{\pi_1} + R_{E_1}(\beta_1 + 1)$$

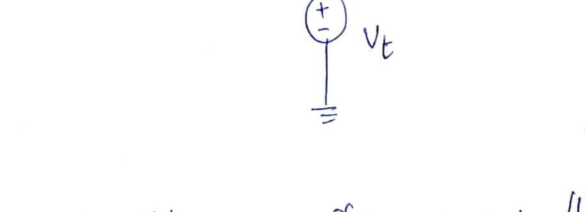
$$\Rightarrow \boxed{R_{in_1} = r_{\pi_1} + \left(\frac{r_{\pi 2} + R_S}{1+\beta_2}\right)(1+\beta_1)}$$

Q9) The simplified circuit,



where $R_{eq2} = \left(r_{\pi 2} \parallel \frac{1}{g_{m2}} \right)$ as derived previously.

The small signal model to calculate R_{in},



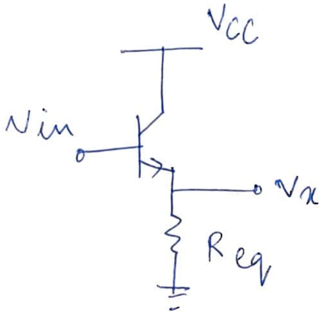
$$v_{\pi 1} = - \frac{v_t}{r_{\pi 1} + R_{eq2}} r_{\pi 1} \quad \ldots \ (1)$$

$$i_t + g_{m1} v_{\pi 1} + \frac{v_{\pi 1}}{r_{\pi 1}} = 0 \quad \ldots \ (2)$$

$$\Rightarrow \frac{v_t}{i_t} = \frac{r_{\pi 1} + R_{eq2}}{\beta_1 + 1}$$

$$\Rightarrow \boxed{R_{in} = \left\{ r_{\pi 1} + \left(r_{\pi 2} \parallel 1/g_{m2} \right) \right\} / (1 + \beta_1)}$$

Q10) The circuit can be divided into two sections since it is a cascade of two different configurations,



where $R_{eq} = R_E \parallel r_{\pi 2}$

R_{eq} can be calculated from the small signal model of Q_2, looking at the impedance into the node at the base of Q_2.

Thus, $$\frac{V_x}{V_{in}} = \frac{R_{eq} \parallel r_{o1}}{\frac{1}{g_{m1}} + R_{eq} \parallel r_{o1}} = \frac{(R_E \parallel r_{\pi 2}) \parallel r_{o1}}{\frac{1}{g_{m1}} + (R_E \parallel r_{\pi 2} \parallel r_{o1})}$$

This is the generalized solution.

For $V_A = \infty$, $r_{o1} \to \infty$.

$$\frac{V_x}{V_{in}} = \frac{(R_E \parallel r_{\pi 2})}{(R_E \parallel r_{\pi 2}) + \frac{1}{g_{m1}}}$$

For the CE configuration, $\frac{V_{out}}{V_x} = -g_{m2} R_C$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_x} \frac{V_x}{V_{in}} = (-g_{m2} R_C) \left\{ \frac{(R_E \parallel r_{\pi 2})}{(R_E \parallel r_{\pi 2}) + \frac{1}{g_{m1}}} \right\}$$

$$\Rightarrow \boxed{A_V = (-g_{m2} R_C) \left\{ \frac{(R_E \parallel r_{\pi 2})}{(R_E \parallel r_{\pi 2}) + 1/g_{m1}} \right\}}$$