Lec-16

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# Voltage Standing Wave Ratio (VSWR)

$$V(\ell) = V^+ e^{j\beta\ell}\left[1 + |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}\right]$$

$$I(\ell) = \frac{V^+}{Z_0} e^{j\beta\ell}\left[1 - |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}\right]$$

$Z_0$ → Characteristic Impedance.

$$\frac{V(\ell)}{I(\ell)} = Z(\ell) = Z_0 \left\{\frac{1 + |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}}{1 - |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}}\right\}$$

$Z(\ell)$ → Function of length

Impedance measured at different points on a Transmission line is different!

$\phi_L - 2\beta\ell$ = Even multiples of $\pi$ → $0, 2\pi, 4\pi \ldots$ ⇒ Maximum Voltage

$\phi_L - 2\beta\ell$ = Odd multiples of $\pi$ → $\pi, 3\pi, 5\pi \ldots$ ⇒ Minimum Voltage

$e^{j\theta} = \cos\theta + j\sin\theta$

Observation

- On a transmission line, the point where magnitude of voltage is maximum, the impedance measured at that point is real.

$$Z_{max} = \max\{Z(\ell)\} = \frac{|V_{max}|}{|I_{min}|} = Z_0\left\{\frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}\right\}$$

$Z_{max}$ → Real number

$$Z_{min} = \min\{Z(\ell)\} = \frac{|V_{min}|}{|I_{max}|} = Z_0\left\{\frac{1 - |\Gamma_L|}{1 + |\Gamma_L|}\right\}$$

$Z_{min}$ → Real

VSWR — $\rho$

$$\rho = \frac{|V_{max}|}{|V_{min}|} = \frac{V^+\{1+|\Gamma_L|\}}{V^+\{1+|\Gamma_L|\}}$$

$$VSWR(\rho) = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

Bounds on VSWR

$-1 \leq \Gamma_L \leq 1$

$1 \leq \rho \leq \infty$

Ideal case

$|V_{max}| = |V_{min}|$

$\rho = 1$ for $|\Gamma_L| = 0$

$\rho = \infty$ for $|\Gamma_L| = 1$

## Short Circuit Transmission Line Standing Wave

$$V(\ell) = V^+ e^{j\beta\ell} + V^- e^{-j\beta\ell}$$

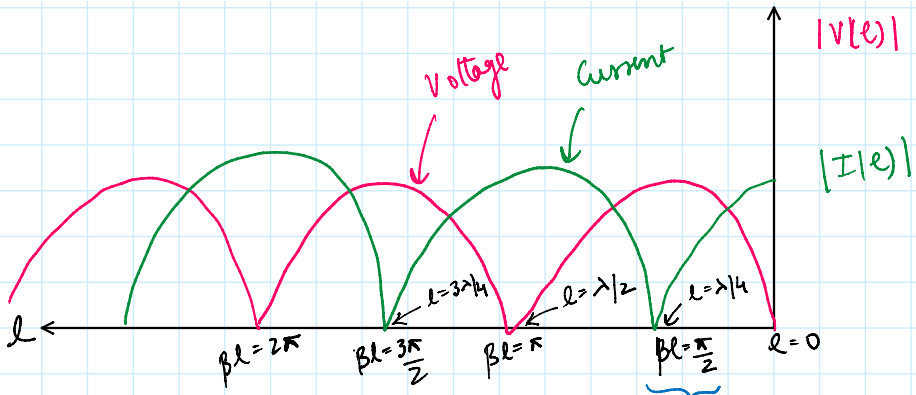
$$I(\ell) = \frac{V^+}{Z_0} e^{j\beta\ell} - \frac{V^-}{Z_0} e^{-j\beta\ell}$$

$V^- = -V^+$ because $\Gamma_L = -1$ {short circuit line}

$$V(\ell) = V^+\{e^{j\beta\ell} - e^{-j\beta\ell}\} \rightarrow 2V^+ j \sin(\beta\ell)$$

$$I(\ell) = \frac{V^+}{Z_0}\{e^{j\beta\ell} + e^{-j\beta\ell}\} \rightarrow \frac{2V^+}{Z_0}\cos(\beta\ell)$$

$$\frac{V(\ell)}{I(\ell)} = j Z_0 \tan(\beta\ell)$$




$\beta\ell = \frac{\pi}{2}$

$\frac{2\pi}{\lambda}\ell = \frac{\pi}{2}$ $\Rightarrow \ell = \lambda/4$